Determination of Optimal District Heating Pipe Network Configuration

M. JAMSEK*, D. DOBERSEK, D. GORICANEC, J. KROPE

*Termoelektrarna Trbovlje d.o.o.
Ob železnici 27, 1420 Trbovlje, SLOVENIA

University of Maribor
Faculty of Chemistry and Chemical Engineering
Smetanova 17, 2000 Maribor, SLOVENIA

miran.jamsek@tet.si, danijela.dobersek@uni-mb.si, darko.goricanec@uni-mb, jurij.krope@uni-mb.si

Abstract: - In this article the developed mathematical model, which based on simplex method is presented. The mathematical model, consisting of the non – linear objective function and system of non – linear equations for the hydraulics limitations is developed. On its basis the computer program for determination optimal tree path with the use of simplex method was solved. For economic estimation the capitalized value method, which consider all costs of investment and operation was used. The simulation is done on a pipe network with 22 pipes and 13 nodes.

Key-Words: - Heating; Pipe network; Optimisation; Non-linear programming; Simplex method

1 Introduction

Nowadays, great emphasises are on search the solutions for savings and rational energy use, due to rapid population growth, increasing environmental problems and reduce energy sources as a result of excessive consumption of natural sources in the past [1, 2, 3].

Climate changes are mainly the consequences of enormous greenhouse gas emissions, which arising from consumer lifestyles in industrialized countries.

In the focus to decrease climate changes we have to start reducing greenhouse gas emissions where they arise. This requires the search for new engineering and scientific solutions in the field of thermal and process engineering.

One of the potential for reducing environmental pollution and lower energy consumption is district heating systems represent in urban settlements. They enable primary energy savings and are acceptable from ecological point of view [4, 5].

District heating has various advantages compared to individual heating systems. Usually district heating is more energy efficient, due to simultaneous production of heat and electricity in combined heat and power generation plants. The larger combustion units also have a more advanced flue gas cleaning than single boiler systems. In the case of surplus heat from industries, district heating systems do not use additional fuel because they use heat which would be disbursed to the environment.

District heating is a long-term commitment that fits poorly with a focus on short-term returns on investment. Benefits to the community include avoided costs of energy, through the use of surplus and wasted heat energy, and reduced investment in individual household or building heating equipment. District heating networks, heat-only boiler stations, and cogeneration plants require high initial capital expenditure and financing. Only if considered as long-term investments will these translate into profitable operations for the owners of district heating systems, or combined heat and power plant operators.

District heating is less attractive for areas with low population densities, as the investment per household is considerably higher. Also it is less attractive in areas of many small buildings; e.g. detached houses than in areas with a few much larger buildings; e.g. blocks of flats, because each connection to a single-family house is quite expensive.

In the goal to find the ways how to reduce CO_2 emissions and ensure better use of energy the optimization of hot water system is presented, which includes economic method of capitalized cost to solve the flow - pressure model with non-linear programming method.

Nowadays, two methods for solving non - linear programmes are used, both of them interactive, searching for a gradually better solution, until the optimum value is achieved. Mostly, simplex methods focused on searching for permissible solutions within the monotonous defined extreme point of the convex polyhedron of possible solutions and a defined base of the vector space, are used.

2 District heating configuration

District heating systems are intended for the distribution of heat energy by a fluid from a heating source to different users [6, 7]. The pressure drop of the incompressible fluid flow from node i to node j is a function of line and local energy losses defined by the non-linear Darcy-Weisbach equation [8]:

$$p_i - p_j = 0.81 \cdot \frac{\rho \cdot q_{\mathrm{vij}}^2}{d^4} \cdot \left(\frac{\lambda \cdot L}{d} + \sum \zeta \right) = K_{\mathrm{ij}} \cdot q_{\mathrm{vij}}^2 \qquad (1)$$

Pressure drop of incompressible fluids can be also determined by some other equations: the Hazen-Williamson, the Manning and the Gauckler – Manning – Srickler equation, but they are less appropriate for calculation.

The Darcy friction coefficient (λ) is a function of the Reynolds number. In the range of laminar flow the pressure losses depend only on the fluid viscosity, but in the range of turbulent flow the decisive factor is the relative pipe roughness coefficient, which depends upon age, corrosion and pipe material.

In laminar region of flow (Re < 2320) all pipes are treated as hydraulically smooth pipes. Friction coefficient is determined by Hagen – Poisseuille equation:

$$\lambda = \frac{64}{\mathrm{Re}} \qquad (2)$$

In turbulent region ($2320 < \mathrm{Re} < 10^5$) the friction coefficient in hydraulic smooth pipes is determined by Bausius equation:

$$\lambda = 0{,}3164 \cdot \mathrm{Re}^{-0{,}25} \qquad (3)$$

For $10^5 < \mathrm{Re} < 5\times10^5$ Nikuradse equation is used:

$$\lambda = 0{,}0032 + 0{,}221 \cdot \mathrm{Re}^{-0{,}237} \qquad (4)$$

For $\mathrm{Re} > 10^5$ Prandtl - Karman equation is used:

$$\frac{1}{\sqrt{\lambda}} = 2 \cdot \log\left(\mathrm{Re}\sqrt{\lambda} - 0{,}8\right) \qquad (5)$$

For determination of friction coefficient in hydraulic rough pipes $\mathrm{Re} \geq 200 \cdot \frac{d}{\sqrt{\lambda} \cdot k}$, implicit Nikuradse equation is used:

$$\frac{1}{\sqrt{\lambda}} = 2 \cdot \log\frac{d}{k} + 1{,}14 \qquad (6)$$

For determination of friction factor in pipes which belong to hydraulic smooth and hydraulic rough, $\mathrm{Re} \leq 200 \cdot \frac{d}{\sqrt{\lambda} \cdot k}$ implicit Prantl – Colebrook equation is used:

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log\left(\frac{2{,}51}{\mathrm{Re} \cdot \sqrt{\lambda}} \cdot \frac{k}{d} \cdot 0{,}269 \right) \qquad (7)$$

The relative pipe roughness coefficient (k/d) is a relation between absolute roughness of the inner pipe wall and the inner pipe diameter.

The heat losses through the pipes are negligible. Optimal thickness of insulation was calculated with the help of developed computer program [9, 10]. The program flow diagram is presented on figure 1 and on figure 2 the difference between static and dynamic economic methods, which are used in the computer program for calculation the optimal insulation thickness is presented.

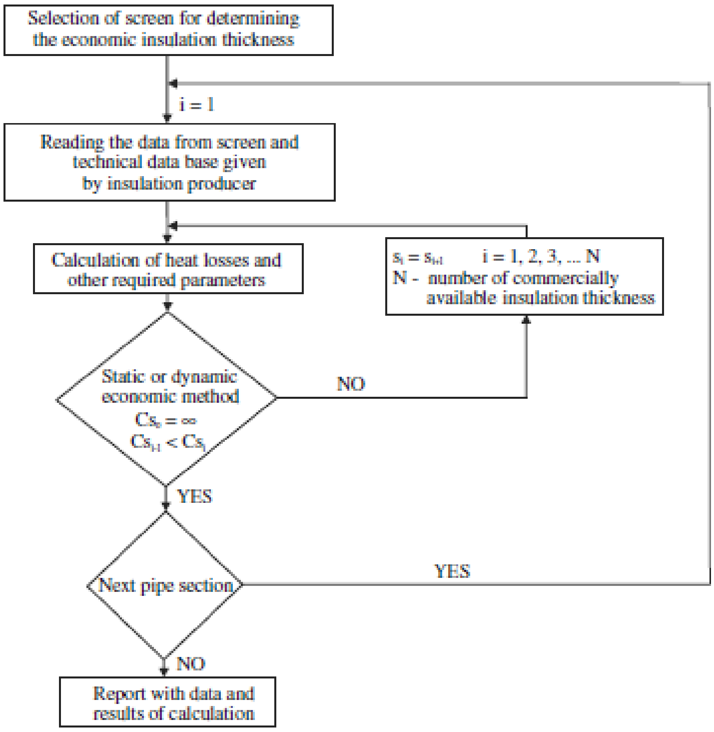

Fig. 1: Simplified flow diagram for optimal thermal insulation thickness (c_s – insulation costs)

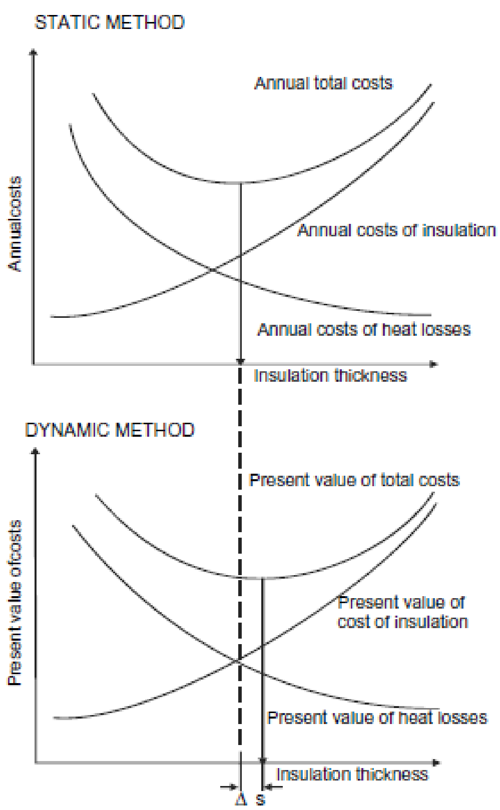

Fig. 2. The difference between the static and dynamic economic methods by determining the economic thermal insulation thickness.

3 The linear programming method

At the beginning the optimisation problem has to be designed. The common configuration of the mathematical model of the linear optimisation problem with m limitations and n variables is:

$$\text{objective function} \quad \min\left(c_1 \cdot x_1 + c_2 \cdot x_2 + \ldots + c_n \cdot x_n\right)$$

limitation

$$\begin{aligned}
&a_{11} \cdot x_1 + a_{12} \cdot x_2 + \ldots + a_{1n} \cdot x_n \quad (\leq, \geq, =)\ b_1 \\
&a_{21} \cdot x_1 + a_{22} \cdot x_2 + \ldots + a_{2n} \cdot x_n \quad (\leq, \geq, =)\ b_2 \\
&\quad\vdots \qquad\qquad\qquad\qquad\qquad\quad \vdots \\
&a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \ldots + a_{mn} \cdot x_n \quad (\leq, \geq, =)\ b_m \\
&x_1, x_2, \ldots, x_m \geq 0
\end{aligned} \tag{2}$$

The mathematical model (2) is soluble with a simplex algorithm.

The condition for solution by the simplex method is that the mathematical model is converted into a standard form [11].

The conditions in the form of non-equations are transferred into equations by introducing additional variables.

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \ldots + a_{1n} \cdot x_n \geq b_1 \tag{3}$$

The possible surplus is subtracted and the non-equation becomes an equation:

$$a_{11} \cdot x_1 + \ldots + a_{1n} \cdot x_n - x_{n+1} = b_1 \tag{4}$$

The additional variable (x_{n+1}) was added and we now have to consider this with conditions, but it has no influence on the objective function due to the fact that its coefficient (c_{n+1}) is zero.

This modified linear programme is complemented with artificial variables (x_{n+m+1}) to find the primary interior point where the simplex calculations are to begin.

$$a_{11} \cdot x_1 + \ldots + a_{1n} \cdot x_n - x_{n+1} + x_{n+m+1} = b_1 \tag{5}$$

We complement the objective function with a new variable and ascribe to it the coefficient (c_{n+m+1}):

$$\begin{aligned}
c = {} & c_1 \cdot x_1 + \ldots + c_n \cdot x_n + c_{n+1} \cdot x_{n+1} + \\
& + \ldots + c_{n+m} \cdot x_{n+m} + c_{n+m+1} \cdot x_{n+m+1}
\end{aligned} \tag{6}$$

The artificial variables serve for determination of the base allowed solution.

In the last step the objective function is transferred from the minimum type to the maximum type:

$$-c = \sum_{j=1}^{n} -c_j \cdot x_j \tag{7}$$

We solve the system of linear equations and come to the primary interior point e.g. the first base solution.

Due to the fact that the first result is not necessarily optimal, we execute the optimality test. If the solution is not optimal a new base is required.

4 District heating optimization

For an effective and accurate description we handle the pipe network as a linear, directed graph, where the fluid entrance points are named as supplied nodes, fluid exit points are user nodes and the nodes where no external inflow or outflow is present as fictive nodes [12].

If we define the number of pipes as N_c and N_v as the number of tree pipe network then the number of nodes is always less than the number of pipes for the structure with loops.

The fluid flow in the net has the direction from the node with higher pressure to the node with lower pressure and is from i to j positive and in the other direction negative. With that we have to consider the first Kirchoff`s law which says that the sum of all flows into a node equals the sum of all outflows [13].

The optimal path with minimal transport costs can be defined by two methods of linear programming, the transport method and the simplex method [14, 15].

The mathematical model for defining the optimal tree path consists of the objective function of capitalised costs $C(q_v)$ that is minimised.

Capitalised costs of each pipe CC_j are the sum of discounted values of investment in pipes, pumps, building expenses and operational costs. As the capitalised costs depend on the pipe diameter, which is discreet variable, and the diameters depend on the flow, the object function is non-linear.

$$\min C(q_v) = \sum_{j=1}^{N_c} CC_j \cdot q_{vj} \tag{8}$$

Further elements of the mathematical model are non-linear equations of hydraulic limitations that represent the continuity of the pipe network and limitations of the simplex method representing the required no- negativity of the simplex variables.

On figure 2 the nonlinear relationship between capitalised costs and volume flow are presented, and on figure 3 the algorithm for determination the optimal branch tree path and pipe diameters are presented.

$$q_{v1} - \sum_{i=2}^{N_v} q_{vi} = 0$$

$$\sum_{j=1}^{N_c} \pm q_{vij} \pm q_{vi} = 0 \left(i = 1,2,3,\ldots,N_v\right) \tag{9}$$

$$q_{vij} \geq 0 \qquad \left(i = 1,2,3\ldots,N_v\ j = 1,2,3,\ldots,N_c\right)$$

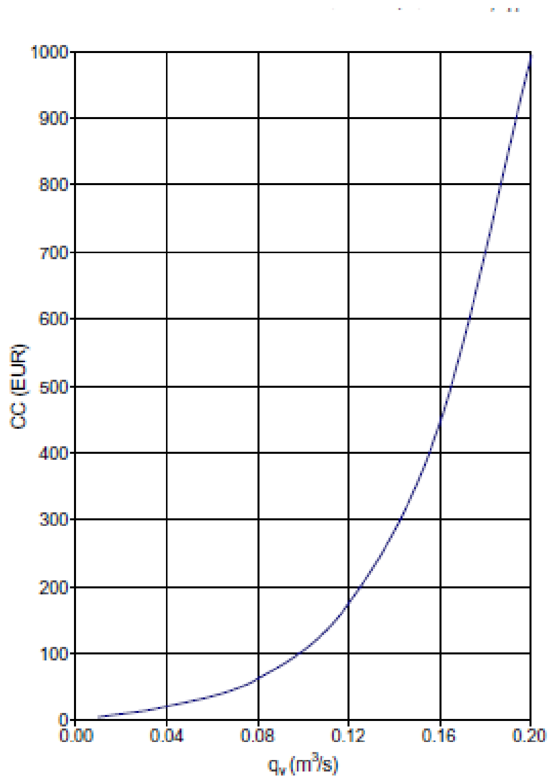

Fig 2.: Relationship between CC and q_v

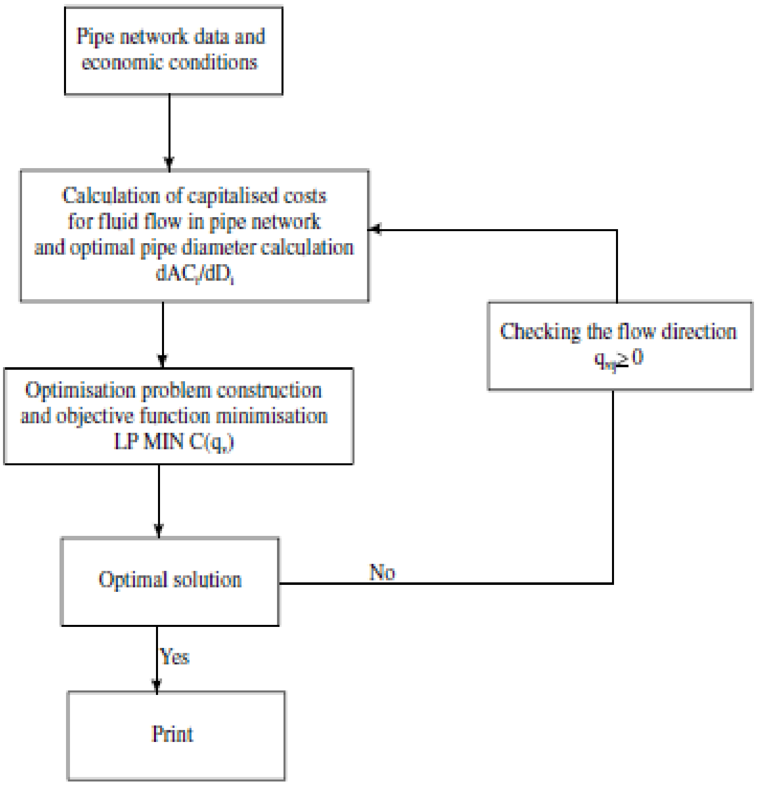

Fig. 3 Algorithm for determination the optimal branch tree path and pipe diameters

3.1 Optimisation problem

The simplex method and the capitalised costs method were used for the optimisation of the district heating system presented on Fig. 4.

The network consists of 22 pipeline sectors and 13 nodes. Hot water flowing at high pressure enters the system in the node 1 and exits in nodes 4, 7, 8, 9, 10, 11, 12, 13, other nodes on fig. 4 are virtual, and represent the branching of pipes.

Data needed for the optimisation of the pipe network are given in tables 1, 2 and 3.

Table 1: Pipe data

Pipe	L (m)	ζ	Pipe	L (m)	ζ
1	200	15	12	100	5
2	200	15	13	100	5
3	150	10	14	200	15
4	90	5	15	200	15
5	170	10	16	160	10
6	150	10	17	130	10
7	110	5	18	80	5
8	90	5	19	70	5
9	220	15	20	40	5
10	50	5	21	40	5
11	80	5	22	30	5

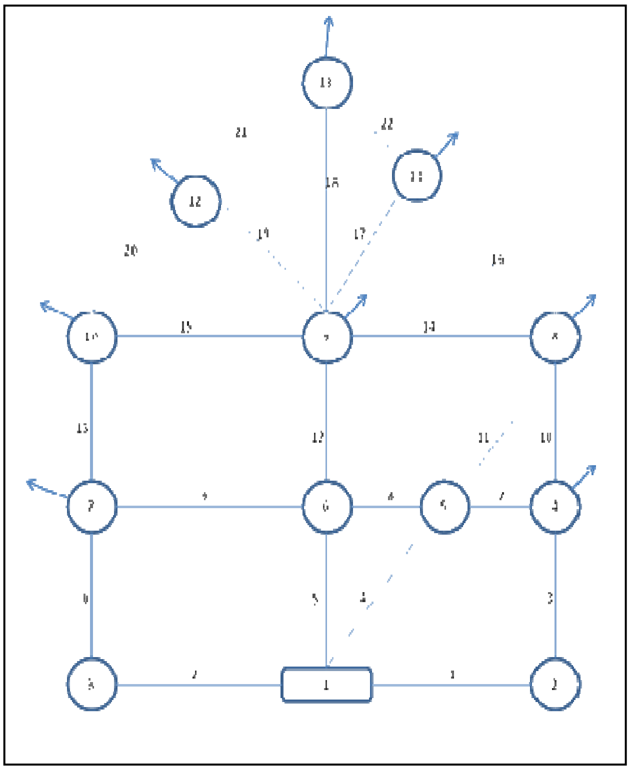

Fig 3: Pipe network with all possible routes for district heating system

Table 2: Data in nodes for input and output

Node	q_v (× 10^{-3} m^3/s)
1	50
2	0
3	0
4	-3,8
5	0
6	0
7	-4
8	-6,2
9	0
10	-5,5
11	-6,0
12	-9,5
13	-15

Table 3: Physical and economical data

Input pressure	10^6 Pa
Input fluid temperature	110°C
Output fluid temperature	70°C
Density	934,8 kg/m^3
Kinematic viscosity	0,226×$10^{-6}$$m^2$/s
Operating time	8760 h/a
Interest rate	0,1
Price of electrical energy	7,5×10^{-5}EUR/Wh
Pump price	0,15 EUR/W
Pumps lifetime	10 year
Pumps efficiency	0,75
Pipe network lifetime	40 year
Pipe roughness	0,4 mm
Coefficient of pipe costs polynomial	
A	18 EUR
B	291 EUR/m
C	229 EUR/m^2
Coefficient of construction costs polynomial	
D	287 EUR
E	310 EUR/m
F	1275EUR/m^2

3.2 Financial analyses

As the lifetime of warm water pipe networks is limited, let us suppose that after that lifetime the network is substituted by a new one. Pumps needed to transport fluids through the pipe network have an essential shorter lifetime than the pipe itself. If we are going to evaluate the project with the future or present value method the overall lifetime of all equipment parts has to be known. The capitalised costs method is used; including all costs of investment and operation to infinity [16]. The capitalised costs are the present value of investment costs repeating to infinity and are presented as formula:

$$CC = B_0 + \frac{B_0 - L_0}{(1+i)^n - 1} + \frac{C_0}{i} \tag{10}$$

Regarding capitalised costs the annuities are defined, based on a discount rate and composed of the sum of all costs and revenues transposed to equal yearly shares. Within defining the annual costs they start from the start of the pipe network operation and end with the closure of the pipe network operation and are presented at the end of each year in equal portions. Between the annuity costs and capitalised costs the following dependence is valid:

$$AC = CC \cdot i \tag{11}$$

Defining the fluid transport costs over the pipe network we have to consider that the initial investment costs are very high. In the first step we define the costs of insulated pipes that are a function of the diameter with the following equation:

$$CC_1 = A + B \cdot d + C \cdot d^2 \tag{12}$$

The conversion on standard diameters is done with the use of economic method and is presented in.

The investment value for pumps depends on the utilisation rate and the pump price calculated per watt of power, the flow volume and the pressure drop that arises with the increase in pump capacity.

$$CC_2 = C_p \cdot P = C_p \cdot \frac{q_v \cdot \Delta p}{\eta} \tag{13}$$

The pumping costs depend on the electrical energy price, the pump power and the operational lifetime of the pipeline.

$$CC_3 = C_e \cdot P \cdot t \tag{14}$$

The construction costs depend on pipe diameter in defined pipe sectors and on the construction site environment of the pipe network.

$$CC_4 = D + E \cdot d + F \cdot d^2 \tag{15}$$

$$\sum CC = CC_1 + CC_2 + CC_3 + CC_4 \tag{16}$$

4 Results

As the optimal solution was chosen by the use of developed computer programme, which based on the simplex optimization method and include capitalized costs the solution presented on figure 5, output pressures in nodes are given in table 4.

The major difference of this optimization process is that all costs are in objective function (CC_1, CC_2, CC_3, CC_4), while at the transition on standard pipe diameters only pipe costs are considered.

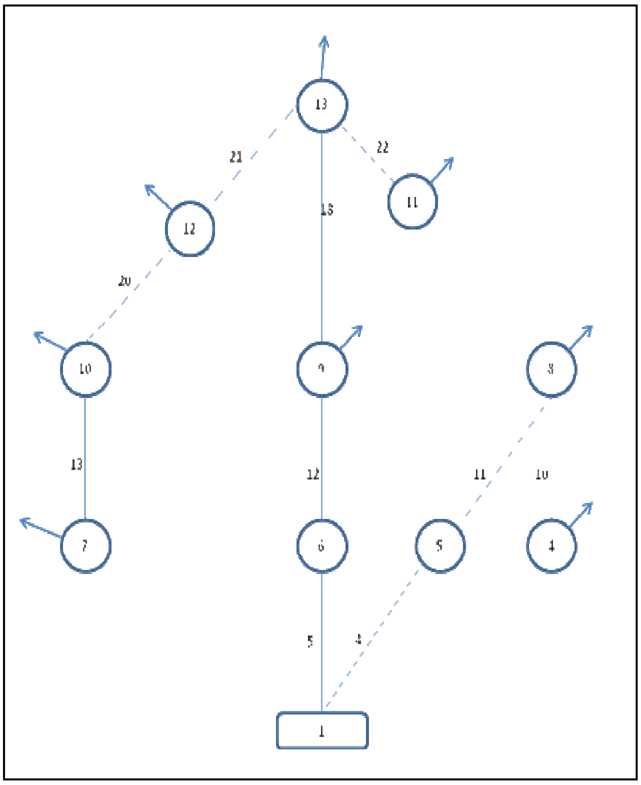

Fig 5: Optimized branch tree path of the district heating system

In table 5 standard pipe diameters are given and flows, velocity and pressure drop are calculated for each pipe section in optimal solution.

All capitalized costs for each section of optimal solution are given in table 6.

Table 4: Output pressure in nodes

Node	p (Pa)
1	1 000 000
2	968 399
3	965 001
4	978 324
5	993 009
6	991 314
7	958 472
8	986 632
9	986 504
10	975 068
11	978 873
12	978 604
13	982 316

Table 5: Optimisation results

Pipe	D(m)	q_v (m^3/s)	v(m/s)	Δp (Pa)
4	0,1270	0,01000	0,789	6692
5	0,2445	0,04000	0,852	8686
10	0,0761	0,00380	0,835	8308
11	0,1270	0,01000	0,789	6376
12	0,2445	0,04000	0,852	4810
13	0,0761	0,00400	0,879	16596
18	0,2445	0,04000	0,852	4187
20	0,1270	0,00950	0,750	3536
21	0,1683	0,01900	0,854	3713
22	0,1016	0,00600	0,740	3443

Table 6: Capitalized costs

Pipe	CC1 (EUR)	CC2 (EUR)	CC3 (EUR)	CC4 (EUR)	ΣCC (EUR)
4	5 400	610	30	31 930	37 960
5	17 880	3 040	110	76 320	97 350
10	2 120	280	10	16 260	18 670
11	4 800	560	20	28 380	33 760
12	10 520	1 690	60	44 890	57 160
13	4 240	580	20	32 520	37 360
18	8 410	1 470	50	35 910	45 850
20	2 400	290	10	14 190	16 900
21	3 000	620	20	15 350	19 000
22	1 530	180	10	10 170	11 890
Σ	60 300	9 320	350	305 930	375 890

4.1 Increased electricity costs

Of some interest are the influences of price changes of construction work, pumps and electrical energy regarding the course and characteristics of the branch tree pipe network.

With equal construction and pump costs, if the costs of electric energy increase by 100% we get the second optimal path, Figure 6.

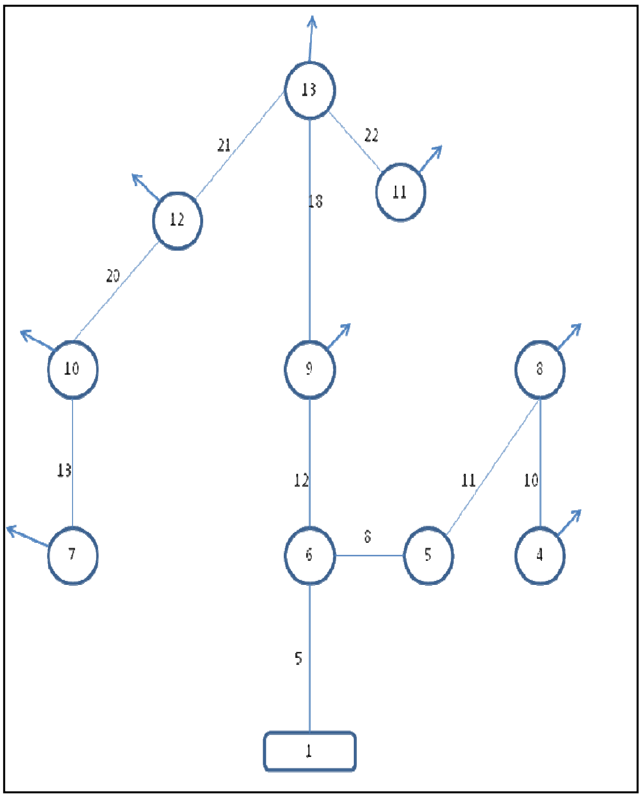

Fig 6: Optimized branch tree path of the district heating system (increased electricity costs)

In table 6 output pressures are presented and in the table 7 standard pipe diameters are given and flows, velocity and pressure drop are calculated for each pipe section in optimal solution from figure 6.

All capitalized costs for each section of optimal solution are given in table 7.

Table 7: Output pressure in nodes

Node	p (Pa)
1	1 000 000
2	985614
3	984069
4	980271
5	989157
6	994710
7	977133
8	984087
9	991553
10	984613
11	986219
12	986857
13	988789

Table 8: Optimisation results

Pipe	D(m)	q_v (m^3/s)	v(m/s)	Δp (Pa)
8	0,2985	0,05000	0,714	5290
8	0,1330	0,01000	0,720	5553
10	0,0889	0,00380	0,612	3815
11	0,1330	0,1000	0,720	5070
12	0,2670	0,04000	0,714	3157
13	0,0889	0,00400	0,644	7480
18	0,2670	0,04000	0,714	2764
20	0,1397	0,00950	0,620	2244
21	0,1937	0,019700	0,645	1932
22	0,1080	0,00600	0,655	2570

Table 9: Capitalized costs

Pipe	CC1 (EUR)	CC2 (EUR)	CC3 (EUR)	CC4 (EUR)	ΣCC (EUR)
8	21780	4630	90	85730	112220
8	5590	970	20	32280	38870
10	2340	250	0	16600	19190
11	4970	890	20	28700	34570
12	11460	2210	40	47110	60820
13	4670	520	10	33200	38400
18	9160	1940	40	37690	48820
20	2580	370	10	14530	17490
21	3390	640	10	16150	20200
22	1600	270	10	10290	12160
Σ	67540	12710	240	322270	402750

4.2 Increased pump and electricity price

With equal construction, if the costs of electric energy increase by 100% and also the pump costs increase for 100% we get the second optimal path presented on Figure 6.

All capitalized costs for each section of optimal solution are given in table 8, for the example if the price of electricity and the price of pumps increase for 100%.

Table 6: Capitalized costs

Pipe	CC1 (EUR)	CC2 (EUR)	CC3 (EUR)	CC4 (EUR)	ΣCC (EUR)
5	21780	4630	170	85730	112310
8	5590	970	40	32280	38880
10	2340	250	10	16600	19200
11	4970	890	30	28700	34590
12	11460	2210	80	47110	60860
13	4670	520	20	33200	38410
18	9160	1940	70	37690	48860
20	2580	370	10	14530	17500
21	3390	640	20	16150	20210
22	1600	270	10	10290	12170
Σ	67540	12710	470	322270	402990

5 Conclusion

Determination by the optimal tree branch path was executed using the simplex method. The basic factors for determination of the path are economy and functionality of the tree pipe network.

When the optimal form of pipe network is searched it is necessary to determine all possible ways, where the pipe could go and with this define the mathematical model. Also all quantities of fluid input and output must be taken into account for each node. In the search for the optimal solution it is absolutely necessary to include economic method to evaluate the costs, which are based on prices of electricity, pipes, construction costs, inflation and discount rates.

Determination of the optimal path tree was done by the mathematical simplex method. Since solving non-linear system requires the use of computers a computer program has been developed, which includes capitalized costs and requests of given case.

We can concluded that the optimal path, defined by the programme is not necessarily the shortest path, due to the fact that the branch tree pipe network is a function of the minimising both the investment and operational costs.

Symbols:

A, B, C, D, E, F	constants
AC	annuity costs
B_0	investment costs
CC	capitalized costs
C_0	operating and maintenance costs
C_1	pipe network expenditure
C_2	pump investment expenditure
C_3	pumping costs
C_4	construction expenditure
C_e	electric price
$C(q_v)$	objective function
C_p	pump price
d	pipe diameter
i	interest rate
k	pipe roughness
K	coefficient of Darcy – Weisbach equation
L	pipe length
L_0	salvage value
n	device lifetime
N_v	number of nodes
N_c	number of pipes
p	pressure

P	power
Re	Reynolds number
t	operating time of pipe network
v	velocity
q_v	flow volume
ρ	density
λ	friction factor
ζ	coefficient of local losses
η	pump efficiency

References:

[1] B. Kulcar, D. Goricanec, J. Krope,. Economy of exploiting heat from low-temperature geothermal sources using a heat pump. *Energy build.*, vol. 40, no. 3, 2008, pp. 323-329.

[2] M. Umberger, T. Krope, J. Krope, Energy economy and the protection of environment with building-in insulated windows, *WSEAS transactions on heat and mass transfer,* vol. 1 no. 1, 2006, pp. 32-38.

[3] J. Krope, D. Dobersek, D. Goricanec, Economic evaluation of possible use of heat of flue gases in a heating plant, *WSEAS transactions on heat and mass transfer,* vol. 1, no. 1, 2006, pp. 75-80.

[4] J. Krope, D. Dobersek, D. Goricanec, Variation of pipe diameters and its influence on the flow-pressure conditions. *WSEAS transactions on fluid mechanics*, vol. 1, no. 1, 2006, pp. 53-58.

[5] D. Dobersek, D. Goricanec, J. Krope, Calibration of pipe networks for district heating using the non-linear optimisation method, *International Journal of Nonlinear Sciences and Numerical Simulation* vol. 7, no. 2, 2006, pp. 225-228.

[6] O. Levenspiel, *Engineering Flow and Heat Exchange,* Revised ed. Plenum Press, 1998.

[7] Garbai L., Börzsönyi, A., Krope, J., The Economic Optimum Of The Setting Up Of Loop District Heating Networks, *Energia és Atomtechnika, XLII. évf. 2. szám.* 1989.

[8] Krope A., Krope J., Ticar I.,. The reduction of friction losses in district-heating pipelines, *Journal of mechanical engineering,* vol. 46, no. 8, 2000, pp. 525-531.

[9] D. Goricanec, J. Krope, A. Jakl, Determining the commercially available thermal insulation thickness of pipe systems, *V. V. International HVAC + R technology symposium*, 2002.

[10] D. Goricanec, J. Krope, A. Krope, A. Jakl, Software application for determining the thermal insulation thickness of pipe systems. V: BOURKAS, P.D. (ur.). *6th IASTED International Conference Power and Energy Systems, July 3-6, 2001, Rhodes, Greece*. Acta Press , 2001, pp. 549-553.

[11] G. Sierksma, Linear and integer programming: Theory and practice*,* Marcel Dekker, Inc., New York, 1996.

[12] Garbai, L., Molnár, L., Optimizing Problems Of Loop District Heating Networks, *Energia és Atomtechnika, XXVIII. évf.,* 1975.

[13] Hetaing Handbook, McGrew Hill, 1999

[14] Saul I. Gass, *Linear Programming Methods and Aplications,* fifth ed., McGrew – Hill, 1985.

[15] Dantzig George B., *Linear Programming and Extensions*, eleventh printing, Princeton, New Jersey Princeton University Press, 1998.

[16] Kurtz M, *Handbook of engineering economics,* McGraw- Hill, 1984.