PERFORMANCE OF THE CONSTANTS COUPLE OF THE SECOND ORDER TURBULENCE MODEL (RSM) APPLIED TO AN AXISYMMETRIC CONFIGURATION

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Abstract

The Reynolds stress model (RSM) is considered as a more sophisticated approach used to modelling the turbulent flows, compared to the other turbulence models. This type of model, used to improve the accuracy of the results, overcomes the turbulent viscosity concept and the assumption of the turbulence isotropy which it means. The pressure-strain correlation term is among the most important terms to be modelled in the exact transport equations of the Reynolds stresses. Different models, used for modelling the pressure-strain correlation term, have been proposed by different authors. The basic model is that proposed by Launder, Reece and Rodi (1975) [1] called LRR-IP, a combination of Rotta's model and the IP model (Isotropisation of Production). The main objective of this study is to test the performance of the constants pair (C_1, C_2) used in such model for modelling the pressure-strain correlation term.

Key words: turbulence, axisymmetric jets, second order modelling, pression-strain correlation.

1. Introduction

The numerical study, which aims to test and develop different models to approach as accurately as possible experience while preserving the universality of developed models, has as a base the turbulence models with second moment closures. This level of closure appears to be the minimum level where a realistic physical approach of the terms present in the equations can be developed, the models of lower order constituting then the simplifications of the second order model. This type of modelling is to recommend in the case of complex flows (shock wave, boundary layer subject to strong adverse pressure gradients, recirculation zone, curvature effects, rotation), especially in the presence of strong anisotropy and for which lower-order models are insufficient.

Several studies on the axisymmetric turbulent jets have been realized by different authors to test the performances of different turbulence models in particular the k-ε model and the second order model (RSM) and extract information on the influence of various parameter such as: the Reynolds number, the velocity ratio U_j/U_e, the shape of the jet nozzle, the density ratio. However, to our knowledge, a specific study of the performances of the constants pair of RSM model, used to modelling the pressure-strain correlation term, has not yet been conducted. However, according to Pope (2000) [2] this term, often considered responsible for compromising the accuracy of the RSM model predictions, represents the important quantity to be modelled. Therefore, an analysis of the performances of this pair of constants, used to modelling the pressure-strain correlation term, was conducted from the comparative study of numerical simulation results by the RSM turbulence model.

2. Governing equations

The equations system obtained, for steady flow, by using the Reynolds averaged and by applying the boundary layer assumptions is as follows:

Continuity equation

$$\frac{\partial}{\partial x}(\overline{\rho}\,\overline{U})+\frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{\rho}\,\overline{V})=0 \qquad (1)$$

Momentum conservation equations

Axial equation:

$$\frac{\partial}{\partial x}(\overline{\rho}\,\overline{U}\,\overline{U})+\frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{\rho}\,\overline{U}\,\overline{V})+\frac{\partial \overline{P}}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{\rho}\,\overline{uv})\text{ - Dif.Mol.}=0 \qquad (2)$$

where Dif.Mol. is the molecular diffusion term. With large turbulence Reynolds numbers, the molecular effects are negligible compared to the turbulent agitation effects.

Radial equation:

$$\frac{\partial}{\partial x}(\overline{\rho}\,\overline{U}\,\overline{V})+\frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{\rho}\,\overline{V}\,\overline{V})+\frac{\partial \overline{P}}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{\rho}\,\overline{vv})-\overline{\rho}\,\frac{\overline{ww}}{r}\text{ - Dif.Mol.}=0 \qquad (3)$$

2.1. Second order modeling

Modelling of the stresses Reynolds equations

$$\frac{\partial}{\partial x_k}(\overline{\rho}\,\overline{U}_k\,\overline{u_i u_j})=P_{ij}+D_{ij}+\phi_{ij}-\frac{2}{3}\delta_{ij}\,\overline{\rho}\,\varepsilon \qquad (4)$$

Production terms:

$$P_{ij}=-\overline{\rho}\,(\overline{u_i u_k}\frac{\partial \overline{U}_j}{\partial x_k}+\overline{u_j u_k}\frac{\partial \overline{U}_i}{\partial x_k}) \qquad (5)$$

It's the exact terms which not need to be modelled. They play an important role in the representation of the various mechanisms of turbulent stresses creation.

Diffusion term:

This term is modeled as follows:

$$D_{ij} = C_S \frac{\partial}{\partial x_k}[(\bar{\rho}\frac{k}{\varepsilon}\overline{u_k u_l})\frac{\partial}{\partial x_l}(\overline{u_i u_j})] \qquad (6)$$

Pressure-strain correlation term:

The modelling of this term is based on the Poisson's equation connecting the pressure fluctuating to the velocity field.

$$\phi_{ij} = \overline{p\,(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})} = \phi_{ij}^{(1)} + \phi_{ij}^{(2)} \qquad (7)$$

The first term called the Rotta term (1951) [3], corresponds to a linear return to isotropy. It is modelled initially by this one as:

$$\phi_{ij}^{(1)} = -C_1 \bar{\rho}\,\varepsilon\,(\frac{\overline{u_i u_j}}{k} - \frac{2}{3}\delta_{ij}) = -C_1 \bar{\rho}\,\varepsilon\, a_{ij} \qquad (8)$$

The second term concerning the interaction between the mean motion and turbulence, also called the rapid term, is modelled by using the IP model (Isotropisation of the Production), and defined by:

$$\phi_{ij}^{(2)} = -C_2 (P_{ij} - \frac{2}{3}\delta_{ij} P_k) \qquad (9)$$

where $P_k = \frac{1}{2} P_{kk} = -\bar{\rho}\,\overline{u_i u_k}\frac{\partial \bar{U}_i}{\partial x_k}$ is the mean rate of turbulent kinetic energy production.

The adjustment of the constants pair (C_1, C_2) is based on some experiments made by certain researchers. According to the figure 1, the range of these is very extensive. Based on various experimental results and calculations, Launder has shown that C_1 and C_2 are related by: $(1-C_1)/C_2 = 0.23$.

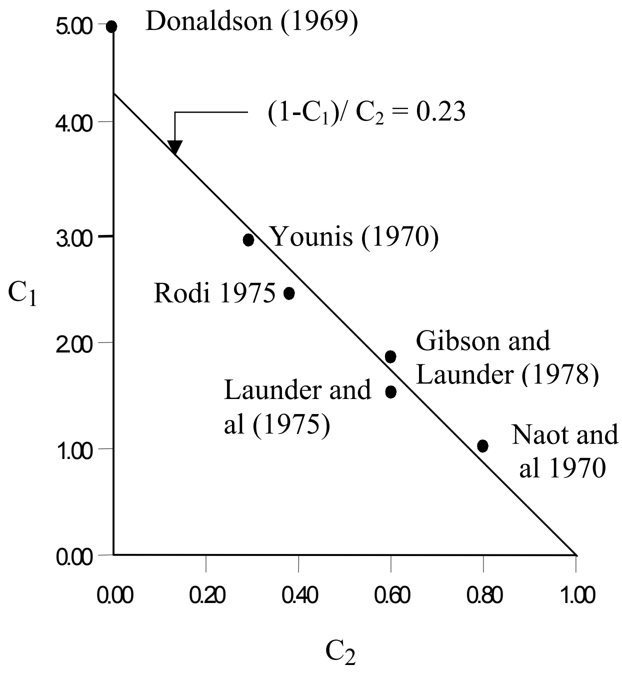

Fig. 1: Cartography of the constants pair (C_1, C_2)

To test the performance of this pair and see its effect on the evolution of the principal parameters characteristic of such flow, we chose six pairs of constants, table 1.

The first two pairs (C_1 = 1.5, C_2 = 0.6) and (C_1 = 1.5, C_2 = 0.76) have been proposed by Launder and al (1975) [1]. Gibson and Launder (1978) [4] suggest using the third pair (C_1 = 1.8, C_2 = 0.6), encountered in the literature. Based on the relationship $(1-C_1)/C_2$ = 0.23 and taking the constant C_1 = 2.3, value used so that the axisymmetric jets are well calculated, then we find, C_2 = 0.47. The fifth pair (C_1= 2.3, C_2 = 0.6) was used by Sanders (1997) [5]. Finally, Hanjalic and Launder (1972) [6] proposed the sixth pair (C_1 = 2.3, C_2 = 0.76).

Table 1: Constants pairs (C_1, C_2), used in the modelling of the pressure-strain correlation term.						
C_1	1.5	1.5	1.5	1.8	2.3	2.3
C_2	0.6	0.76	0.78	0.6	0.6	0.76

Modelling of dynamic dissipation equation

The modelled equation is following form:

$$\frac{\partial}{\partial x_k}(\bar{\rho} U_k \varepsilon) = D_\varepsilon + \bar{\rho}\frac{\varepsilon^2}{k}\psi(\varepsilon) \qquad (10)$$

The term $\psi(\varepsilon)$ contains the effects of production and destruction of dynamic dissipation. This term is modeled by using the form proposed by Launder, Reece and Rodi (1975) [1]:

$$\psi(\varepsilon) = C_{\varepsilon 1}\frac{P_k}{\bar{\rho}\varepsilon} - C_{\varepsilon 2} \qquad (11)$$

The modeling of the diffusion term D_ε is given by the following relation:

$$D_\varepsilon = C_\varepsilon \frac{\partial}{\partial x_k}[\bar{\rho}\frac{k}{\varepsilon}\overline{u_k u_l}\frac{\partial}{\partial x_l}(\varepsilon)] \qquad (12)$$

2.2. Expression of the generalized equation

The transport equations quoted previously and in a cylindrical coordinates system can be put in the following parabolic general form:

$$\frac{\partial}{\partial x}(\bar{\rho} U \bar{\Phi}) + \frac{1}{r}\frac{\partial}{\partial r}(r\bar{\rho} V \bar{\Phi}) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{\rho} D\frac{\partial \bar{\Phi}}{\partial r}\right) + S_\Phi \qquad (13)$$

The model coefficients used in the present study are given in the table 2 and The sources terms S_Φ in table 3, where $P = -\bar{\rho}\,\overline{uv}\frac{\partial \bar{U}}{\partial r}$.

Table 2: model coefficients in the Reynolds stress models (RSM).			
C_s	$C_{\varepsilon,1}$	$C_{\varepsilon,2}$	C_ε
0.22	1.45	1.90	0.18

Table 3: Source terms for the second order model in the generalized equation (13). The turbulent diffusion coefficient D is $C_{diff}\frac{k}{\varepsilon}\overline{vv}$. For all the Reynolds stresses, $C_{diff} = C_s$ and for dynamic dissipation rate $C_{diff} = C_\varepsilon$

Variable	S_Φ
$\overline{uu}$	$-2(1-C_2)\bar{\rho}\,\overline{uv}\frac{\partial \bar{U}}{\partial r} - \frac{2}{3}\bar{\rho}\varepsilon - C_1\bar{\rho}\frac{\varepsilon}{k}(\overline{uu} - \frac{2}{3}k) + \frac{2}{3}C_2 P$
$\overline{vv}$	$-\frac{2}{3}\bar{\rho}\varepsilon - C_1\bar{\rho}\frac{\varepsilon}{k}(\overline{vv} - \frac{2}{3}k) + \frac{2}{3}C_2 P - \frac{2C_s}{r^2}\frac{k}{\varepsilon}\bar{\rho}\,\overline{ww}(\overline{vv} - \overline{ww})$
$\overline{ww}$	$-\frac{2}{3}\bar{\rho}\varepsilon - C_1\bar{\rho}\frac{\varepsilon}{k}(\overline{ww} - \frac{2}{3}k) + \frac{2}{3}C_2 P + \frac{2C_s}{r^2}\frac{k}{\varepsilon}\bar{\rho}\,\overline{ww}(\overline{vv} - \overline{ww})$
$\overline{uv}$	$-(1-C_1)\bar{\rho}\,\overline{vv}\frac{\partial \bar{U}}{\partial r} - C_1\bar{\rho}\frac{\varepsilon}{k}\overline{uv} - \frac{C_s}{r^2}\frac{k}{\varepsilon}\bar{\rho}\,\overline{ww}\,\overline{uv}$
ε	$\frac{\varepsilon}{k}(C_{\varepsilon,1}P - C_{\varepsilon,2}\bar{\rho}\varepsilon)$

3. Numerical method

The equations to be solved: the Navier-Stocks, energy conservation, to which are added those corresponding to the model of turbulence modelled above are all the equations of convection diffusion type with sources terms. Each equation of the variable Φ for a two-dimensional steady flow can thus be written in the form of the equation (13). This equation is integrated with the appropriate boundary conditions according to the finite volume method described by Patankar (1980) [7] over a finite control volume. For the best possible resolution, the equations are discretized using starggered arrangement (starggered grid) where the velocity are localised on the faces of the control volume, turbulent shear stress on the peaks while the scalar variables are on the central points (nodes).

In order to reduce the computation time several numerical schemes are possible as, the upwind, hybride and the power law schemes, Patankar (1980) [7] and Luppes (2000) [8]. Among these schemes, the power law difference scheme (PLDS) is not that which gives the best gain time but it is that which offers the closest solution to the exact solution given by the exact scheme (LEDS, for Locally Exact Difference Scheme), Ruffin (1994) [9].

3.1. Studied configuration

In this study, we will focus on the nonreacting axisymmetric confined turbulent jets. We have adapted our computational code on the less complex configuration and more common in theoretical or experimental studies. This configuration is that shown in figure 2.

The velocity of principal jet (Air) is fixed at Uj=38 m/s and that of the secondary jet (coflow) is Ucf=1 m/s. The value of the Reynolds number at the nozzle exit is Rej ≈ 21000, which corresponds to a fully turbulent flow.

3.2. Boundaries conditions

Upstream, the entry profiles are given by experiment of Chassaing (1979) [10]. The profile of dissipation rate of turbulent kinetic energy is given by the following relationship, based on the assumption of an equilibrium turbulent boundary layer, Pope (2000) [2]:

$$\varepsilon = 0.09^{(3/4)} \times \frac{k^{3/2}}{0.43\,(0.5\,D_j - r)} \times C\,.$$

To take into account the density variation in the turbulent jets with variable density, we introduced the coefficient $C = 0.66 + 0.26 \ln R_d$, Ruffin (1994) [9]. On the symmetry axis, most variables admit perpendicularly to the axis a zero derivative. At the exit, we impose a zero gradient for each variable. Taking into account the wall is obtained by the use of wall functions. This approach, widely used in the flows with high Reynolds number, is economical, robust, and reasonably accurate.

Finally the value of the scalar F is equal to 1 in the tube and 0 elsewhere.

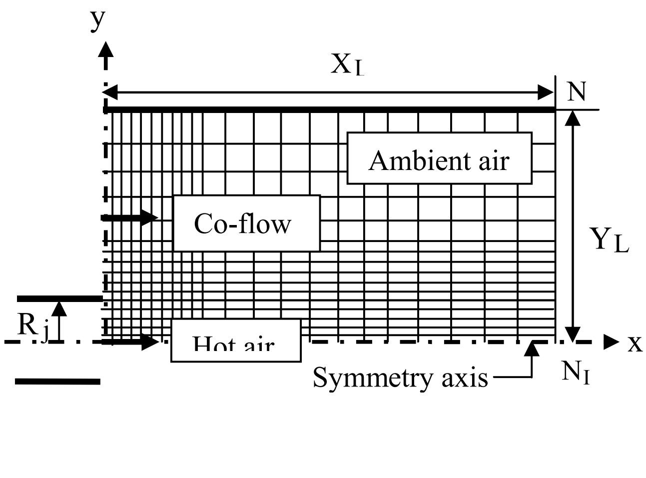

Fig. 2: Configuration of the axisymmetric jets.

4. Numerical results and discussion

The numerical results obtained by our model are discussed in comparison with those resulting from the experiments of Djeridane (1994) [11] and the numerical simulation of Sanders (1997) [5].

To more easily interpret the results, all variables are non-dimensionalized by the greatness imposed at the ejection section of the jet (U_j and D_j).

4.1. Velocity field

Figure 3 shows the axial profiles of the mean longitudinal velocity of the jet. We can see that the axial development of the mean longitudinal velocity is very influenced by the choice of the constants pair (C_1, C_2), a significant difference between the different used pairs. However, it may be remarked that the difference between the three profiles of the three pairs (C_1 = 1.8, C_2 = 0.6), (C_1 = 2.3, C_2 = 0.47) and (C_1 = 1.5, C_2 = 0.6) is negligible.

As shown in figure 3, the pair (C_1 = 1.5, C_2 = 0.76), is that which gives the best prediction.

4.2. Root mean square of velocity fluctuations

Figures 4 and 5 show the axial evolution of the root mean square of longitudinal and radial velocity fluctuations (u' and v'). Both quantities follow developments almost similar in function of the constants pair (C_1, C_2). The influence of the choice of the constants pair is again very important, especially for u'. The maximum values reached are very different, in amplitude and axial position. However, the difference in amplitude of the maximum values of the v' component is negligible compared to the u' component.

It is also noted that, these values are practically reached at the same axial position with the same amplitude for both pairs (C_1 = 1.8, C_2 = 0.6) and (C_1 = 2.3, C_2 = 0.47). Same remark for the two pairs (C_1 = 1.5, C_2 = 0.76) and (C_1 = 2.3, C_2 = 0.6).

The three profile of the radial fluctuating velocity v' associated to the three pairs (C_1 = 1.5, C_2 = 0.6), (C_1 = 1.8, C_2 = 0.6) and (C_1 = 2.3, C_2 = 0.47), figure 5, are identical and predict better, in general, the experimental results compared to others profile and those of Sanders (1997) [5], especially in the zones located at an axial distance higher than or equal to 20 ($X/Dj \geq 20$). This is not the case of profile of the longitudinal fluctuating velocity u', figure 4, where the best prediction, is given, apparently, by the pair (C_1 = 2.3, C_2 = 0.6), especially in the zones which are very close to the nozzle exit ($X/Dj \leq 8$).

Overall, the behaviours of the axial and radial fluctuating velocity are correctly reproduced in those zones by the two pairs (C_1 = 1.5, C_2 = 0.76) and (C_1 = 2.3, C_2 = 0.6) and in the zones located at an axial distance higher than or equal to 20 ($X/Dj \geq 20$) by the three pairs (C_1 = 1.5, C_2 = 0.6), (C_1 = 1.8, C_2 = 0.6) and (C_1 = 2.3, C_2 = 0.47). In the intermediate zones ($8 \leq X/Dj \leq 20$), the numerical simulation results obtained by the six pairs of constants (C_1, C_2) over predict the experimental results. The intensity of this over-prediction, which is important in the v' evolution compared to that of u', depends on the chosen pair.

4.3. Turbulent kinetic energy

Figure 6 shows the evolution of the turbulent kinetic energy on the jet axis as a function of axial position for the six pairs of constants. This evolution reflects the same effects observed on the axial evolution of the root mean square of longitudinal and radial velocity fluctuations.

Comparison with the experimental results (figure 6) shows a better prediction of all profiles, except that relating to the pair (C_1 = 2.3, C_2 = 0.76), compared to that of Sanders (1997) [5]. A difference of the maximum values in amplitude and axial position, much lower than that obtained by Sanders.

4.4. Dissipation rate of the turbulent kinetic energy

The evolution of the dissipation rate of the turbulent kinetic energy is shown in the figure 7. We can notice a significant effect of the choice of the constants pair on this evolution compared to that observed on the turbulent kinetic energy evolution. There is an important difference, in amplitude, of the maximum values reached and less important one in the axial position.

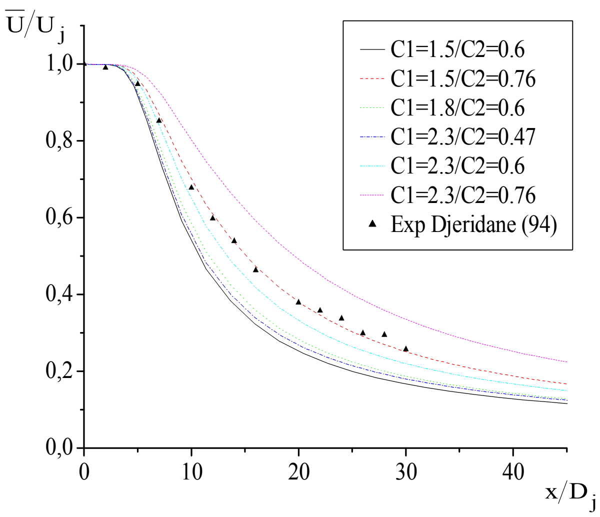

Fig. 3: Evolution of mean longitudinal velocity on the jet axis.

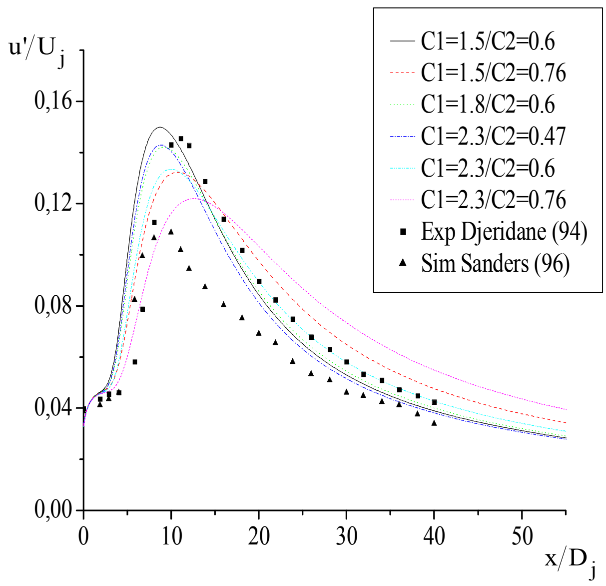

Fig. 4: Evolution of root mean square of longitudinal velocity v on the jet axis.

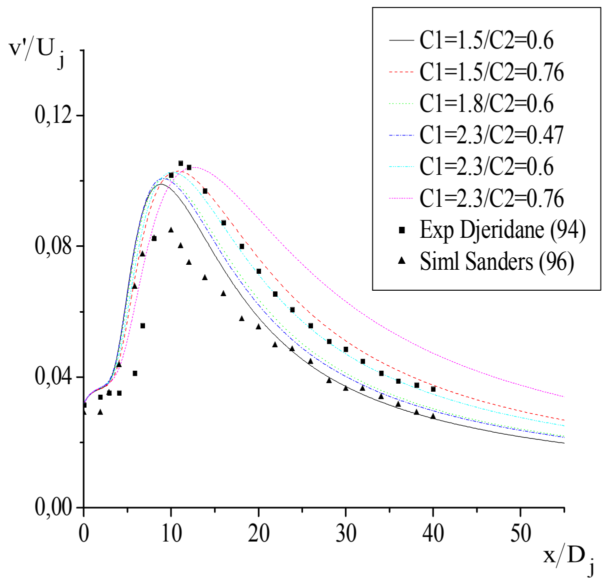

Fig. 5: Evolution of the root mean square of radial velocity fluctuations on the jet axis.

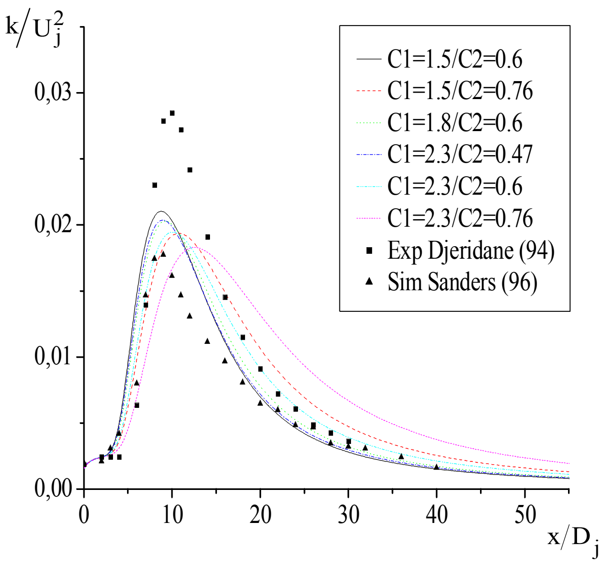

Fig. 6: Evolution of turbulent kinetic energy on the jet axis.

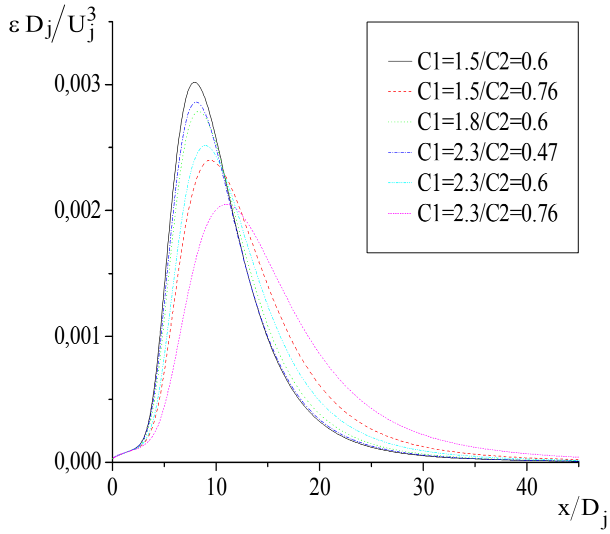

Fig. 7: Evolution of dissipation rate of the turbulent kinetic on the jet axis.

5. Conclusion

The principal objective of this work developed in this present paper is to test the performance of the constants pair (C_1, C_2), used in the modelling of the pressure-strain correlation term proposed by Launder, Reece and Rodi (1975) [1]. We analyzed the effect of the choice of this pair on the dynamic field development of a confined axisymmetric turbulent jet by using the RSM model. The study is performed, for turbulent jets by maintaining constant the exit velocity. Thus, on the basis of obtained result, we can draw the following conclusions:

- The initial development of the dynamic field of an axisymmetric turbulent jet is very influenced by the choice of the constants pair (C_1, C_2).
- This influence starts at a distance very near to nozzle exit (X/Dj≈3), in the core region (X/Dj ≤ 5) and becomes increasingly important away from the nozzle exit.
- Both profile relating to the two pairs (C_1 = 1.8, C_2 = 0.6) and (C_1 = 2.3, C_2 = 0.43) are almost the same.
- Both profile relating to the two pairs (C_1 = 1.5, C_2 = 0.76) and (C_1 = 2.3, C_2 = 0.6) reproduced better the experimental profile in the regions very close to the nozzle exit (X/Dj ≤ 8). In the far regions (X/Dj ≥ 20), the best prediction, is given by the three pairs (C_1 = 1.5, C_2 = 0.6), (C_1 = 1.8, C_2 = 0.6) and (C_1 = 2.3, C_2 = 0.47). Between these two regions, the experimental results are over-predicted by the numerical simulation. The intensity of this over-prediction depends on the chosen pair. In general, we can say that the two pairs (C_1 = 2.3, C_2 = 0.6) and (C_1 = 2.3, C_2 = 0.47) can be considered as an intermediate solution between the six pairs used.
- Numerical simulation results predicted better, in general, the experimental results compared to those of Sanders [5].
- Finally, we can say that the difference which exists between the results obtained by the RSM model, using the different constants pairs, and

those obtained by the Djeridane experiment (1994) [11] could result from different effects, present in the ejection section or within the tube. The modelling used in this present study is also a factor to taken into account.

Nomenclature

a_{ij} : Anisotropy tenseur

C_1 : Constant of return to isotropy

C_2 : Constant in the IP model

D_{ij} : Turbulent diffusion of Reynolds stresses

D_ε : Turbulente diffusion of dissiption rate

D_j : Diameter of nozzle exit

k :Turbulent kinetic energy

P, p : Instantaneous Pressure and its fluctuation

P_{ij} : Production of R_{ij} due to mean strain

R_{ij} : Reynolds stresses

R_{ej} : Reynolds number of the jet ($R_{ej} = \frac{U_j D_j}{\upsilon_j}$)

r : Radial distance

R_d : Density ratio ($R_d = \rho_j / \rho_a$)

u_i : Component i of the Reynolds fluctuations of U_i

U, V : Instantaneous axial and radial velocity

x : Axial distance

ε : Dissipation rate of turbulent kinetic energy

Φ : Generalized variable

ν : Kinematic viscosity

ρ : Density

— : Reynolds average (Conventional average)

$(.)'$: Root mean square

$(.)_a$: Ambient fluid

$(.)_{ij}$: Tensorial notation with summation on the repeated indices

$(.)_j$: Reference to the exit of the tube (x=0)

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