

Numerical Modeling of Extended Mild Slope Equation with Modified Mac Cormack Method

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Abstract The transformation of waves is one of the important subjects in coastal engineering studies. Refraction, diffraction, shoaling, reflection can be analysed with the mild slope equation over mild sloped topographies. But the extended mild slope equation can be applied to the rapidly varying topographies since it includes higher order bottom effects such as square of bottom slope and bottom curvature. In this study, extended mild slope equation has been solved with finite difference method using Mac Cormack and Point Gauss Seidel Methods together. The nonlinear wave celerity and group velocity have been used. The numerical model has been tested on elliptic shoaling area and compared with the physical experiment measurements given in literature. The predictions are in consistency with the measurements. As a result of error analysis, it is decided that the numerical can be used for the simulation of wave propagation. Numerical model has been applied to a coastal area in the Kocaeli Bay in the Marmara Sea in Turkey.

Key-words: Extended mild slope equation, Mac Cormack Method, Point Gauss Seidel Method, wave refraction, diffraction, finite difference method, CFD

1 Introduction

Mild slope equation has been used to simulate combined refraction diffraction based on a mild slope assumption [1]. The mild slope equation proposed by Berkhoff has been shown in equation (1) [2].

$$\frac{\partial}{\partial x}(CC_g \frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y}(CC_g \frac{\partial \phi}{\partial y}) + \sigma^2 \frac{C_g}{C} \phi = 0 \quad (1)$$

ϕ is velocity potential function, C is wave celerity, C_g is group velocity, σ is wave frequency. x and y are horizontal coordinates. The major characteristic of mild slope equation is analysing wave refraction and diffraction together. The mild slope equation is valid when $|\nabla h|/kh \ll 1$. h is the water depth, k is the wave number and ∇ is horizontal operator. This assumption means that the bottom slope or the change of water depth per wave length is very small The

mild slope equation includes wave refraction, diffraction, shoaling and reflection.

Mild slope equations can be classified in four categories based on Hsu et al. [3, 4]:

- i. The original mild slope equation
- ii. The original mild slope equation with seabed slope and curvature terms
- iii. The original mild slope equation with special treatments for seabed discontinuities
- iv. The original mild slope equation with seabed slope and curvature terms plus a second equation derived from the seabed boundary condition

The example studies of the four categories based on Liu and Shi [4] has been given with extensions. The original mild slope

equation has been investigated by Biesel [1], Berkhoff [2], Lozano and Liu [5], Booij [6], Liu [7], Behrendt and Jonsson [8], Kirby [9], Ebersole [10], Balas and Inan [11], Madsen and Larsen [12], Li and Anastasiou [13], Dingemans [14], Tang et al. [15], Zhang et al., [16].

The original mild slope equation with seabed slope and curvature terms has been studied by Massel [17], Chamberlain and Porter [18], Chandreskera and Cheung [19], Miles and Chamberlain [20], Lee and Suh [21], Lee et al. [22], Maa et al. [23].

The original mild slope equation with special treatments for seabed discontinuities has been examined by Porter and Staziker [24], Porter [25], Kim and Bai [26].

On the original mild slope equation with seabed slope and curvature terms plus a second equation derived from the seabed boundary condition has been worked by Athanassoulis and Belibassakis [27], Chandreskera and Cheung [28], Arduin and Herbers [29], Chen et al. [30], Hsu et al. [3].

In recent years, the researchers have worked on the decrease of the limitations of the mild slope equation and adding the other phenomenas like wave breaking, harbor resonance and bottom friction effects in the mild slope equation.

Lee and Yoon investigated how the refraction of water waves is affected by the higher- order bottom effect terms proportional to the square of bottom slope & to the bottom curvature in the extended mild slope equations [31].

Kim et al. developed techniques of numerical wave generation in the time dependent extended mild slope equations for random waves using a source function method. The source function method avoids rereflection problems at the offshore boundary. They verified that the source function of the Helmholtz equation may be

used in generating waves in any refraction diffraction equation models [32].

Bellotti et al. applied a numerical model based on the mild slope equation to reproduce the propagation of small amplitude transient waves. The results of two available experimental studies on tsunamis generated by landslides are used to validate the model, which appears to be able of reproducing the effects of the frequency dispersion [33].

Liu et al. developed a numerical model to solve the MSE on the basis of a self adaptive finite element model combined with an iterative method to determine the wave direction angle to the boundary and thus to improve the treatment of the boundary conditions. Developed numerical model represents effectively wave absorption at the absorbing boundaries & simulates multidirectional wave propagation [34].

Chen et al. accomplished finite element wave model based on the an extended mild slope equation wave-current equation that includes wave breaking. Improved boundary conditions are used to provide more accurate forcing & to minimize spurious wave reflections from the boundaries [35]

In this study, the extended mild slope equation suggested by Maa et al. [23] has been solved numerically. This equation examines wave refraction, diffraction, shoaling, reflection, wave breaking and bottom friction dissipations and harbour resonance. The nonlinear wave celerity and group velocity have been used in the calculations to examine the nonlinear effects more accurately.

Finite difference method has been applied to the problem. Finite difference method is used very often in a wide variety field of computational fluid dynamics [36, 37, 38]. Mac Cormack Method and Point Gauss Seidel have been handled in numerical solution procedure. The numerical model has been tested on the elliptical shoaling

area given in literature and applied to a coastal region in Kocaeli Bay of Marmara Sea in Turkish coasts.

2 Theory

The extended mild slope equation proposed by Maa et al. has been given in the equation (2) [23]. It includes harbor resonance and dissipations due to wave breaking & bottom friction beside wave refraction, diffraction, shoaling and reflection. Furthermore it is applicable to the rapidly varying topographies since it includes higher order bottom effects such as bottom curvature and square of the bottom slope.

$$\nabla \cdot (CC_g \nabla \phi) + k^2 CC_g (1 + if_{bd}) \phi + [f_1 g \nabla^2 h + f_2 (\nabla h)^2 g k] \phi = 0 \quad (2)$$

∇h is bottom slope, $\nabla^2 h$ is bottom curvature, f_{bd} is the sum of bottom friction dissipation factor and energy dissipation factor after breaking. g is the gravitational acceleration. f_1 is the bottom curvature coefficient and f_2 is the coefficient of square of bottom slope. They are functions of wave number and water depth.

$$f_1 = \frac{-4kh \cos(kh) + \sinh(3kh) + \sinh(kh) + 8(kh)^2 \sinh(kh)}{8 \cosh^3(kh) [2kh + \sinh(2kh)]} - \frac{kh \tanh(kh)}{2 \cosh^2(kh)} \quad (3)$$

$$f_2 = \frac{\sec^2 h^2(kh)}{6 [2kh + \sinh(2kh)]^3} \left\{ \begin{array}{l} 8(kh)^4 + 16(kh)^3 \sinh(2kh) - \\ 9 \sinh^2(2kh) \cos(2kh) + \\ 12(kh) [1 + 2 \sinh^2(kh)] [kh + \sinh(2kh)] \end{array} \right\} \quad (4)$$

Bottom friction dissipation factor (f_b) has been calculated with the equation (5). Here, f_w is wave friction factor [23].

$$f_b = \frac{4f_w}{3\pi} \frac{a\sigma^2}{ng \sinh^3 kh} \quad (5)$$

$$n = \frac{1}{2} \left(1 + \frac{2kh}{\sinh 2kh} \right) \quad (6)$$

Jonsson and Carlsen [39] recommended the equation (7) to calculate wave friction factor.

$$\frac{1}{4\sqrt{f_w}} + \log_{10} \frac{1}{4\sqrt{f_w}} = m_f + \log_{10} \frac{a_{1m}}{k_N} \quad (7)$$

a_{1m} is semi distance of the movement of the fluid particle on bottom and k_N is the Nikuradse roughness coefficient. After empirical studies, m_f had been calculated as -0.08 by Jonsson and Carlsen [39]. If a_{1m}/k_N is less than 2, wave friction factor f_w is 0.24. Otherwise the value calculated in the equation (7) is used in the numerical solution [39, 40].

Wave breaking dissipation factor (f_d) has been calculated with the equation (8) [23, 41]. Γ and K are empirical constants and $\Gamma=0.4$, $K=0.15$ [41].

$$f_d = \frac{\Gamma}{kh} \left(1 - \frac{K^2}{4\gamma^2} \right) \quad (8)$$

Breaker index (γ_b) is calculated with the formulation [42] given in the equation (9).

$$\gamma_b = 0.53 - 0.3 \exp(-3\sqrt{h/L_0}) + 5 \tan \beta^{3/2} \exp[-45(\sqrt{h/L_0} - 0.1)^2] \quad (9)$$

γ is the ratio between wave amplitude and water depth ($\gamma=a/h$). γ and γ_b are calculated in each step and compared. If γ is less than γ_b , f_d is equalized to zero. Otherwise f_d is calculated with the equation (9) [40]. The couple of breaker index and wave breaking dissipation factor has been selected because of the error analysis of different couples in literature based on Hsu and Wen [43]. This couple gives minimum error in the numerical analysis [43].

2.1 Nonlinear Wave Celerity and Group Velocity

Nonlinear effects are especially important in shallow regions where refraction is dominant. Nonlinearity should be taken into account to obtain more accurate results in

wave propagation problems. Kirby and Dalrymple recommended a dispersion relationship given in the equation (10) to determine nonlinear wave celerity and group velocity [14, 44]. It is valid either in deep sea or in shallow regions.

$$\sigma^2 = gk \left(1 + f_1'(kh) \varepsilon_*^2 D \right) \tanh(kh + \varepsilon_* f_2'(kh)) \quad (10)$$

$$\varepsilon_* = ka \quad (11)$$

$$D = \frac{\cosh(4kh) + 8 - 2 \tanh^2(kh)}{8 \sinh^4(kh)} = \frac{9 - 12 \tanh^2(kh) + 13 \tanh^4(kh) - 2 \tanh^6(kh)}{8 \tanh^4(kh)} \quad (12)$$

$$f_1'(kh) = \tanh^5(kh) \quad (13)$$

$$f_2'(kh) = \left(\frac{kh}{\sinh(kh)} \right)^4 \quad (14)$$

After the calculation of σ , the nonlinear wave celerity and group velocity can be obtained simply.

$$C = \frac{\sigma}{k} \quad \text{and} \quad C_g = \frac{d\sigma}{dk} \quad (15)$$

2.2 Boundary Conditions

General boundary condition used in coastal engineering problems referring radiation, partial and full reflection conditions is given in the equation (16) [45].

$$\frac{\partial \phi}{\partial n} + \alpha^* k \phi = 0 \quad (16)$$

$\alpha^* (= \alpha_1 + i\alpha_2)$ is complex transmission coefficient and depend on energy transfer on boundary, wave height, wave phase and reflection coefficient. α_1 and α_2 are calculated with the equations (17) and (18), respectively.

$$\alpha_1 = \frac{2K_R \sin \beta \cos \theta}{1 + K_R^2 + 2K_R \cos \beta} \quad (17)$$

$$\alpha_2 = \frac{(1 - K_R^2) \cos \theta}{1 + K_R^2 + 2K_R \cos \beta} \quad (18)$$

Here, K_R is reflection coefficient, β is the phase difference between incident and approaching waves, θ is the angle between boundary normal and incident wave. Since β is so small, it is assumed to be zero in the calculations.

Total potential function on the boundary of incident and reflected waves is given in the equation (19).

$$\phi = A \left\{ \exp[ik(x \cos \theta + y \sin \theta)] + K_R \exp[-ik(x \cos \theta - y \sin \theta) + i\beta] \right\} \quad (19)$$

Velocity potential of a wave with the height (H) and period (T) is calculated using the equation (20) with linear theory. They have been used as input values.

$$\phi_g = \frac{igH}{2\sigma} e^{is} \quad (20)$$

$$k_x = k \cos \theta \quad (21)$$

$$k_y = k \sin \theta \quad (22)$$

$$s = k_0 \cos \theta_0 x - k_0 \sin \theta_0 y - \sigma t \quad (23)$$

Wave number vector is related to wave phase.

$$\vec{k} = \nabla s \quad (24)$$

Phase function (s) is determined with the equation (21) and wave angle is a function of wave phase. The details can be found in the study of Hsu and Wen [42].

$$s = \tan^{-1} \left(\frac{\text{Im}(\phi)}{\text{Re}(\phi)} \right) \quad (25)$$

$$\theta = \tan^{-1} \left(\frac{\partial s / \partial y}{\partial s / \partial x} \right) \quad (26)$$

3 Numerical Model

Mac Cormack Method and Point Gauss Seidel Method have been used together in the numerical model. It can be said that a modified form of Mac Cormack method has been applied to the problem. Since Mac

Cormack Method is a multistep method, more stable results can be obtained. In Mac Cormack method, predictor and corrector values are calculated, respectively. In predictor step, forward finite difference method in the order of $(O(\Delta x)$ and $O(\Delta y)$) has been used for the first order derivatives and central finite difference method in the order of $(O(\Delta x^2)$ and $O(\Delta y^2)$) has been used for the second order derivatives. Backward finite difference method in the order of $(O(\Delta x)$ and $O(\Delta y)$) has been applied to the first order derivatives in corrector step. The second order derivatives have been calculated with the same method used in predictor step. The irregular mesh has been used so the grid sizes in the domain can vary due to the bottom configurations. The calculated nodes have been taken into account rapidly because of the use of Point Gauss Seidel Method. So iteration number has been minimized. Mac Cormack Method has been applied to the governing equation (2).

Predictor Step

$$\begin{aligned} & \left(\frac{\phi_{i+1,j}^k - \phi_{i,j}^*}{\alpha_2 \Delta x} \right) \left[C_{i,j} \left(\frac{C_{gt+1,j} - C_{gt,j}}{\alpha_2 \Delta x} \right) + C_{gt,j} \left(\frac{C_{i+1,j} - C_{i,j}}{\alpha_2 \Delta x} \right) \right] + \\ & \left(\frac{\phi_{i,j+1}^k - \phi_{i,j}^*}{\beta_2 \Delta y} \right) \left[C_{i,j} \left(\frac{C_{gt,j+1} - C_{gt,j}}{\beta_2 \Delta y} \right) + C_{gt,j} \left(\frac{C_{i,j+1} - C_{i,j}}{\beta_2 \Delta y} \right) \right] + \\ & C_{i,j} C_{gt,j} \left[\frac{2\phi_{i+1,j}^k}{\alpha_2(\alpha_1 + \alpha_2)\Delta x^2} - \frac{2\phi_{i,j}^*}{\alpha_1\alpha_2\Delta x^2} + \frac{2\phi_{i-1,j}^{k+1}}{\alpha_1(\alpha_1 + \alpha_2)\Delta x^2} \right] + \\ & C_{i,j} C_{gt,j} \left[\frac{2\phi_{i,j+1}^k}{\beta_2(\beta_1 + \beta_2)\Delta y^2} - \frac{2\phi_{i,j}^*}{\beta_1\beta_2\Delta y^2} + \frac{2\phi_{i,j-1}^{k+1}}{\beta_1(\beta_1 + \beta_2)\Delta y^2} \right] + \\ & \left. \left[\begin{aligned} & k_{i,j}^2 C_{i,j} C_{gt,g} (1 + if_{i,j}) + \\ & f_1 g \left[\frac{2h_{i+1,j}}{\alpha_2(\alpha_1 + \alpha_2)\Delta x^2} - \frac{2h_{i,j}}{\alpha_1\alpha_2\Delta x^2} + \frac{2h_{i-1,j}}{\alpha_1(\alpha_1 + \alpha_2)\Delta x^2} \right] + \\ & \left[\frac{2h_{i,j+1}}{\beta_2(\beta_1 + \beta_2)\Delta y^2} - \frac{2h_{i,j}}{\beta_1\beta_2\Delta y^2} + \frac{2h_{i,j-1}}{\beta_1(\beta_1 + \beta_2)\Delta y^2} \right] \\ & f_2 g k_{i,j} \left(\frac{h_{i+1,j} - h_{i,j}}{\alpha_2 \Delta x} + \frac{h_{i,j+1} - h_{i,j}}{\beta_2 \Delta y} \right)^2 \end{aligned} \right] \right\} = 0 \end{aligned} \tag{27}$$

$$\phi_{i,j}^{k+1/2} = \frac{1}{2} (\phi_{i,j}^k + \phi_{i,j}^*) \tag{28}$$

$\phi_{i,j}^{k+1/2}$ will be used in the calculation of corrector step.

Corrector Step

$$\begin{aligned} & \left(\frac{\phi_{i,j}^{k+1} - \phi_{i-1,j}^{k+1}}{\alpha_1 \Delta x} \right) \left[C_{i,j} \left(\frac{C_{gt,j} - C_{gt-1,j}}{\alpha_1 \Delta x} \right) + C_{gt,j} \left(\frac{C_{i,j} - C_{i-1,j}}{\alpha_1 \Delta x} \right) \right] + \\ & \left(\frac{\phi_{i,j}^{k+1} - \phi_{i,j-1}^{k+1}}{\beta_1 \Delta y} \right) \left[C_{i,j} \left(\frac{C_{gt,j} - C_{gt,j-1}}{\beta_1 \Delta y} \right) + C_{gt,j} \left(\frac{C_{i,j} - C_{i,j-1}}{\beta_1 \Delta y} \right) \right] + \\ & C_{i,j} C_{gt,j} \left[\frac{2\phi_{i+1,j}^k}{\alpha_2(\alpha_1 + \alpha_2)\Delta x^2} - \frac{2\phi_{i,j}^{k+1}}{\alpha_1\alpha_2\Delta x^2} + \frac{2\phi_{i-1,j}^{k+1}}{\alpha_1(\alpha_1 + \alpha_2)\Delta x^2} \right] + \\ & C_{i,j} C_{gt,j} \left[\frac{2\phi_{i,j+1}^k}{\beta_2(\beta_1 + \beta_2)\Delta y^2} - \frac{2\phi_{i,j}^{k+1}}{\beta_1\beta_2\Delta y^2} + \frac{2\phi_{i,j-1}^{k+1}}{\beta_1(\beta_1 + \beta_2)\Delta y^2} \right] + \\ & \left. \left[\begin{aligned} & k_{i,j}^2 C_{i,j} C_{gt,g} (1 + if_{i,j}) + \\ & f_1 g \left[\frac{2h_{i+1,j}}{\alpha_2(\alpha_1 + \alpha_2)\Delta x^2} - \frac{2h_{i,j}}{\alpha_1\alpha_2\Delta x^2} + \frac{2h_{i-1,j}}{\alpha_1(\alpha_1 + \alpha_2)\Delta x^2} \right] + \\ & \left[\frac{2h_{i,j+1}}{\beta_2(\beta_1 + \beta_2)\Delta y^2} - \frac{2h_{i,j}}{\beta_1\beta_2\Delta y^2} + \frac{2h_{i,j-1}}{\beta_1(\beta_1 + \beta_2)\Delta y^2} \right] \\ & f_2 g k_{i,j} \left(\frac{h_{i,j} - h_{i-1,j}}{\alpha_1 \Delta x} + \frac{h_{i,j} - h_{i,j-1}}{\beta_1 \Delta y} \right)^2 \end{aligned} \right] \right\} = 0 \end{aligned} \tag{29}$$

The input values for the numerical solution are water depth, deep water wave height, deep water incidence wave angle and wave period. An identity matrix has been created to determine the location of nodal point. So it is decided if the nodal point is inside the domain, or outside the domain or on the boundary. Firstly, wave characteristics have been obtained with the linear theory as input for the calculation of the governing equations (2). After determination of the complex velocity potential, wave phase and wave angle at each nodal point have been calculated with the equations (25) and (26), respectively. As the grid sizes decreases, either geometric and bathymetric properties or the effects of diffraction can be determined more accurately. The grid sizes in each axis should be less than 1/10 of wave length [23].

4 Applications of the Numerical Model

Numerical model has been tested on the elliptical shoaling area given in literature. After obtaining acceptable results, the numerical model has been applied to a coastal region in Kocaeli Bay of Marmara Sea in Turkey.

4.1 Elliptic Shoaling Area

Berkhoff et al. [2] prepared a topography model to model wave propagation physically. The bottom has a slope 1/50 and elliptical shoaling area. Wave period (T) is 1 sec. Incident wave amplitude (a₀) at x=0 is 0.0232m.

The angle between bathymetry lines and y-axis is 20°. Elliptical shoaling area had been builded to observe the diffraction effects better.

The mathematical relationship between (x, y) and (x', y') coordinate axis have been given in the equations (30) & (31).

$$x' = (x - 10.5) \cos 20^\circ - (y - 10) \sin 20^\circ \quad (30)$$

$$y' = (x - 10.5) \sin 20^\circ + (y - 10) \cos 20^\circ \quad (31)$$

The point (x', y') = (0, 0) is the shoaling center. The bottom topography without elliptical shoaling area can be modeled with the equations (32) and (33).

$$h = 0.45m \quad x' < -5.82m \quad (32)$$

$$h = 0.45 - 0.02(5.82 + x')m \quad x' \geq -5.82m \quad (33)$$

The boundaries of the elliptical shoaling area has been given in the equation (34).

$$\left(\frac{x'}{3}\right)^2 + \left(\frac{y'}{4}\right)^2 = 1 \quad (34)$$

The water depths in shoaling area have been calculated with the the equation (35).

$$h = h_m - 0.5 \left[1 - \left(\frac{x'}{3.75}\right)^2 - \left(\frac{y'}{5}\right)^2 \right]^{1/2} + 0.3 \quad (35)$$

The bathymetry of the elliptical shoaling area has been shown in the Fig. 1. The water depths shown in Fig.1 are in meters.

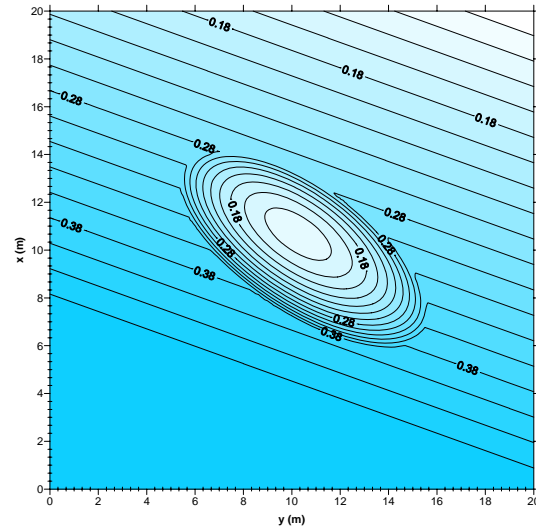


Fig. 1: Bathymetry of the experiment [2]

In Fig. 2, the increase of the wave amplitude due to shoaling can be observed.

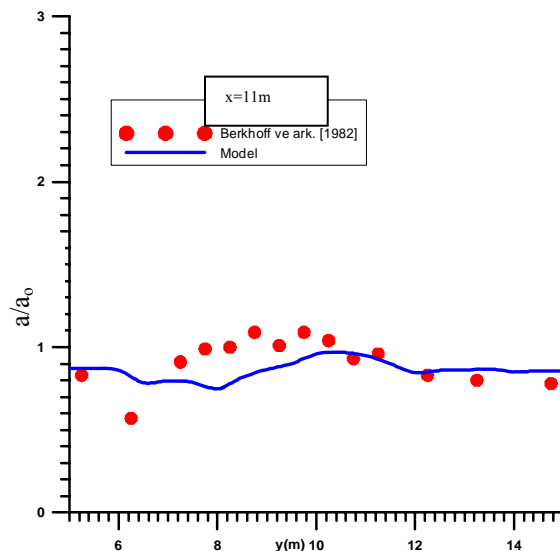


Fig. 2: Relative wave amplitude at x=11m

In Fig. 3, the shoaling and diffraction effects are dominant when the wave leaves the elliptical shoaling area. Diffraction at this caotic region can not be determined by linear theory.

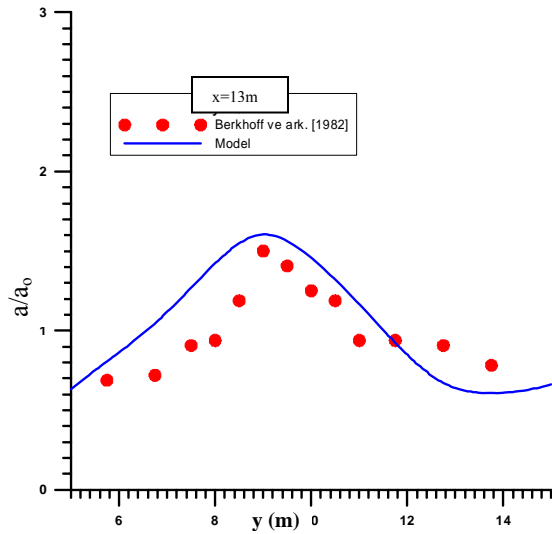


Fig. 3: Relative wave amplitude at $x=13m$

The RMSE and BIAS values obtained from the physical experiment and numerical model have been given in Table 1.

Table 1: RMSE and BIAS values

	RMSE	BIAS
$x=11m$	0,1588	0,0550
$x=13m$	0,2434	-0,1264

4.2 A Coastal Region in Kocaeli Bay

Kocaeli is located in Marmara Region where the northern branch of the North Anatolian Fault Zone passes through. North Anatolian Fault has potential to trigger submarine ground failures that can be the cause of occurrence tsunami [46]. Since the earthquake held on 17 August 1999 when thousands of people died, the Kocaeli Bay and the sea of Marmara are thought by researchers as natural laboratory. Furthermore, Kocaeli is one of the important industrial cities of Turkey. There are industrial harbor, many factories and oil pipelines under the sea. Therefore it is critical region point of view of coastal engineering. The location of studied region has been shown in Fig. 4.



Fig. 4. Location of Kocaeli [47]

The dominant wave direction is W. The wave approaching angle in deep water is set to be zero. The significant wave height is $H_0=3m$ and wave period is $T=8sec$ under the effect of the wind velocity $U=13m/sec$. Bathymetry of the computed coastal region has been given in Fig. 5 and the water depths are in meters.

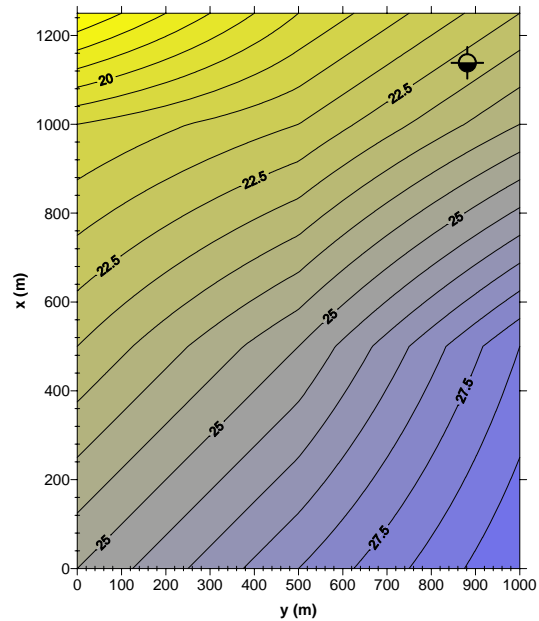


Fig. 5: Bathymetry of the computed coastal region in Kocaeli Bay

The wave height distributions obtained by the numerical model has been shown in Fig. 6. The wave heights are in consistency with the bottom contours.

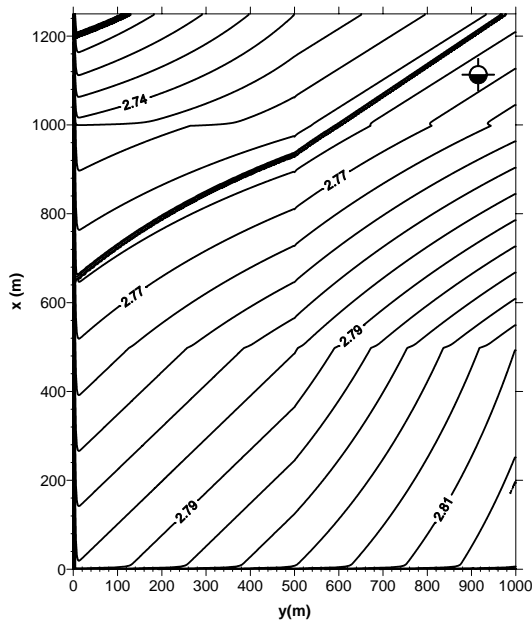


Fig. 6: Wave height distribution (Wave heights are in meters)

5 Conclusion

Determination of wave transformations is an important subject in coastal engineering field. Wave height distribution in any coastal region must be known for any coastal studies. Extended mild slope equation is a successful tool to define wave propagation from deep sea to shallow water and includes wave refraction, diffraction, shoaling, reflection, harbor resonance and dissipations due to wave breaking and bottom friction. Extended mild slope equation can be applied to rapidly varying topographies because higher order bottom effects such as the square of bottom slope and bottom curvature. Extended mild slope equation has been solved numerically using Modified Mac Cormack Method. It is the use of Mac Cormack Method and Point Gauss Seidel together. Since Mac Cormack is a multi step method, a stable solution can be obtained. New calculated nodal points are used immediately in Point Gauss Seidel Method so the iteration number decreases. Furthermore, the nonlinear wave celerity and group velocity have been taken into account in the numerical model. It is an advantage especially in shallow areas where refraction is important. The application of

nonlinear wave celerity and group velocity to the extended mild slope equation is a new approach. The numerical model has been compared with the physical experiment given in literature and obtained acceptable results. The numerical model can be used for the simulation of wave transformation in coastal engineering studies. In future studies, harbor resonance can be examined in details and wave-current interaction can be taken into account so the numerical model can be applied to the coastal areas where currents are significant and wave spectrum can be used for irregular waves.

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