# Flow Analysis of Annular Jets by Proper Orthogonal Decomposition. Active Control on in the Initial Zone of Large Diameter Ratio Annular Instabilities with Acoustic Excitations.

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Abstract: - Proper Orthogonal Decomposition (P.O.D.) is a technique used for analysis of vortex structures in a turbulent flow. In this study, complex shear flows are observed by P.I.V. measurements (Particle Image Velocimetry) of a large diameter ratio annular jet. The annular jet is an example of complex shear flow situations. Two axisymmetric shear layers, originating at the jet exit, one at the nozzle lip and the other at the centre body, eventually meet downstream or interact with each other. The main aim of this study is to observe and analyze the effects of active control using acoustic waves on an annular jet with a great diameter ratio (r= 0.91), in order to find a new way to reduce jet instabilities. This contribution discusses the application of Proper Orthogonal Decomposition to the P.I.V. (Particle Image Velocimetry) velocity fields of an annular jet and on a statistic of time resolved tomographic images of the initial zone of the annular jet. Acoustic waves are then applied on the annular jet with different frequencies (fundamental, first harmonic...). Measurements are conducted with a Reynolds number  $Re_{Do}=107800$ . The fluctuation frequency of the stagnation point is known for this Reynolds number. The Strouhal number corresponding to this frequency is  $St_{Do} = 0.27$ . The P.O.D. analysis applied on a natural annular jet and an excited annular jet enables us to see the importance of the triggering of the acoustic wave with the stagnation point motion. An active control is therefore necessary to use acoustic excitation to reduce instabilities in the initial zone of these turbulent jets. Active control has already been used with round jets and has given promising results, but only a few studies have been conducted on annular jets in this field. This work will permit us essentially to have a better knowledge of annular jets and to meet manufacturers' needs.

Key-Words: - Particle Image Velocimetry, Proper Orthogonal Decomposition, annular jet, control, acoustics excitations

# **1** Introduction.

Annular jet is used in the industrial domain, in combustion (burners, bluff bodies...) or in industrial treatment processes. The geometry of the annular jet is determined by the ratio  $D_i/D_o$ , where  $D_i$  represents the internal diameter and Do the external diameter. The flow characteristics of the initial region of an annular jet discharging into stationary air have been investigated previously. Chigier [1], in 1964, studied them as a limiting case of coaxial jets. Because the configuration of the annular jet often adopted by authors was without after body or bullet at the nozzle exit, an internal recirculating region is formed downstream of the interface. This recirculation zone is the result of the lack of any air supply in the center and the entrainment of air from the main stream of the annular jet. The interests of the flow here are the interaction between the jet and the recirculation zone near the nozzle of the annular jet. Many authors, like Ko [2][3](fig 1), work on small

diameter ratio annular jets, with a ratio  $D_i/D_o$  smaller than 0.7. Only the fully-developed merging zone of a diameter ratio larger than 0.7 have been studied by Aly [4] in 1991. The scientific interest in the study of turbulence has led to the development of a P.I.V. postprocessing, which is able to bring out the inner driving mechanism of the flow [5]. In other words, the structure responsible for the development or the maintenance of flow instability has to be reached. So, in this context, Lumley [6] [7] has proposed a method for identifying coherent and instantaneous structures in turbulent flow. The method is the Proper Orthogonal Decomposition (P.O.D.). This method provides a base for the modal decomposition of a set of functions, such as data obtained in the course of the experiments. The most striking property of this decomposition is optimality: it provides the most efficient way of capturing the dominant components of an infinite-dimensional process with only a few functions [5].



**Fig 1:** Shape of the flow of a basic annular jet with a large diameter ratio [2]

That is why the P.O.D. process has been applied in different turbulent flows to analyze experimental P.I.V. data with a view to extracting dominant features and trends : coherent structures [6][7][8][9]. P.O.D. provides an optimal set of basis functions for an ensemble of data. The present study relies on an experimental investigation of the initial zone of a large diameter ratio annular air jet by the use of Particle Image Velocimetry (P.I.V.) and fast tomography imagery. The intention of the present investigation is to dissociate the oscillation and velocity fluctuations due to the turbulent flow behavior. So it is useful to establish the link between the first P.O.D. eigenfunctions and the motion of the stagnation point at the end of the annular jet recirculation zone. Then the effect of acoustic waves on instabilities of the stagnation point of an annular jet will be discussed and will be explained by application of P.O.D. on P.I.V. measurements of active control.

# 2 Experimental set-up.

### 2.1 Annular jet

The nozzle used for this study was designed in the CORIA laboratory. Its diameter ratio r is equal to 0.91. The outer and inner diameters are, respectively, Do=53.88mm and Di=48.75mm. The contraction area ratio of the nozzle is equal to 102. The nozzle that we use is made of many components which reduce air flow rate fluctuations inside the nozzle in order to obtain a top-hat velocity profile at the nozzle outlet with a fluidic bed, silk filter layers, a bee nest and a convergent.



Fig. 2 Structure of the annular jet nozzle

A diagram of annular jet is shown in fig 3. The thickness of the jet e is thus equal to 2.565 mm. In our set-up,  $D_i/D_o$  is equal to 0.91. The exit velocity  $U_o$  is equal to 30m/s which correspond to the value of the Reynolds number Re, based on the thickness e, of 7680. At a distance from the nozzle of 50 µm, the cross sectional profile of the longitudinal, measured with hot wire anemometry, shows a "top hat" distribution (fig 3b).





Fig. 3 a) annular jet nozzle b) normalized longitudinal exit velocity profile for  $U_o$  measured at a distance of 0.025Do.

### 2.2 Cross correlation PIV set-up

The experimental set-up of the P.I.V. measurements is presented in fig 2. A double pulsed Nd - Yag laser is used to set up the light sheet. The output energy is nearly 30 mJ for each laser pulse. The wavelength is 532 nm. The laser beams are focused onto a sheet across the median plane of the annular jet by one cylindrical lens (f=-0.02 m) and one spherical lens (f=0.5 m). The time delay between the two pulses, which depends on the exit velocity  $U_o$ , is 10 µs. The observation field is 2.8\*2.5 cm<sup>2</sup>.

In this study, the video images are recorded by a LAVISION Flow Master 3S camera. The frame grabber, using a pixel clock, digitizes the analogue video signal to an accuracy of 12 bits. In the frame grabber, each field is digitized in 1280\*1024 pixels with grey levels. The acquisition frequency is 4 Hz.

Interrogation of the recorded images is performed by two-dimensional digital cross correlation analysis using "Davis 6.2.2."



Fig 4: experimental PIV set-up

For all velocity fields, the sampling window has a size of 16 by 16 pixels (0.377 by 0.377 mm) and there is a 50 % overlap with the next window. 400 P.I.V. images have been recorded. We observed less than 1 % of false vectors calculated in the flow.

### 2.3 Acoustics Acoustic forcing device

As the time delay between two couples of images depends on the camera frequency (8Hz), if we want to have access at all acts of the acoustic wave on the jet, we have to use a P.I.V. device which could be connected to acoustic forcing. Then we have used a TTL casing in order to control the triggering of the camera according to the acoustic wave (fig 5). When the acoustic wave exceeds a threshold fixed by the casing features (13mV), the casing triggers the camera and the laser. A delay can be added between the receiving of the signal issued from the TTL casing and the triggering of the acquisition. Measurements have been carried out with different delays between the acoustic wave and the acquisition. We have scanned 16 phases of the wave and for each phase (phases from 0 to 15), 100 images have been acquired.



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Fig 5: (a) Diagram of the phased set-up used for this for the study of active control, (b) Principle of the phased P.I.V. acquisition set-up.

The majority of works concerning the active control of the annular jets are carried out with measurements by hot wire anemometry or visualizations. Nakazona and Al [10], in 1991 work on annular jets of diameter ratio ranging between 0.2 and 0.8. They discover that for a frequency of excitation equal to the frequency of formation of the swirls, the recirculation zone can be influenced and the length of this one can even decrease. Travnicek and Tesar [11] work on an annular impinging jet of diameter ratio 0.95 and notice that they can, in the absence of wall, influence the initial zone by using acoustic excitations of frequencies corresponding to frequencies of formation of the swirls. They note that the harmonic and double excitation harmonic cause the increase or the suppression of the appearance of the swirls. The Strouhal numbers for this study are between 0.38 and 2.47.

The loudspeaker used for this set-up is a boomermedium of 130mm (AUDAX HM130CO). The range of frequencies of this device extends from 100 Hz to 10kHz.Its sensitivity is equal to 90dB and its nominal power is 50W. We have used only one loudspeaker placed in two different positions : face to the camera (position 1) and perpendicular to the camera (position 2), as it is schematized on the figure 3a.

The other parameters studied are the frequencies of the excitation (fundamental and double harmonic) and the amplitude (0.9V and 3.62V).

The active control is observed for a basic annular jet developed with a Reynolds number  $Re_{Do}$  based on the outer diameter equal to 107 760 (for a velocity at the outlet of the nozzle U0=30 m.s<sup>-1</sup>). With this Reynolds number, the frequency associated to the periodic beat of

the stagnation point is 150Hz. It corresponds to a Strouhal number StDo equal to 0.27. As we want to know jet responses to excitations of frequency equal to the fundamental frequency of the stagnation point beat but also equal to the double harmonic frequency, the forcing frequencies applied on the jet are respectively 148Hz or 152Hz (St<sub>Do</sub>=0.27) and 298Hz or 302Hz (St<sub>Do</sub>=0.54).

### 2.4 Fast tomographic imaging set-up

The laser sheet is obtained by a 4W, 515nm Argon-Ion continuous laser. The tomography system is composed of a high-speed digital camera Kodak Ektapro 8 bits. For the annular jet we used 256x256 pixels at 4500 images/s. For the round jet, we used 128x256 pixels, 0.13mm/pixels at 9000 images/s.

#### 2.5 Flow seeding

Flow seeding is one of the most important aspects of P.I.V. measurements. The intake air is seeded with 2-3  $\mu$ m diameter olive oil droplets. These were generated in an atomizer by passing air through a bath of olive oil. The air pressure varied from 1.5 to 2.5 bars (fig 5a). The particle seeding density was controlled by the flow rate of air through the atomizer.

# **3** Proper Orthogonal Decomposition.

Coherent structures are present in turbulent flow and P.I.V. is able to highlight those on the largest scale at any given moment. The scientific interest in the study of turbulence has led to the development of P.I.V. post-processing, which is able to bring out the inner driving mechanism of the flow [5].

Lumley [6] [7] has proposed a method for identifying coherent and instantaneous structures in turbulent flow. The method is Proper Orthogonal Decomposition (P.O.D.). P.O.D. provides an optimal set of basis functions for a set of data, Delville [12], Graftieux [13], Sirovish [13], Patte-Rouland [14], [15], [16], [17]. [18].

It is optimal in the sense that it is the most efficient way of extracting the most energetic components of an infinite dimensional process with only a few modes. The proper orthogonal decomposition is a linear procedure, which decomposes a set of signals in modal base. The P.O.D. analysis is introduced in this context by the use of physical functions with finite kinetic energy equivalent to the square integrable function. Coherent structures should be the structures that have the largest mean square projection on the velocity field. If it represents the candidate structure, then:

$$\frac{\left\langle (U.\Phi)^2 \right\rangle}{\left\langle (U.U)^2 \right\rangle} = M_{\Psi} \frac{\left\langle (U.\Psi)^2 \right\rangle}{\left\langle (U.U)^2 \right\rangle} \tag{1}$$

Where  $\langle \rangle$  represents an ensemble average. That is, the field that maximises the inner product with the velocity field is found. Maximising this inner product leads to the solution of the following eigenvalue problem:

$$\iint_{D} \left\langle U(x)U^{*}(x')\right\rangle \Phi(x')dx' = \lambda \cdot \Phi(x)$$
(2)

Or in another way,

$$\iint_{D} R_{ij}(x,x') \cdot \Phi_{j}^{(n)}(x') dx' = \lambda^{(n)} \Phi_{i}^{(n)}(x)$$
(3)

D represents the two dimensional domain of the velocity fields. R is the averaged two-point correlation tensor defined by:

$$R_{ij}(x,x') = \left\langle U_i(x) . U_j(x') \right\rangle \tag{4}$$

The solutions of (3) represent a set  $\Phi_k$  of base functions where each velocity field  $U_i$  is:

$$U_{i} = \sum_{k=0}^{N-1} a_{k,i} \Phi_{k}$$
 (5)

Then the eigenfunctions  $\Phi$  can be of the following form:

$$\Phi = \sum_{k=0}^{M-1} A_k U^{(k)}$$
(6)

We will order the eigenvalue by  $\lambda_i i > \lambda_{i+1}$  and since R is non negative we can be sure that  $\lambda_i > 0$  ( $\forall i \in N$ ).

Each velocity fields can be decomposed as a linear combination of proper mode  $\phi$  such that:

$$u_i(x,y) = \sum_{k=0}^{N-1} a_{i,k} \phi_k(x,y) \text{ and } \delta_{kk'} \lambda_k = \langle a_k a_{k'}^* \rangle$$
(7)

Where  $\lambda_k$  is the energy contained in the mode k. The computation has been done with 400 velocity fields. This number is sufficient for a good representation of the flow.

#### **Snapshot P.O.D. method**

In the case of analysis of velocity fields, classic P.O.D. method meets a real difficulty. The size of the correlations tensor R is directly linked with the number of points contained in the analysis domain. If we consider a set of N velocity fields solved on a grid described by PxP points (with P>N), the size of R will be (2PxP).(2PxP). P.O.D. decomposition will then be very long in term of calculation time. To take into account the turbulent phenomenon a large number of scales is necessary and the memory of usual calculators may be too low to resolve this study. Sirovich [13] in 1987 purposes a solution by resolving the system on the set of velocity fields (called snapshots) instead of points grid. The new correlation tensor used for this method in our case will be then of a dimension of NxN which reduce considerably the calculation time.

# Parameters of P.O.D. analysis of experimental P.I.V. results

P.O.D. analysis has been used to study the flow of a natural annular jet of air without any forcing then it has been applied on this annular jet excited by an acoustic wave created by only one loudspeaker at the nozzle exit. To study an excited annular jet, the acquisition of 100 velocity fields has been triggered for 16 different time delays (phases) between the start of the acoustic wave and the illumination of the seeding flow.

The number of snapshots used has been chosen equal to the number of velocity fields. In this way, any particular field is used to select snapshots. P.O.D. calculation results would be then totally objective since any structure is privileged.

# 4 Velocity characteristics of annular jet.

# 4.1 Aerodynamic characteristics of the initial zone of annular jet.

For the annular jet, it appears that the stagnation point is put through important radial fluctuations and, axially, the maximal fluctuations are localized on the external-mixing layer. This was also observed by Ko and Chan [3], but they used a hot wire and this cannot measure null velocity. Therefore, for a spatial quantification, the P.I.V. technique has been used [9]. Figure 6 shows an example of the mean velocity field with the corresponding Reynolds decomposition fluctuation fields. The initial merging zone extends from the jet exit to the tip of the potential core. This zone contains a recirculation zone. The stagnation point which marks the end of the recirculating region is located at x/Do=0.5.



Fig 6: Aerodynamic characteristics of the annular jet:  $Re_e = 7680$ . a) Average velocity field calculated with 400 P.I.V. fields. b) Reynolds decomposition radial velocities c) Reynolds decomposition axial velocities.

# 4.2 Results of Proper Orthogonal Decomposition application

In this study the P.O.D. is applied on P.I.V. velocity fields of the recirculation zone of this annular jet. Reynolds decomposition velocity fluctuations show two effects: the importance of oscillations for the stagnation point and the air entrainment in the annular jet.





Fig 7 : Influence of mode 1 on Reynolds decomposition radial velocity fluctuations: a) Radial velocity fluctuations b) Radial velocity fluctuations reconstructed with mode 0 and 1. c) Radial velocity fluctuations reconstructed without mode 1.

Thus, to evaluate the importance and the influence of each mode on an instantaneous velocity field, each instantaneous field reconstructed by choosing P.O.D. modes, and then the Reynolds decomposition radial fluctuations are calculated. A reconstruction with the first mode and the  $k^{th}$  mode, using the projection value of the instantaneous field on the modes, automatically shows what instability is represented by this  $k^{th}$  mode.

Fig 7 shows the reconstruction of all the instantaneous fields with the first two modes and without mode 1. So, it is clear that with modes 0+1 the position and the intensity of the Reynolds decomposition radial fluctuations are the same as those calculated for all modes, contrary to the reconstruction without mode 1. So mode 1 is responsible for the radial fluctuations of the stagnation point.

# 5 Control of the radial fluctuations of the stagnation point.

### Control by acoustic excitation

There are many methods to control instabilities in the jet. In this study, we have chosen to present an active control method using acoustic excitations because of disadvantages of passive control which consists in modifying the nozzle geometry. This modification is not always easily to set up for some industrial processes. This part of this study has the ambition to establish a control method of instabilities of an annular jet, by applying acoustic waves created by a loudspeaker place outside of the jet. PIV measurements have been carried out with a new set-up using a TTL casing in order to control the triggering of the camera according to the acoustic wave. Measurements have been carried out with different delays between the acoustic wave and the acquisition of images. The jet have been subjected to different excitation frequencies corresponding to the frequency of the natural beat of the stagnation beat which is equal to 150Hz for a Reynolds number Re, based on the thickness e, of 7680 or corresponding to the

double harmonic frequency of this frequency inherent of the jet (for 300 Hz).

The board of figure 8 shows the results obtained for a scanning of 16 phases over one period of an acoustic wave of frequency 148Hz, with amplitude of 3.62 V for the loudspeaker placed in front of the camera. This board enables us to see that fields of Reynolds decomposition radial velocities are different from a phase to another one in an irregular way. It seems normal that the results are different according to the studied phase.

If the acoustic wave comes at the good time on the stagnation point to then vibrate in an opposite way to this point the acoustic wave pushes back the jet so that the point of stagnation transversely remains motionless. On the other hand, if the wave comes on the stagnation point in a way, which actuates the jet with it, the wave accentuates the oscillation of the stagnation point which is then in resonance with the acoustic wave. The difference in phase between the beat of the jet and the acoustic wave is thus a significant parameter to take into account to establish a control of the oscillations of the stagnation point. We can see that radial fluctuations of the stagnation point are much more significant for phase 2 than for phase 3.



Fig 8: a scanning of 16 phases over one period of an acoustic wave of frequency 148Hz, with amplitude of 3.62 V.



Fig 9: Field of Reynolds decomposition radial velocities: (a) without acoustics, (b) with acoustic excitations (f=148Hz), (c) with acoustic excitations (f=298Hz).

The fields of radial fluctuations calculated for the various phases with amplitude, frequency and position of the loudspeaker fixed, show significant differences: the radial fluctuations of the stagnation point are more or less strong according to the studied phase. The comparison of the radial fluctuations, for a given position of the loudspeaker, and different frequencies of excitation enables us to see that the radial fluctuations of the stagnation point are considerably reduced when the frequency of the acoustic wave is 148 Hz and are, on the other hand, larger for a frequency of 298 Hz. An example of these comparisons is illustrated on figure 9.

# P.O.D. analysis of a natural annular jet

P.O.D. analysis has been applied on a natural annular jet (fig 10) and results were discussed in a previous study in order to put the stress on the importance of the stagnation point oscillations [10]. After using P.O.D. calculation, each instantaneous velocity field has been reconstructed by choosing P.O.D modes. After this step, the Reynolds decomposition radial fluctuations were estimated. This method enables to evaluate the influence of each mode on an instantaneous field. Mode 0 is the most energetic mode (it represents about 70% of the total kinetic energy of the flow) and is similar to the average field. When we combine modes with their associated weight in the flow (their projection value of the instantaneous field), we can extract the mode responsible for particular instabilities. In this way it is shown on the figure 6 that the mode 1 is the major actor for stagnation point radial fluctuations.



a)















e)



Fig. 10 P.O.D. analysis of a natural annular jet : (a) mode 0, (b) mode 1, (c) mode 2, (d) mode 3, (e) mode 4, (f) mode 5.

### P.O.D. application on active control results

We applied Proper Orthogonal Decomposition on PIV fields obtained for the study of the active control of instabilities of the annular jet by using acoustic waves. The eigenvalues spectrum (fig 11) enables us to see the energy distribution in modes. We can see that the ten first modes are sufficient to represent a great part of the energy of PIV fields.



Fig 11: Eigenvalues spectrum of an annular jet subjected to different excitation frequencies.

The figure 11 shows that the energy in the mode 0, which is associated to the mean field of flow, when we apply an acoustic wave on the jet, is increased. In contrary to this first observation, for the modes superior to mode 1, energy is less than without excitation. For the mode 1 acoustic wave have no influence on the contributed part of energy. So we can assume that acoustic waves reduce small vortices and bring more contribution to the great scale vortices which have a higher energy. P.O.D. is thus an efficient method to obtain a kind of energy mapping of the flow subjected or not to a forcing. This technique enables us to determine with a higher accuracy the action of the acoustic control thanks to the sharp identification of the components sensitive to this particular excitation.

Figure 12 shows another contribution of the P.O.D. towards the explanation of the effects of active control. This graph represents the reconstruction coefficients standard deviation for each mode of a jet subjected to different frequencies. We can see that the standard deviation is smaller in a forced jet than in a natural jet for the mode 1. This parameter is characteristic of the span of the stagnation point fluctuations at the end of the recirculation zone. Higher the value of the stagnation point are. So results of the figure 9 indicate that acoustic waves reduce considerably stagnation point fluctuations.



Fig 12: Reconstruction coefficient standard deviation of modes in a jet subjected to different frequencies.

# 6 Conclusion.

The study describes the recirculation zone of an annular jet by Particle Image Velocimetry. The Proper Orthogonal Decomposition has been applied to find the relationship of these radial fluctuations to the inner structures of the instantaneous P.I.V. fields. This statistical method can help to find the overall behavior of the flow and to link fluctuations with typical modes. The P.O.D. analysis has shown that the flow could be decomposed into four main modes (90% of the total kinetic energy). Each one is responsible for a characteristic motion of the recirculation zone. The mode 0 represents the carrier flow, which is the most energetic. The space fluctuation of the stagnation point is principally due to the mode 1. Indeed, when the instantaneous fields are reconstructed with the modes without the first eigenfunction, the statistical analysis of these fields does not show local fluctuations.

The study of active control by acoustic excitations is delicate with PIV measurements: it's necessary, in this case, to find a relation between the beat jet frequency, the frequency of the acoustic wave and the frequency of the images acquisition in order to know the real influence of the excitation on the flow. By putting the system of illumination and acquisition in phase with the acoustic wave, we can observe the response of a basic annular jet subjected to excitations of frequency equal to the natural frequency of stagnation point beat. These transversal oscillations are reduced for this frequency of excitation. A series of measurements coupling the techniques of P.I.V. and of hot wire Anemometry is considered by sweeping a frequency band.

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