Application of Euler-Euler Model for Numerical Simulation of a Radial Turbine Working in the Two-Phase Flow Regime

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Abstract: - This paper deals with the numerical calculation of a single stage radial turbine with nucleation in the impeller. The calculations are performed with Ansys CFX 12.1 and are based on the Euler-Euler model. In terms of velocity and temperature, the flow is inhomogeneous. The Euler-Euler model is combined with a nucleation model based on the classical nucleation theory. The working fluid is steam calculated with the IAPWS formulation. First a validation of the used numerical code with experimental results is performed. On the investigated geometry different parameter variations such as different inlet conditions and change in rotational velocity are performed. The goal of the work is to show the influence of this parameters on the water film formation. It will be shown that a high proportion of water is at the blades even if the nucleation occurs in the impeller and the area averaged wetness at the outlet is low.

Keywords: - Euler-Euler, inhomogeneous, two-phase flow, nucleation

1 INTRODUCTION

In many processes an expansion of steam into the two-phase flow regime is required to achieve high efficiency and maximum power output. With the expansion into the two-phase regime different problems occur. It is well known that droplet impact can cause high erosion rates. Prior research shows the effect of droplet impact on turbine blades. From the economic point of view it is important to know the efficiency and the power output of the turbine. Baumann [1] and many different authors show that the efficiency of a turbine is reduced in a range of one percent by one percent wetness at the turbine outlet. Depending on the amount of water in the machine this effects will be more or less important in the engineering process.

The process of condensation, the so called nucleation, is described within the classical nucleation theory [2], [3]. The classical nucleation theory was used to describe the nucleation process in steam turbines by Gyarmathy [4] and Gerber [5]. For the calculation of two-phase flows in turbo machinery with CFD, the Euler-Euler and Euler-Lagrange models are common. The Euler-Lagrange model is valid for small volume fractions of the dispersed

phase and is mostly used if the initial conditions like velocity, particle diameter and the place of origin of the particles are known and time independent. For flows, where the dispersed phase has a high volume fraction and phase change occurs within the flow, the Euler-Euler flow is the more general approach.

The Euler-Euler model is a volume-averaged approach. Each governing equation is multiplied with the phase volume fraction. The values for velocity, density and enthalpy are then calculated for each phase in each numerical cell. From the governing equations it is not known of which shape the droplets are. Also mass transfer, heat transfer and drag cannot be calculated directly from the governing equations. This terms must be calculated by using empirical correlations.

Finally the equations are closed by assuming that the phases share one pressure field.

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2 NOMENCLATURE

Arabic and Greek letters

G	Gibbs-Free-Energy	
h	enthalpy	
k	thermal conductivity	
k _B	Boltzmann constant	
р	pressure	
Е	energy interaction term	
F	impuls interaction term	
U	velocity vector	
М	mass interaction term	
n	rotational speed	
r	radius	
R	gas constant	
S	subcooling p/p _{sat}	
Т	temperature	
3	volume fraction	
μ	dynamic viscosity	
σ	surface tension	
ρ	density	

Indices

c	continuous phase
d	dispersed phase
Ν	any phase
sat	saturation conditions
stat	static conditions
tot	total conditions
in	conditions at inlet
out	conditions at outlet

Definitions

wetness (mass water / total mass) x 100

3 GOVERNING EQUATIONS

The numerical model is based on the Euler-Euler approach. For each phase a set of volume-averaged governing equations is solved.

$$\frac{\partial}{\partial t}(\varepsilon_{N}\rho_{N}) + \nabla(\varepsilon_{N}\rho_{N}U_{N}) = M_{N}$$
(1)

$$\frac{\partial}{\partial t} \left(\epsilon \rho_{\rm N} U_{\rm N} \right) + \nabla \left(\epsilon \rho_{\rm N} U_{\rm N} U_{\rm N} \right) = \\ -\epsilon \frac{\partial p}{\partial x} + \nabla^2 \left(\epsilon \mu U_{\rm N} \right) + F_{\rm W, x} + F_{\rm M, x}$$
(2)

$$\frac{\partial \left(\epsilon_{N} \rho_{N} h_{tot, N}\right)}{\partial t} - \epsilon \frac{\partial p}{\partial t} + \nabla \left(\epsilon_{N} \rho_{N} U_{N} h_{tot, N}\right) =$$
(3)
$$\nabla \left(\epsilon_{N} k_{N} \nabla T_{N}\right) + \nabla \left(\epsilon_{N} U_{N} \tau_{N}\right) + E_{W} + E_{M}$$

The governing equations are connected with interaction terms to describe the mass, heat and impulse transfer between the phases. The transport of droplets is calculated with an additional transport equation (4).

$$\frac{\partial \left(\varepsilon_{d} \rho_{d} N_{d}\right)}{\partial t} + \nabla \left(\varepsilon_{d} \rho_{d} U_{d} N_{d}\right) = N_{Nd}$$
(4)

4 INTERACTION TERMS

The onset of condensation is calculated with the classical nucleation theory. For the formation of a droplet equilibrium is needed between the surface energy and the volume energy. The number of molecules needed to create an droplet or initial cluster at a given subcooling is calculated with the change in Gibbs-Free-Energy, this change is given by equation (5).

$$\Delta G = -\frac{4}{3}\pi \cdot r^3 \cdot \rho_{d} \cdot R \cdot T_{c} \cdot \ln(S) + 4\pi \cdot r^2 \cdot \sigma$$
(5)

At low subcooling the number of molecules needed to create a cluster is high compared to the number at higher subcoolings. At the local maximum in the Gibbs-Free-Energy a droplet can exist and grow further. The droplet radius at this point is calculated with equation (6).

$$r_{\rm d}^* = \frac{2\sigma}{\rho_{\rm d} \cdot \mathbf{R} \cdot \mathbf{T}_{\rm c} \cdot \ln(\mathbf{S})} \tag{6}$$

The probability of formation of a cluster with a high number of molecules is of course lower than the formation of a cluster with only a few molecules. In the classical nucleation theory the probability distribution is calculated with the Boltzmann equation. With the change in Gibbs-Free-Energy and the Boltzmann distribution for the probability of cluster formation, the nucleation rate is given by equation (7).

$$J = A \cdot exp\left(\frac{-\Delta G}{k_{B} \cdot T_{C}}\right)$$
(7)

The nucleation coefficient A is calculated with equation (8), which was for example published by Gerber [5].

$$A = \frac{1}{1+\eta} \left(\frac{2\sigma}{\pi \cdot m^3}\right)^{0.5} \frac{\rho_c}{\rho_d}$$
(8)

$$\eta = 2 \frac{\kappa - 1}{\kappa + 1} \frac{\Delta h_{cd}}{R \cdot T_c} \left(\frac{\Delta h_{vl}}{R \cdot T_c} - \frac{1}{2} \right)$$
(9)

The mass transfer between the continuous phase and an existing droplet is calculated using the "Thermal Phase Change Model". Condensation is driven by the temperature difference between the continuous phase and the surface temperature of an existing droplet. For both, small and large droplets, the surface temperature is calculated following the equations from Gyarmathy [4] and Gerber [5]. The surface temperature of a small droplet is calculated by equation (10) and the surface temperature of a larger droplet is expected to remain at saturation conditions.

$$T_{d} = T_{sat}(p) - \frac{r_{d}^{*}}{r_{d}} \left[T_{sat}(p) - T_{c} \right]$$
(10)

Through using equation (11) the heat transfer is calculated.

$$q_{c} = \frac{k_{c} N u_{c}}{2r_{d}} A_{d} \left(T_{c} - T_{sat} \right)$$
(11)

In this equation the Nusselt number for small bubble $d \le d$ is calculated using the Knudsen correction (12), which can be found in the publications of Gyarmathy [4].

$$Nu_{c} = \frac{2}{1 + c \cdot Kn}$$
(12)

For bubbles larger than 1 μ m the heat transfer is calculated using the Ranz-Marshall correlation [6].

$$Nu_{c} = 2 + 0.6 \operatorname{Re}_{cd}^{0.5} \operatorname{Pr}_{c}^{0.3}$$
(13)

At the surface of the bubbles the energy fluxes from liquid to vapour and from vapour to liquid must be equal.

The flux contains of the energy transported with the mass across the surface and molecular transport mechanism at the surface. The balance equation is then:

$$q_d + \dot{m}_{cd} \cdot h_d = q_c + \dot{m}_{cd} \cdot h_c$$
(14)

From this equation the mass flow across the surface can be calculated with equation (15).

$$\dot{m}_{cd} = \frac{q_d + q_c}{h_c - h_d}$$
(15)

The drag between the continuous phase and the droplets is calculated with the Schiller-Naumann [7] correlation which is applicable for spherical droplets at low particle Reynolds numbers.

$$C_{W} = \frac{24}{\text{Re}_{cd}} \left(1 + 0.150 \,\text{Re}_{cd}^{0.687} \right)$$
(16)

At the given Eötvös, Morton and Reynolds number it can be shown from a diagram from Clift et al. [8] that the droplets are spherical. The measurement values, for the drag exposed to a sphere at Reynolds numbers below 800, fit with high accuracy the experimental results.

5 VALIDATION OF THE SOLVER

Before starting with an investigation on a real geometry problem the CFD Code CFX 12 used is validated using the experimental results from Bakhtar et al. [9] and White et al. [10]. In the experiment of Bakhtar a cascade was investigated consisting of seven translational disposed blades, of which the middle channel was used for the measurements. To validate the CFD code the experimental and numerical results are compared using the surface pressure on the middle blade. The geometry with the boundary conditions used for the CFD calculation is shown in Fig. 1.



FIG. 1 – BC'S OF BAKHTAR-CASCADE

The surface pressure is shown in Fig. 2 where the red dots represent the measurements and the black line represents the numerical results. The calculated surface pressure fits very well the experimental results. The rise in pressure at the onset of nucleation is well predicted with the numerical code. On the pressure side the location of pressure rise is at the trailing edge and at the suction side the rise in pressure is at 85% of axial chord.



FIG. 2 - COMPARISON OF SURFACE PRESSURE

Besides the calculated surface pressure, the numerical calculation gives some more results, which will be presented in the next section.

The vapour subcooling is shown in Fig. 3. The maximum subcooling reaches a value of 27 K, this means that the vapour temperature is 27 K below the saturation temperature.



FIG. 3 - SUBCOOLING BAKHTAR-CASCADE

In Fig. 4 the droplet diameter is shown, in which the dark blue coloured area represent regions where no droplets exist. It can be easily seen that not far away from the onset of nucleation the droplet diameter is one magnitude higher than at the birth of the nuclei. To the cascade outlet the droplet growth slows down, this is caused by an decrease in temperature difference between vapour and liquid.



droplet diameter [µm]

FIG. 4 - DROPLET DIAMETER BAKHTAR-CASCADE

The wetness in the cascade reaches a maximum of 7% at the outlet, which is shown in Fig. 5. Shortly behind the trailing edge the system returns to thermodynamic equilibrium, what can be seen, because no further condensation exists.



FIG. 5 - WETNESS BAKHTAR-CASCADE

Based on the experimental results of White et al. [10] a second test case is used for a validation of the numerical code. Here the test case named D1 from White is used. The geometry is like the Bakhtar Cascade a translational cascade. The geometry with the boundary conditions is shown in Fig. 6. Again one blade is investigated numerically and the influence of the other blades is described with periodic boundary conditions.



FIG. 6 - BOUNDARY CONDTIONS WHITE-CASCADE

In Fig. 7 the Wilson-Line determined from the experimental results on the left hand side is compared with the numerical solution on the right hand side. The onset of nucleation is identical with the experimental results at the pressure side trailing edge and nearly identically with the results at the suction side. Also the shape of the Wilson-Line shows good agreement between the experiments and the numerical results.



FIG. 7 - COMPARISION OF WILSON-LINE

The maximum subcooling is here in a small area at the trailing edge 62 K. The average subcooling along the Wilson Line is about 40 K, which can be seen in Fig. 8. In contrast to the Bakhtar cascade the nucleation did not occur within the blade channel but at the end of it.



FIG. 8 - SUBCOOLING WHITE-CASCADE

The comparison of the experimental results with the numerical results shows for both investigated cases, Bakhtar and White Cascade, a good agreement. The pressure rise caused by the nucleation is well predicted and the position and shape of the Wilson-Line shows good agreement with the experimental results.

In addition to the validation of the software, this section has shown that the location of the Wilson line and the maximum droplet diameter between the two cascades have strong differences. An overview of the characteristic sizes is in Table 1. In the results of the Bakhtar cascade it is found, that the nuclei have a much smaller diameter, than in the White cascade, which is due to the higher subcooling rate, which in turn is caused by a faster expansion.

	Cascade	
	Bakhtar	White
max. subcooling [K]	40 K (62 K*)	26.6 K
min. droplet diameter [µm]	0.007	0.04
max. droplet diameter [µm]	0.035	0.14
* small area at the trailing edge		

TAB. 1 - COMPARISON BAKHTAR-WHITE CASCADE

6 INVESTIGATION OF A RADIAL TURBINE

The influence of the Wilson-Point and the rotational speed is investigated in a radial turbine with water as working fluid. At the inlet temperature of 393 K the working fluid is subcooled at the inlet of the turbine stage. Such inlet conditions occur, if a fast expansion exists before the inlet. For example, caused by a previous stage. At the other inlet temperatures, the steam is superheated. The turbine consists of a stator with 36 blades and an impeller with 18 blades. The geometry and the boundary conditions are shown in Fig. 9. The outlet is at atmospheric pressure of 0.1 MPa.



FIG. 9 - BOUNDARY CONDITIONS

7 VARIATION OF INLET TEMPERATURE

The behaviour of the water droplets is investigated for different inlet temperatures. By varying the inlet temperature the position of the Wilson-Line changes and so the influence of different effects like centrifugal force caused by machine rotation and centrifugal force caused by the curved blades is varied. The inlet temperature is changed stepwise from 433 K to 393 K in 10 K steps, other parameters are caped constant.

In Fig. 10 the subcooling is shown for an inlet temperature of 393 K at the middle plane of the blading, the nucleation occurs in the stator blading and the temperature of the vapour is 14 K below the saturation temperature at the Wilson-Point. After the first droplet exist the temperature falls very fast to the saturation temperature.



FIG. 10 - SUBCOOLING FOR $T_{IN} = 393$ K

In Fig. 11 the growth of the droplets is shown. The growth rate corresponds to the fall in subcooling level obtained from Fig. 10. In regions of high temperature difference the droplet growth is faster than in regions of low temperature difference, which can be easily seen from equation (15) and (11). The droplet size increases in a short time by two magnitudes, which is shown in Fig. 11.



FIG. 11 - DROPLET GROWTH FOR $T_{IN} = 393$ K

With increasing inlet temperature the Wilson-Line is shifted into the impeller. At an inlet temperature of 433 K the Wilson-Line is nearly at the trailing edge of the impeller, which can be seen in Fig. 12.



In Fig. 13 the position of the Wilson-Line for different inlet temperatures is marked by hand. With the investigated parameter variation a wide range of positions of the Wilson-Line is covered.



FIG. 13 - WILSON-LINES FOR DIFFERENT $T_{\rm IN}$

Depending on the position of nucleation the behaviour of the droplets is quite different. For an inlet temperature of 393 K the wetness distribution is shown in Fig. 14. Numerical cells with wetness above 10% are highlighted with a rainbow scalar that represents the wetness fraction.



FIG. 14 - WETNESS DISTRIBUTION AT $T_{IN} = 393$ K

The droplets move to the shroud at about 10% of axial chord and the wetness is above 5%. At the suction side of the blades is the highest amount of water. This result is caused by the rotation of the machine, therefore the droplet trajectory is strongly dependent on the centrifugal force and the coriolis force. The centrifugal force caused the water to move to the shroud and the coriolis force moves the water to the suction side. At about 20% of axial chord the water distribution is nearly homogenous over the shroud. Then the water is moved because of the machine rotation to the suction side of the blades. At 40% axial chord a high amount of water is only

at the suction side of the blade. In the middle between two blades the wetness is below 5 %.

At an inlet temperature of 403 K the water accumulation is higher at the shroud and at the blades then at an inlet temperature of 393 K. Never the less the averaged wetness at the turbine outlet is higher for the lower inlet temperature like it is expected. For an inlet temperature of $T_{in} = 393$ K the averaged wetness at the outlet is 4.3 % and at $T_{in} = 403$ K the averaged wetness is 3.7 %. The wetness distribution is more homogenous for the lower inlet temperature, the reason is a lower particle diameter, maximum diameter of 0.5 µm at $T_{in} = 393$ K and 1.0 µm at $T_{in} = 403$ K. Which in turn is caused by the higher subcooling in the stator as an consequence of the faster expansion. This could be also derived from equation (6).

At the inlet temperature of 403 K a high amount of water is at the shroud. At 10 % axial chord the wetness is above 5 %. Now not only at the shroud accumulation of water exist but also at the blading over the full span from hub to the shroud. The water at the blading is no longer at the suction side but on the pressure side. This leads to the conclusion that the movement of the droplets is influenced by the centrifugal force caused by the curved streamlines as a reason of the curved blades.



FIG. 15 - WETNESS DISTRIBUTION AT $T_{IN} = 403$ K

With increasing inlet temperature and corresponding to that, the Wilson-Line is shifted in direction of the impeller trailing edge, more water accumulates at the blading, especially at the hub region. Caused by inertial forces the droplets cannot follow the main flow and are deflected towards the hub. The amount of water at the shroud is getting more and more negligible. Anyway, if the nucleation occurs at the trailing edge or even after the trailing edge, no water is at the impeller parts.



FIG. 16 - WETNESS DISTRIBUTION AT T_{IN} = 413 K

From this investigations the result of water accumulation can be described by naming three positions of water accumulation: at the hub, the shroud and at the blades.

8 VARIATION OF ROTATION SPEED

The rotational speed is varied for an inlet temperature of Tin = 403 K. Varying the rotational speed leads to two different effects that influence the droplet behaviour. The first is the difference in centrifugal force at different rotational speeds. The second effect is the change of the incidence angle at different rotational speeds. In Fig. 17 the direction of the relative velocity is shown for the investigated rotational speeds.



FIG. 17 - INCIDENCE ANGLE FOR DIFFERENT N

The results are shown in appendix "Variation of rotational speed". Like in the previous section numerical cells with wetness fraction above 5% are highlighted with a rainbow scalar. With increasing rotational speed, respectively positive incidence angle, a water film exist right at the pressure side leading edge. With decreasing rotational velocity, respectively with negative incidence

angle, the water droplets are shifted away from the blades through the middle of the blading channel. In all investigated cases water accumulation occurs manly at the shroud. It can be seen that both with increasing and decreasing rotational speed water accumulation over the full span from hub to shroud at the blades is reduced. But the reasons for that are quite different. For the first case, with increasing rotational speed, the water is moved towards the shroud because of the centrifugal force. In the second case, with decreasing rotational velocity, no water accumulation exists at the blades because of the negative incidence angle.

9 CONCLUSION

In the first part of the paper it was shown that the used numerical code predicts with high accuracy the Wilson-Line and the pressure rise at the onset of nucleation. The droplet behaviour was investigated in a radial turbine. It was shown that the water accumulation is highly dependent on the location of the nucleation.

The incidence angle has an high influence on the droplet behaviour.

Overall the conclusion can be drawn that even at low area averaged mass fractions at the outlet a high amount of water accumulates at the blades and at the shroud.

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ANNEX A

VARIATION OF INLET TEMPERATURE











 $N = 1.3 * N_{NOM}$