# Void fraction fluctuations in two phase flow: theoretical models

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*Abstract:* The knowledge on the structure and dynamical behavior of the two phase flow is important for a great number of processes, including the coolant behavior in a nuclear power reactor. Previous studies performed on gas liquid phase or in experimental devices specially designed for that purpose, have introduced theoretical models that describe the void fraction fluctuations in a two phase flow. Nevertheless, they only partially explained the measured effects or have a doubtful applicability. In this work, from data obtained from a BWR nuclear power plant, we apply different mathematical models in order to study the change of the intensity of the void fraction fluctuations along the channel. By comparing these models, we conclude that the bimodal two-phase flow model is a more realistic one for void fraction estimation. This model can be applied for high void fraction rate and converges to the Poisson model for low void fraction rate. The expected contribution of this work is, by means of a proper use of neutron signal analysis, to obtain a better understanding of the dynamics and structure of the two phase flow in a nuclear power reactor.

*Key-Words:* Two phase flow, Bimodal model, Neutron noise, Coolant, Nuclear reactor.

## 1 Introduction

When the coolant is a two phase mixture of liquid and vapor, the coolant flow pattern can become quite complicated. The knowledge on the structure and dynamical behavior of the two phase flow is important for a great number of processes, including the coolant behavior in a nuclear power reactor.

In the design and operation of industrial systems, in which two-phase flows occur, it is essential to know the flow patterns. The heat and mass transfer, pressure drop and chemical reaction rate are strongly affected by the way in which phases are distributed in the flow.

In the simultaneous transport of gas-liquid petroleum compounds in a single pipeline, the changes of pressure loss occur if the flow pattern changes over the flow lines. Therefore, in order to estimate the pressure losses along the flow pipes it is necessary to know the flow patterns.

In some industrial natural circulation boilers, due to the formation of large bullet-shaped vapor bubbles in the pipes exposed to high heat flux, it is important to choose the flow conditions in order to avoid the occurrence of slug flow.

In chemical, biochemical, environmental, and food process, the bubble rise characterization becomes relevant for the design of heat and mass transfer operations. Air bubbles are used for improving the heat and mass transfer. The overall mass transfer is affected by the bubble size, pressure inside the gas phase, interaction between bubbles, rise velocity and trajectory [1, 2].

In some chemical processes, it is useful to increase the contact surface between gas-liquid phases in order to improve the mass transfer for a better chemical reaction. In these processes finely distributed bubbles are necessary to maximize the interfacial area between the two phases.

In nuclear power reactors, an important task of the nuclear designers is to predict two-phase flow transients under various operating conditions or under the loss-of-coolant-accident conditions. The majority of the modern thermal hydraulics codes, used for calculating steady state and transient flow conditions, are based on the use of separate flow equations, for the vapor and liquid phases. In this way, separate time dependent conservation equation for mass, momentum and energy are applied for each phase.

Under conditions of thermal and mechanical non equilibrium, the transfer of mass, momentum and energy between the phases depends on the distribution of the phases (saturated steam in contact with subcooled liquid or superheated steam carrying

droplets of water at saturation temperature flowing with different velocities).

The above mentioned situations are few examples, among others, where constitutive relationships, that are required to solve the equations of the system, are very sensitive to the topology of the flow.

The understanding and prediction of the two phase flow phenomena plays a relevant role in the safety analysis of present and future light water reactors. In this case, the complexity of single or two phase flows requires physically detailed modeling in order to take the strong inhomogeneity of two phase flows into account. In particular, natural convection flow phenomena for the removal of the decay heat from a nuclear reactor core out of a reactor pressure vessel or containment. Various innovative nuclear reactor safety concepts are based on passive devices for decay heat removal with no need for pumps [3].

It is extremely important for the safety and efficiency of Boiling Water Reactor (BWR) the monitoring of the local two-phase flow conditions in individual fuel bundles. These conditions are not only axially space dependent, but also radially. The reason lays on the basis that the control rod pattern, different fuel assemblies types and burn-up, the radial distribution of the total circulation flow rate over the core and the assembly flow rate are highly non-uniform, therefore, the power distribution will be non-uniform too. For many years, the researchers have concentrated on the development of reliable methods for estimating two-phase flow variables. Thinking about a reliable, and low costing measuring technique, the analysis of the noise in-core neutron detectors seems to be a good one in order to study the local two phase flow in a BWR.

Many significant properties of the flow can be characterized by analysis of the induced neutron fluctuations. We are able to classify these characteristics as propagation, structure and dynamics of the flow.

Propagation is characterized by the transit time between two axially displaced neutron detectors. The propagating quantity is the density fluctuation of the flow.

It should be noted here that estimation of time delay by means of cross-correlation analysis is a well-known and established technique, if the quality of the observed signals is reasonably good. In our analysis with signals measured at a BWR, the measured variable is neutron fluctuation. Because of experimental limitations we can not install ordinary flow monitors (optical, impedance, etc.) inside the reactor. The neutron signal can be disturbed not only by bubbles in the coolant flow but also by other quantities, e.g. temperature non-uniformity, flow regime, flow distribution within the coolant channel, etc. Detailed treatment of the fluctuations is needed in this regard.

In this work, from data obtained from a nuclear power plant, we apply different mathematical models in order to study the change of the intensity of the void fraction fluctuations along the channel.

By comparing these models, we obtain a model that is a more realistic one for void fraction estimation, in a sense that it can be applied for high void fraction rate.

The expected contribution of this work is to obtain a better understanding of the dynamics and structure of the two phase flow in a nuclear power reactor by means of a proper use of neutron signal analysis.

## 2 Theory

The number of characteristics flow patterns and the names used for them vary somewhat from researcher to researcher.

### 2.1 Flow pattern in a diabatic flow

When a gas and a liquid are forced to flow together inside a pipe, there are at least seven geometrical configurations, or flow regimes that are considered to be observed. These patterns are illustrated in Figure 1 (taken from [4]).

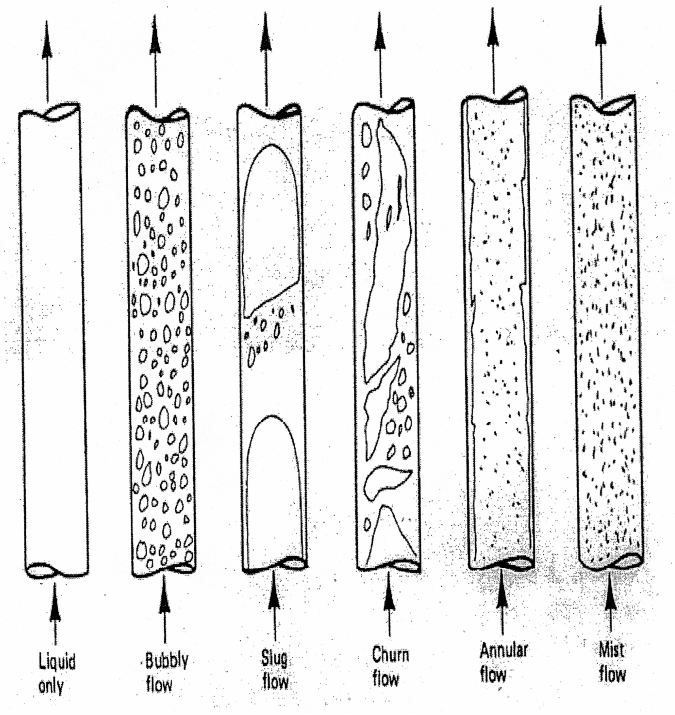


Figure 1: Typical flow regime patterns in a vertical pipe

From single phase liquid to single vapor or gas flow are: bubbly flow, plug flow, stratification, wave flow, slug flow, annular flow, dispersed flow and fog or mist flow. The regime depends on the fluid properties, the size of the conduit and the flow rates of each of the phases.

The flow regime can also depend on the configuration of the inlet; the flow regime may take some distance to develop and it can change with distance while another parameter (perhaps the pressure which affects the gas density) changes.

For fixed fluid properties and conduit, the flow rates are the independent variables that when adjusted will often lead to changes in the flow regime.

In order to increase clarity, the number of flow regimes can be reduced to only three major characteristic basic flow patterns in both adiabatic and diabatic (without and with heat addition) flow: bubbly flow, slug flow and annular flow.

### 2.1.1 Bubbly flow

In this case, the liquid phase is continuous and the vapor phase is discontinuous. The liquid flow rate is high enough to break up the gas into bubbles, but it is not high enough to cause the bubbles to become mixed well within the liquid phase.

As the channel is oriented vertically, the phase orientation would be symmetric, but they would likely slip between the phases and the gas would not move at the same speed as the liquid.

This flow pattern, where the gas is uniformly distributed in the form of discrete bubbles in a continuous liquid phase, is considered to be observed at low void fraction.

### 2.1.2 Slug flow

In slug flows, most of the gas is concentrated in large bullet shaped bubbles. These gas bubbles move uniformly upward and are separated by slugs of continuous liquid that span the entire channel. These liquid slugs may or may not contain small entrained gas bubbles. They travel at a speed that is a substantial fraction of the gas velocity and occur intermittently. They cause large pressure and liquid flow rate fluctuations. The slug can be compared with a large flow surge, or a large wave.

Liquid may or may not coat the entire pipe because there will be substantial atomization. This flow is considered to be observed at moderate void fractions and relatively low flow velocities.

This flow pattern seems to occur at the transition between the bubbly and the annular flow.

### 2.1.3 Annular flow

In annular flow, the liquid phase is continuous in an annulus along the wall and the vapor phase is continuous in the central region. Discontinuous liquid phase is present in the center as droplets, while discontinuous vapor phase appear as bubbles in the annulus. The liquid coats the wall. However, because of gravity, the liquid distribution is not symmetric. The velocity of the gas is large enough to cause waves to form in the liquid and also to atomize some liquid.

Annular flow is a general term for a class of flow patterns that includes annular, spray annular, dispersed, fog and mist flows because all of them have a liquid phase moves upwards, partly as wavy liquid film and partly in the form of droplets entrained in the gas core.

This flow pattern generally occur at high void fractions and high flow velocities.

### 2.1.4 Diabatic flow

In a boiling flow, the void fraction increases along the channel; so the flow patterns are expected to be diabatic flow.

The flow pattern of a boiling flow moving vertically upward in a heated channel are postulated to develop as it is shown in Figure 2 (taken from [5]) for the case of steadily increasing void fraction.

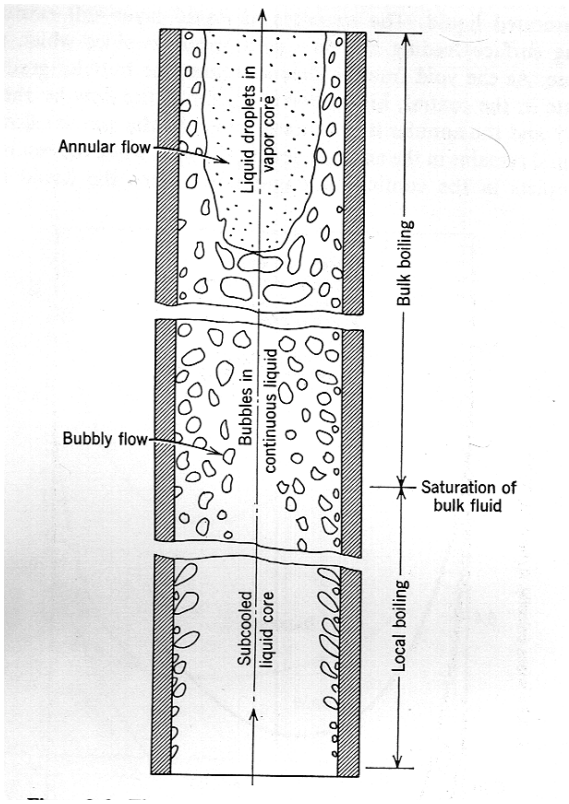


Figure 2: Flow patterns in a vertical heated channel

In the local boiling section, a superheated liquid layer exists next to the wall, while the bulk liquid may be subcooled. The bubbles in the superheated liquid layer nucleate and grow adjacent to a two phase bubble layer where, either still attached to the wall or carried along the stream, are collapsing.

## 2.2 On the state of the coolant in a BWR

In a BWR the operating pressure level is low compared to Pressurized Water Reactor (PWR) and two phase coolant flow takes place in the core.

The structure of the coolant flow is different at different axial elevations. After an initial warming up region with single phase coolant, first occurs bubbly flow, followed by slug and (possibly) annular flow regimes.

A transit time can be associated with the axial propagation of the coolant density fluctuations.

It is empirically clear that one can obtain a value of time delay, or velocity, by means of the cross-correlation technique. In this application, void fraction fluctuations in the coolant perturb the parameters of the reactor core and generate neutron flux changes, which in turn cause fluctuations in the detectors signals. However, the physical interpretation of the measured velocity has been a controversial topic during the last three decades. It is not clear which parameter is measured.

In relation to this matter, Lübbesmeyer [6] posed the following question: “what kind of velocity of the two-phase flow do we really measure?”

The opinions of the researchers in this point are still diverse and discrepancies are significant. It can be the steam velocity, the water velocity or something in between (the volumetric flux).

Kósaly [7] investigated the local component of the neutron fluctuations in the reactor core of the Muhleberg BWR (Switzerland) both in a frequency range below 10 Hz and in the region between 4 and 50 Hz. He concluded that the measured transit time (velocity because we know the distance between the neutron detectors) of propagating coolant density fluctuations is to be identified with the axial propagation velocity of the steam phase. He claims that his assumption is valid not only in the lower part of the core but also in the upper part, where annular flow is assumed to be dominant.

Seifritz and Cioli [8], Analytis [9] and Naito [10] also assumed that the fluid velocity inferred from transit time measurements is steam velocity.

Based on the above theory, in an annular flow regime, one would not expect to be able to measure a delay time using noise analysis because in order to obtain this, the presence of a bubble that moves from one position to the other is required. In fact, Atta [11] found a poor agreement between his results and the annular flow model at the top of the channel.

He explained that, near the top of the core, they no longer measure the effect of the bubbles, but instead measure the effect of liquid droplets entrained in steam flow.

This theory appears to refute the conjecture of Kósaly [7] where the steam and the droplets move with a velocity that is nearly the same. Based on this assumption, he concluded that their measurement still provide an estimation of the steam velocity near the top of the core.

On the other hand, Lübbesmeyer [6], from phenomenological considerations assumed that the velocity measured is the so-called volumetric flux of the two phase mixture (J) which is given by the relation

$$J = \alpha . v_g + (1-\alpha).v_l \qquad (1)$$

where $\alpha$ is the radially averaged steam void fraction and $v_g$ and $v_l$ are the velocity of vapor and liquid, respectively. He checked the validity of his assumption by measurements performed at air-water two phase flow test facility Fredli-II. In this experiment, the signals of two light beams were cross-correlated to get the transit time and the velocity of the two phase flow.

Chaudhary [12] made transit time measurement between axially separated local power range monitors (LPRM) in a BWR1 of Tarapur Atomic Power Station (India) and also showed that is reasonable to identify the fluid velocity with the volumetric flux.

Additionally, it was reported in [13] that two kinds of transit time, which corresponded to vapor and liquid velocities were observed.

Somewhat different interpretation of the transit time measurement is that the velocity corresponds to propagation of void fraction fluctuations or void wave and belongs to Jones and Zuber [14] and Kósaly [15].

In this context, velocity of void fraction fluctuations is formulated as

$$\Omega = \frac{\delta\alpha}{\delta t} + C_k \frac{\delta\alpha}{\delta z} \qquad (2)$$

where $\Omega$ is a source term of void and the velocity of the void fraction fluctuations is related to other parameters as

$$C_k = v_g + \alpha.\left[ J.\frac{\delta C_0}{\delta \alpha} + \frac{\delta v_{gi}}{\delta \alpha} \right] \quad (3)$$

with $v_{gi}$ the drift velocity, $C_0$ a distribution parameter and $J$ the volumetric velocity.
$C_0$ and the vapor velocity $v_g$ are related as follows

$$v_g = v_{gi} + C_0 \times J \quad (4)$$

Based on the amount of local power range monitors (LPRM) noise data acquired in a wide range of operating conditions, Mori et al. [16] stated that these very different results seem to be extracted from data in limited conditions.
They evaluated the accuracy of the measurement of local and total core flow rates using in-core neutron noise signals, which were measured in operating BWR plants, BWR-5 type (Japan). They investigated the characteristics of two phase flow velocity profiles and calculated the transit time by the weighting of vapor and liquid velocities. The weighting factor $w(\alpha)$ can be empirically determined as a function of the void fraction of the lower detector position. Its expression is as follows

$$w(\alpha) = 1 - 1.7\alpha \quad (5)$$

Finally, Stekelenburg [17], in their review on the measurements of the two phase flow variables in a BWR by analysis on in-core neutrons detectors noise signals, conclude that a general agreement has not yet been reached in regard to the interpretation of the transit time. He found that the neutronics involved in neutron noise processes is quite well understood, the researchers were able to predict the response of a neutron detector to arbitrarily distributed density fluctuations. Nevertheless, the research on the fluctuations in two phase flow is still ongoing, ([16] to [22], among others).
In order to get more understanding of the theoretical models related to the void fraction fluctuations in two phase flow, from now on, we will apply two models (the Poissonian model and the modified binomial one) to an operating BWR reactor and try to characterize the void fraction fluctuations.

## 2.3 Characterization of Fluctuations Sources

Quantitative characterization of the noise intensity over a given frequency range can be given by the normalized root mean square (NRMS) neutron noise.
The NRMS neutron noise over frequencies $f_1$ to $f_2$ is defined as follows

$$NRMS^2 = \frac{1}{I^2} \int_{f_1}^{f_2} APSD(f)\, df \quad (6)$$

where $I$ is the direct current (dc) component of the detector signal and the APSD is the auto power spectral density of the detector response.
The LPRM data have been represented by the corresponding probability density function (pdf).
This function is the relative density with which the value $x$ appears in the collection of data, and is an estimate of the rate of change of probability with magnitude.
The NRMS over all frequency band is equal to the normalized standard deviation of the pdf.

### 2.3.1 Poisson Statistics of bubbly flow

Before going into details about the Poisson´s model, we will explain briefly the Poisson distribution (for more details see [23, 24]). This distribution describes cases where there are particular outcomes but no idea of the number of trials; these are sharp events occurring in continuum.
Poisson statistics is applicable to a kind of experiments in which the number of occurrences in a given period of time is recorded. Such experiments are sometimes referred to as counting experiments.

### 2.3.2 Poisson model of bubbly flow

Taking into account the void fraction fluctuations, in the Poisson model of bubbly flow, Kósaly [7] introduced the following assumptions:
a) The production of bubbles is a Poisson process, in other words, there is no spatial or temporal correlation between distinct bubbles, which means there is no correlation between different radial positions and the bubble spectrum is white.
b) Bubbles neither coalesce nor collapse while they pass through the core.
c) It is only considered bubbly flow with low void fraction (up to 30%)
He developed a theoretical model for the normalized detector void strength of a neutron detector. This *NRMS* depends on the void fraction $\alpha$ and the average steam velocity $v$ which can be both a function of the axial detector position *r*

$$NRMS^2(r, f_1, f_2) = \frac{m\alpha(r)}{v(r)} + C \quad (7)$$

where $m$ and $C$ depend on the position and the frequency but are independent of $\alpha$ and $v$.

In this model, for the neutronic transfer of the fluctuations to the detector signal, he also considered that the fluctuations are filtered by the finite detector length and the finite sensitive volume of the detector.
The results of the Poisson theory given in Equation (6) have been successfully applied by Kósaly [7], Van Dam [25] and Mittef [26].
Moreover, Kósaly [7] used these results for the interpretation of neutron noise measurements with void fraction near 60%. As expected, the measurements deviated from the theoretical predicted behavior.
The assumption concerning spatial and temporal independence of bubbles is definitely not valid in slug flows, due to the presence of a well-defined spatial and temporal correlation in the void fraction fluctuation signal. Therefore, the Poisson model is only valid for low void fraction.

#### 2.3.3 Bernoulli Statistics of bubble dynamics

The binomial distribution describes a process with a given number of identical trials, with two possible outcomes. This distribution is used for repetitive experiments in which only the occurrence or non occurrence of an event is recorded.

#### 2.3.4 Poisson model and bimodal one

In a subsection before we explain the Poisson model of bubbly flow, in which the spectrum of void fraction fluctuations does not depend on the frequency and there is no correlation between void fraction fluctuations at different positions.
Kozma [27] developed theoretical models for higher void fractions, one based on the Bernoulli statistics of bubble dynamics and one based on the dynamics of a bimodal flow in which certain time-correlations are incorporated. In this model, the flow has two basic modes. These two theoretical models lead to a second-order polynomial function for the variance of the density fluctuations and the void fractions

$$NRMS^2(r, f_1, f_2) = m'\alpha(r)[1-\alpha(r)] + C' \quad (8)$$

where $m'$ and $C'$ depend on the position and the frequency but are independent of $\alpha$ and $v$.
We applied both the model used by Kósaly [7] and the binomial modified model of Kozma [27] in order to see changes of the intensity of the void fractions fluctuations along the channel.
The bimodal two-phase flow model is determined by the following set of parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ which are the expected values and variances of the first and second mode respectively.
Assuming that these values satisfy the Bernoulli statistics of boiling noise, we can write the firs and second mode respectively

$$\sigma_1^2 = Na^2\alpha_1(1-\alpha_1) + \sigma_{1bg}^2 \quad (9)$$

$$\sigma_2^2 = Na^2\alpha_2(1-\alpha_1) + \sigma_{2bg}^2 \quad (10)$$

where $\sigma_{ibg}^2$ is the background component, $N$ is the maximum possible number of bubbles in the sensitivity volume of the detector.
Also, for the expected values we have

$$\mu_1 = Na\,\alpha_1 \quad (11)$$

$$\mu_2 = Na\,\alpha_2 \quad (12)$$

with $a$ being the change in the detector signal caused by a single bubble.

## 3 Results and Discussion

The data were taken from a 900MW BWR nuclear power plant. The measurement conditions were: reactor power: 64%, core flow: 4220 Kg/sec, frequency sampling = 5 Hz. LPRM D, LPRM C, LPRM B and LPRM A are in the same string; LPRM D is in the top of the core as it is shown in Figure 3. The measured string (06) is located in a non-peripheral place which allows us to make general conclusions.

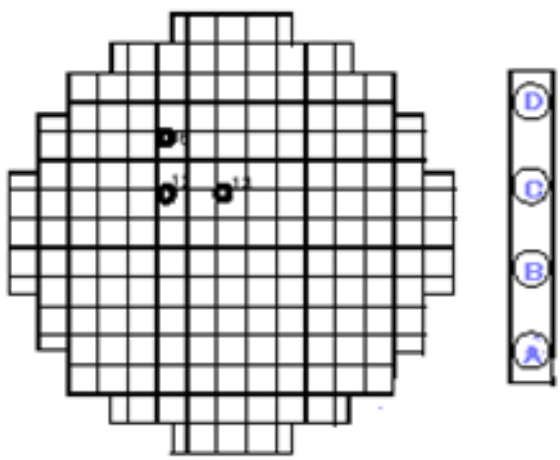


Figure 3: Reactor core and detector string

Figure 4 shows part of the analyzed data corresponding to the string 06. In this Figure, in the y-axis is represented the neutron fluctuation in arbitrary units and, in x-axis, the number of counts per second of the neutron detectors located in position A, B, C, and D respectively.

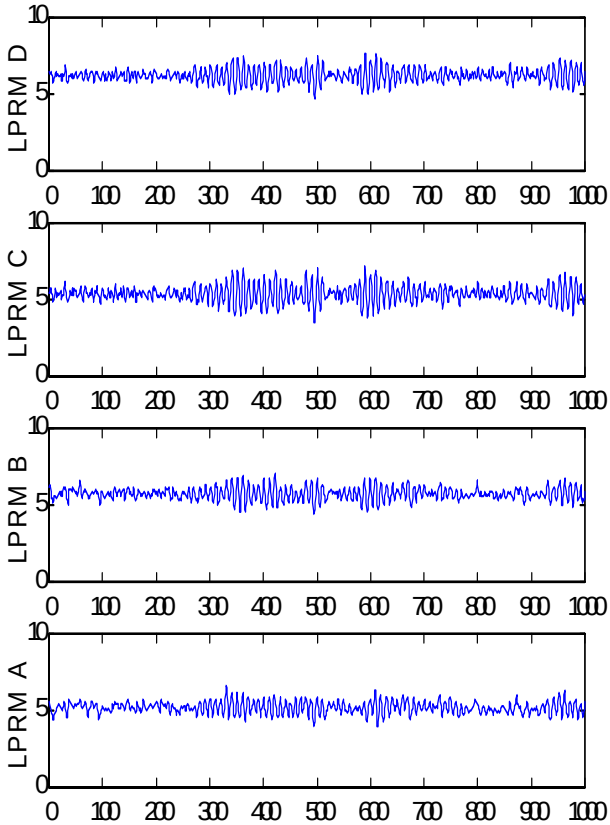


Figure 4: Part of the analyzed data in string 06.

## 3.1 Method of fitting

By making use of the above mentioned parameters, the following expressions are obtained for the expected value $\mu_{BM}$ and variance $\sigma^2_{BM}$ of the bimodal mixture

$$\mu_{BM} = \alpha_L \mu_1 + (1-\alpha_L)\mu_2 \qquad (13)$$

$$\sigma^2_{BM} = \alpha_L \sigma_1^2 + (1-\alpha_L)\sigma_2^2 + \alpha_L(1-\alpha_L).(\mu_1-\mu_2)^2 \qquad (14)$$

The variance of the bimodal mixture is the sum of weighted variances of the separate modes and an additional term, which depends on the difference between the expected values of the two modes and on their relative frequency of occurrence $\alpha_L$.

Bubbly and annular flows have unimodal character, i.e. their pdf is single peaked. Slugs flows are classified as bimodal and they have pdf with two peaks. The modality of the flows is related to the moments of the pdf.

For performing the fitting of the pdf we first defined a function $S(x)$ as a sum of two Gaussians with $\mu_1, \mu_2$ and $\sigma_1$, $\sigma_2$ the expected value and variance respectively. We also had to define some constrains in order to give the problem a physical meaning.

Our problem is to find the constrained minimum of a scalar function of several variables, $S(x)$, starting at an initial estimate. We performed this constrained non-linear optimization in the MatLab environment.

In our specific case, given input data, $x_i$ (relative signal magnitude), the observed output, $y_0$ (pdf magnitude counts), using an algorithm for solving the system with a Sequential Quadratic Programming in MatLab, we found coefficients $x$ that best-fit the equation $F(x)$

$$\min_x \sum_j (F(x) - y_0 j)^2 \qquad (15)$$

where $x_i$ and $y_0$ are vectors and $F(x)$ is a vector valued function.

It is necessary to start at an initial estimate value of $x$.

The values of $x$ that minimize $F(x)$ are the expected values $\mu_1, \mu_2$ and variances $\sigma_1^2, \sigma_2^2$ of the two Gaussians.

## 3.2 Fitting results for LPRM data

The results from the Gaussian fitting performed to the four LPRM are summarized in Table1

| Detectors | $\mu_1$ | $\mu_2$ | $\sigma_1$ | $\sigma_2$ |
|---|---|---|---|---|
| D | -0.54 | 1.00 | 1.05 | 0.99 |
| C | -0.36 | 1.30 | 1.12 | 0.97 |
| B | -0.22 | 1.60 | 1.20 | 0.97 |
| A | 0.04 | 1.80 | 1.35 | 1.02 |

Table 1: Gaussian fitting of the four detectors

Figure 5 illustrates an example of the binormal fit of the pdf of LPRM C for one set of data points.

In this Figure, crosses denote points of the calculated pdf having 50 channels and solid lines denote the binormal fit and the two Gaussian components of the fit.

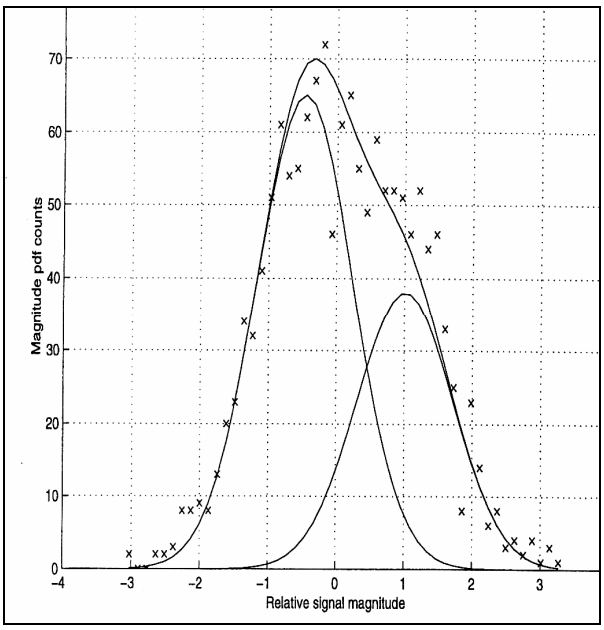


Figure 5: Fitting for LPRM-C in string 06.

These fitting results have been tested using a Lilliefors test for goodness of fit to a normal distribution. This test evaluates the hypothesis that $x$ has a normal distribution with unspecified mean and variance, against the alternative that $x$ does not have a normal distribution.

Table 2 shows the results of the normal distribution test. In this case $l$ is the Lilliefors test statistic and the *p*-value is the probability of obtaining the test statistic value. When the *p*-value is not between the intervals from 0.01 to 0.2 we denote $p\rangle\rangle$.

| Detectors | $l$ | $p$ |
|---|---|---|
| D | 0.0095 | $p\rangle\rangle$ |
| C | 0.0112 | 0.068 |
| B | 0.0133 | 0.026 |
| A | 0.0093 | $p\rangle\rangle$ |

Table 2: Lilliefors test values *l* and *p*

In order to develop an objective indicator, eliminating the characteristic constant and the background noise of each detector, we calculate

$$z = \frac{(\mu_1 - \mu_2)^2}{\sigma_1^2 - \sigma_2^2} \tag{16}$$

Based on this simplified model we can see that $z$ is a two phase flow structure parameter and is a function of the void fraction of the two modes.

The dependence of this parameter to the position in the channel, for two different sets of data points, is shown in Table 3

| Detectors | $z_1$ | $z_2$ |
|---|---|---|
| D | 19.2 | 17.9 |
| C | 9.09 | 11.9 |
| B | 6.6 | 3.5 |
| A | 4.4 | 3.2 |

Table 3: Parameter *z* and Lilliefors test values

We can see from Table 4 that the relative quadratic deviation for the bimodal fitting is lower compared to the Poisson model, specially for the case of LPRM C and D.

| Detectors | Bimodal | Poisson |
|---|---|---|
| D | 0.018 | 0.025 |
| C | 0.030 | 0.033 |
| B | 0.013 | 0.015 |
| A | 0.020 | 0.021 |

Table 4: Relative deviation for the two models

### 3.3 Integration of bimodal approach and empirical results

The bimodal model explains the apparent inconsistency found in [16]. They show that the relationship between the *NRMS* and the void fraction divided by the square root of void velocity is not linear. This result contradicts the conclusions of the Poisson theory introduced before in Equation (7) which predict a linear relationship between $NRMS^2$ and $\alpha / v$.

Moreover, applying the Bernoulli statistics of boiling noise, in which certain time correlation are incorporated, we have a parabolic relationship between $NRMS^2$ and $\alpha$ in Equation (8), instead of a linear dependence, which let us explain the obtained empirical results in [16].

## 4 Conclusion

The Poisson model is a very much simplified model and is only applicable for low void fraction values (up to 30%), whereas the modified Bernoulli model is also valid for higher void fraction values. Comparing these models for different heights in the channel of a reactor core, we can conclude that the bimodal two phase flow model is the more suitable. It is a more realistic model for void fraction estimation, in a sense

that it can be applied for high void fraction rate and also shows the best adjustment to the obtained experimental results. For low void fraction rate, this model also converges to the Poisson one.

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