Analytic Solutions to a Boundary Layer Problem for Non-Newtonian Fluid Flow Driven by Power Law Velocity Profile

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Abstract: - In this paper the similarity solutions of the Prandtl boundary layer equations describing a non-Newtonian power law fluid past an impermeable flat plate, driven by a power law velocity profile $U = By^{\sigma}$ $(B > 0)$ have been investigated. It is shown that there are analytical solutions for any $n > 0,\ n \neq 2$ and any $-1/2 \leq \sigma < 0$. We give a method for the determination of the power series solutions to the momentum equation and we estimate the convergence radius of the proposed solutions.

Key-Words: - Similarity solution, power series solution, boundary value problem, non-Newtonian fluid flow

1 Introduction

The problems of heat and mass transfer in two-dimensional boundary layers on continuous stretching surfaces, moving in a fluid medium, have attracted considerable attention during the last few decades. There are numerous applications in industrial manufacturing processes, such as rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning.

For Newtonian fluids, the laminar boundary layer to en exterior power law velocity profile of the form $U = By^{\sigma}$ was investigated by Weidman et al. [23] for a large range of the power law parameter σ. An analytical solution of the momentum equation in terms of Airy function was proposed for the case $\sigma = -1/2$. The power law velocity profile form $U = By^{\sigma}$ was proposed by Barenblatt [3] for the mean velocity to fully developed turbulent shear flows, and in [4] Barenblatt and Protokishin proved that $\sigma = 3/(2\ln \mathrm{Re})$.

Recently, Magyari et al. [21] have examined the effect of a lateral suction/injection of the fluid for the existence of similarity solutions in the Newtonian case. It was shown that while for $\sigma = -2/3$ the flow over an impermeable plate to power law shear is not possible, the presence of suction allows for a family of boundary layer solutions. In the case $\sigma = -1/2,$ the solutions were found both for suction and injection, and the skin friction parameter is independent of the suction/injection parameter.

For both Newtonian fluids [13] and non-Newtonian fluids [14] Guedda has given a theoretical analysis of the existence of the boundary layer similarity flows for a range of exponents σ and B.

The study of non-Newtonian fluid flows has considerable interests, this is primarily because of the numerous applications in several engineering fields. One particular non-Newtonian model which has been widely studied is the Ostwald-de Waele power law model, which relies the shear stress to the strain rate by the expression

$$\tau_{xy} = \mu_0 \left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y}, \qquad (1)$$

where μ_0 is a positive constant, and $n > 0$ is called the power law index. The case $n < 1$ is referred to pseudo-plastic or shear-thinning fluid, the case $n > 1$ is known as dilatant or shear-thickening fluid. The Newtonian fluid is a special case where the power law index $n = 1$. For Newtonian fluid with $\sigma = 0$ the problem of laminar boundary layer problem is described by the famous Blasius equation [5].

Our interest has been motivated by the work of Cossali [11]. In this paper the similarity flow over

an impermeable flat plate driven by a power law velocity profile for Newtonian fluid has been considered, for which power series solutions were found for all the allowed range of the parameter σ.

The aim of the present paper is to present analysis for the steady-state laminar boundary layer flow, governed by the Ostwald-de Waele power law model of an incompressible non-Newtonian fluid driven by a power law velocity profile. A generalization of the usual Blasius similarity transformation is used to find similarity solutions. Using a shooting method, we establish the existence of analytic solutions, i.e., solutions are expandible in convergent power series to the momentum boundary layer equation under the general case of the power law velocity profile, thus extending the classical Blasius result for the shear driven case and Cossali's results for non-Newtonian fluid flow when $n \neq 2$. Some properties of the solutions are discussed depending on the viscosity power law index.

2 Derivation of the problem

Consider a steady two-dimensional laminar flow of an incompressible fluid of density ρ, past a semi-infinite flat plate. Let (x, y) be the Cartesian coordinates of any point in the flow domain, where x -axis is along the plate and y -axis is normal to it. The continuity and momentum equations can be simplified, within the boundary-layer approximation, into the following equations [2]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

and

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}, \tag{3}$$

where u and v represent the components of the fluid velocity in the direction of increasing x and y, τ_{xy} denotes the shear stress. Equations (2) and (3) are accompanied by the boundary conditions

$$u(x,0)=0,\quad v(x,0)=0 \quad \text{and} \quad \lim_{y\to\infty} u(x,y)=U, \tag{4}$$

where $U = By^{\sigma}$ as $y \to \infty$. In term of the stream-function ψ, which satisfies

$$u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x}, \tag{5}$$

equations (2) and (3) can be reduced to the equation

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=\mu_c \frac{\partial}{\partial y}\left[\left|\frac{\partial^2 \psi}{\partial y^2}\right|^{n-1}\frac{\partial^2 \psi}{\partial y^2}\right], \tag{6}$$

where $\mu_c = \mu_0 / \rho,$ with the boundary conditions

$$\frac{\partial \psi}{\partial y}(x,0)=0,\quad \frac{\partial \psi}{\partial x}(x,0)=0, \tag{7}$$

and

$$\lim_{y\to\infty}\frac{\partial \psi}{\partial y}(x,0)=U. \tag{8}$$

To look for similarity solutions we define the stream function ψ and similarity variable η as

$$\psi = bx^{-\alpha} f(\eta),\quad \eta = ax^{-\beta} y, \tag{9}$$

where a, b, α and β are constants to be determined, and $f(\eta)$ denotes the dimensionless stream function. Using (6) and (9) we find that the profile function f satisfies

$$\begin{aligned}&\mu_c a^{2n+1} b^{2n} x^{-(\alpha+2\beta)n-\beta}\left(\left|f''\right|^{n-1} f''\right)'\\&-\alpha a^2 b^2 x^{-2(\alpha+\beta)-1} f f''\\&+(\alpha+\beta) a^2 b^2 x^{-2(\alpha+\beta)-1} f'^2 = 0\end{aligned} \tag{10}$$

Equation (10) is an ordinary differential equation if and only if

$$(2-n)\alpha+(1-2n)\beta=1, \tag{11}$$

$\alpha + \beta = M$; the scaling relation, i.e.,

$$\alpha=\frac{M(2n-1)-1}{n+1},\quad \beta=\frac{M(2-n)+1}{n+1}, \quad (12)$$

and the parameters a and b satisfy

$$\mu_c a^{2n-1}b^{n-2}=1. \quad (13)$$

Moreover,

$$M=-\frac{\sigma}{(2-n)\sigma+(n+1)}. \quad (14)$$

So, function f satisfies the following boundary value problem

$$\left(\left|f''\right|^{n-1}f''\right)'-\alpha ff''+Mf'^2=0, \quad (15)$$

$$f(0)=0,\ f'(0)=0,\ \lim_{\eta\to\infty}f'(\eta)=A\eta^{\sigma}, \quad (16)$$

where the prime denotes the differentiation with respect to the similarity variable η, and

$$A=B/(a^{1+\sigma}b),\quad \sigma+1=-\alpha/\beta. \quad (17)$$

With the choice $a=1$ we get that

$$b=\mu_c^{1/(2-n)},\quad A=B\mu_c^{-1/(2-n)},\quad n\neq 2, \quad (18)$$

and the non-dimensional velocity components are obtained by f as follows:

$$u(x,y)=\mu_c^{1/(2-n)}x^{-M}f'(\eta), \quad (19)$$

$$v(x,y)=x^{-(\alpha+1)}\left[\alpha f(\eta)+\beta\eta f'(\eta)\right]. \quad (20)$$

For $\sigma=0$, equation (15) is referred to as generalized Blasius equation [8] and for the Newtonian case, equation (15) coincides with the well-known Blasius equation

$$f'''+\frac{1}{2}ff''=0. \quad (21)$$

If $\sigma=0$, then (16) is reduced to

$$f(0)=0,\ f'(0)=0,\ \lim_{\eta\to\infty}f'(\eta)=A. \quad (22)$$

The case $n=2$ is not considered in this paper. We note that for $\sigma=0$, $n=2$, numerical results were obtained in [18] and [20].

3 Analytic solutions

The Blasius function is defined as the unique solution to the boundary value problem (21)-(22) with $A=1$. Blasius [5] derived power series expansion which begins

$$f(\eta)\approx\frac{1}{2}\gamma\eta^2-\frac{1}{240}\gamma^2\eta^5+\frac{11}{161280}\gamma^3\eta^8-\frac{5}{4257792}\gamma^4\eta^{11}+\ldots, \quad (23)$$

where γ is the curvature of the function at the origin. A closed form for the coefficients is not known. However, the coefficients can be computed for

$$f(\eta)=\eta^2\sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k\frac{A_k\gamma^{k+1}}{(3k+2)!}\eta^{3k}, \quad (24)$$

from the recurrence

$$A_k=\sum_{j=0}^{k-1}\binom{3k-1}{3r}A_rA_{k-r-1},\ \text{if}\ \ k\geq 2, \quad (25)$$

with $A_0=A_1=1$. Numerous papers were published on the numerical determination of γ. E.g., Howarth [16] has obtained $\gamma\approx 0.332057$. Abbasbandy [1] proposed an Adomians's decomposition method to the Blasius's problem and obtained $\gamma=0.333329$ with a 0.383% relative error, and Tajvidi et al. [22] used a modified rational Legendre method, to show that $\gamma=0.33209$ with a 0.009% relative error. By the fourth-order Runge--Kutta method γ is determined as $\gamma\approx 0.33205733621519630$ (see [9]). A fully analytical solution to the Blasius problem has been

found by Liao [19] using the homotopy analysis method.

Our objective is to show the existence of analytic solutions to the boundary value problem (15)-(16) and to determine the approximate local solution $f(\eta)$. We use the shooting method and replace the condition at infinity by one at $\eta = 0$. Therefore, (15)-(16) is converted into an initial value problem of (15) with initial conditions

$$f(0) = 0, \quad f'(0) = 0, \quad f''(0) = \gamma. \tag{26}$$

We suppose that $n > 0$, $n \neq 2$ and f'' is positive in the neighborhood of zero. We consider (15) as a system of certain differential equations, namely, the special Briot-Bouquet differential equations. For this type of differential equations we refer to the book by Hille [15], and by Ince [17]. In order to establish the existence of a power series representation of $f(\eta)$ about $\eta = 0$ we apply the following theorem:

Briot-Bouquet Theorem [10]: Let us assume that for the system of equations

$$\left.\begin{aligned} \xi \frac{dz_1}{d\xi} &= u_1(\xi, z_1(\xi), z_2(\xi)), \\ \xi \frac{dz_2}{d\xi} &= u_2(\xi, z_1(\xi), z_2(\xi)), \end{aligned}\right\} \tag{27}$$

where functions u_1 and u_2 are holomorphic functions of ξ, $z_1(\xi)$, and $z_2(\xi)$ near the origin, moreover

$$u_1(0,0,0) = u_2(0,0,0) = 0, \tag{28}$$

then a holomorphic solution of (27) satisfying the initial conditions $z_1(0) = 0$, $z_2(0) = 0$ exists if none of the eigenvalues of the matrix

$$\begin{bmatrix} \left.\frac{\partial u_1}{\partial z_1}\right|_{(0,0,0)} & \left.\frac{\partial u_1}{\partial z_2}\right|_{(0,0,0)} \\ \left.\frac{\partial u_2}{\partial z_1}\right|_{(0,0,0)} & \left.\frac{\partial u_2}{\partial z_2}\right|_{(0,0,0)} \end{bmatrix} \tag{29}$$

is a positive integer.

The Briot-Bouquet theorem ensures the existence of formal solutions

$$z_1 = \sum_{k=0}^{\infty} a_k \xi^k, \quad z_2 = \sum_{k=0}^{\infty} b_k \xi^k \tag{30}$$

to system (27), and also the convergence of formal solutions.

This theorem and the method presented here have been successfully applied to the determination of local analytic solutions of different nonlinear initial value problems (see [6]-[8]).

Let us consider the initial value problem (15)-(26), and take its solution in the form

$$f(\eta) = \eta^2 Q(\eta^\delta), \ \eta \in (0, \ \eta_c), \tag{31}$$

where function $Q \in C^2(0, \eta_c)$ for some positive value η_c. Substituting (31) into (15) one can get

$$\begin{aligned} &\delta^3 \eta^{3\delta-1} Q''' + 3\delta^2(\delta+1)\eta^{2\delta-1} Q'' \\ &+ \delta(\delta+1)(\delta+2)\eta^{\delta-1} Q' \\ &+ \frac{M}{n}\left(2\eta Q + \delta\eta^{\delta+1} Q'\right)^2 \\ &\left[2Q + \delta(\delta+3)\eta^\delta Q' + \delta^2\eta^{2\delta} Q''\right]^{1-n} \\ &- \frac{\alpha}{n}\eta^2 Q\left[2Q + \delta(\delta+3)\eta^\delta Q' + \delta^2\eta^{2\delta} Q''\right]^{2-n} = 0. \end{aligned} \tag{32}$$

Let us introduce the new variable ξ such as $\xi = \eta^\delta$ and function Q as follows

$$Q(\xi) = a_0 + a_1\xi + a_2\xi^2 + z(\xi), \tag{33}$$

where a_0, a_1, a_2 are real constants, and $z \in C^2(0, \eta_c^\delta)$, with $z(0) = 0$, $z'(0) = 0$, $z''(0) = 0$. Then Q fulfills the following properties $Q(0) = a_0$, $Q'(0) = a_1$, $Q''(0) = 2a_2$, $Q'''(\xi) = z'''(\xi)$. From the initial condition $f''(0) = 2Q(0) = \gamma$ we have

$$a_0 = \frac{\gamma}{2}. \tag{34}$$

We restate the third order differential equation in (32) as a system of three equations

$$\left.\begin{aligned} u_1(\xi, z_1(\xi), z_2(\xi), z_3(\xi)) &= \xi z_1'(\xi) \\ u_2(\xi, z_1(\xi), z_2(\xi), z_3(\xi)) &= \xi z_2'(\xi) \\ u_3(\xi, z_1(\xi), z_2(\xi), z_3(\xi)) &= \xi z_3'(\xi) \end{aligned}\right\} \quad (35)$$

with choosing

$$\left.\begin{aligned} z_1(\xi) &= z(\xi) \\ z_2(\xi) &= z'(\xi) \\ z_3(\xi) &= z''(\xi) \end{aligned}\right\} \quad \text{with} \quad \left.\begin{aligned} z_1(0) &= 0 \\ z_2(0) &= 0 \\ z_3(0) &= 0 \end{aligned}\right\} \quad (36)$$

as follows

$$\begin{aligned} &u_1(\xi, z_1(\xi), z_2(\xi), z_3(\xi)) = \xi z_2, \\ &u_2(\xi, z_1(\xi), z_2(\xi), z_3(\xi)) = \xi z_3, \\ &u_3(\xi, z_1(\xi), z_2(\xi), z_3(\xi)) = \xi z_3' \\ &= -\frac{1}{n(n+1)\delta^3}\xi^{\frac{3}{\delta}-2} \\ &\left(a_0 + a_1\xi + a_2\xi^2 + z_1(\xi)\right)K^{2-n}(\xi) \\ &-3\frac{\delta+1}{\delta}\left(2a_2 + z_3(\xi)\right) \\ &-\frac{4M}{\delta^3}\left(a_0 + a_1\xi + a_2\xi^2 + z_1(\xi)\right)^2 \\ &-\frac{4M}{\delta^3}\left(a_0 + a_1\xi + a_2\xi^2 + z_1(\xi)\right) \\ &\cdot\left(a_1 + 2a_2\xi + z_2(\xi)\right) \\ &-\frac{M}{\delta^3}\left(a_1 + 2a_2\xi + z_2(\xi)\right)^2 \\ &-\frac{(\delta+1)(\delta+2)}{\delta^2}\frac{1}{\xi}\left(a_1 + 2a_2\xi + z_2(\xi)\right), \end{aligned} \quad (37)$$

where

$$\begin{aligned} &K(\xi) = 2a_0 + (\delta+1)(\delta+2)a_1\xi \\ &+ 2(\delta+1)(2\delta+1)a_2\xi^2 \\ &+ 2z_1(\xi) + \delta(\delta+3)\xi z_2(\xi) + \delta^2\xi^2 z_3(\xi). \end{aligned} \quad (38)$$

We apply the Briot-Bouquet theorem for the system of three equations in (37). In order to satisfy conditions

$$u_1(0,0,0,0) = u_2(0,0,0,0) = u_3(0,0,0,0) = 0 \quad (39)$$

in the corresponding Briot-Boquet theorem the following connection yields

$$\frac{3}{\delta} - 2 = -1, \quad (40)$$

therefore $\delta = 3$, and for the coefficients of ξ^{-1} and ξ^0:

$$\frac{\alpha}{27n}a_0(2a_0)^{2-n} - \frac{4M}{27}a_0 - \frac{20}{9}a_1 = 0, \quad (41)$$

$$\begin{aligned} &\frac{\alpha}{27n}a_1(2a_0)^{2-n}(21-10n) \\ &-\frac{20M}{27}a_1a_0 - \frac{112}{9}a_2 = 0, \end{aligned} \quad (42)$$

respectively. Applying (34) for (41) and (42)

$$a_1 = \frac{1}{5!}\left(\frac{\alpha}{n}\gamma^{3-n} - 2M\gamma^2\right), \quad (43)$$

$$a_2 = \frac{1}{8!}\left(\frac{\alpha(21-10n)}{n}\gamma^{2-n} - 10M\gamma\right)\left(\frac{\alpha}{n}\gamma^{3-n} - 2M\gamma^2\right). \quad (44)$$

Therefore, the eigenvalues of matrix

$$\begin{bmatrix} \partial u_1/\partial z_1 & \partial u_1/\partial z_2 & \partial u_1/\partial z_3 \\ \partial u_2/\partial z_1 & \partial u_2/\partial z_2 & \partial u_2/\partial z_3 \\ \partial u_3/\partial z_1 & \partial u_3/\partial z_2 & \partial u_3/\partial z_3 \end{bmatrix} \quad (45)$$

at $(0,0,0)$ are 0. Since all three eigenvalues are non-positive, referring to the Briot-Bouquet theorem we obtain the existence of unique analytic solutions z_1, z_2 and z_3 near zero. Thus, there exists a formal solution

$$f(\eta) = \eta^2 \sum_{k=0}^{\infty} a_k \eta^{3k}, \quad (46)$$

where the first three coefficients are known (see (34), (43), (44)).

For the determination of coefficients a_k, $k > 2$, we shall use the J.C.P. Miller formula (see [16]):

$$\left[\sum_{k=0}^{L} c_k x^k\right]^{p+1} = \sum_{k=0}^{(p+1)L} d_k(p) x^k, \quad (47)$$

where $d_0(p)=1$ for $c_0=1$, and

$$d_k(p)=\frac{1}{k}\sum_{j=0}^{k-1}[(p+1)(k-j)-j]d_j(p)c_{k-j},\quad (k\geq 1). \tag{48}$$

From (46)

$$[f''(\eta)]^{2-n}=\left[\sum_{k=0}^{\infty}(3k+2)(3k+1)a_k\eta^{3k}\right]^{2-n}=\sum_{k=0}^{\infty}A_k\eta^{3k}, \tag{49}$$

$$[f''(\eta)]^{1-n}=\left[\sum_{k=0}^{\infty}(3k+2)(3k+1)a_k\eta^{3k}\right]^{1-n}=\sum_{k=0}^{\infty}C_k\eta^{3k}, \tag{50}$$

where coefficients A_k, C_k can be expressed in terms of a_k $(k=0,1,\ldots)$. Substituting them into equation (15) we get

$$\sum_{k=0}^{\infty}(3k+5)(3k+4)(3k+3)a_{k+1}\eta^{3k}-\frac{\alpha}{n}\sum_{k=0}^{\infty}a_k\eta^{3k}\sum_{k=0}^{\infty}A_k\eta^{3k}+\frac{M}{n}\left[\eta\sum_{k=0}^{\infty}(3k+2)a_k\eta^{3k}\right]^2\sum_{k=0}^{\infty}C_k\eta^{3k}=0. \tag{51}$$

Applying the recursion formula (48) for the determination of A_k and comparing the proper coefficients in (51) one can have the necessary values of a_k for some given values of n, M, α. We note that the coefficients obtained by this method for $n=1$, $\sigma\neq 0$ are the same as the coefficients of the power series approximation given by Cossali [11]. Moreover, if $n\neq 1$ and $\sigma=0$, coefficients a_k are fully consistent with the result obtained in [8]. If $n=1$, $\sigma=0$, the coefficients coincide with the Blasius results given in (43).

4 Some special cases

In this section we show some numerical results obtained for three different values of n (0.5, 1, 3) and three different values of σ ($-1/2$, $-1/3$, 0). Fig. 1-3 represent the effect of power-law index on $f'(\eta)/\eta^{\sigma}$ for $\sigma=-1/2$, $\sigma=-1/3$, $\sigma=0$. Fig. 4-6 exhibit how the graph of $f'(\eta)/\eta^{\sigma}$ changes for different values of n ($n=0.5$, $n=1$, $n=3$).

The power series approximations can be determined as it was shown in Section 3. Using formula (51) after the comparison of the proper coefficients one can determine a_k for $k>0$.

(i.) For $\sigma=0$, we refer to the paper [8], when γ is obtained by a method using Töpfer-like transformation from the initial value value problem ($g=g(\bar{\eta})$)

$$\left(|g''|^{n-1}g''\right)'+\frac{1}{n+1}gg''=0,\quad g(0)=0,\ g'(0)=0,\ g''(0)=1. \tag{52}$$

Solving problem (52) numerically, one gets γ as

$$\gamma=\left[\lim_{\bar{\eta}\to\infty}g'(\bar{\eta})\right]^{-\frac{3}{n+1}} \tag{53}$$

(see Table 1.) Note, that for $n=1$ we get $\gamma=0.332057$ as it is known [5]. The radius of convergence η_c for the series solution can be found by applying the ratio test. The first 10 terms of the sequence were evaluated and the numerical results for η_c are presented in Table 1.

For $\sigma=0$, coefficients a_1 and a_2 can be formulated as 1mm

$$a_1=-\frac{\gamma^{3-n}}{5!\,n(n+1)},$$

$$a_2 = \frac{21-10n}{8!\,n^2(n+1)^2}\gamma^{5-2n}$$

and further coefficients can be obtained from (51).

n	γ	η_c
0.5	0.33123	3.579
1	0.33206	5.688
3	0.46243	10.225

Table 1. Case $\sigma = 0$

(ii.) We remark that the Töpfer-like transformation can be applied can be to (15) for the determination of γ by solving

$$\begin{aligned} &\left(|g''|^{n-1}g''\right)' - \alpha g g'' + M g'^2 = 0, \\ &g(0) = 0,\ g'(0) = 0,\ g''(0) = 1 \end{aligned} \tag{54}$$

for $g(\bar{\eta})$ and

$$\gamma = \left[\lim_{\bar{\eta}\to\infty} \frac{g'(\bar{\eta})}{\bar{\eta}^{\sigma}}\right]^{-\frac{3}{n+1+\sigma(2-n)}}. \tag{55}$$

Then one can find the similarity solution by $f(\eta) = \gamma^{\frac{2n-1}{3}} g(\gamma^{\frac{2-n}{3}}\eta)$.

For $\sigma = -1/3$ the numerical results are presented in Table 2 and

$$a_1 = -\frac{2}{5!\,n(4n+1)}(\gamma^{3-n} + n\gamma^2),$$

$$a_2 = \frac{4}{8!\,n^2(4n+1)^2}$$

$$\cdot[(21-10n)\gamma^{5-2n} + n(26-10n)\gamma^{4-n} + 5n^2\gamma^3]$$

.

n	γ	η_c
0.5	0.3975	1011
1	0.3817	109
3	0.5087	13

Table 2. Case $\sigma = -1/3$

(iii.) Case $\sigma = -1/2$ is a special case, when $\gamma = \left(\frac{2}{3n}\right)^{\frac{1}{n}}$ (see [14]). Applying (43), (44) for the determination of the coefficients a_k from (51) we obtain

$$a_1 = -\frac{1}{3\,5!\,n^2}\left(\frac{\gamma^{3-n}}{n} + 2n\gamma^2\right),$$

$$a_2 = \frac{1}{9\,8!\,n^4}$$

$$\cdot\left((21-10n)\gamma^{5-2n} + 4n(13-5n)\gamma^{4-n} + 20n^2\gamma^3\right)$$

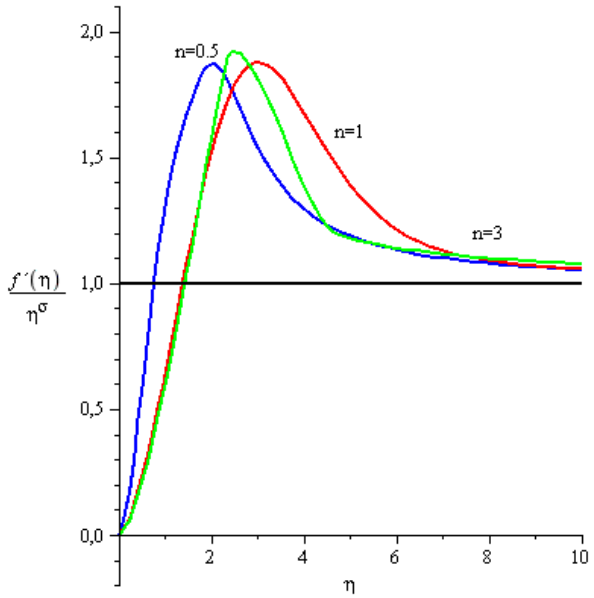

Figure 1. $\sigma = -1/2$

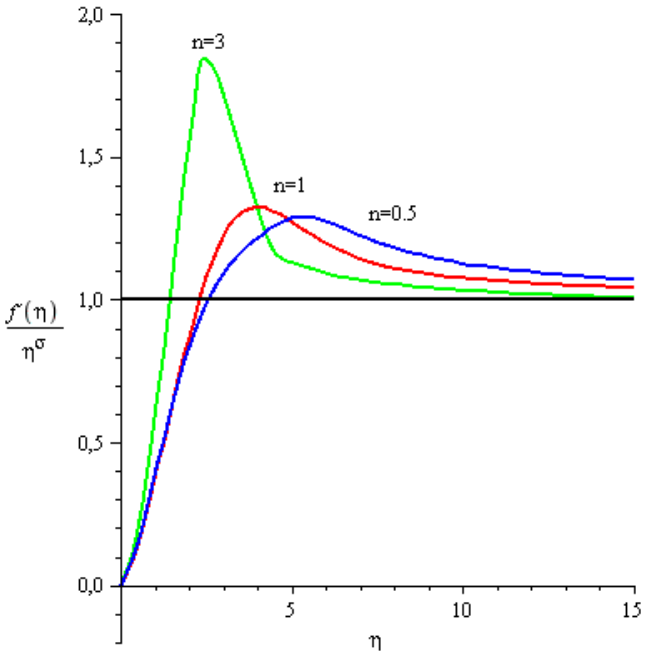

Figure 2. $\sigma = -1/3$

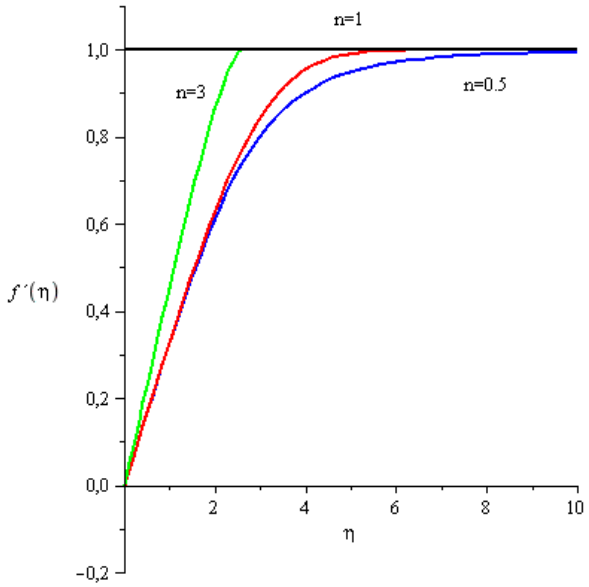

Figure 3. $\sigma = 0$

n	γ	η_c
0.5	1.7778	1.439
1	0.6667	4.465
3	0.6057	6.951

Table 3. Case $\sigma = -1/2$

For some special values of n with $\sigma = -1/2$ the following series approximations are valid:

n = 0.5;

$$f(\eta) = \eta^2 (0.8888888885 - 0.8193872876 \cdot 10^{-1}\eta^3 + 0.1522200957 \cdot 10^{-1}\eta^6 - 0.3355516709 \cdot 10^{-2}\eta^9 + 0.9224307230 \cdot 10^{-3}\eta^{12} - 0.2690360667 \cdot 10^{-3}\eta^{15})$$

n = 1;

$$f(\eta) = \eta^2 (0.33333333 - 0.3703703704 \cdot 10^{-2}\eta^3 + 0.5144032922 \cdot 10^{-4}\eta^6 - 0.4272260788 \cdot 10^{-6}\eta^9 + 0.4644272544 \cdot 10^{-8}\eta^{12} - 0.5218400155 \cdot 10^{-10}\eta^{15})$$

n = 1.5;

$$f(\eta) = \eta^2 (0.2911934882 - 0.1804899031 \cdot 10^{-2}\eta^3 + 0.1059595232 \cdot 10^{-4}\eta^6 - 0.8568983478 \cdot 10^{-8}\eta^9 + 0.1872900943 \cdot 10^{-10}\eta^{12} - 0.5576286295 \cdot 10^{-13}\eta^{15})$$

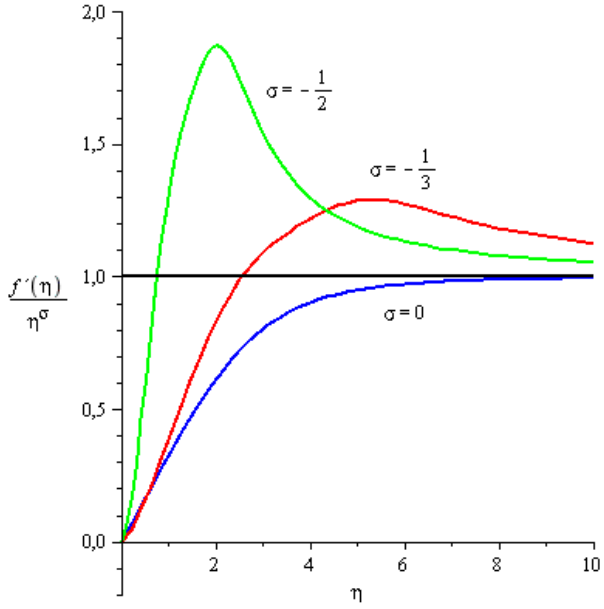

Figure 4. $n = 0.5$

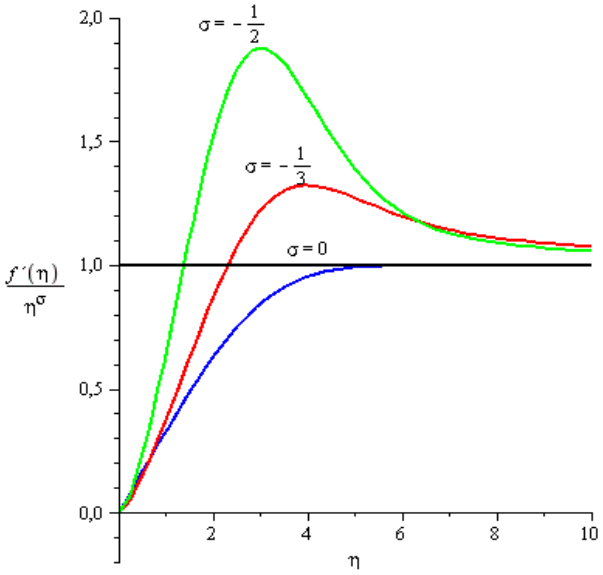

Figure 5. $n = 1$

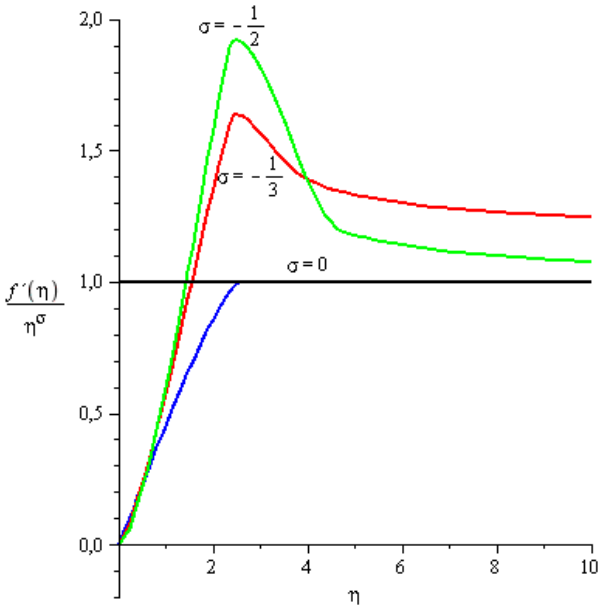

Figure 6. $n = 3$

4 Conclusion

In this paper a generalization of the usual Blasius Transformation is applied to find similarity solutions to non-Newtonian fluid flow over an impermeable flat plate driven by a power law velocity profile. We have generalized the power series formulation of the similarity solution of the Newtonian flow obtained by G.E. Cossali [11] to non-Newtonian fluid flow with Ostwald de Waele power law nonlinearity when for the power law index the condition $n \neq 2$ holds. The coefficients of the more general problem coincide with the coefficients obtained for problems related to special values of the parameters. The effect of parameters n and σ are exhibited by numerical results.

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