Inverse Box-Counting Method and Application: A Fractal-Based Procedure To Reclaim a Michigan Surface Mine

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Abstract: Planners and designers are interested in replicating biospheric landscape patterns to reclaim surface mines to match existing natural landscape patterns. One approach that shows promise is the use of fractal geometry to generate biospheric landscape patterns. While the measurement of the actual fractal dimension of a landscape can be difficult, a box-counting method was developed at AgroCampus Ouest, Angers, France which approximates the spatial patterns of biospheric landscapes. Essentially the procedure entails covering a natural object/pattern with a regular grid of size r and then one simply counts the number of grid boxes, N(r), that contain some part of the object. The boxes are subdivided and the value of r is progressively reduced and N(r) is similarly re-measured until some of the boxes become empty (containing no landscape objects of interest). Then the fractal dimension of the object is approximated to be the log(N(r))/log(1/r). We illustrate this procedure by measuring and replicating a stand of trees in the Upper Peninsula of Michigan and applying the method for a planting plan on a surface mine. Our study revealed a fractal number of 1.017 ($p \leq 0.01$), with a mean of 77.4 trees per 100 m by 100 m stand, and a standard deviation of 34.87 trees per stand.

Key-words: - landscape architecture, landscape planning, physical geography, landscape ecology, landscape science, plant ecology

1 Introduction

Planners, designers, and environmental specialists are interested in assessing the spatial composition of landscape features such as the distribution of hills, arrangement of vegetation, and shapes of water bodies to blend disturbed landscapes with natural landscapes. However natural looking compositions were difficult to mathematically replicate. Typical approaches employed to replicate landscapes included gestalt methods and ecological field laboratory methods. The gestalt method was heuristic in nature where an individual would artistically blend and integrate patterns together. The ecological field laboratory method employed the measures of frequency, density, and size to construct patterns. A different approach evolved that relied upon the concept of fractals to quantify spatial patterns in the landscape.

1.1 Origin of Fractals

Fractals were first noticed/observed at the end of the 19th century. Although the term "fractal" was only attributed later, the Peano curves seem to be the very first examples of fractal objects, first described by Guiseppe Peano (1858 – 1932). These were curves that could, through a series of iterations and a few simple rules, fill a space [12]. Such mathematical objects have been considered as mere mathematical curiosities for a long time.

Fractals have been the heart of a new branch of mathematics only in the second half of the 20th century, thanks to the work of the French mathematician Benoît Mandelbrot. While researching "econometry" (mathematics applied to economy), he discovered that there is no difference in the shape/pattern of the curves of predicting short-term and long-term prices. He presents a comprehensive description of the curves following this property and invented the word fractal (coming from the Latin word *fractus*, meaning broken) to name the objects where irregularity distinguishes them from the Euclidian geometry curves. Since their discovery, the use and application of fractals have spread. They are now used in many sciences including geology, biology and econometrics.

1.2 Further Descriptions Illustrating Fractals

To illustrate the concept of fractals, imagine a tour along the French coastline of Brittany, a rugged rocky coastline. What is the actual length of this coastline? To determine the length of the coastline, one can look at two forms for resolution:

1. a series of pictures from 10,000 meters high and calculating the visible length of the coast.

2. a second series of pictures from 500 m high and observing details of the coastline one meter by one meter.

After calculating the length, one will discover the coast is more precisely known in the second case and the calculated length is actually longer. If one examines the coast at an even higher resolution, new details appear and the length of the coast will increase even more. The more precise the measuring instrument is, the more the length of the coast increases, because any one section of the coastline is equally as complex at any scale or resolution. The Brittany coastline example introduces a fundamental understanding of the fractal world. The complexity of the Brittany coast (being unable to be described with Euclidian geometry) makes it a fractal object. In the landscape, fractals are everywhere.

A useful conceptual definition of a fractal is a "geometrical shape resulting from infinite regular fragmentation of a given form." It is indeed possible to describe a fractal as a repetition of the same operation on each part of the curve. An essential property results from this kind of internal homothetia: self-similarity. If one looks closely at a piece of the curve, it looks like the whole curve itself. The von Koch's snowflake illustrates this property. This von Koch's snowflake fractal, as most all the fractals, is easy to design even if the resulting shape is complex. The von Koch's snowflake has the geometric property where as the construction iteration process increases towards infinity, the total length L increases towards infinity. Therefore, the length of the curve is infinite. Here lies a paradox: the area of the von Koch's snowflake A is a finite measure (see equations 1 and 2).

$$L = \lim_{n \to \infty} 3 \times \left(\frac{4}{3}\right)^n = \infty$$

$$A = \lim_{n \to \infty} \left[\sqrt{\frac{4}{3}} + 3 \times \sqrt{\frac{3}{20}} \times \left(1 - \left(\frac{4}{9}\right)^n\right)\right] = 2 \times \sqrt{\frac{3}{5}} \qquad (1), (2)$$

In many respects there is little difference between the mathematics of fractals and descriptive statistics.

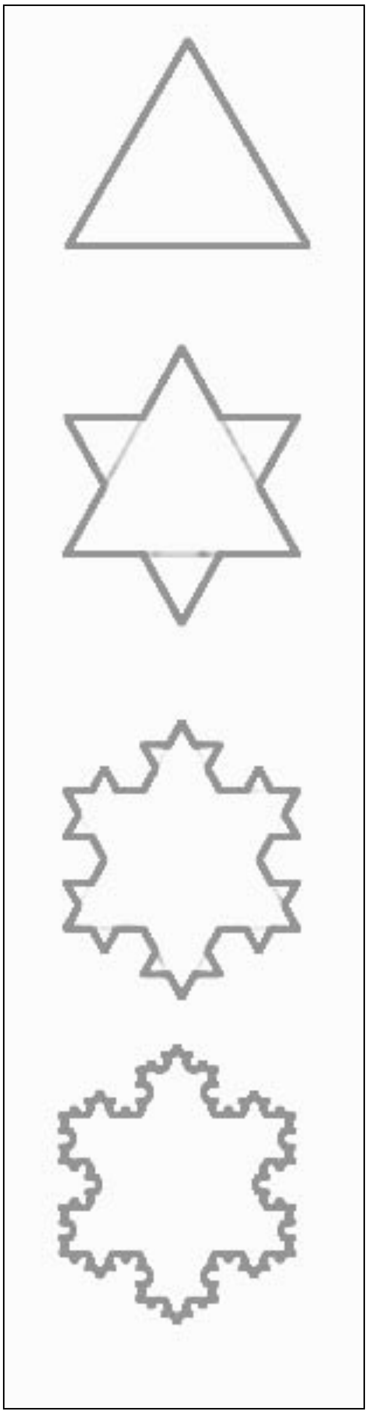

Figure 1. Four iterations of the Koch Snowflake. At an infinite number the perimeter of the snowflake approaches infinity but the area is finite.

1.3 Geometric Properties of Fractals

Geometric properties of fractals are used in many models and numerous sciences [8] [9] [15]. For example in economics, fractals are used in complex random phenomena, such as in finance to represent the variations of the prices on the trade market. In climatology, fractal models can also be applied to understand the turbulence of atmospheric movements. In geology, they can be used for modeling the earth relief or rock porosity. For computer sciences, fractals assist in finding the optimal arrangement of electronic components, to avoid crossings of circuit tracks. In chemistry, they are used to design new materials. The fractal nature of such materials gives them exceptional properties, such as a very high thermal cooling power.

1.4 Planning and Design Applications

There is a belief that fractals may have an application to recreate complex landscape patterns that are difficult to describe with typical Euclidian approaches because the landscape is full of fractals: rivers, trees, landscape networks in general [1]. Fractals are highly detailed, complex geometric shapes and one measure of their complexity is fractal dimension [12]. Thus several authorities have examined fractals in landscape planning and design including studies by Diaz-Delgado, Lloret, and Pon; DiBari; Griffith, Martinko, and K.P. Price; Li; Milne; Palmer; and Thomas, Grankhauser, and Biernacki; [3] [4] [7] [10] [13] [14] [16]. However, the use of fractals seems to be looking for a practical application. For example in describing landscapes, it has always been easy to calculate an existing pattern, but difficult to replicate the pattern. In this paper we present an approach to replicate the pattern and possibly a practical approach in the use of fractals.

2 Methodology

The approach in the methodology is related to the dimensions of fractals. Both Euclidian geometry and fractal geometry have dimension number. In Euclidian geometry, the point (the elementary unit in geometry) is of Euclidian dimension 0. Lines or curves are of dimension 1. Areas are of dimension 2, such as a circle or rectangle. Volumes are of dimension 3, such as ball or cube. Euclidian dimensions are also call topological dimensions and are named in honor of Euclidian geometric objects such as a circle or a square. Fractal objects have dimensions too.

2.1 Fractal Dimensions

To illustrate fractal dimensions, consider the Brittany coastline. If one needs to measure 1 m length of a relatively straight line with a 20 cm ruler, this ruler will be used 5 times, 10 times for a 10 cm ruler, 20 times for a 5 cm ruler. Let’s suppose now that the line one needs to measure is highly variable and curved. One will not be able to follow the coastline precisely with the ruler and one will under-estimate the real length. But, the smaller the ruler is, the more accurate the result. To analyze this phenomenon in a mathematical way, one can say that the result tends towards the exact length of the line when the ruler is small when compared to the curvature of this line. If one can divide the length of a ruler of an infinite small size by "n," one has to use this ruler n times more (same as if the line were straight). This property can define the topological dimension of the curve or line as we have (Equation 3):

$$D_{topological} = \frac{\log(n)}{\log(n)} = 1 \tag{3}$$

Replicating the process again with a surface, one can use a square where the length of the side is L. To measure its area, one can use a smaller square where the length of the side is L/2, then you will need 4 of them, 16 with an L/4 square, and so on. So, if the length of the side of the measuring square is divided by "n," the number of such squares used is multiplied by "n" (Equation 4):

$$D_{topological} = \frac{\log(n^2)}{\log(n)} = 2 \times \frac{\log(n)}{\log(n)} = 2 \tag{4}$$

Similar results can be obtained for volumes and the topological dimension of a Euclidian geometric object with a fractal dimension of 3.

In the relatively simple case of self-similar fractal objects (meaning they seem the same whichever zooming factor is used), resulting in a constant iterative factor "k," the fractal dimension is (Equation 5):

$$D_{fractal} = \frac{\log(n)}{\log(k)} \tag{5}$$

Where:

n = is the number of the subsets counted during the scaling process using a factor 1/k (self-similarity factor).
k = is the number of iterations

The von Koch’s snowflake illustrates how to calculate the fractal dimension of self-similar fractal objects. Call L the initial length of the triangle (the snowflake starts as an equilateral triangle). If one uses a ruler of length L and applies it on the snowflake, one can only measure the initial triangle and find a length of 3L for the snowflake. If one uses a smaller ruler of size L/3, we can follow the snowflake more precisely and apply it 12 times. One can continue by dividing again the size of the ruler by 3 (the snowflake presents an infinite number of spikes, with smaller and smaller sizes), it will be applied 48 times, and so on. In other words, each time the size of the ruler is divided by 3, the number of times it is applied on the snowflake is multiplied by 4. This process can be carried on indefinitely. Then according to the same reasoning one can calculate the fractal dimension of the von Koch's snowflake (Equation 6):

$$D_{fractal} = \frac{\log(4)}{\log(3)} \approx 1.262 \tag{6}$$

Therefore, we can only conclude that the fractal dimension of this strange curve is not 1 as any of classic linear geometrical curves. The von Koch's snowflake has a topological dimension equal to 1 (it's a broken line), but a fractal dimension strictly greater than 1, and moreover, which is not an integer but a real number.

2.2 Inverse box-counting method: a tool for replicating landscapes

The fractal dimension is not easy to calculate but can be estimated by several methods. The box-counting method is one of the easier and more popular methods to implement (Figure 2): the natural object is covered with a regular grid of size r and one simply counts the number of grid boxes, N(r), that contain some part of the object. The value of "r" is progressively reduced and N(r) is similarly measured. As "r" trends to very small values (0 in a theoretical way) one finds that log(N(r))/log(1/r) becomes the fractal dimension of the object.

In our study, we illustrate the application of fractals in the planting pattern of trees in the Upper Peninsula of Michigan in Iron and Dickinson counties (Figure 3). The location of trees can be placed on a map (100 meters by 100 meters) derived from an aerial photograph and measured. This set of points (location of trees) can be viewed as a complex and fractal object in the landscape. The box-counting method is a simple way to characterize the complexity of this planting through the value of its fractal dimension. The greater the value of the fractal dimension (2 is the maximum value in a plane), the less the complexity of the planting pattern (in terms of scale, alignment, structure, etc.). This method was developed by Duchesne *et al.* [5] and computed by Durandet in the Landscape Department of the National Institute of Horticulture and Landscape (Angers, France), now the Unité de Recherche Paysage; AgroCampus Ouest [5] [6]. By using the inverse box-counting method one is able to control the randomness of a planting of trees or other natural landscape pattern with several parameters: the fractal dimension (D), the average minimum distance between two trees (ε_{min}) and the average maximum size of the glades (ε_{max}). Figure 4 illustrates some of the initial patterns for European vegetation generated by Unité de Recherche Paysage; AgroCampus Ouest.

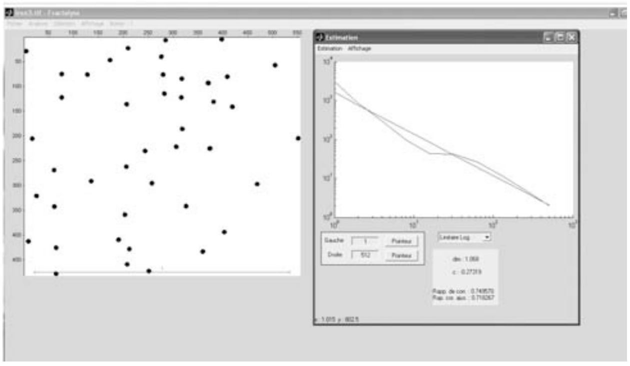

Figure 2. Example of fractal pattern for a distribution of points and the plot, forming a regression line, supplied by Cyril Fleurant, Unité de Recherche Paysage; AgroCampus Ouest.

Figure 3. Location of the study areas of Iron and Dickinson Counties in the Upper Peninsula of Michigan.

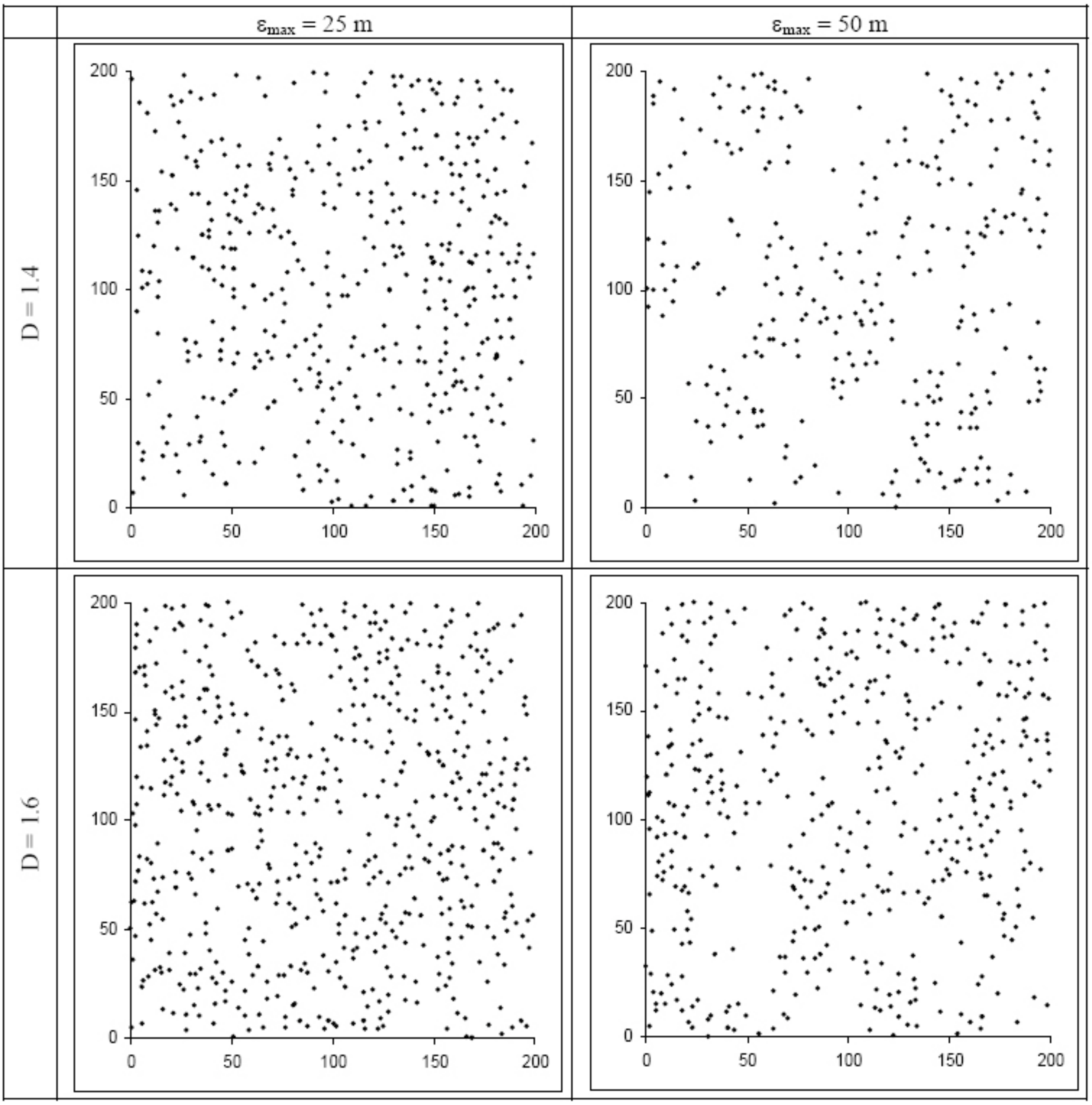

Figure 4. Fractal patterns of vegetation in European stands as supplied by Cyril Fleurant, Unité de Recherche Paysage; AgroCampus Ouest.

In the process, the pairs of values r and the number of boxes N(r), start with a value of r being 100 meters, and N(r) being one. Then r is divided in half and r is 50 meters, while N(r) can range from one to four, depending upon how many boxes contain trees. The pairs for the regression analysis start with the first pair where at least one box is empty and end when only one tree is found in any box. The slope of the regression equations represents the fractal number.

We selected five 100 meter by 100 meter boxes in Iron County and five 100 meter by 100 meter boxes in Dickinson County [11] [17]. The areas that we selected to measure were rocky and dry xeric northern forests, an environment similar to waste rock piles on a surface mine where a fractal planting plan for dry forests might be appropriate [2]. These forests are predominantly composed of about 16% red pine (*Pinus resinosa* Sol. Ex Aiton), 21% jack pine (*Pinus banksiana* Lamb.), 15% Eastern white pine (*Pinus strobus* L.), and 12% northern pin oak (*Quercus ellipsoidalis* E.J. Hill), plus a scattering of other trees such as 7% quaking aspen (*Populus tremuloides* Michx.), 3% red maple (*Acer rubrum* L.), 4% paper birch (*Betulus papyrifera* Marsh.), 4% northern red oak (*Quercus rubra* L.), 7% white oak (*Quercus alba* L.) and 7% of bigtooth aspen (*Populus grandidentata* Michx.).

3 Results

Figure 5 presents an aerial photograph of trees distributed in the study area of Iron County, Michigan; while Figure 6 illustrates the results related to one of the aerial plots, Iron County 2. Figure 7 and 8 illustrate a stand in Dickinson County, Michigan.

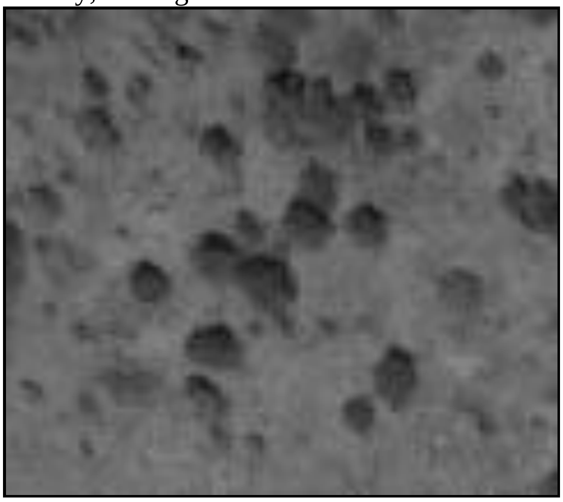

Figure 5. An aerial photograph from Iron County, Michigan with dimensions 100 meters by 100 meters.

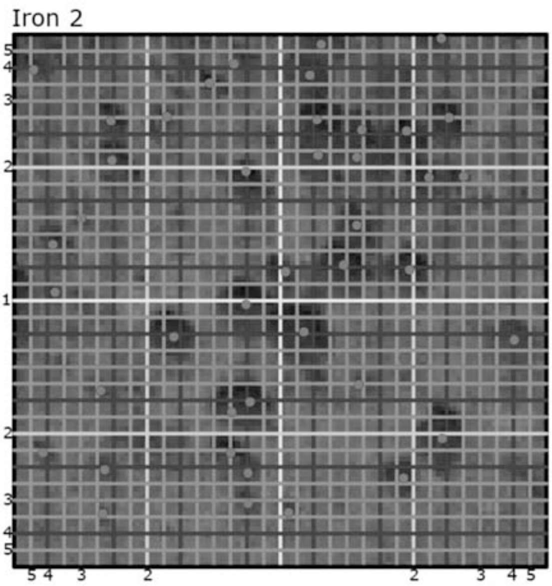

Figure 6. The same aerial photograph from Iron County in Figure 5 now divided into grids with the location of trees.

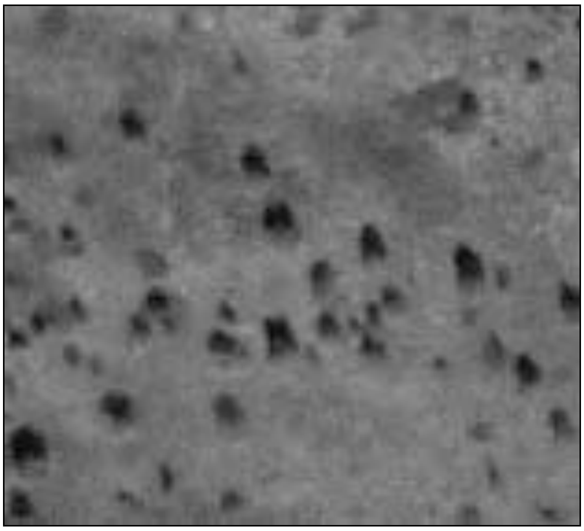

Figure 7. An aerial photograph from Dickinson County, Michigan with dimensions 100 meters by 100 meters.

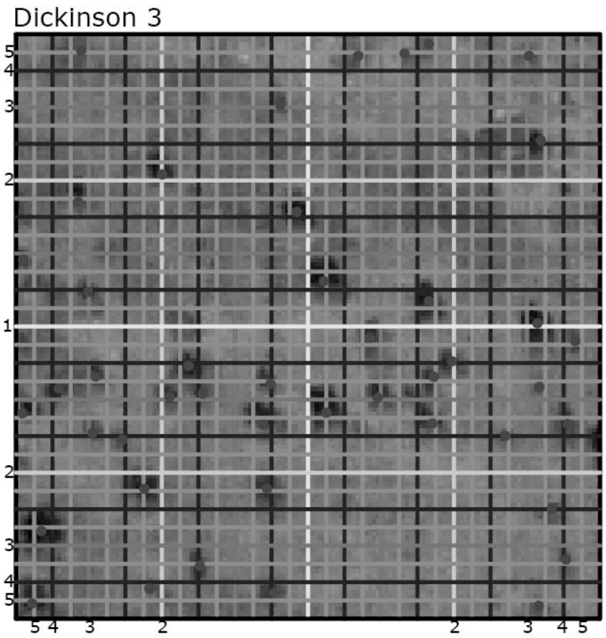

Figure 8. The same aerial photograph from Iron County in Figure 7 now divided into grids with the location of trees.

From the 10 plots of trees, 43 pairs of numbers were derived (Table 1). The regression analysis revealed an adjusted r-square of 0.792, with a significant regression ($p \leq 0.01$), a significant constant ($p \leq 0.01$) and a significant predicator Ln(1/r) ($p \leq 0.01$). The regression is expressed in Equation 7. The slope of the line expressed in Equation 7 is 1.017. This suggests that the fractal dimension is nearly a line in typology.

$$\mathrm{Ln}(N(r)) = 1.017\,\mathrm{Ln}(1/r) + 5.875 \qquad (7)$$

Where:

N(r) = number of boxes with trees
r = length of one side of box

The investigation revealed that each stand contained an average of 77.4 trees and a standard deviation of 34.87 trees per stand.

Table 1. Pairs of numbers for regression analysis.

Country	Ln(1/r)	Ln(N(r))
Iron 1	-3.219	2.773
	-2.526	4.043
	-1.833	4.521
	-1.139	4.787
Iron 2	-3.219	2.773
	-2.526	3.434
	-1.833	3.738
Iron 3	-4.605	0.000
	-3.912	1.099
	-3.219	2.485
	-2.526	3.044
	-1.833	3.526
	-1.139	3.714
Iron 4	-3.219	2.773
	-2.526	3.951
	-1.833	4.575
	-1.139	4.796
Iron 5	-3.912	1.386
	-3.219	2.708
	-2.526	3.219
	-1.833	3.367
Dickinson 1	-3.219	2.773
	-2.526	4.060
	-1.833	4.533
	-1.139	4.727
Dickinson 2	-3.219	2.773
	-2.526	3.912
	-1.833	4.489
	-1.139	4.700
	-0.446	4.718
Dickinson 3	-3.912	1.386
	-3.219	2.708
	-2.526	3.526
	-1.833	3.807
	-1.139	3.829
Dickinson 4	-3.219	2.773
	-2.526	3.871
	-1.833	4.407
	-1.139	4.443
Dickinson 5	-3.219	2.773
	-2.526	3.714
	-1.833	4.382
	-1.139	4.190

4 Discussion & Conclusion

To apply the inverse box-counting approach to this area in the landscape one would then follow these procedures:

A. Divide the landscape to be planted in 100 meter grids.
B. Divide each 100 meter grid into grids with sides equal to 3.125 meters (the size of the smallest boxes in Figures 6 and 8).
C. Randomly fill the 100 meter grids with an average of 77.4 trees per grid and a standard deviation of 35 trees. The number of trees per grid can be increased proportionally if the mortality rate of the trees is known, such as a 20% mortality rate means that the grids should be planted with an average of 97.75 trees.
D. The composition of the stands should be about:
   21% jack pine (*Pinus banksiana* Lamb.),
   16% red pine (*Pinus resinosa* Sol. Ex Aiton),
   15% Eastern white pine (*Pinus strobus* L.),
   12% northern pin oak (*Quecus ellipsoidalis* E.J. Hill)
   7% quaking aspen (*Populus tremuloides* Michx.),
   7% bigtooth aspen (*Populus grandidentata* Michx.),
   7% white oak (*Quercus alba* L.),
   4% paper birch (*Betulus papyrifera* Marsh.)
   4% northern red oak (*Quercus rubra* L.),
   3% red maple (*Acer rubrum* L.),
   4% assorted list of 24 trees by Curtis [2].

This approach is illustrated with Table 2 and with Figure 9, where 7.5 percent of random numbers were assigned to boxes with a 3.125 meter grid on a 100 m by 100 m site located at the surface mine in the Upper Peninsula of Michigan. In the Upper Peninsula of Michigan, a typical mine site contains waste rock, with environmental conditions similar to xeric forest sites in the region (Figure 10). The process generated 46 boxes for planting trees. 46 boxes are within one standard deviation ($\pm$35) of the average of 77.4, so 46 boxes were deemed acceptable. Then each box was randomly assigned a tree species based upon the percentage of composition indicated by Curtis [2]. Table 3 lists the composition of the planting area. Notice that because random numbers are employed, the composition may not be exactly the same as the percentages noted by Curtis [2]. The result will be that each planted stand will have variation.

The planting scheme can be accomplished with seedlings being planted by hand or even with machine planting, as long as the tree is placed in the correct designated box.

Table 2. Boxes with trees. Both box number and tree species are randomly selected.

Box Number	Tree
38	paper birch
87	jack pine
137	assorted trees
154	northern pin oak
160	Eastern white pine
180	jack pine
202	big tooth aspen
205	Eastern white pine
214	big tooth aspen
232	northern red oak
266	red pine
317	big tooth aspen
327	red maple
366	jack pine
379	Eastern white pine
385	white oak
401	quaking aspen
417	red pine
423	northern pin oak
502	northern pin oak
525	assorted trees
544	jack pine
545	jack pine
561	red pine
570	northern pin oak
584	jack pine
585	jack pine
596	paper birch
625	Eastern white pine
665	red pine
697	white oak
706	white oak
708	quaking aspen
712	red pine
719	jack pine
729	Eastern white pine
738	northern pin oak
743	quaking aspen
806	Eastern white pine
878	quaking aspen
890	Eastern white pine
911	Eastern white pine
931	Eastern white pine
956	northern pin oak
963	big tooth aspen
972	jack pine

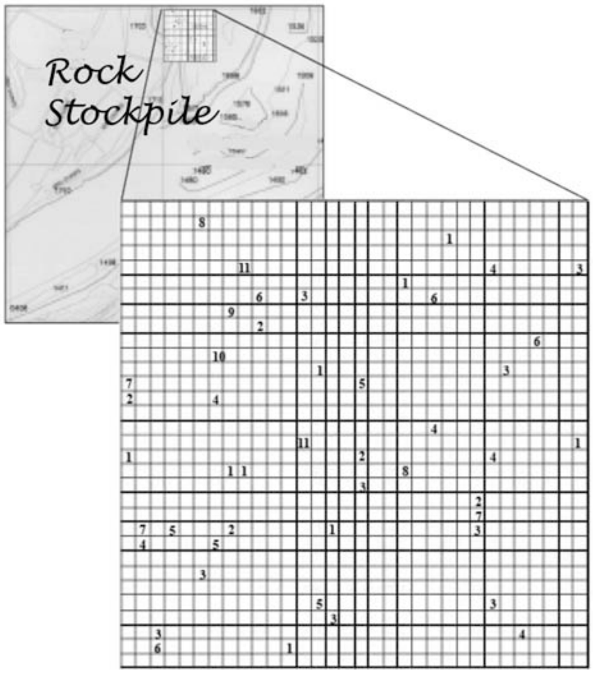

Figure 9. A planting plan example based in the methodology described in this paper: 1) jack pine, 2) red pine, 3) Eastern white pine, 4) northern pin oak, 5) quaking aspen, 6) big tooth aspen, 7) white oak, 8) paper birch, 9) northern red oak, 10) red maple, and 11) assorted trees from Curtis [2].

Figure 10. An example of a waste rock pile in the Upper Peninsula on Michigan (Used by permission of Jon Bryan Burley ©2007, all rights reserved).

The inverse box-counting process illustrates that it is possible to use the fractal pattern to create a stand of vegetation. The process employs calculating the fractal score of an existing pattern and employing the inverse box process to apply the pattern to a landscape. However the inverse box-counting process is a reverse process, as opposed to a forward process when investigators first began calculating the fractal scores of objects. The reverse process takes an existing score to create something new. Currently there is no mathematical proof that this process is truly reversible.

Table 3. Tree composition in the planting plan.

Species	Plan	Curtis [2]
jack pine	20%	21 %
red pine	11%	16 %
Eastern white pine	20%	15 %
northern pin oak	13%	12 %
quaking aspen	9%	7 %
bigtooth aspen	9%	7 %
white oak	7%	7 %
paper birch	4%	4 %
northern red oak	2%	4 %
red maple	2%	3 %
assorted list of 24 trees	3 %	4 %

While this process has been employed with vegetation, we believe that it is possible to replicate fractal patterns of hills, waterways, and complex multi-species patterns. We expect to explore this potential in the future. In our study we did not differentiate various species of vegetation. With more careful on-site study, it may be possible to gather multi-species data and construct patterns with numerous species (Figure 11).

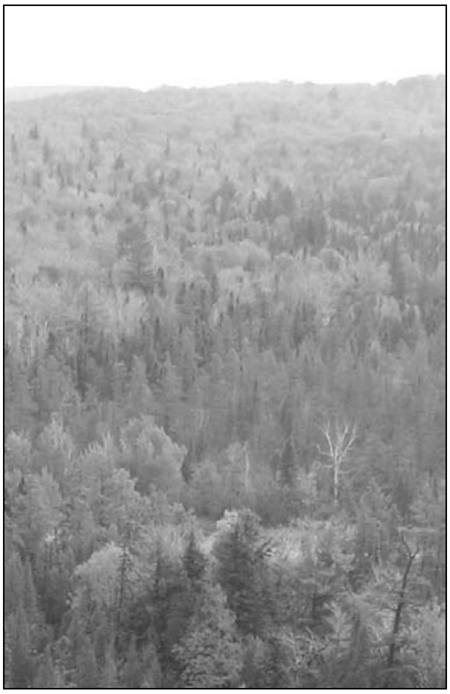

Figure 11. This is a picture of the forest vegetation in the Upper Peninsula of Michigan. Notice the interspersion of tree species. Each species may have its own fractal number in the forest.

We encourage reclamation and restoration planning and design specialists to explore the inverse box-counting method to create biospheric landscapes.

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