Universal solutions of unsteady two-dimensional MHD boundary layer on the body with temperature gradient along surface

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Abstract: - In this paper, we consider multi-parametric method for solution of unsteady temperature two-dimensional MHD laminar boundary layer. Outer magnetic field induction is assumed as function of longitudinal coordinate and time with force lines perpendicular to the body on which boundary layer forms. Temperature varies along body surface with longitudinal coordinate, but not with time. Further, electric field is neglected and value of magnetic Reynolds number is significantly less then one i.e. problem is considered in induction-less approximation. According to temperature differences under 50^{o}C physical properties of fluid are constant. Introducing new variables and then similarity parameters, starting equations are transformed into universal form. Obtained universal equations and corresponding boundary conditions do not contain explicit characteristics of particular problems. Appropriate approximations of obtained equations are solved numerically in this paper, and a part of obtained results is given in the form of figures and corresponding conclusions.

Key-Words: - **MHD, multi-parametric method, boundary layer, similarity parameters, temperature, universal solutions.**

1 Introduction

The problem of boundary layer separation and control has attracted considerable attention over several decades because of the fundamental flow physics and technological applications. Prandtl [1] has addressed some of the essential ideas related to boundary layer separation and the need to prevent the same from occurring. For a long time following methods was used for boundary layer control: admit the body motion in stream-wise direction, increasing the boundary layer velocity, boundary layer suction [2], second gas injection, body cooling…

The interest in effects of outer magnetic field on heat-physical processes appeared sixty years ago [3]. The research in MHD flows was stimulated by two problems: the protection of space vehicles from aerodynamic overheating and destruction during the passage through the dense layers of the atmosphere; the enhancement of the operational ability of the constructive elements of high temperature MHD generators for direct transformation of heat energy into electric. The first problem showed that the influence of a magnetic field on ionized gases is a convenient control method for mass, heat and hydrodynamic processes. Solutions of mentioned problems were followed with rapid increase of analytical papers and experimental procedures about heat transfer in MHD boundary layer [4], [5], [6]. The field of MHD prospecting extended gradually to new applied problems and nowadays research in viscoelastic fluids [7], Marangoni convection under the influence of magnetic filed [8], magneto-biological processes in medicine [9]…

2 Problem Formulation

In this paper, for the sake of richness of mentioned research, unsteady temperature two-dimensional laminar MHD boundary layer of incompressible fluid is studied. Outer magnetic field is still in relation to fluid in outer flow. It is assumed that outer magnetic filed induction is function of longitudinal coordinate and time with force lines perpendicular to the body surface on which boundary layer forms. Assuming absence of outer electric filed and value of magnetic Reynolds number significantly lower then one, considered problem is in induction-less approximation.

Velocity of flow is considered much lower then speed of light and usual assumption in temperature boundary layer calculation that temperature difference is small (under 50°C) is used, accordingly characteristic properties of fluid are constant (viscosity, heat conduction, electro-conductivity, magnetic permeability, mass heat capacity …). Temperature varies along body surface with longitudinal coordinate. Obtained system of partial differential equations can be solved for every particular case using modern numerical methods and computer.

In this paper, quite different approach is used based on ideas in papers [10], [11], [12], [13] that is extended in papers [14], [15], [16]. Essence of this approach is in introducing adequate transformations and sets of parameters in starting equations of laminar two-dimensional unsteady temperature MHD boundary layer of incompressible fluid, which transform the equations system and corresponding boundary conditions into form unique for all particular problems and this form is considered as universal.

Accordingly, in this paper universal equations of observed problem with universal boundary conditions are obtained. Obtained equations do not contain characteristic values of observed problem, which distinguish particularly cases, and in that sense, they are universal. Solutions of universal system of equations obtained with numerical integration are used in paper to yield general conclusions about boundary layer development which are valid for all particular cases. Results can be also used for calculations of particular cases and this task will be subject of future research. Arbitrary particular problems can be also solved numerically using some modern numerical method for example genetic algorithms in the variational methods for boundary value problems [17]. Comparison of results for the case of classical boundary layer on circular cylinder obtained using parametric method and other methods [11], [12] depict suitability of this method. According to our research we expect to retain benefits of described parametric method for observed case and in future research.

2.1 Mathematical model

Described two-dimensional problem of unsteady MHD temperature boundary layer is mathematically presented with:
continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0; \tag{1}$$

momentum equation:

$$\begin{aligned}&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\\&=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}\left(u-U\right);\end{aligned} \tag{2}$$

energy equation:

$$\begin{aligned}&\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\\&=\frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2}{\rho c_p}\left(u-U\right)^2;\end{aligned} \tag{3}$$

and corresponding boundary and initial conditions:

$$\begin{aligned}&u=0,\, v=0,\, T=T_w\left(x\right) \text{ for } y=0;\\&u\to U\left(x,t\right),\, T\to T_\infty \text{ for } y\to\infty;\\&u=u_0(x,y),\, T=T_0\left(x,y\right) \text{ for } t=t_0;\\&u=u_1(t,y),\, T=T_1\left(t,y\right) \text{ for } x=x_0.\end{aligned} \tag{4}$$

In previous equations and initial and boundary conditions the parameter labelling used is common for the theory of MHD boundary layer: x, y-longitudinal and transversal coordinate respectively, t-time, u, v-longitudinal and transversal velocity component in boundary layer respectively, $U\left(x,t\right)$-velocity on outer edge of boundary layer, ν-fluid kinematics viscosity, σ-fluid electro-conductivity, ρ-density of fluid, $B(x,t)$-magnetic field induction, T-fluid temperature, λ-thermal conductivity, c_p-mass heat capacity, μ-fluid viscosity, $T_w(x)$-body surface temperature, T_∞-temperature on outer boundary layer edge, $u_0\left(x,y\right)$ and $T_0\left(x,y\right)$-longitudinal velocity and fluid temperature respectively at moment $t=t_0$, $u_1\left(x,y\right)$ and $T_1\left(x,y\right)$- longitudinal velocity and fluid temperature respectively in cross section $x=x_0$.

For further consideration stream function, $\Psi\left(x,y,t\right)$ is introduced with following relations:

$$\frac{\partial \Psi}{\partial x}=-v,\ \frac{\partial \Psi}{\partial y}=u; \tag{5}$$

which satisfies equation (1) identically and transform momentum equation (2) into equation:

$$\frac{\partial^2\Psi}{\partial t\partial y}+\frac{\partial\Psi}{\partial y}\frac{\partial^2\Psi}{\partial x\partial y}-\frac{\partial\Psi}{\partial x}\frac{\partial^2\Psi}{\partial y^2}= =\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^3\Psi}{\partial y^3}-\frac{\sigma B^2}{\rho}\left(\frac{\partial\Psi}{\partial y}-U\right); \tag{6}$$

and energy equation into equation:

$$\frac{\partial T}{\partial t}+\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+ +\frac{\mu}{\rho c_p}\left(\frac{\partial^2\Psi}{\partial y^2}\right)^2+\frac{\sigma B^2}{\rho c_p}\left(\frac{\partial\Psi}{\partial y}-U\right)^2. \tag{7}$$

Boundary and initial conditions are transformed into conditions:

$$\Psi=0,\frac{\partial\Psi}{\partial y}=0;\ T=T_w(x)\ \text{ for }\ y=0;$$
$$\frac{\partial\Psi}{\partial y}\to U(x,t);\ T\to T_\infty\ \text{ fo r }\ y\to\infty;$$
$$\frac{\partial\Psi}{\partial y}=u_0(x,y),\ T=T_0(x,y)\ \text{ for }\ t=t_0;$$
$$\frac{\partial\Psi}{\partial y}=u_1(t,y),\ T=T_1(t,y)\ \text{ for }\ x=x_0. \tag{8}$$

Equation (6) does not depend from equation (7) and it can be solved independently. Solution of equation (6) is used for solving of equation (7).

3 Universal Solution

In order to analyze described flow problem following new variables are introduced:

$$x=x;\ t=t;\Phi(x,t,\eta)=\frac{D\Psi(x,y,t)}{U(x,t)h(x,t)}; \quad \eta=\frac{D\cdot y}{h(x,t)};\Theta(x,t,\eta)=\frac{T_w-T}{T_w-T_\infty}; \tag{9}$$

where D is normalizing constant, η-dimensionless transversal coordinate, $h(x,t)$-characteristic linear scale of transversal coordinate in boundary layer, $\Psi(x,y,t)$-dimensionless stream function (ratio of velocities in boundary layer and on outer edge of boundary layer) and $\Theta(x,t,\eta)$-dimensionless temperature difference.

According to introduced variables, equations (6) and (7) are transformed into following system:

$$D^2\frac{\partial^3\Phi}{\partial\eta^3}+f_{1,0}\left(\Phi\frac{\partial^2\Phi}{\partial\eta^2}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2+1\right)+ +\frac{1}{2}(F\Phi+\eta g)\frac{\partial^2\Phi}{\partial\eta^2}+\left(f_{0,1}+g_{1,0}\right)\left(1-\frac{\partial\Phi}{\partial\eta}\right)= =z\frac{\partial^2\Phi}{\partial t\partial\eta}+UzX(\eta;x);$$

$$\frac{D^2}{P_r}\frac{\partial^2\Theta}{\partial\eta^2}-D^2E_c\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-E_cg_{1,0}\left(1-\frac{\partial\Phi}{\partial\eta}\right)^2+ +(1-\Theta)l_1\frac{\partial\Phi}{\partial\eta}+\frac{1}{2}\eta g\frac{\partial\Theta}{\partial\eta}+ +\frac{1}{2}\left(F+2f_{1,0}\right)\Phi\frac{\partial\Theta}{\partial\eta}=z\frac{\partial\Theta}{\partial t}-UzY(x;\eta); \tag{10}$$

where for the sake of shorter expression, the notations are introduced:

$$z=\frac{h^2}{\nu};\ g=\frac{\partial z}{\partial t};\ N=\frac{\sigma B^2}{\rho};\ g_{1,0}=Nz;\ F=U\frac{\partial z}{\partial x};$$
$$f_{1,0}=z\frac{\partial U}{\partial x};\ f_{0,1}=\frac{z}{U}\frac{\partial U}{\partial t};\ l_1=\frac{Uz}{T_w-T_\infty}\frac{dT_w}{dx};$$
$$X(x_1;x_2)=\frac{\partial\Phi}{\partial x_1}\frac{\partial^2\Phi}{\partial\eta\partial x_2}-\frac{\partial\Phi}{\partial x_2}\frac{\partial^2\Phi}{\partial x_1\partial\eta};$$
$$Y(x_1;x_2)=\frac{\partial\Phi}{\partial x_1}\frac{\partial\Theta}{\partial x_2}-\frac{\partial\Phi}{\partial x_2}\frac{\partial\Theta}{\partial x_1};$$
$$Pr=\frac{\nu\rho c_p}{\lambda}\text{-Prandtl number;}$$
$$E_c=\frac{U^2}{c_p(T_w-T_\infty)}\text{-Eckert number.} \tag{11}$$

Now we introduce sets of parameters in following order dynamical, magnetic, temperature and constant parameter:

$$f_{k,n}=U^{k-1}\frac{\partial^{k+n}U}{\partial x^k\partial t^n}z^{k+n} \quad (k,n=0,1,2,...;k\vee n\neq 0); \tag{12}$$

$$g_{k,n}=U^{k-1}\frac{\partial^{k-1+n}N}{\partial x^{k-1}\partial t^n}z^{k+n} \quad (k,n=0,1,2,...,;k\neq 0); \tag{13}$$

$$l_k = \frac{U^k}{q}\frac{\partial^k q}{\partial x^k} z^k \;, \left(k = 1,2,...\right); \tag{14}$$

$$g = \frac{\partial z}{\partial t} = const. \tag{15}$$

where:

$$q = T_w - T_\infty . \tag{16}$$

Introduced sets of parameters reflect the nature of velocity change on outer edge of boundary layer, alteration characteristic of variable N and temperature change along body surface, and a part from that, in the integral form (by means of z and $\partial z / \partial t$) pre-history of flow in boundary layer.

Further, using the parameters (12)-(15) as new independent variables and differentiation operators for x and t:

$$\frac{\partial}{\partial x} = \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left(\frac{\partial f_{k,n}}{\partial x}\frac{\partial}{\partial f_{k,n}} + \begin{cases}0, for\ \Phi\\ \dfrac{\partial l_k}{\partial x}\dfrac{\partial}{\partial l_k}, for\ \Theta\end{cases}\right) + \sum_{\substack{k=1\\ n=0}}^{\infty}\frac{\partial g_{k,n}}{\partial x}\frac{\partial}{\partial g_{k,n}};$$

$$\frac{\partial}{\partial t} = \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left(\frac{\partial f_{k,n}}{\partial t}\frac{\partial}{\partial f_{k,n}} + \begin{cases}0, for\ \Phi\\ \dfrac{\partial l_k}{\partial t}\dfrac{\partial}{\partial l_k}, for\ \Theta\end{cases}\right) + \sum_{\substack{k=1\\ n=0}}^{\infty}\frac{\partial g_{k,n}}{\partial t}\frac{\partial}{\partial g_{k,n}}; \tag{17}$$

respectively, where parameter derivates along coordinate x and time t are obtained by differentiation of Eqs. (11)-(14):

$$\frac{\partial f_{k,n}}{\partial x} = \frac{1}{Uz}\left\{\left(k-1\right) f_{1,0} f_{k,n} + \left(k+n\right) F f_{k,n} + f_{k+1,n}\right\} = \frac{1}{Uz} Q_{k,n};$$

$$\frac{\partial f_{k,n}}{\partial t} = \frac{1}{z}\left\{\left(k-1\right) f_{0,1} f_{k,n} + \left(k+n\right) g f_{k,n} + f_{k,n+1}\right\} = \frac{1}{z} E_{k,n};$$

$$\frac{\partial g_{k,n}}{\partial x} = \frac{1}{Uz}\left\{\left(k-1\right) f_{1,0} g_{k,n} + \left(k+n\right) F g_{k,n} + g_{k+1,n}\right\} = \frac{1}{Uz} K_{k,n};$$

$$\frac{\partial g_{k,n}}{\partial t} = \frac{1}{z}\left\{\left(k-1\right) f_{0,1} g_{k,n} + \left(k+n\right) g g_{k,n} + g_{k,n+1}\right\} = \frac{1}{z} L_{k,n};$$

$$\frac{\partial l_k}{\partial x} = \frac{1}{Uz}\left\{\left[k f_{1,0} - l_1 + kF\right] l_k + l_{k+1}\right\} = \frac{1}{Uz} M_k;$$

$$\frac{\partial l_k}{\partial t} = \frac{1}{z}\left\{k\left(f_{0,1} + g\right) l_k\right\} = \frac{1}{z} N_k; \tag{18}$$

where $Q_{k,n}; E_{k,n}; K_{k,n}; L_{k,n}; M_k; N_k$ are terms in curly brackets in obtained equations. It is important to notice $Q_{k,n}; K_{k,n}; M_k$ beside the parameters depend on value $U\,\partial z/\partial x = F$. Using parameters (12)-(14), operators (17) and terms (18) system of equations (10) is transformed into system:

$$\Im_1 = \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left[Q_{k,n} X\left(\eta; f_{k,n}\right) + E_{k,n}\frac{\partial^2 \Phi}{\partial \eta \partial f_{k,n}}\right] + \sum_{\substack{k=1\\ n=0}}^{\infty}\left[L_{k,n}\frac{\partial^2 \Phi}{\partial \eta \partial g_{k,n}} + K_{k,n} X\left(\eta; g_{k,n}\right)\right];$$

$$\begin{aligned}\Im_2 = &\sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left[Q_{k,n} Y\left(\eta; f_{k,n}\right) + E_k\frac{\partial \Theta}{\partial f_{k,n}}\right] + \\ &+\sum_{k=1}^{\infty}\left[N_k\frac{\partial \Theta}{\partial l_k} + M_k Y\left(\eta; l_k\right)\right] + \\ &+\sum_{\substack{k=1\\ n=0}}^{\infty}\left[L_{k,n}\frac{\partial \Theta}{\partial g_{k,n}} + K_{k,n} Y\left(\eta; g_{k,n}\right)\right];\end{aligned} \tag{19}$$

where the following markings have been used for shorter statement: $\Im_1$ -left side of first equation of system (10), $\Im_2$ - left side of second equation of system (10).

In order to make system (19) universal it is necessary to show that value F which appears in terms for $Q_{k,n}; K_{k,n}; M_k$ can be expressed by means of introduced parameters. In order to prove mentioned we start from impulse equation of described problem:

$$\frac{\partial}{\partial t}\left(U\delta^*\right) + \frac{\partial}{\partial x}\left(U^2\delta^{**}\right) + U\left(\frac{\partial U}{\partial x} + N\right)\delta^* - \frac{\tau_w}{\rho} = 0; \tag{20}$$

where δ^* is displacement thickness:

$$\delta^*(x,t) = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \, ; \qquad (21)$$

δ^{**} -momentum thickness:

$$\delta^{**}(x,t) = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \, ; \qquad (22)$$

τ_w-friction stress on the body:

$$\tau_w(x,t) = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} . \qquad (23)$$

Introducing dimensionless characteristic functions:

$$H^*(x,t) = \frac{\delta^*}{h} ; H^{**}(x,t) = \frac{\delta^{**}}{h} ; \xi(x,t) = \frac{\tau_w h}{\mu U} ; \quad (24)$$

which, according to Eqs. (9) and (21)-(23), can be expressed in the following form:

$$H^*(x,t) = \frac{1}{D}\int_0^\infty \left(1 - \frac{\partial \Phi}{\partial \eta}\right) d\eta \, ;$$

$$H^{**}(x,t) = \frac{1}{D}\int_0^\infty \frac{\partial \Phi}{\partial \eta}\left(1 - \frac{\partial \Phi}{\partial \eta}\right) d\eta \, ;$$

$$\xi(x,t) = D \left.\frac{\partial^2 \Phi}{\partial \eta^2}\right|_{\eta=0} . \qquad (25)$$

After transition to new independent variables values H^*, H^{**}, ξ become functions only from parameters $f_{k,n}$, $g_{k,n}$, l_k and g.

Now, using parameters as new independent variables and derivative operators from impulse equation (20) after simple transformation next equation is obtained:

$$F = \frac{P}{Q} ; \qquad (26)$$

where, P and Q are:

$$P = \xi - f_{1,0}\left(2H^{**} + H^*\right) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}g\right)H^* -$$
$$\sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} \left\{ E_{k,n}\frac{\partial H^*}{\partial f_{k,n}} + \left[(k-1) f_{1,0} f_{k,n} + f_{k+1,n}\right]\frac{\partial H^{**}}{\partial f_{k,n}} \right\}$$
$$-\sum_{\substack{k=1 \\ n=0}}^{\infty} \left\{ L_{k,n}\frac{\partial H^*}{\partial g_{k,n}} + \left[(k-1) f_{1,0} g_{k,n} + g_{k+1,n}\right]\frac{\partial H^{**}}{\partial g_{k,n}} \right\};$$

$$Q = \frac{1}{2}H^{**} + \sum_{\substack{k,n=0 \\ k \vee n \neq 0}}^{\infty} (k+n) f_{k,n}\frac{\partial H^{**}}{\partial f_{k,n}} + \sum_{\substack{k=1 \\ n=0}}^{\infty} (k+n) g_{k,n}\frac{\partial H^{**}}{\partial g_{k,n}} . \qquad (27)$$

Last two equations define function F in terms of values, which depends only from introduced parameters. System of equations (10) is now universal system of described problem. Boundary conditions, also universal, are given with terms:

$$\Phi = 0, \frac{\partial \Phi}{\partial \eta} = 0, \Theta = 0 \text{ for } \eta = 0 ;$$

$$\Phi \to 1, \Theta \to 1 \quad \text{for} \quad \eta \to \infty ;$$

$$\Phi = \Phi_0(\eta), \Theta = \Theta_0(\eta) \text{ for } \left\{ \begin{array}{l} f_{k,n} = 0, (k,n = 0,1,2,...k \vee n \neq 0) \\ g_{k,n} = 0 \quad (k,n = 0,1,2,...,k \neq 0) \\ l_k = 0 \quad (k = 1,2,...) \\ g = 0 \end{array} \right\} ; \qquad (28)$$

where $\Phi_0(\eta)$-Blasius solution for stationary boundary layer on the plate, $\Theta_0(\eta)$ is solution of following equation:

$$\frac{D^2}{Pr}\frac{d^2\Theta_0}{d\eta^2} - D^2 Ec\left(\frac{d^2\Phi_0}{d\eta^2}\right)^2 + \frac{\xi_0}{H^{**}}\Phi_0\frac{d\Theta_0}{d\eta} = 0. \quad (29)$$

Universal system of equations (19) with boundary conditions (28) are strictly for wide class of problems in which $z = At + C(x)$, where A is arbitrary constant and $C(x)$ some function of longitudinal coordinate. For other problems this equations are approximated "universal" equations.

System of equations (19) is integrated in m-parametric approximation once for good and all. Obtained characteristic function can be used to yield general conclusions about development of described boundary layer and to solve any particular problem. Before integration for scale of transversal coordinate in boundary layer $h(x,t)$ some characteristic value is chosen. In this case $h=\delta^{**}$ and accordingly to Eq. (24) $H^{**}=1$, $H^{*}=\delta^{*}/\delta^{**}=H$ and equality (26) now have form:

$$F=2\left[\xi-f_{1,0}(2+H)-\left(f_{0,1}+g_{1,0}+\frac{1}{2}g\right)H-\right.$$
$$\left.-\sum_{\substack{k,n=0\\k\vee n\neq 0}}^{\infty}E_{k,n}\frac{\partial H}{\partial f_{k,n}}-\sum_{\substack{k=1\\n=0}}^{\infty}L_{k,n}\frac{\partial H}{\partial g_{k,n}}\right]. \qquad (30)$$

Taking parameters $f_{k,n}=0$, $g_{k,n}=0$, $g=0$ first equation of the system (19) is simplified into form:

$$\frac{d^3\Phi_0}{d\eta^3}+\frac{\xi_0}{D^2}\Phi_0\frac{d^2\Phi_0}{d\eta^2}=0 \qquad (31)$$

and if $D^2=\xi_0$ then previous equation became well-known Blasius equation. According to previous statement for normalizing constant D value 0,47 must be chosen.

For selected value h equation (29) for determining variable Θ_0 became:

$$\frac{1}{Pr}\frac{d^2\Theta_0}{d\eta^2}+\Phi_0\frac{d\Theta_0}{d\eta}-Ec\left(\frac{d^2\Phi_0}{d\eta^2}\right)^2=0. \qquad (32)$$

In this paper approximated system of equations (19) is solved in which influence of parameters $f_{1,0}$, $f_{0,1}$, $g_{1,0}$, l_1, and g are detained and influence of parameters $f_{0,1}$, l_1 derivatives are disregarded.

In this way system is simplified into following form:

$$\Im_1=Ff_{1,0}X(\eta;f_{1,0})+gf_{1,0}\frac{\partial^2\Phi}{\partial\eta\partial f_{1,0}}+$$
$$+Fg_{1,0}X(\eta;g_{1,0})+gg_{1,0}\frac{\partial^2\Phi}{\partial\eta\partial g_{1,0}};$$

$$\Im_2=Ff_{1,0}Y(\eta;f_{1,0})+gf_{1,0}\frac{\partial\Theta}{\partial f_{1,0}}+$$
$$+Fg_{1,0}Y(\eta;g_{1,0})+gg_{1,0}\frac{\partial\Theta}{\partial g_{1,0}}; \qquad (33)$$

and function F is obtained from Eq. (30) in same approximation:

$$F=2\left[\xi-f_{1,0}(2+H)-gg_{1,0}\frac{\partial H}{\partial g_{1,0}}-\right.$$
$$\left.-\left(f_{0,1}+g_{1,0}+\frac{1}{2}g\right)H-gf_{1,0}\frac{\partial H}{\partial f_{1,0}}\right]. \qquad (34)$$

Boundary conditions which coincide to system of equations are conditions:

$$\Phi=0,\frac{\partial\Phi}{\partial\eta}=0,\Theta=0 \text{ for } \eta=0;$$

$$\Phi\to 1,\Theta\to 1 \quad \text{for} \quad \eta\to\infty;$$

$$\Phi=\Phi_0(\eta),\Theta=\Theta_0(\eta) \text{ for}$$
$$f_{1,0}=0,\ f_{0,1}=0,\ g_{1,0}=0,\ l_1=0,\ g=0; \qquad (35)$$

which is obtained from condition (28), using same simplifications like as equations.

First equation of system (33) is four-parametric once localized approximation and second is five-parametric twice-localized approximation of system of equations (19).

In this paper, system of equations (33) with appropriate boundary conditions (35) is solved using three-diagonal method, known in Russian literature as the "progonka" method. Obtained results of system integration are given in next section in the form of diagrams and conclusions.

4 Results

In this section, part of results obtained with numerical integration of equation system (33) with boundary conditions (35) is given on figures 1 to 11. Fig. 1 presents the variations of values F and ξ in function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$ and value of magnetic parameter $g_{1,0}=0.10$. It may be noted that value F decrease with increase of dynamic parameter. Value F is higher for the case of decelerated outer flow, and lower for the accelerated outer flow in relation to stationary outer flow.

Same figure shows that ξ have higher values for the case of accelerated outer flow and lower for the case of decelerated flow in relation to stationary case. This remark lead to conclusion that accelerated outer flow postpones the boundary layer separation and decelerated flow has quite opposite influence.

Fig. 2 shows the value H (ratio of boundary layer displacement thickness and momentum thickness) in function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$, while magnetic parameter is set to $g_{1,0} = 0.10$. Ratio H decrease with increase of dynamic parameter $f_{1,0}$. It may be noted also that for the same value of dynamic parameter ratio H is higher for the case of decelerated outer flow and lower for the case of outer flow acceleration. According to derived conclusions it may be observed that accelerated outer flow have positive influence on boundary layer development.

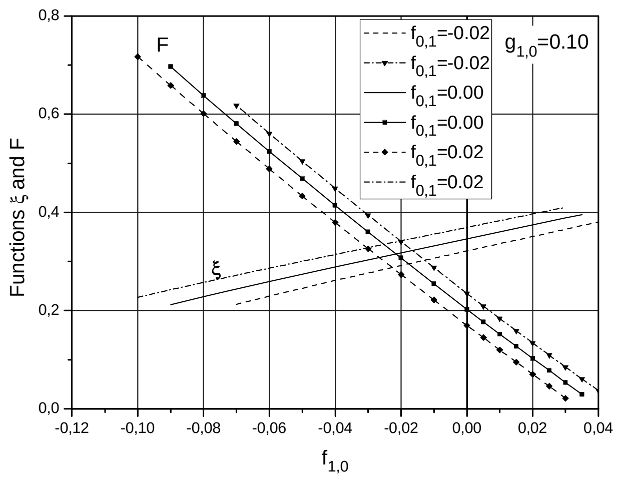

Fig. 1 Variations of quantities F and ξ in function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$ ($P_r = 1, E_c = 0.3$)

The effect of magnetic parameter $g_{1,0}$ on quantities F and ξ for different values of dynamic parameter is shown on Fig. 3. Figure presents the case of accelerated outer flow ($f_{0,1} = 0.02$). Quantity F decrease and ξ increase with increasing of dynamic parameter $f_{1,0}$. It is interesting to note that for the same value of dynamic parameter quantity F decrease and quantity ξ increase with increasing of magnetic parameter $g_{1,0}$. This remark lead to conclusion that magnetic field postpone the boundary layer separation and greater postponement is achieved with increasing of magnetic parameter $g_{1,0}$.

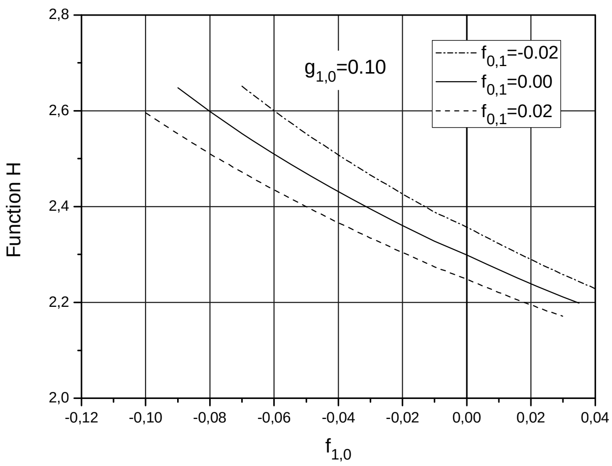

Fig. 2 Variations of quantity H in function of dynamic parameter $f_{1,0}$ for different values of unsteadiness parameter $f_{0,1}$ ($P_r = 1, E_c = 0.3$)

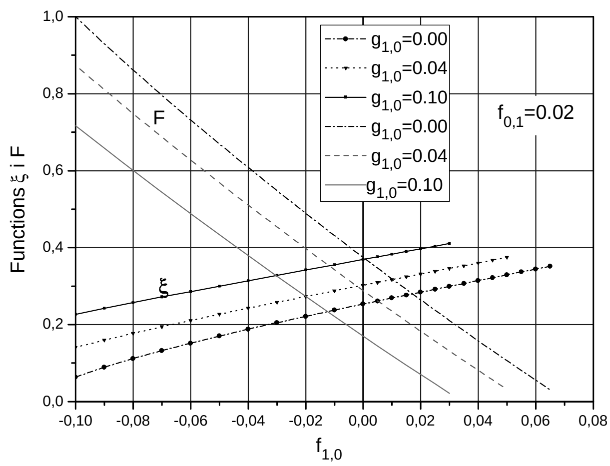

Fig. 3 Variations of quantities F and ξ in function of dynamic parameter $f_{1,0}$ for different values of magnetic parameter $g_{1,0}$ ($P_r = 1, E_c = 0.3$)

Fig. 4 present the results obtained for ratio H in function of dynamic parameter for different values of magnetic parameter $g_{1,0}$, while unsteadiness parameter is set to $f_{0,1} = 0.02$. According to Fig. 4 ratio of boundary layer displacement thickness and momentum thickness decrease with increase of dynamic parameter. This ratio also decreases while magnetic parameter increases for the same value of dynamic parameter.

Fig. 5 and 6 present the variations of quantities F, ξ and H in function of dynamic parameter $f_{1,0}$ for different values of magnetic parameter $g_{1,0}$.

In this case, outer decelerated flow is analyzed in order to compare prior derived conclusions.

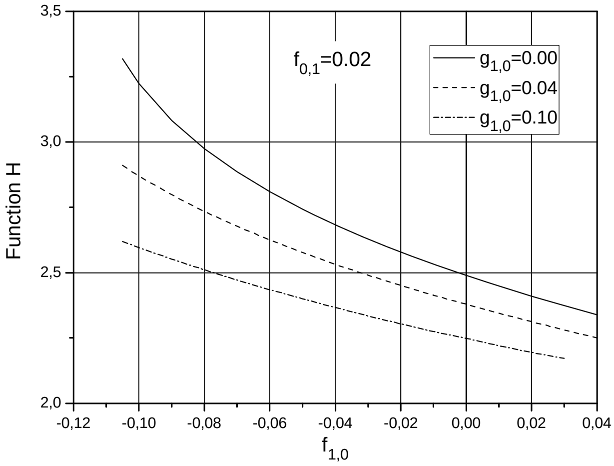

Fig. 4 Variations of quantity H in function of dynamic parameter $f_{1,0}$ for different values of magnetic parameter $g_{1,0}$ ($P_r = 1, E_c = 0.3$)

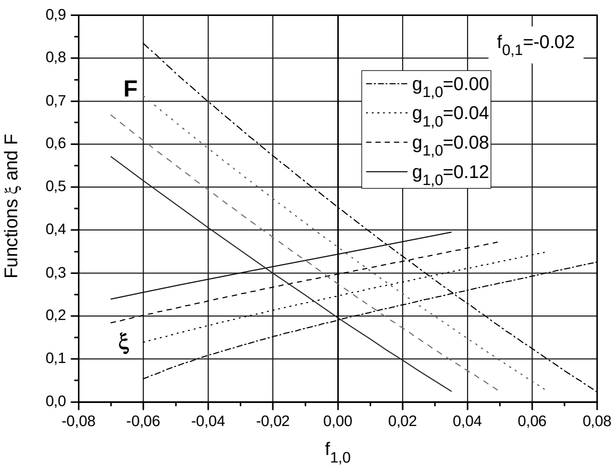

Fig. 5 Variations of quantities F and ξ in function of dynamic parameter $f_{1,0}$ (decelerated outer flow case $f_{0,1} = -0.02$)

According to obtained results, it may be noted that magnetic field has positive influence on boundary layer development and this conclusion is valid for the cases of accelerated and decelerated outer flow.

Ratio of velocities in boundary layer and at the outer edge of boundary layer (dimensionless stream function Φ) is shown in the Figure 7 in function of dimensionless transversal coordinate η for different values of unsteadiness parameter, while values of dynamic parameter and magnetic parameter are set to $f_{1,0} = -0.04$, $g_{1,0} = 0.06$. It may be noted that velocity in boundary layer faster tends to velocity on outer edge of boundary layer for the case of accelerated outer flow and slower for the case of decelerated outer flow compared with stationary outer flow.

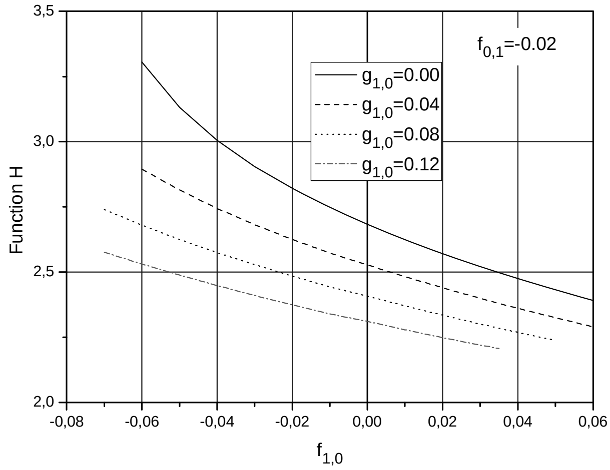

Fig. 6 Variations of quantity H in function of dynamic parameter $f_{1,0}$ for different values of magnetic parameter $g_{1,0}$ (decelerated outer flow case $f_{0,1} = -0.02$)

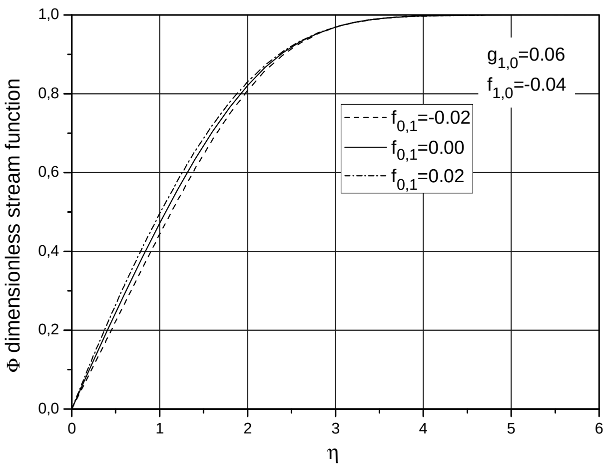

Fig. 7 Dimensionless stream function Φ in function of dimensionless transversal coordinate η for different values of unsteadiness parameter $f_{0,1}$

Fig. 8 describe temperature distribution in function of dimensionless transversal coordinate η for different unsteadiness parameter $f_{0,1}$ values with dynamic parameter $f_{1,0} = -0.04$, magnetic parameter $g_{1,0} = 0.06$ and temperature parameter $l_{1,0} = -0.02$. Solid line presents the case of stationary outer flow. It may me noted that temperature function Θ faster tends to value on outer edge of boundary layer for the case of decelerated outer flow.

In the Fig. 9 variations of dimensionless stream function Φ (ratio of velocities u/U) in function of value η for different values of magnetic parameter $g_{1,0}$ and $f_{0,1} = -0.02$, $f_{1,0} = 0.03$. It may be concluded that with increase of magnetic parameter longitudinal velocity in boundary layer faster tends to the velocity on outer edge of boundary layer. This conclusion holds also for the case of accelerated outer flow ($f_{0,1} > 0$).

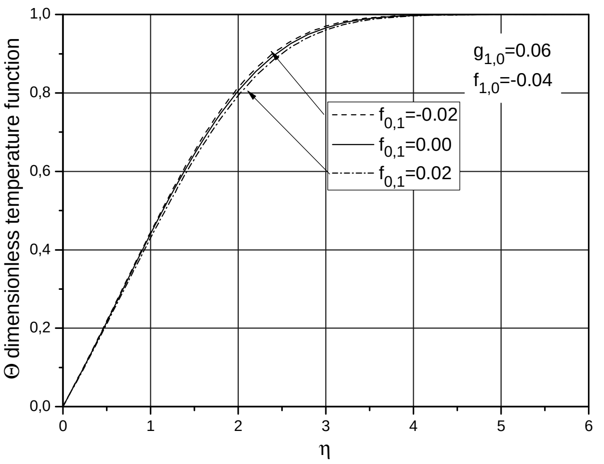

Fig. 8. Temperature distributions in function of dimensionless transversal coordinate η for different values of unsteadiness parameter $f_{0,1}$

Fig. 10 shows the variations of temperature function Θ in function dimensionless transversal coordinate η for different values of magnetic parameter $g_{1,0}$ while unsteadiness, temperature and dynamic parameters are set to $f_{0,1} = -0.02$, $l_1 = 0.02$, $f_{1,0} = 0.03$. With increase of magnetic parameter temperature function Θ slower tends to the value on outer edge of boundary layer. This conclusion holds also for the cases of accelerated outer flow ($f_{0,1} > 0$) and temperature decreasing along the body ($l_1 < 0$).

The effect of temperature parameter l_1 on dimensionless temperature function Θ is presented on the Fig. 11 in function of dimensionless transversal coordinate η. In this case like for the all other Prandtl number P_r and Eckert number E_c are set to 1.0 and 0.3 respectively. Magnetic, unsteadiness, temperature and dynamic parameters are set to: $g_{1,0} = 0.1$ $f_{0,1} = 0.01$, $f_{1,0} = 0.02$.

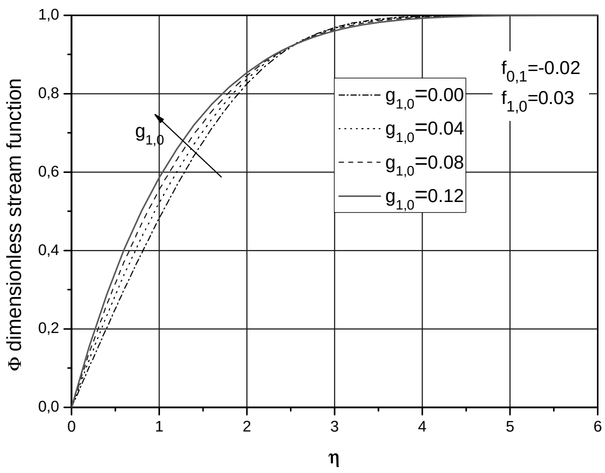

Fig. 9 Variations of dimensionless stream function Φ in function of value η for different values of magnetic parameter $g_{1,0}$

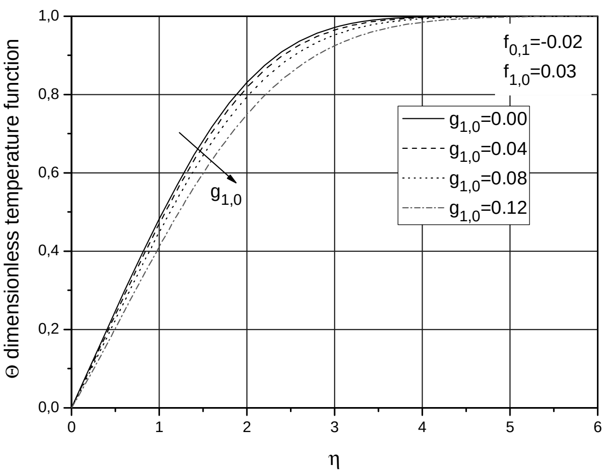

Fig. 10 Dimensionless temperature function Θ for different values of magnetic parameter $g_{1,0}$

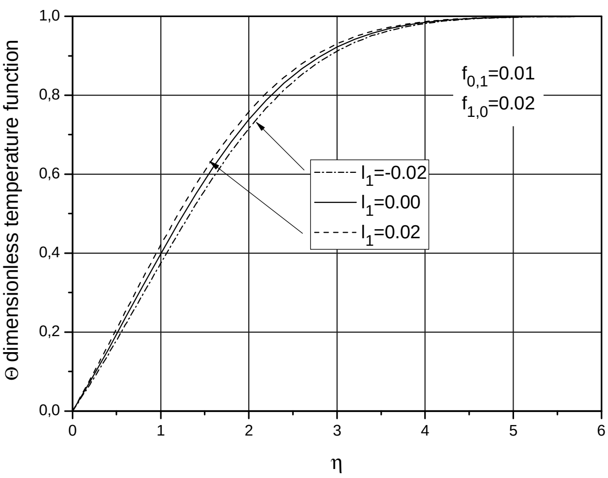

Fig. 11 Variations of temperature function for different values of temperature parameter l_1

It may be noted that for the case of temperature decreasing along the body this value slower achieve value on outer edge of boundary layer, while for the case of temperature increasing along the body surface this tends is faster. For comparison is used the case of constant body surface temperature. This conclusions is valid for the cases of accelerated and decelerated outer flow.

5 Conclusion

In this paper, unsteady two-dimensional MHD boundary layer on the body with temperature gradient along surface is considered. This problem can be analyzed for every particular case i.e. for given outer flow characteristics. Here is used quite different approach in order to use benefits of multi-parametric method and universal equations of observed problem are derived. These equations are solved numerically in some approximation and integration results are given in the form of diagrams and conclusions. Obtained results are used to yield general conclusions about developing of described temperature MHD boundary.

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