Modelling of Flow and Tracer Dispersion over Complex Urban Terrain in the Atmospheric Boundary Layer

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Abstract: A new mathematical model of polutant plume dispersion in an urban environment is presented. The model uses parameters that explicitly take into account turbulent flow close to the ground and the urban canopy parameters enabling an analytic calculation of the plume concentration profiles and concentration fluctuations. Model predictions are compared with some recent experimental data, showing a close match. The model developed can be used as an analytical tool for predicting CBR plume behaviour in complex urban environments, or as a prototype and performance check for a new generation of dispersion models.

Key–Words: Plume, Pollutant, Dispersion, Tracer, Urban Canopy, Concentration Fluctuations, Intermittency, Model.

1 Introduction

There has been a growing interest in recent years in the modelling of hazards arising from the atmospheric dispersion of chemical, biological and radiological (CBR) agents in the environment, and the threat that they pose to the population and military forces. This is a particularly challenging problem in an urban setting. Dispersion of CBR tracer in an atmospheric boundary layer (ABL) over a heterogeneous (urban) canopy is a complex process to be described by advanced methods of fluid dynamics, turbulence theory, diffusion and statistics. Using comprehensive modelling is computationally intensive and too time consuming when applied to operational problems when a reliable outcome has to be produced within a limited time frame. Plume characterisation requires the development of simplified analytical models of turbulent dispersion based on physical assumptions and "first principles" physics considerations. These models must still be simple enough to be easily treated numerically in an operationally viable way. Such models can also provide a theoretical foundation for "backtracking" problems, i.e. finding a CBR source in a complex canopy under various meteorological conditions. The purpose of this paper is to summarise the recent research conducted by DSTO in the development of such models.

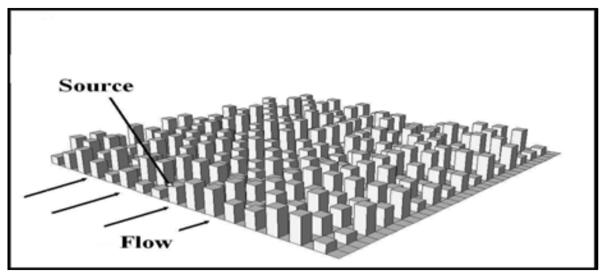

Figure 1: An example of one of the obstacle arrays used in the experiment.

2 A New Modelling Framework

2.1 Mean Flow in a Complex Canopy

The flow model in a surface layer within and above a canopy should correctly describe the average (i.e. non-fluctuating) velocity field. The traditional model for the ABL velocity profile is the celebrated log-law profile

$$U(z) = \frac{v_*}{\kappa} \ln\left(\frac{z-d}{z_0}\right), \tag{1}$$

where $U(z)$ is the horizontal velocity, z is the distance from the ground, v_* is the friction velocity, $\kappa = 0.4$ is Von Karman's constant, z_0 is the roughness height and d is the so-called displacement height. For the real ABL flow over the canopy both d and z_0 should be considered as fitting parameters.

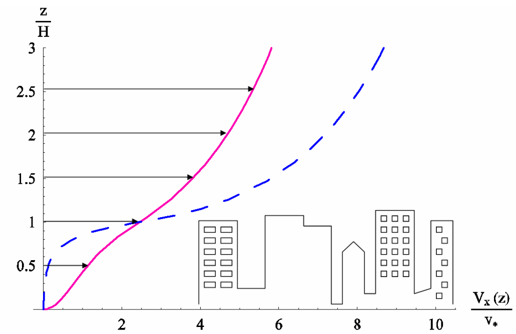

Figure 2: The average horizontal velocity profile. The pink/solid line is for a sparse canopy ($\epsilon = 0.5$) and the blue/dashed line is for a dense canopy ($\epsilon = 2$).

It has been known for a long time (dating back to Prandtl, see [9]) that the ABL mean velocity profile can be fairly approximated by a power-law function:

$$U(z) = v_0 \left(\frac{z-d}{z_0}\right)^m . \tag{2}$$

In general v_0, z_0, d, and m may be considered as "fitting" parameters (given or derived from observations). Obviously expression (2) makes sense only for $z \geq d$. It is important to note that for the ABL over a flat smooth surface $d = 0$ and the "fitting" parameters v_0 and m can be expressed in terms of the main flow parameters (see [2])

$$v_0 = a v_*, \quad a = \frac{\ln Re}{\sqrt{3}} + \frac{5}{2}, \quad m = \frac{3}{2\ln Re}, \tag{3}$$

where Re is the Reynolds number of the flow in the ABL and the power-law model becomes completely defined. Observed values of m in the atmosphere range from nearly 0 in very unstable conditions, representing perfect mixing and a uniform velocity profile, to nearly 1 in very stable conditions, approaching the Couette linear profile of laminar motion over a plane surface. For neutral conditions $m \approx 1/7$ [9]. The value of m also depends on surface roughness: roughness promotes mixing near the surface, which reduces the velocity gradient at small z and thus leads to larger variation in m. A simple estimate for a can also be derived from the condition $U(\delta) = U_0$, where δ is the boundary layer thickness and U_0 is the velocity of unobstructed flow. Thus

$$a \approx \frac{U_0}{v_*}\left(\frac{z_0}{\delta}\right)^m \tag{4}$$

If δ is used as a reference height (i.e. $z_0 = \delta$) we can deduce that $a \approx \sqrt{2/c_x}$, where c_x is the drag coefficient of the underlying surface (canopy).

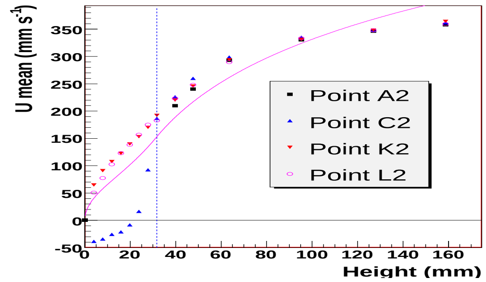

Figure 3: Measured velocity profiles for a simulated urban canopy at different positions relative to canopy objects.

Based on the so-called distributed drag approach it has been recently shown [6] that the entire influence of the canopy on the ABL flow (2) can be described by only one parameter that describes the ratio of the canopy surface area to the total area. For an array of identical cylinders (similar to Fig. 1) this parameter is approximately equal to:

$$\epsilon = \frac{2H}{r_0}\frac{1-\lambda_p}{\lambda_p}, \tag{5}$$

where all parameters in this formula are determined by the canopy morphology (H is the canopy height, r_0 is the radius of the cylinders and λ_p is the packing density of canopy elements). The limiting values of ϵ correspond to sparse ($\epsilon \gg 1$) and dense ($\epsilon \ll 1$) canopies. We have developed a consistent theoretical framework that allows us to derive a "modified" velocity profile $U(z)$ (2) for a given value of ϵ, i.e. for a given canopy. Our approach is based on "smooth" matching of the two solutions of momentum balance (below and above the canopy) near the canopy top (for details see [8]). In this way we have derived an algebraic system for functions $d(\epsilon)$, $z_0(\epsilon)$ to account for the effect of the canopy:

$$\frac{d}{H} = (1-\sigma), \quad \frac{z_0}{H} = \frac{\sigma}{e}, \quad \frac{1}{\kappa}\sqrt{\frac{\epsilon\sigma}{2}} = \tanh\left(\sqrt{\frac{2\epsilon\kappa^2}{\sigma}}\right). \tag{6}$$

The solution of the last equation in system (6) provides a value of $\sigma(\epsilon)$ that should be substituted in the first two equations to obtain $d(\epsilon)$, $z_0(\epsilon)$ and hence a "canopy-modified" profile $U(z)$ (2), (1). We found that for a large ϵ value $d(\epsilon) \rightarrow 0$, $z_0(\epsilon) \rightarrow 0$ as a power law (i.e. rather slowly) and $d(\epsilon) = z_0(\epsilon) = 0$ if $H = 0$. It should be emphasized that in the proposed framework, the entire morphological variety of canopies manifests itself only in different values of the parameter ϵ.

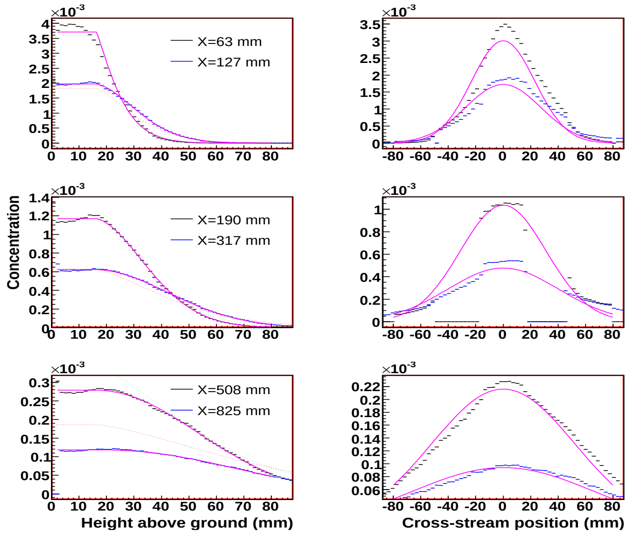

Figure 4: Concentration data fit to the "clipped" model for different downstream distances (C_z for the left column and C_y for the right column). The horizontal axis is the distance from the ground (left column) and cross-stream distance from the source position (right column).

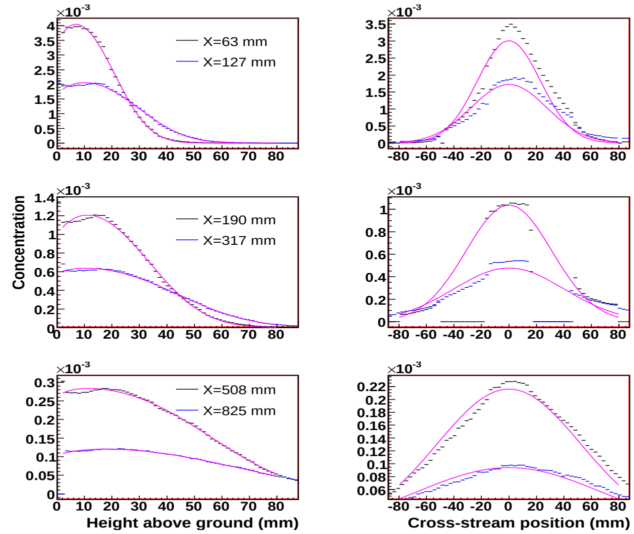

Figure 5: Concentration data fit to "variable α" model (see text). All notations are as in Fig 4.

Examples of velocity profiles calculated with (2), (6) are presented in Fig. 2. In Fig. 3 we present our experimental data from a water channel experiment ([5]). The urban canopy was modelled by an array of cubic obstacles that were packed in regular or random patterns (see Fig. 1). The velocity measurements were conducted in various positions within a canopy cell (including wake areas). The solid line in Fig. 3 is our model prediction, which represents an average velocity profile for the whole cell. This is to be compared to the individual point measurements of velocity within each cell, which vary significantly from point to point. The point C2 corresponds to the position directly behind the obstacle (wake area) with a clearly visible reverse flow (negative velocity). Our simplified models attempt to capture the "averaged-over-cell" behaviour. For a variety of obstacle array configurations, we observed a reasonably good agreement between our model and the measured velocity profiles.

2.2 Mean Concentration Profile

For the derived velocity profile in and above the canopy we computed the mean concentration field from the advection-diffusion equation for mean concentration C. It is known that for the power-law mean wind velocity profile (2), an analytical solution can be represented by

$$\overline{C}(x, y, z) = \overline{C}_y(x, y)\overline{C}_z(x, z), \tag{7}$$

with analytical expressions for $\overline{C}_y$ and $\overline{C}_z$ derived from the advection-diffusion equation of turbulent dispersion [9], [10], [11]. Here $\overline{C}_y(x, y)$ is the concentration profile in the y direction (crosswind), $\overline{C}_z(x, z)$ is the vertical concentration profile.

Under the assumption that the source is on the ground, adopting the velocity profile (2), $\overline{C}_y(x, y)$ has a Gaussian shape (see [2], [9], [10], [11])

$$\overline{C}_y(x, y) = \frac{1}{\sqrt{2\pi}\sigma_y} \exp(-\eta^2/2), \tag{8}$$

where $\eta = (y - y_0)/\sigma_y$. Parameter $\sigma_y(x)$ is a plume crosswind spread that increases with the downwind distance (see (13)) and the vertical concentration obeys a stretched-exponential profile (see [9], [10], [11]):

$$\overline{C}_z(x, z) = C_0 \exp(-\zeta^\alpha). \tag{9}$$

Here $\zeta = f_z B z/\sigma_z$, $\sigma_z(x)$ is a vertical plume spread (defined below);

$$B = \frac{\Gamma(2/\alpha)}{\Gamma(1/\alpha)}, \quad f_z = \frac{\Gamma(3/\alpha)}{\Gamma(1/\alpha)} \frac{1}{B^2}, \tag{10}$$

and $\Gamma(\cdot)$ is the gamma function,

$$C_0 = G \frac{Q_0}{v_0 \sigma_z} \left(\frac{z_0}{\sigma_z}\right)^m, \tag{11}$$

where Q_0 is an emission rate of the source,

$$G = \frac{\alpha B}{\Gamma(1/\alpha)} \frac{f_z^{1+m}}{f_u}, \quad f_u = \Gamma\left(\frac{1+m}{\alpha}\right)\left[\Gamma\left(\frac{1}{\alpha}\right) B^m\right]. \tag{12}$$

There are two conventional models of turbulent diffusion for which the value of the parameter α in (9) can be derived analytically: the model with a conjugate diffusivity profile ($\alpha = 1 + 2m$) and the model with a linear diffusivity profile ($\alpha = 1 + m$), for details see [11], [9]. For neutral stability conditions $m \ll 1$ and the two profiles are almost identical. Overall we found that the model of conjugate diffusivity profile provides a better agreement with our experimental data.

For the plume spread we assume [10]

$$\sigma_z(x) = Hz_0 \left(\frac{x - x_0}{z_0}\right)^{1/\alpha}, \quad \sigma_y(x) = D\sigma_z(x), \tag{13}$$

with $D = \sigma_v/\sigma_w \approx 1.74$ [9] (σ_v, σ_w are the standard deviations of velocity fluctuations in the vertical and horizontal directions near the surface) and

$$H = \frac{1}{q}\left[\kappa q(1+m)/a\right]^{\frac{1}{1+m}}, \; q = \left[\alpha \frac{\Gamma(2/\alpha)}{\Gamma(1/\alpha)}\right]^{\frac{1}{1-\alpha}}, \tag{14}$$

where a is defined in (3).

It should be emphasized that the model (8), (9) explicitly takes into account the flow shear (parameter p), the underlying surface influence (no-flux boundary condition $\partial_z \overline{C} = 0$ at $z = 0$) and the strong turbulence anisotropy in the surface layer (parameter D). As such this model is an evident generalization of the well known Gaussian plume models. Note however that even in the limiting case $m = 0$ (the wind speed independent of the height), the adopted model (9) becomes exponential and thus is different from the celebrated Gaussian plume model.

It is evident that the solution given in (9) is a valid representation for the concentration profile above the canopy top (i.e. for $z \gg d$). In order to have a consistent profile for all z it should be matched with the tracer concentration modelled within the canopy (i.e. for $z \le d$). Two models of the concentration profile within the canopy were validated. The first model was a "clipped" profile, where we simply assumed that $\overline{C}_z(z) = \overline{C}_z(d)$ for $z \le d$. The justification for such a model is the strong process of turbulent mixing that occurs within the canopy that should "smooth out" all concentration gradients. The data fit to the "clipped" model for different downstream positions is presented in Fig. 4. The left column is the vertical concentration profile and the right column is the lateral structure of the plume with a Gaussian fit.

The second evaluated concentration model was based on allowing the variation of α with height to provide the best data fit i.e. $\alpha = \alpha(z)$. The rationale behind this framework was the known limiting values of α: $\alpha = 1 + 2m$ for $z \gg d$, and $\alpha = 2$

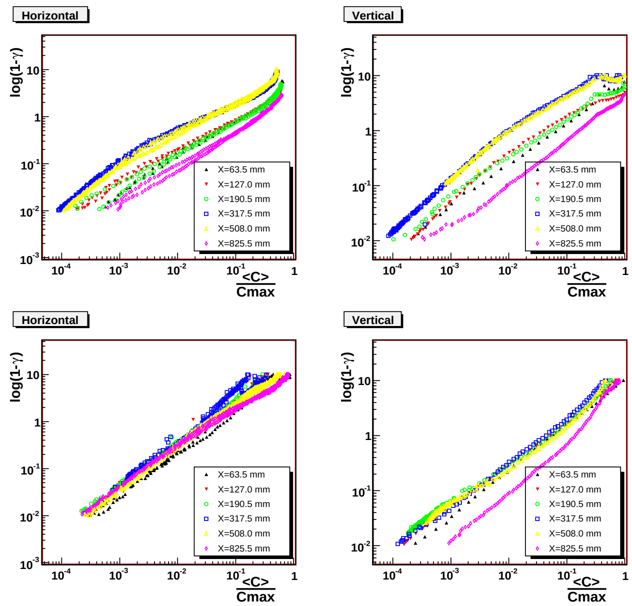

Figure 6: Examples of intermittency profiles as a function of inverse mean concentration: slices across the centre of the plume for different downstream positions (top row - no-obstacle canopy, bottom row - $1H$ regular obstacle array). Straight lines correspond to (18).

for $z \le d$ (Gaussian diffusion in stagnation areas near the canopy floor). As a reasonable approximation we proposed that

$$\alpha(z) = (1 + 2m)(1 + \phi \exp(-z/d)), \tag{15}$$

where $\phi = (1 - 2m)/(1 + 2m)$ is a function of the velocity profile parameter m and canopy parameter ϵ (since $d = d(\epsilon)$). It is worth noting that $\overline{C}_z$ in (9) depends on α in rather a convoluted way (not only through an exponential power) causing a complex deformation of the concentration profile even with minor change in $\alpha(z)$.

The data fit to the "variable α" model is presented in Fig. 5 (left column). As in Fig. 4, the right column is the lateral structure of the plume with a Gaussian fit. In general we observe that both models are in good agreement with the experimental data. The data fits are nearly indistinguishable downstream of the source. Closer to the source the "variable α" model seems to be a better representation of the vertical plume structure. The better performance of the "variable α" model can be attributed to the more adequate description of the process of turbulent mixing in the canopy layer (i.e. mixing is changing within the canopy with height). The "clipped" model corresponds to constant mixing in the canopy. For dense canopies, where near the ground is a stagnation flow

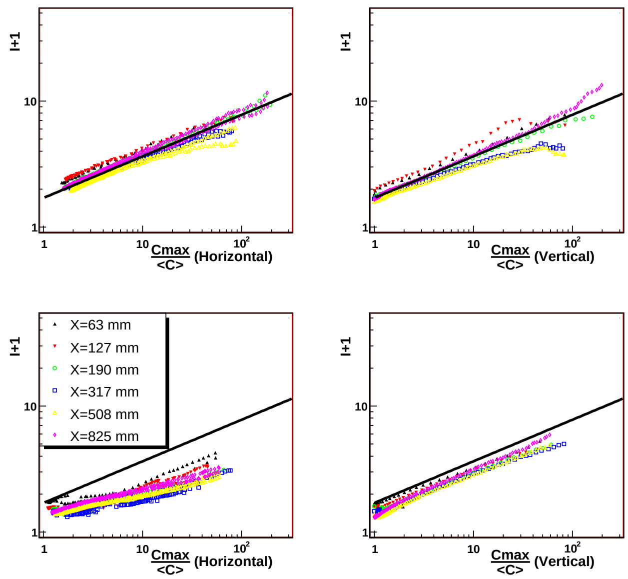

Figure 7: Examples of concentration intensity profile as a function of inverse mean concentration: slice across the centre of the plume, different downstream positions (top row - no-obstacle canopy, bottom row - $1H$ regular obstacle array). Solid line is (19) with $\omega = 1/3$.

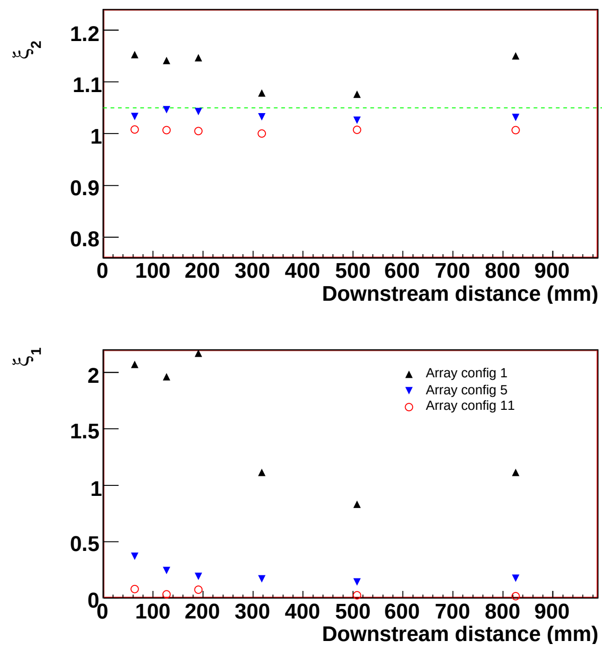

Figure 8: Parameters $\xi_2 = \overline{Z_c^2}/\overline{Z_c}^2$ (top graph) and $\xi_1 = \overline{Y_c^2}/\overline{Z_c}^2$ (bottom graph), see text.

zone, height dependence is not so important (see [6]) and both models produce very similar results.

2.3 Concentration Fluctuations

The approach outlined above models the development of a "mean plume" within a complex environment. This is the time averaged behaviour of a real dispersing plume, or equivalently, the average pattern that would be seen if an identical release of material was performed many times. Model analysis of CBR events also requires the development of "concentration realisation" models that give a statistically sound representation of possible instantaneous patterns of plume dispersion. This is important to enable the investigation of uncertainty or risk in CBR hazard assessments, as well as to provide realistic synthetic environments for operational analysis studies.

It is well-known that tracer fluctuations are very intermittent, so a correct description of intermittency is an important step in building realistic models of tracer fluctuations (see [5], [13]). Intermittency (i.e. the fraction of time when concentration is zero) manifests itself as a "singular" term in the PDF (Probability Density Function) of tracer concentration (see [5])

$$f(C) = \gamma\psi(C) + (1-\gamma)\delta(C), \qquad (16)$$

where $\delta(\cdot)$ is the delta function, γ is the intermittency parameter ($\gamma = 1$ corresponds to the no-intermittency case), $\psi(C)$ is any "regular" concentration PDF (see [5]).

A simple model of intermittency can be developed based on the following physics-based consideration. Let us assume that tracer is advected in the form of consolidated structures (blobs, sheets [4]). This is a realistic assumption since the system is far from equilibrium, so the tracer distribution is far from being perfectly mixed. The existence of discrete structures of tracer suggest that Poisson statistics may provide a useful framework for describing tracer fluctuation behaviour, with "an individual event" being attributed to the "tracer blob" passing a particular point of the flow. Then for the probability of k occurrences of such an event we can apply the celebrated formula of the Poisson distribution

$$Prob(k,\lambda) = \frac{\exp(-\lambda)\lambda^k}{k!}, \qquad (17)$$

where λ is the average number of occurrences. Then for the "no-occurrences" event we simply get $Prob(0,\lambda) = \exp(-\lambda)$ irrespective of any statistical properties of "blobs distribution". This results in the following formula:

$$1-\gamma \sim \exp\left(-\beta\frac{C}{C_{max}}\right), \qquad (18)$$

where we have used the obvious relationships: $\gamma = Prob(0, \lambda)$, $\lambda = \beta C/C_{max}$, $\beta = -\ln(1 - \gamma_{max})$. Thus we may expect that intermittency factor γ exponentially decreases with $\overline{C}$ and that this behavior holds for a wide range of surface flow regimes (canopies) since it is not based on a specific mean concentration profile $\overline{C}$. We also expect that the parameter β has a rather universal value. Our "data fitting" estimate provides $\beta \approx 0.75$.

The formula (18) was validated with our data from a water channel experiment. The results are presented in Fig. 6 where $\ln(1 - \gamma)$ is plotted against C_{max}/C on a log-log scale, so straight lines correspond to (18). The solid line corresponds to the value $\beta = 0.75$. As we can see the relationship (18) holds for a wide range of parameter values: concentrations, downstream positions and types of obstacle arrays.

Our next step was to developed a simple model for concentration intensity $i = \overline{C^2}/\overline{C}^2 - 1$ which is a universal parameter describing tracer fluctuations. In order to have a closed model we needed to express the concentration intensity i in terms of the mean concentration $\overline{C}$ (7). An important result was found in [5] where it was shown that for Gifford's isotropic meandering plume model

$$i + 1 \sim \left(\frac{C_{max}}{C}\right)^{\omega}, \qquad (19)$$

with $0 \leq \omega \leq 1$.

A similar formula can be derived in another simplified case - tracer dispersion in a surface layer with a strong velocity gradient. As a reasonable approximation in this case we can neglect any "weak" dependency on cross-stream direction and consider only the two-dimensional problem (dispersion from a point source in the vertical plane). For such a problem all concentration moments can be described by the same universal profile [7], so $\overline{C^2} \sim \overline{C}$. Then for i we arrive at the same expression as (19), but now with $\omega = 1$. These results provide a solid foundation for an assumption that the functional relationship of (19) should hold in a more general case (i.e. when the plume is strongly anisotropic and 3D effects cannot be neglected). We also expect the "shape" parameter ω to have a rather universal value.

The above assumption was validated with our experimental data and the results are presented in Fig. 7. The value of $i + 1$ is plotted against C_{max}/C in the log-log scale, so straight lines should correspond to (18). Again, we can see that the relationship (18) holds over a wide range of the model parameters. The solid line corresponds to the "fitting" value $\omega = 1/3$.

A more advanced model of plume concentration fluctuations (including generation of a "synthetic" time series [16]) has been developed based on the so-called "fluctuating plume" approach, where overall fluctuations are represented as the combined effect of slowly oscillating plume meander and fast in-plume fluctuations. Thus, for the conditional PDF of concentration in the absolute frame f the following general representation was adopted ([5]):

$$f(C,\ \mathbf{r}) = \int f_r(C,\ \mathbf{r} - \mathbf{R}_c) f_c(\mathbf{R}_c) d\mathbf{R}_c, \qquad (20)$$

where f_c is the PDF of centroid meander, $\mathbf{r} = \{y, z\}$, f_r is the concentration PDF in the relative frame (associated with plume centroid), $\mathbf{R}_c(t)$ is the position of the centroid. Development of realistic models for f_r and f_c requires the application of rather complicated statistical methods and are described in detail in our other publications (see [5], [13], [12]). The mean centroid position can be calculated from (7) by (see [5])

$$\overline{\mathbf{R}}_c = \frac{\int \mathbf{r}\, \overline{C}(x, \mathbf{r})\, d\mathbf{r}}{\int \overline{C}(x, \mathbf{r})\, d\mathbf{r}}, \qquad (21)$$

It is well-known that the PDF for horizontal meander is always close to Gaussian [5], so it can be completely characterized by its first two statistical moments (or by mean $\overline{Y_c}$ and standard deviation σ_{cy}). Because of the "averaged" symmetry of the ABL flow in the cross stream direction $\overline{Y_c} \approx 0$ and there is only one free parameter in this model [5]. We proposed the following model

$$\sigma_{cy} = F \sigma_y (z_0/\overline{Z_c})^{1/2}, \qquad (22)$$

where $F \approx 0.2$ (data fit), $\overline{Z_c}$ is defined by (21).

A functional form of the PDF for vertical centroid meander is less straightforward since it is skewed due to anisotropy caused by underlying surface and flow gradients. Based on Large Deviation Theory we have proposed a model where the PDF for vertical plume meander Z_c can be described by a Gamma distribution [12]:

$$f_{cz}(\zeta) \sim \zeta^{a-1} \exp\left(-\zeta\right), \qquad (23)$$

where $\zeta = Z_c/b$ and, based on the properties of the Gamma distribution [1], parameters a and b can be expressed in terms of the the first two moments of Z_c: $ab = \overline{Z_c}$, $ab^2 = \overline{Z_c^2} - \overline{Z_c}^2 \equiv \sigma_{cz}^2$. Since a mean centroid position Z_c can be calculated from (21) the distribution (23) effectively has only one free parameter ($\xi_2 = \overline{Z_c^2}/\overline{Z_c}^2$). For plumes in the ABL the estimate $\xi - 1 \ll 1$ holds over a wide range of parameters [9], so the value of ab^2 seems to be rather small. In Fig. 8 we presented values of the parameter ξ_2 derived from our water channel experiment. We observed that

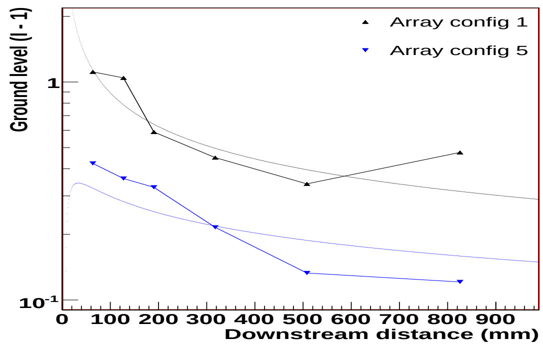

Figure 9: Fluctuation intensity i as function of downstream position.

a typical value of $\xi_2 \approx 1.05$. To describe the statistical properties of horizontal centroid meander we calculated a similar parameter $\xi_1 = \overline{Y_c^2}/\overline{Z_c}^2$ and also presented it in Fig. 8. We found that the value of this parameter was rather universal for a given obstacle array and may be strongly affected by its morphological structure.

For in-plume fluctuations we used the model proposed in [5]. The PDF f_r for in-plume concentration C_r we expressed in a form similar to (23) with $\zeta = C_r/\overline{C_r}$ using profile (8) for C_r (with different "relative" spreads $\sigma_{ry}(x),\ \sigma_{rz}(x)$). The "relative" spreads $\sigma_{ry}(x), \sigma_{rz}(x)$ can be determined from a "squared" rule [5]: $\sigma_z^2 = \sigma_{cz}^2 + \sigma_{rz}^2,\ \ \sigma_y^2 = \sigma_{cy}^2 + \sigma_{ry}^2$. Parameter a in (23) can be expressed in terms of the relative fluctuation intensity i defined previously: $a = 1/i^2$ (for details see [5]). For the downstream evolution of i we used an approach developed in [15] (based on two-particle statistics). Thus we obtained

$$i(x) = \left(\frac{\overline{x}}{\overline{x}_0}\right)^s \frac{1}{(1 + (\overline{x}/\overline{x}_0)^p + J(\overline{x})^w)^q} - 1, \quad (24)$$

where $s = pq$, $p = (2 + 2m)/(q\beta)$, $\beta = 1 + 5m/3$, $w = 2m/(q\beta)$, $\overline{x} = x/z_0$, $\overline{x}_0 = x_0/z_0$, x_0 is a scale of the tracer source. The value of parameter q depends on the regime of relative tracer dispersion in the plume and is always within the range $2 \leq q \leq 5$. For a classical Richardson regime $q = 3$, for a surface layer turbulence regime $q = 2$ [9] and for a convective layer regime $q = 5$ [14]. In general q is defined by $r^2 \sim t^q$, where r is the average distance between tracer particles. The value of J cannot be estimated analytically and was derived from data fit ($J \approx -1.5$).

Obviously this model is valid only until $i \geq 0$ and further downstream we need a modified expression for i. Nevertheless for short-range dispersion modelling (24) seems to be sufficient. The data fit for the analytical form (24) is presented in Fig. 9 with parameter $q = 3$.

Equations (23) and (24) provide a simple yet universal parametrization of tracer plume statistics with given mean flow and canopy parameters (v_0, z_0, m, ϵ).

3 Conclusions

Physics based models of a plume in an urban canopy allow a simplified (but still adequate) analytical description of pollution transport in a complex environment which is particularly useful for CBR applications. The proposed theoretical framework has been validated against our experimental data (water channel experiment) and has provided a close match. The proposed framework can help to validate and justify some more empirically based and heuristic assumptions of some operational dispersion models. Our modelling framework can thus be used as a valuable performance check of such models, or be extended to an operational model prototype, able to be linked to larger modelling systems.

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