Proper Orthogonal Decomposition Analysis for Unsteady Rotor-Stator Interaction in a Low Pressure Centrifugal Compressor

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Abstract: - In this paper, Proper Orthogonal Decomposition (POD) is applied to the analysis of the unsteady rotorstator interaction in a low-pressure centrifugal compressor. Numerical simulations are carried out through finite volumes method using the Unsteady Reynolds-Averaged Navier-Stokes Equations (URANS) model. Proper Orthogonal Decomposition allows an accurate reconstruction of flow field using only a small number of modes; therefore, this method is one of the best tools for data storage The POD results and the data obtained by the Adamczyk decomposition are analyzed. Both decompositions show the behavior of unsteady rotor-stator interaction, but the POD modes allow quantifying better the numerical errors.

Key-Words: - Unsteady Rotor-Stator Interaction, Adamczyk decomposition, POD, CFD, Compressor, URANS, Flow Field Reconstruction

1 Introduction

In the centrifugal compressors the fluid flow has a very complicated due the unsteady and turbulence effects, having time scales that vary considerably. This complexity makes difficult, both experimental and numerical, analysis. Usually, in the practical applications, in the reference frame linked to the studied row, a steady flow is assumed. Furthermore, one can decompose the flow in two components: the main flow and the secondary flow respectively. The first flow corresponds to the physical flow with non-zero rotor velocity. In the secondary flow, vortices generate the losses due to the entropy increase, leading to threedimensional behavior of the flow.

C. Dano [1] indentifies the sources of unsteady phenomena in turbomachinery flows. Because the rotorinteraction can affect dramatically the stator turbomachinery performance, we paid it a special attention in this paper. The majority of researchers that studied this interaction from the numerical point of focused their research view on transonic turbomachinery; therefore, there is very few information about the rotor-stator interaction for low velocity turbomachinery. Moreover, a recent study [2] showed important discrepancies between experimental and numerical results for a low-pressure centrifugal stage. Unfortunately, this study did not succeed to identify the effects that caused the major discrepancies between experimental and numerical results.

Up to now, the Fourier transform is a common tool

for the analysis of periodic and non-periodic signals [3, 4, 5]. Some recent studies [6, 7] clearly showed that POD is a more efficient method to extract the dominant modes involved in unsteady flow field. Unfortunately, these studies applied POD only for one-dimensional decompositions. In order to take the full advantage of POD method, we have applied it for decomposition of full three-dimensional flow field.

For this reason, we have considered that it is useful to study the rotor-stator interactions in a low-pressure centrifugal stage, using both Adamczyk and proper orthogonal decomposition.

2 Nomenclature

e internal energy (J/kg)

 f_e external acceleration (m/s²)

 F_x, F_y, F_z vectors of convective components of flux

 G_{x}, G_{y}, G_{z} vectors of diffusive components of flux

- h static enthalpy (J/kg)
- *I* rothalpy (m^2/s^2)
- *p* static pressure (Pa)
- r radius (m)
- R gas constant $(J/(kg \cdot K))$
- *S* vector of source term
- *T* static temperature (K)
- t time (s)
- u, v, w Cartesian components of velocity (m/s)
- *V* absolute velocity (m/s)

W relative velocity (m/s)

- κ thermal conductivity (W/(m·K))
- $\mu \qquad \ \ dynamic \ viscosity \ (kg/(m\cdot s))$
- μ_t eddy viscosity (kg/(m·s))
- θ azimuthal (circumferential) angle (rad)
- ρ static density (kg/m³)
- τ shear stress tensor (Pa)
- Ω angular velocity (rad/s)

Subscript

- *R* rotor
- t turbulent

Superscript

eff effective (laminar + turbulent)

3 Governing Equations

For a three-dimensional rotating Cartesian coordinate system, the unsteady Reynolds-averaged Navier-Stokes equations using the Favre averaging (a mass-weighted averaging) could be written in the conservative form as [8, 9]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = \rho f_i + \frac{\partial}{\partial x_j}(\tau_{u_j}^{eff} - p\delta_{ij})$$
(2)

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_j}(\rho u_j I) = \rho f_j u_j + q_v + \frac{\partial}{\partial x_j} \left(\kappa^{eff} \frac{\partial T}{\partial x_j} + \tau^{eff}_{ij} u_i\right)$$
(3)

where energy *E* and rothalpy *I* are defined by:

$$E = e + \frac{w_i^2}{2} - \frac{v_{ri}^2}{2}$$
(4)

$$I = h + \frac{w_i^2}{2} - \frac{v_{ri}^2}{2}$$
(5)

$$w_i = u_i - v_{ri}, v_{ri} = \varepsilon_{ijk} \Omega_j x_k$$
(6)

where ε_{iik} is the Levi-Civita symbol.

According to the Boussinesq hypothesis and Stokes postulates and hypothesis for a Newtonian fluid, the shear stresses τ^{eff} may be written as:

$$\tau_{ij}^{eff} = \left(\mu + \mu_{t}\right) \left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}}\right) - \frac{2}{3} \left(\mu + \mu_{t}\right) \frac{\partial u_{k}}{\partial x_{k}} \delta_{ij}$$
(7)

The Sutherland's formula could be used to determine the dynamic viscosity μ as function of temperature, while the eddy viscosity μ_t is computed with a turbulence model.

For gases, the external force f_i due to the gravitational acceleration is very small, therefore it can be neglected. Moreover, we can assume that the thermal conductivity is the single heat source.

The pressure is obtained from the equation of state, $p = \rho RT$ (8)

4 Turbulence Model

In 1992, P. Spalart and S. Allmaras [10] publish an original turbulence model. The model includes one differential transport equation for turbulent viscosity. To include the wall effects, the apparent viscosity is affected by a damping function:

$$\mathbf{v}_t = f_{vl} \mathbf{\tilde{v}} \tag{9}$$

and the transport equation for $\widetilde{\nu}\,$ is:

$$\frac{\partial}{\partial t}(\widetilde{\mathbf{v}}) + U_k \frac{\partial}{\partial x_k}(\widetilde{\mathbf{v}}) = C_{b1} \widetilde{S} \widetilde{\mathbf{v}} - C_{w1} f_w \left(\frac{\widetilde{\mathbf{v}}}{d}\right)^2 +$$

$$+ \frac{1}{2} \partial \left[(u + \widetilde{\mathbf{v}}) \partial \widetilde{\mathbf{v}} \right] + C_{b2} \partial \widetilde{\mathbf{v}} \partial \widetilde{\mathbf{v}}$$
(10)

$$+\frac{1}{\sigma}\frac{\partial}{\partial x_{k}}\left[\left(\nu+\widetilde{\nu}\right)\frac{\partial\widetilde{\nu}}{\partial x_{k}}\right]+\frac{C_{b2}}{\sigma}\frac{\partial\widetilde{\nu}}{\partial x_{k}}\frac{\partial\widetilde{\nu}}{\partial x_{k}}$$

The first term in the right hand represents the production term:

$$P_{\widetilde{v}} = C_{b1} \widetilde{S} \widetilde{v} \tag{11}$$

$$\widetilde{S} = \overline{S} + \frac{v}{\kappa^2 d^2} f_{v2}$$
⁽¹²⁾

where κ este Kármán's constant ($\kappa = 0.41$), *d* is the wall distance and:

$$\Omega = \sqrt{2\Omega_{ij}\Omega_{ij}} \quad \Omega_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right)$$
(13)

The second term in the right hand:

quantifies the viscous dissipation in the wall vicinity. The model dumping functions are:

$$f_{\nu 1} = \frac{\chi^{3}}{\chi^{3} + C_{\nu 1}^{3}}, f_{\nu 2} = 1 - \frac{\chi}{1 + \chi f_{\nu 1}},$$

$$f_{w} = g \left[\frac{1 + C_{w3}^{6}}{g^{6} + C_{w3}^{6}} \right]^{1/6}$$

$$\chi = \frac{\widetilde{\gamma}}{g} - g = r + C_{w3} - (r^{6} - r) - r = -\frac{\widetilde{\gamma}}{\gamma}$$
(15)

$$\chi = \frac{\mathbf{v}}{\mathbf{v}}, \ g = r + C_{w2}(r^6 - r), \ r = \frac{\mathbf{v}}{\widetilde{S}\kappa^2 d^2}$$

and the closure constants: $C_{b1} = 0.1355$, $C_{b2} = 0.622$,

$$C_{v1} = 7.1, \quad \sigma = \frac{2}{3} C_{w1} = \frac{C_{b1}}{\kappa^2} + \frac{1 + C_{b2}}{\sigma}, \quad C_{w2} = 0.3,$$

 $C_{w3} = 2, \ \kappa = 0.41$ (16)

The boundary conditions for the transport equation (10) refer to the vanishing of the turbulent viscosity at the wall: for d = 0, $\tilde{v} = 0$.

The wall friction stress is obtained by imposing the law of the wall in the nearest grid points (Fig. 1).



Fig. 1. Computational grid near the wall

$$\frac{U_P}{u_{\tau}} = \frac{u_{\tau} y_P}{v}, y_P^+ = \frac{u_{\tau} y_P}{v} < 3$$
(17)

$$\frac{U_P}{u_\tau} = \frac{1}{\kappa} \ln \left(E \frac{u_\tau y_P}{v} \right), \ E = 9,793, \ y_P^+ > 30$$
(18)

Applications and detailed analyses of the Spalart-Allmaras model can be found in the papers of: W.H. Jou and all [11], P.R. Spalart and S.R. Allmaras [12], E. Shima and all [13], V.A. Sai and F.M. Lutfy [14], C.L. Rumsey and all [15], H.Y. Wong [16] etc. Improvements concerning the model accuracy for different applications are published by L. Lee şi G.C. Paynter [17], S. Catris and B. Aupoix [18] and J. Dacles-Mariani and all [19] The last ones propose for the component \overline{S} in the source term (12):

$$\overline{S} = \Omega + C_{\text{prod}} \min(0, S - \Omega)$$
⁽¹⁹⁾

$$S = \sqrt{2S_{ij}S_{ij}} , \ S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \tag{20}$$

and C_{prod} is a closure constant, $C_{\text{prod}} = 2.0$.

So, including the rotation effect the diminishing of the total production is obtained.

5 Numerical Simulation

The numerical simulations of the three-dimensional viscous flow were carried out on a centrifugal compressor designed, manufactured and tested by COMOTI, with commercial CFD code FLUENT that is based on finite volume method where each unknown takes an average value on each discretization cell. The computational domain generated in Gambit was split into eight blocks to facilitate the building of a fully structured mesh as shown in Fig. 2. The mesh for which, the results are given, has about 253 000 hexahedral cells for the impeller passage and 127 000 hexahedral cells for the vaned diffuser passage.

In order to decrease the computational time, impressively, the time discretization is made with a backward implicit first order scheme and multigrid technique is used. To take into account the physical properties of flow, the convective fluxes are discretized with the Roe scheme, which is a Godunov-type scheme Sterian Danaila, Mihai Leonida Niculescu

[8, 9]. Because the turbulence is not a critical issue of this study, we used the Spalart-Allmaras model, which is a one-equation turbulence model.

At the inlet, a uniform stagnation pressure (96 310 Pa) and temperature (300 K) are imposed, turbulent viscosity ratio μ_t/μ is 10 and the flow is normal to inlet. At the outlet, a uniform static pressure (156 000 Pa) is imposed. At the left and right sides of computational domain, the rotational periodic boundary conditions are imposed. All the walls have been assumed adiabatic. The shaft speed of impeller is 14 915 rpm.



Fig. 2 Computational domain of centrifugal compressor

6 Adamczyk Decomposition

Non-uniformities and unsteadiness due to the rotorstator interaction introduce major complexity in the analysis of the turbomachinery flow field. This problem can be considerably simplified if we apply the method of Adamczyk [7, 20] that proposed the decomposition of an arbitrary field variable u associated to a turbomachinery in four contributions through the successive application of averaging operators:

$$\begin{bmatrix} u(z,r,\theta,t) = \underbrace{\overline{u}(z,r)}_{1} + \underbrace{\overline{u}(z,r,\theta)}_{2} + \underbrace{\overline{u}(z,r,\theta_{R})}_{3} + \underbrace{u^{*}(z,r,\theta,t)}_{4} \\ \theta_{R} = \theta - \Omega t \end{bmatrix}$$
(21)

Starting from an arbitrary field u expressed in an inertial reference frame attached to the stationary row, the first averaging has as objective to extract the axisymmetric field independent by time and azimuthal coordinate. The second averaging is a time averaging in the inertial reference frame and it extracts from the remained field, the flow structures attached to the stationary row while the third averaging also is a time averaging but in the rotating reference frame and it

extracts from the remained field, the flow structures attached to the rotating row. Therefore, the third contribution is steady in the rotating reference frame. Finally, after three averaging, the residual field (fourth contribution) represents the unsteady part of initial field u in the inertial and rotating reference frame associated to stationary and rotating row, respectively. This contribution characterizes purely unsteady phenomena of turbomachinery flow. In order to understand better, the unsteady rotor-stator interaction, the fourth contribution was decomposed with POD technique as shown in the next section.

As it follows, we will give some results for some control points placed in a section at mid height of blade of vaned diffuser, at the middle distance between the blade and the right periodic as shown in Fig. 3. The numbering of these control points is from upstream to downstream.









The first component of Adamczyk decomposition for static pressure and absolute velocity, at considered control points is shown in Fig. 4. One sees that the compression process is smooth while the absolute velocity has big variations especially in the first part of vaned diffuser where the strong deceleration triggers a huge jet-wake region accompanied by boundary layer separation on suction side of vaned diffuser blade. These phenomena generate huge nonuniformities in the absolute velocity field as shown in Figs. 5 and 6, which induce important total pressure losses. For this reason, the compression process is very slow in the last part of vaned diffuser. Furthermore, the rectangular trailing edge of vaned diffuser blade generates additional nonuniformities, which are shown in Fig. 6 and losses. The homogenization process of flow begins after the trailing edge of vaned diffuser blade and it is accompanied by significant total pressure losses. For this reason, the air compression is very weak downstream of the trailing edge. The second component of Adamczyk decomposition for static pressure clearly shows the stagnation point, the rarefaction near leading edge, as well as the interaction among the blades of vaned diffuser in the region where the distance among blades is small as shown in Fig. 7.

The Adamczyk decomposition clearly shows that this classical vaned diffuser with circular arc blades generates a huge jet-wake zone and important pressure losses because the channel is extremely divergent in the first part of vaned diffuser. In order to obtain better compressor performance, it is necessary to renounce single circular arc vaned diffuser.



Fig. 5 Second and third component of Adamczyk decomposition for static pressure normalized by inlet static pressure and absolute velocity normalized by inlet absolute velocity



Fig. 6 Isolines of second and third component of Adamczyk decomposition for absolute velocity in the section from the middle height of vaned diffuser



Fig. 7 Isolines of second and third component of Adamczyk decomposition for static pressure in the section from the middle height of vaned diffuser

7 **Proper Orthogonal Decomposition**

In the field of fluid mechanics, two approaches have been used for the POD. Historically the method of Continuous POD (or the classical method) of Lumley [21] proceeded by the Snapshot POD of Sirovich [22]. More information regarding the application of the proper orthogonal decomposition in the analysis of turbulent flows together with a detailed bibliography are given in [23, 24]. In this paper, we used the Snapshot POD because it is much more efficient from the numerical point of view.

The POD is a method that reconstructs a data set from its projection onto an optimal base. Besides using an optimal base for reconstructing the data, the POD does not use any prior knowledge of the data set. It is because of this that the basis is only data dependent and this is reason that the POD is used also in analyzing the natural patterns of the flow field.

For the reconstruction of the dynamic behavior of a system the POD decomposes the data set in two parts: a time dependent part, $a_k(t)$, that forms the orthonormal amplitude coefficients and a space dependent part, $\psi_k(x)$, that forms the orthonormal basis. The reconstructed data set is:

$$u(x,t) = \sum_{k=1}^{M} a_k(t) \cdot \psi_k(x)$$
(22)

where M is the number of time instant observations in the data set.

We denote the error of the reconstructed data set as:

$$\varepsilon(x,t) = u(x,t) - \sum_{k=1}^{m} a_k(t) \cdot \psi_k(x)$$
(23)

The base from which the data set is reconstructed is said to be optimal in the sense that the average least squares truncation error is minimized for any given number ($m \le M$) of basis functions over all possible sets of orthogonal functions:

$$\varepsilon_m = \left\langle \left(\varepsilon, \varepsilon\right) \right\rangle \tag{24}$$

where the $\langle . \rangle$ is the ensemble average and (.,.) is the standard Euclidian inner product.

It was shown that the minimization condition for error $\varepsilon(x,t)$ translates into maximum condition for:

$$\lambda = \frac{\left\langle \left| u, \psi \right|^2 \right\rangle}{\left(\psi, \psi \right)} \tag{25}$$

This maximization can be proven to take place if the time independent base functions $\psi(x)$ are obtained from the Fredholm integral equation:

$$\sum_{j=1}^{M} \int R_{ij}(x,x') \cdot \psi_j(x') dx' = \lambda \psi_i(x)$$
(26)

where R_{ij} is the correlation kernel. In this way, we transform this into an eigenvalue problem and λ_k is the eigenvalue corresponding of the eigenvector ψ_k . Because we can consider the inner product as being the equivalent of an "energy", the value of λ_k is linked to the energy contained in mode ψ_k and the optimization process involved can be summarized as: the data set is projected onto a basis that maximizes the energy content. While in the classical approach of Lumley [21], the correlation matrix is constructed as a space correlation matrix and solving the eigenvalue problem, we obtain directly the eigenvectors as the spatial modes and then use them in order to obtain the time-dependent coefficients

$$a_k(t) = \left(u(x,t), \psi_k(x)\right) \tag{27}$$

in the Snapshot POD of Sirovich [22], the correlation matrix is a time correlation matrix:

$$C = \frac{1}{V} \int_{V} u(x,t) \cdot u(x,t') dV$$
(28)

which is of the size of the square of the number of snapshots. From the time correlation matrix, we get the eigenvalues λ_k and time dependent eigenvectors $\phi_k(t)$. The spatial eigenmodes that are time independent, are computed according to the formula:

$$\psi_k(x) = \frac{1}{\mu_k} \int_t^t \phi_k(t) u(x,t) dt$$
⁽²⁹⁾

where

$$\mu_k = \sqrt{|\lambda_k|} \tag{30}$$

For the reconstruction of u(x,t), we take into account only a small number of modes that contain the most energy:

$$u(x,t) = \sum_{k=1}^{m} \mu_k \phi_k(t) \psi_k(x)$$
(31)

The processed data are the variations of absolute velocity magnitude and static pressure fields, which represent the fourth term of Adamczyk decomposition according to Eq. 21. These variations were obtained from numerical simulations using the commercial CFD code Fluent. For each period, we took 20 snapshots and the time between adjacent snapshots is of $\Delta t =$ 9.5781µs; therefore, the Snapshot POD of Sirovich yields 20 eigenmodes for each considered field.

The very high efficiency of the proper orthogonal decomposition is clearly underlined by Table 1. The sum of the first two modes represents 90.5% and 95.5% of the total energy, respectively for the variations of static pressure and absolute velocity magnitude fields while the sum of modes 6 to the last mode represents only 0.163% and 0.236% of the total energy, respectively for the variations of static pressure and absolute velocity magnitude fields. Therefore, both variations of static pressure and absolute velocity magnitude fields can be accurately reconstructed using only the first four modes. Furthermore, these results confirm that the base from which the data set is reconstructed is indeed optimal.

Table 1	Fraction of	f total	energy
for the	most energy	getic n	nodes

for the most energetic models			
Mode	Fraction of total	Fraction of total	
	energy for variation	energy for variation	
	of static pressure	of absolute velocity	
1	6.40E-01	5.49E-01	
2	2.65E-01	4.06E-01	
3	6.63E-02	2.13E-02	
4	1.72E-02	1.58E-02	
5	8.97E-03	5.73E-03	
6	1.41E-03	7.57E-04	
7	6.29E-04	4.50E-04	
8	1.60E-04	8.52E-05	
9	4.58E-05	7.30E-05	
10	4.02E-05	5.25E-05	





Mode 1 for variation of static pressure



Fig. 9 Isolines of mode 1 for variation of static pressure in the section from the middle height of vaned diffuser

Mode 2 for variation of static pressure



Fig. 10 Isolines of mode 2 for variation of static pressure in the section from the middle height of vaned diffuser

The sum of the first three most energetic modes of variation of static pressure field is 97.1% of the total energy. These modes are physical because they show how the potential and wake effects affect the flow, especially in the impeller region. More exactly, the first mode that contains 64% of the total energy shows especially the potential effects that affect the flow in the impeller region. For this reason, the peak of this mode is located on the interface between impeller region and vaned diffuser region as shown in Figs. 8 and 9. The second mode contains 26.5% of the total energy and it shows especially, the interaction between the wakes due to the circumferential Coriolis force and blunt trailing edge of impeller blade and potential effects. The peak of this mode is also placed near the interface between impeller region and vaned diffuser region as shown in Figs. 8 and 10 because, at the middle distance between rows, this interaction is usually maximal. The third mode has 6.6% of the total energy and it represents mainly, the potential effects and the interaction between the wakes due to the circumferential Coriolis force and blunt trailing edge of impeller blade and potential

effects that cannot be captured by the first two modes. The last modes contain only 2.9 of the total energy and represent mainly, the numerical errors that occur at the interface between rotating region and stationary region and due the rotational periodicity condition that is not too correct. Fortunately, they contain little energy (information).



Fig. 11 The first four most energetic modes of variation of absolute velocity magnitude field

Mode 1 for variation of absolute velocity



Fig. 12 Isolines of mode 1 for variation of absolute velocity magnitude in the section from the middle height of vaned diffuser





Fig. 13 Isolines of mode 2 for variation of absolute velocity magnitude in the section from the middle height of vaned diffuser

The first two most energetic modes of variation of absolute velocity magnitude field contain as much as 95.5% of the total energy. The first mode has 54.9% of the total energy and it represents the interaction between wakes due to the circumferential Coriolis force and blunt trailing edge of impeller blade and potential effects. According to theory of characteristics, this interaction affects especially the vaned diffuser region and its peak is located near the middle distance between impeller and vaned diffuser as shown in Figs. 11 and 12. The second mode contains 40.6% of total energy and it represents the interaction between wakes and potential effects in the vaned diffuser region as well as the propagation of potential effects in the impeller region as shown in Figs. 11 and 13. The third and fourth modes have 3.7% of the total energy and they contain both physical and numerical information. From the physical point of view, they contain the information regarding the interaction between wakes and potential effects as well as the influence of potential effects in the impeller region. From the numerical point of view, they represent the numerical errors that occur at the interface between rotating region and stationary region and due the rotational periodicity condition. Furthermore, one sees that the value of the third mode is not close to zero at the outlet boundary of computational domain because we imposed a uniform static pressure on this frontier and this is not too correct according to the theory of characteristics [8, 9].

8 Reconstruction

Because the POD is a method that reconstructs a data set from its projection onto an optimal base, we need only four modes to rebuild the variations of static pressure and absolute velocity fields, accurately. We will rebuild them for two snapshots placed at half the period (the number of impeller blades is equal to the number of vaned diffuser blades) as shown in Figs. 14 and 15.



Fig. 14 The first snapshot for which, the reconstruction is built and isolines of static pressure computed with commercial code Fluent



Fig. 15 The second snapshot for which, the reconstruction is built and isolines of static pressure computed with commercial code Fluent



Fig. 16 Reconstruction of variation of absolute velocity magnitude field for snapshot 1







Fig. 18 Reconstruction of variation of absolute velocity magnitude field for snapshot 2



Fig. 19 Reconstruction of variation of static pressure field for snapshot 2

Analyzing the Figs. 16-19, one observes that only four modes are enough to reconstruct accurately the variations of static pressure and absolute velocity magnitude fields. Furthermore, the points where the field variable u(x,t) has high absolute values are better reconstructed than the points with small absolute values because the data set is projected onto a basis that maximizes the energy content. In other words, points with high energy (information) are reconstructed more accurately than points with low energy.

9 Conclusions

Both Adamczyk and proper orthogonal decomposition have been successfully applied to the decomposition of fully three-dimensional static pressure and absolute velocity magnitude fields obtained from numerical simulations using the commercial CFD code Fluent.

The Adamczyk decomposition clearly shows that the single circular arc vaned diffuser generates a huge jet-

wake region and important pressure losses because the channel is highly divergent in the first part of vaned diffuser. In order to obtain better compressor performance, it is necessary to renounce circular arc vaned diffuser.

Both variations of static pressure and absolute velocity magnitude fields can be accurately reconstructed using only the first four modes; therefore, the proper orthogonal decomposition method is a very efficient method for the data storage of unsteady flows. Moreover, POD technique is able to capture the relevant features of the unsteady rotor-stator interaction, especially, the potential effects and the interaction between wakes due to the circumferential Coriolis force and blunt trailing edge of impeller blade and potential effects. Furthermore, the POD method clearly shows the numerical errors such as those errors that occur at the interface between rotating region and stationary region because the information exchange does not use the characteristic variables, the reflection of numerical waves at rotational periodic and outlet boundaries as well as their magnitude. In order to obtain more accurate results [6, 25], we should impose the phaselagged condition, which is not yet available in Fluent, on the left and right sides of computational sides, instead of the rotational periodicity condition.

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