

Air Quality Indices and their Modelling by Hierarchical Fuzzy Inference Systems

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Abstract: - The paper presents the overview of current methods for air quality evaluation, i.e. air stress indices and, especially, air quality indices. Traditional air quality indices are determined as mean values of selected air pollutants. Thus, air quality evaluation depends on strictly given limits without taking into account specific local conditions and synergic relations between air pollutants and other meteorological factors. The stated limitations can be eliminated e.g. using systems based on fuzzy logic. Therefore, the paper presents a design of air quality indices based on hierarchical fuzzy inference systems. Tree and cascade hierarchical fuzzy inference systems of Mamdani type are proposed as alternative air quality indices. For selected localities, they provide both the resulting class of air quality and the degree of membership to each class.

Key-Words: - Air quality index, air quality, fuzzy logic, hierarchical fuzzy inference systems.

1 Introduction

In order to meet additional requirements, e.g. information of regional and urban planning authorities or information of the public on the status of the ambient air quality, air stress indices and air quality indices were developed to assess the integral ambient air pollution. They are determined as (weighted) mean values of selected air pollutants. Strictly given limits are set for air pollutants. Local conditions and synergic relations between air pollutants and other meteorological factors are not taken into account. The stated limitations can be eliminated e.g. using systems based on fuzzy logic.

Fuzzy sets allow expressing object attributes which can have non-numeric values as numeric. Numeric nature of values can deeply influence model design. Currently, application of fuzzy sets is moving from technical sciences to economic, environmental and social sphere [1,2]. That allows processing semantics of natural language in these science branches. The main characteristics of natural language semantics is its uncertainty. Uncertainty in fuzzy sets theory can be quantified [3,4]. Communication in management and decision-making is often realized based on natural language that is why it is vague and uncertain. This fact leads to solving the uncertainty by transforming speech meaning, given by natural language semantics, to a set of real numbers by fuzzy sets. Simultaneously, it allows learning the computer to understand natural language.

Classification of the i -th district $o_i^t \in O$, $O = \{o_1^t, o_2^t, \dots$

, $o_n^t\}$ in time t to the j -th class $\omega_{i,j}^t \in \Omega$, $\Omega = \{\omega_{1,j}^t, \omega_{2,j}^t, \dots, \omega_{i,j}^t, \dots, \omega_{n,j}^t\}$ can be realized by fuzzy inference systems (FISs). Based on FIS it is possible to define hierarchical fuzzy inference systems (HFISs) as the classification process becomes more efficient and it is better interpretable.

The paper is structured as follows. First, current approaches to air quality indices are introduced. Then the parameters are designed which are consequently applied for the modelling by a HFIS. Finally the analysis of results including the comparison to classical air quality indices is provided.

2 Air Quality Indices

Both the air stress indices (ASIs) and air quality indices (AQIs) consider relevant air pollutants (e.g. CO, NO₂, O₃, PM₁₀ and SO₂) frequently monitored at long-term stations within air pollution monitoring networks. Air stress indices aggregate relative concentrations of different air pollutants, i.e. per air pollutant the ratio of ambient concentration and reference value [5].

As a result, ASIs do not show a pronounced relation to people. In contrast, AQIs quantify the impacts of a mixture of air pollutants, which is typical of the ambient air, on well-being and health of people in a graded way, i.e. they are impact related with respect to people. Current AQIs show some differences, which are summarised by [6], e.g. different number of index classes or different class boundaries. A well-known index is the AQI developed by Environmental Protection

Agency (EPA) [7]. It is based on the Pollution Standards Index (PSI) which was initially established in response to a dramatic increase in the number of people suffering respiratory irritation due to the deteriorating air quality. The PSI was revised, renamed to the AQI, and subsequently implemented in 1999 by the US EPA. It is defined with respect to the five main common pollutants: carbon monoxide (CO), nitrogen dioxide (NO₂), ozone (O₃), particulate matter (PM₁₀) and sulphur dioxide (SO₂). Modified versions of the AQI of EPA were developed by [8] taking into consideration the limit values ruling in Europe.

However, even though AQI has completely replaced PSI in the US, a greater part of the world still could not adopt the AQI system, mainly because the lack of PM_{2.5} measurement capability. Similarly, PM₁₀ total mass measurement may be not sufficient as air quality index due to its complex composition since the metal content of PM₁₀ is not related to its total mass, especially in sites with industrial activities [9].

The Revised AQI (RAQI) is derived from the AQI, and is a background arithmetic mean index and a background arithmetic mean entropy index [10].

While effects of short-term AQI's exposures have been emphasized, research has also shown that long-term exposures to lower concentrations of air pollutants can result in adverse health effects. An aggregate index was developed by [11] that represents long-term exposure to these pollutants, using monitoring data for metropolitan areas obtained from the US EPA's Aerometric Information Retrieval System.

A uniform indexing scale using well pre-established air quality standards and, at the same time, accounting for local conditions assessed via statistical analysis of data recorded at each monitoring station was proposed by [12] and implemented at the Athens metropolitan area. AQIs with unlike goals are also reported in the literature. A daily quality index was proposed by [13] to show exceeding limit values.

An aggregate AQI based on the combined effects of five criteria pollutants (CO, SO₂, NO₂, O₃ and PM₁₀) taking into account the European standards was developed by [14].

The example of the development of an alternative air quality index (AQI) is used by [15] to illustrate issues related to quantifying the public health burden attributable to air pollution. These issues include according to [15]: (1) appropriately representing the weight of evidence; (2) extrapolation of risk measures over time and space; (3) attribution of health effects to air pollution versus other risk factors and to individual pollutants versus the rest of the mix; (4) application of complementary approaches from health economics; and

(5) effective risk communication. A no-threshold, multipollutant AQI was developed, based on the relationship of CO, NO₂, O₃, SO₂, and PM_{2.5} with mortality in Canadian cities in a daily time-series study.

The common air quality index (CAQI) proposed by [6] is a set of two indices: one for roadside monitoring sites and one for average city background conditions.

The AQI uses both the direct numerical expression and the linguistic description. The values of air pollutants are transformed into a dimensionless number characterizing the state of air pollution.

Based on the value of the AQI the state of air pollution can be classified into six classes. A sample of classes $\omega_{i,j}^t \in \Omega$ for the AQI of the Czech National Institute of Public Health (CNIPH) is presented in Table 1.

Table 1 AQI classes $\omega_{i,j}^t \in \Omega$ of the CNIPH

AQI	$\omega_{i,j}^t$	Class description
<0,1)	$\omega_{1,1}^t$	Clean air, very healthy environment.
<1,2)	$\omega_{1,2}^t$	Satisfactory air, healthy environment.
<2,3)	$\omega_{1,3}^t$	Slightly polluted air, acceptable environment.
<3,4)	$\omega_{1,4}^t$	Polluted air, environment dangerous for sensitive population.
<4,5)	$\omega_{1,5}^t$	High polluted air, environment dangerous for the whole population.
<5,6)	$\omega_{1,6}^t$	Very high polluted air, harmful environment.

Another AQI used in the Czech Republic was developed by the Czech Hydro-meteorological Institute. The AQI is based on the results of weight concentrations measures of substances in the air (Table 2). The evaluation takes the possible influence of human health into account [16].

Table 2 AQI of the Czech Hydro-meteorological Institute

Air quality	SO ₂	NO ₂	CO	O ₃	PM ₁₀
	1h [$\mu\text{g}\cdot\text{m}^{-3}$]	0-25	8h [$\mu\text{g}\cdot\text{m}^{-3}$]	1h [$\mu\text{g}\cdot\text{m}^{-3}$]	
Very good	0-25	0-25	0-1.10 ³	0-33	0-15
Good	25-50	25-50	1.10 ³ -2.10 ³	33-65	15-30
Favourable	50-120	50-100	2.10 ³ -4.10 ³	65-120	30-50
Satisfactory	120-250	100-200	4.10 ³ -1.10 ⁴	120-180	50-70
Bad	250-500	200-400	1.10 ⁴ -3.10 ⁴	180-240	70-150
Very bad	>500	>400	>3.10 ⁴	>240	>150

However, due to inconsistency and distinction of each air pollutant, there is a vagueness or fuzziness in air quality. It must be observed that the values of breakpoint concentration derived from epidemiological researches are affected by some uncertainty and change frequently as more information on health effects become available. At the same time standards or guidelines for air quality

introduced by public authorities become always more stringent.

Fuzziness makes the use of sharp boundaries in classification schemes hard to justify. A small increase/decrease in pollutant data, near its boundary value, will change its class. Moreover, different breakpoint concentration values and air quality standards are reported in the literature [17]. Further, it would be significant to consider local conditions when defining breakpoint concentrations. As a matter of fact, different areas of the world are characterised by different climatic conditions influencing the effect of atmospheric pollutants on human health and also the response of population to air pollution could be different. This fuzziness led some environmental researchers to look for advanced assessment methods based on fuzzy logic [2,18].

Another issue is coupled to the fact that there are several pollutants presented simultaneously in the atmosphere. Therefore, the effects on human health due to the simultaneous presence of different pollutants in the atmosphere should be considered. Knowledge of the effects of a mixture of air pollutants on human health is at present limited. However, some basic relations between pollutants are known [15]. An attempt in considering these and other effects in the evaluation of pollution indexes was proposed by [19] using a constant elasticity of substitution function but the absence of epidemiological data did not allow the assumption of the proper values for the parameters contained in the function.

3 Problem Formulation

Harmful substances in the air represent the parameters of air quality modelling. They are defined as the substances emitted into the external air or generated secondarily in the air which harmfully influence the environment directly, after a physical or chemical transformation or eventually in the interaction with other substances. Except the harmful substances, other components influence the overall air pollution. For example, ozone, solar radiation, the speed or the direction of wind, air humidity and air pressure represent these components. Both the parameters concerning the harmful substances in the air and the meteorological parameters influence air quality development. The interaction of both types of parameters can cause an increase of air pollution and influence the human health this way. The design of the parameters, based on previous correlation analysis and recommendations of notable experts, can be realized as presented in Table 3.

Based on the presented facts, the following data matrix **P** can be designed

$$\mathbf{P} = \begin{matrix} & \begin{matrix} x_1^t & \dots & x_k^t & \dots & x_m^t \end{matrix} \\ \begin{matrix} o_1^t \\ \dots \\ o_i^t \\ \dots \\ o_n^t \end{matrix} & \begin{matrix} \left[\begin{matrix} x_{1,1}^t & \dots & x_{1,k}^t & \dots & x_{1,m}^t \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{matrix} \right] \omega_{i,j}^t \end{matrix} \end{matrix} ,$$

where $o_i^t \in O$, $O = \{ o_1^t, o_2^t, \dots, o_i^t, \dots, o_n^t \}$ are objects (districts) in time t , x_k^t is the k -th parameter in time t , $x_{i,k}^t$ is the value of the parameter x_k^t for the i -th object $o_i^t \in O$, $\omega_{i,j}^t \in \Omega$ is the j -th class assigned to the i -th object $o_i^t \in O$, $\mathbf{p}_i^t = (x_{i,1}^t, x_{i,2}^t, \dots, x_{i,k}^t, \dots, x_{i,m}^t)$ is the i -th pattern, $\mathbf{x}^t = (x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t)$ is the parameters vector.

Table 3 Parameters design for air quality modelling

Parameters	
Harmful substances	$x_1^t = \text{SO}_2$, SO_2 is sulphur dioxide.
	$x_2^t = \text{O}_3$, O_3 is ozone.
	$x_3^t = \text{NO}$, NO_2 (NO_x) are nitrogen oxides.
	$x_4^t = \text{CO}$, CO is carbon monoxide.
	$x_5^t = \text{PM}_{10}$, PM_{10} is particulate matter (dust).
Meteorological	$x_6^t = \text{SW}$, SW is the speed of wind.
	$x_7^t = \text{DW}$, DW is the direction of wind.
	$x_8^t = \text{T}_3$, T_3 is the temperature 3 meters above the Earth's surface.
	$x_9^t = \text{RH}$, RH is relative air humidity.
	$x_{10}^t = \text{AP}$, AP is air pressure.
	$x_{11}^t = \text{SR}$, SR is solar radiation.

New limits specified in Government Order of the Czech Republic No: 350/2002 Coll. (No: 429/2005 Coll.) sets, except the limits of pollutants presented in Table 2, the conditions and the procedure of air quality's monitoring, evaluation and management. These limits are set for health protection, vegetation, and ecosystems protection separately. The dispersion conditions depend on the horizontal and vertical airflow especially [9].

The monthly values of parameters $\mathbf{x}^t = (x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t)$, $m=11$, for $o_i^t \in O$, $O = \{ o_1^t, o_2^t, \dots, o_i^t, \dots, o_n^t \}$, districts in the city of Pardubice, Czech Republic, (Fig. 1) represent the data set **P**.

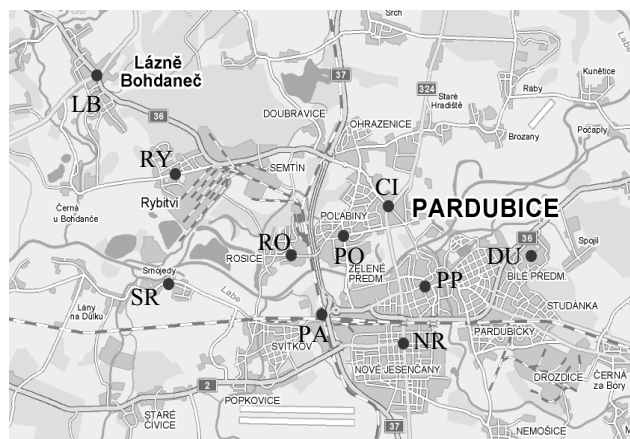


Fig. 1 The map of the districts (black points)

Legend: Bus stops: (Cihelna (CI), Dubina (DU), Polabiny (PO), Rosice (RO), Rybitví (RY), Srnojedý (SR)), crossroads: (Palacha-Pichlova (PP), Square of Republic (NR)), Spa Bohdaneč (LB), chemical factory of Paramo (PA).

Basic statistics on data set P are presented in Table 4. It is obvious that the values of harmful substances are much more unstable than the values of meteorological parameters. This is caused mainly due to the fact that the Pardubice region is a lowland region with mild climate.

Table 4 Basic statistics on data set P

Parameter	MIN	MAX	MEAN	Standard Deviation
x_1^t	7.16	43.77	15.57	8.34
x_2^t	30.33	75.84	50.18	9.76
x_3^t	17.45	266.64	72.41	57.30
x_4^t	0.22	1.56	0.54	0.38
x_5^t	33.32	138.82	56.03	18.85
x_6^t	0.57	2.52	1.26	0.41
x_7^t	118.52	256.76	190.26	25.48
x_8^t	9.80	14.76	12.39	1.16
x_9^t	59.20	74.12	65.67	2.87
x_{10}^t	984.84	992.43	989.57	1.81
x_{11}^t	137.21	347.81	239.89	43.83

For more information on data, it is useful to analyze the time series of the daily values of parameters $x_1^t, x_2^t, \dots, x_m^t$, for each district $o_i^t \in O$. As an example, the parameters $x_1^t, x_2^t, \dots, x_m^t$, for the crossroad PP are presented in Appendix 1. Especially the seasonal effects seem to be important.

4 Fuzzy Inference Systems

General structure of FIS [4,20] (Fig. 2) contains a fuzzification process of input variables by membership functions, design base of IF – THEN rules (BRs) or automatic IF – THEN rules extraction from input data, operators (AND, OR, NOT) application in rules, implication and aggregation within these rules and process of defuzzification of gained values to crisp values. In the process of defuzzification, standardization of inputs and their transformation to domain of values of membership function takes place. Inference mechanism in based on operations of fuzzy logic and implication within IF – THEN rules [4,20]. Based on aggregation process, transformation of outputs of individual IF – THEN rules to the output fuzzy set occurs. In process of defuzzification conversion of fuzzy values to expected crisp values is realized.

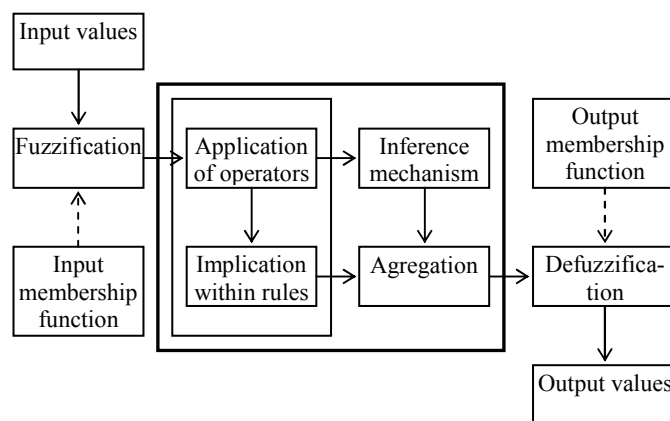


Fig. 2 General structure of fuzzy inference system

There is no general method for designing form, number and parameters of input and output membership functions, e.g. clustering algorithms can be applied if data matrix P is available. The input to fuzzification process is a crisp value given by the universum (reference set). Output of fuzzification process is the membership function value. The BRs design can be realized by extraction of IF – THEN rules from historic data, provided that they are available. The IF – THEN rules are used for creating conditional proposition, which represent the base of FIS. Based on the general structure of FIS, three fundamental types of FIS can be designed, i.e. Mamdani type [21], Takagi-Sugeno type [22] and Tsukamoto type [4]. The FISs of Mamdani type are suitable for air quality evaluation as both the inputs and the outputs of the FIS are represented by the values of linguistic variables. The IF-THEN rules of the FIS of Mamdani type can be defined as follows.

Let $x_1^t, x_2^t, \dots, x_m^t$ be input variables defined on reference sets $X_1, X_2, \dots, X_i, \dots, X_m$ and let y be an input variable defined on reference set Y . Then FIS has

m input variables and one output variable. Further, each set X_i , $i=1,2, \dots, m$, can be divided into p_j , $j=1,2, \dots, n$ fuzzy sets $\mu_1^{(i)}(x_1^t), \mu_2^{(i)}(x_1^t), \dots, \mu_{p_j}^{(i)}(x_1^t), \dots, \mu_n^{(i)}(x_1^t)$. Individual fuzzy sets represent mapping of linguistic variables values, which are related to sets X_i . Similarly, set Y is divided to p_k , $k=1,2, \dots, o$ fuzzy sets $\mu_1(y), \mu_2(y), \dots, \mu_{p_k}(y), \dots, \mu_o(y)$. Fuzzy sets $\mu_1(y), \mu_2(y), \dots, \mu_{p_k}(y), \dots, \mu_o(y)$ represent mapping of linguistic variables values to set Y . Then IF – THEN rule in the FIS of Mamdani type can be defined in following form [4,20]

$$\text{IF } x_1^t \text{ is } A_1^{(i)} \text{ AND } x_2^t \text{ is } A_2^{(i)} \text{ AND } \dots \text{ AND } x_m^t \text{ is } A_{p_j}^{(i)} \text{ AND } \dots \text{ AND } x_n^t \text{ is } A_n^{(i)} \text{ THEN } y \text{ is } B, \quad i=1,2, \dots, m; \quad j=1,2, \dots, n, \quad (1)$$

where $A_1^{(i)}, A_2^{(i)}, \dots, A_{p_j}^{(i)}, \dots, A_n^{(i)}, B$ represent the values of linguistic variable. Interpretations of IF – THEN rules consist of three parts. Those are fuzzification of inputs and application of fuzzy operators. Result of these two steps is the output value of antecedent part. Next step is application of gained result in consequent (implication). If antecedent of IF – THEN rule consists of more parts, then all antecedent parts computed simultaneously are transformed to value from interval $[0,1]$ by means of logical operators. Input, while applying fuzzy operations, are two or more membership function values and output is a value from $[0,1]$ interval. This number represents the result of the antecedent part of IF – THEN rule. It is subsequently used in consequent. Operator AND between elements of two fuzzy sets ($A^{(1)}$ AND $A^{(2)}$) can be generalized by t–norm [3]. It implies from interpretation of conditions, which t-norm meets that intersection of two fuzzy sets can only lead to exclusion of elements (values of membership function will be zero) or to preservation of current membership function value. Analogous to operator AND, operator OR between elements of two fuzzy sets ($A^{(1)}$ OR $A^{(2)}$) can be generalized by s–norm [3].

5 Hierarchical Fuzzy Inference Systems Design

Let there exist the FIS of Mamdani type defined in [4]. Then the number of IF – THEN rules $p_{FIS}=k^m$, where k is the number of membership functions, m is the number of input variables. For a great number m of input variables, the FIS of Mamdani type may be inefficient due to the increase in the number p_{FIS} of IF – THEN rules. One of the ways to reduce the number p_{FIS} of IF – THEN rules is to design the FIS of Mamdani type with a hierarchical structure. The aim of HFIS design is to reach efficiency and ability to interpret (i.e. with small number p_{FIS} of IF – THEN rules with small number of variables m , and with a small number k of membership functions for each

variable). Reducing the number p_{FIS} of IF – THEN rules leads to a reduction in computing demand of the system. This way, it comes to be more effective [23,24].

Let there exist the HFIS of Mamdani type defined in [20]. Then the number p_{HFIS} of IF – THEN rules is given as

$$p_{HFIS} = \left(\frac{m - v}{v - 1} + 1 \right) \times k^v, \quad (2)$$

where v is the number of variables in each layer. The minimum number p_{HFIS} of IF-THEN rules is achieved if each subsystem in the HFIS has only 2 inputs ($v=2$).

Comparison of the number of p_{FIS} IF – THEN rules for FIS and the number of p_{HFIS} IF – THEN rules for HFIS is shown in Table 5.

Table 5 Number of p_{FIS} and p_{HFIS} IF – THEN rules for FIS and HFIS

m	FIS			
	k=3	k=4	k=5	k=6
2	9	16	25	36
3	27	64	125	216
4	81	256	725	1296
5	243	1024	3625	7776
6	729	4096	18125	46656
m	HFIS			
	k=3	k=4	k=5	k=6
2	Not defined			
3	18	18	18	18
4	27	27	27	27
5	36	36	36	36
6	45	45	45	45

Basic types of HFISs are a tree and cascade HFIS [20]. Based on HFIS types mentioned, it is possible to design various different (hybrid) HFISs.

Let $x_1^t, x_2^t, \dots, x_i^t, \dots, x_m^t$ be input variables, and let $y_\mu^{1,1}, y_\mu^{1,2}, \dots, y_\mu^{q,1}$ be the outputs of subsystems $FIS_\mu^{1,1}, FIS_\mu^{1,2}, \dots, FIS_\mu^{q,1}$, where μ are membership functions. Then the IF – THEN rules $R^{h_{1,1}}, R^{h_{1,2}}, \dots, R^{h_{q,1}}$ of the tree HFIS, presented in Fig. 3, where q is the number of layers, can be defined as follows:

$$\begin{aligned} \text{Layer 1: } & FIS_\mu^{1,1} \quad R^{h_{1,1}} : \text{IF } x_1^t \text{ is } A_1^{h_{1,1}} \text{ AND } x_2^t \text{ is } A_2^{h_{1,1}} \\ & \quad \quad \quad \text{THEN } y_\mu^{1,1} \text{ is } B^{h_{1,1}}, \\ & FIS_\mu^{1,2} \quad R^{h_{1,2}} : \text{IF } x_3^t \text{ is } A_3^{h_{1,2}} \text{ AND } x_4^t \text{ is } \\ & \quad \quad \quad A_4^{h_{1,2}} \text{ THEN } y_\mu^{1,2} \text{ is } B^{h_{1,2}}, \\ & \quad \quad \quad \dots \\ \text{Layer q: } & FIS_\mu^{q,1} \quad R^{h_{q,1}} : \text{IF } y_\mu^{q-1,1} \text{ is } B^{h_{q-1,1}} \text{ AND } y_\mu^{q-1,2} \\ & \quad \quad \quad \text{is } B^{h_{q-1,2}} \text{ THEN } y_\mu^{q,1} \text{ is } B^{h_{q,1}}, \end{aligned} \quad (3)$$

where: $h_{1,1}=h_{1,2}=\dots=h_{q,u}=\{1,2, \dots, k^m\}$, $u=1,2$, $A_1^{h_{1,1}}$, $A_2^{h_{1,1}}, \dots, A_n^{h_{q,1}}$ are linguistic variables corresponding to fuzzy sets represented as $\mu_1^{h_{1,1}}(x_1^t), \mu_2^{h_{1,1}}(x_2^t), \dots, \mu_m^{h_{q,1}}(x_m^t)$, $B^{h_{1,1}}, B^{h_{1,2}}, \dots, B^{h_{q,1}}$ are linguistic variables corresponding to fuzzy sets represented as $\mu^{h_{1,1}}(y_1^{1,1}), \mu^{h_{1,2}}(y_1^{1,2}), \dots, \mu^{h_{q,1}}(y_n^{q,1})$, $\mu_{B^{h_{1,1}}}(y_j^{1,1}), \mu_{B^{h_{1,2}}}(y_j^{1,2}), \dots, \mu_{B^{h_{q,1}}}(y_j^{q,1})$ are membership function μ values of aggregate fuzzy set for outputs $y_j^{1,1}, y_j^{1,2}, \dots, y_j^{q,1}$.

The outputs $y_j^{1,1}, y_j^{1,2}, \dots, y_j^{q,1}$ of particular subsystems $FIS_{\mu}^{1,1}, FIS_{\mu}^{1,2}, \dots, FIS_{\mu}^{q,1}$ of the tree and cascade HFIS can be expressed by using defuzzification method Center of Gravity (COG) [4] as follows

$$y_{\mu}^{1,1}(B^{h_{1,1}}) = \frac{\sum_{j=1}^q y_j^{1,1} \times \mu_{B^{h_{1,1}}}(y_j^{1,1})}{\sum_{j=1}^q \mu_{B^{h_{1,1}}}(y_j^{1,1})}, \quad (4)$$

$$y_{\mu}^{1,2}(B^{h_{1,2}}) = \frac{\sum_{j=1}^q y_j^{1,2} \times \mu_{B^{h_{1,2}}}(y_j^{1,2})}{\sum_{j=1}^q \mu_{B^{h_{1,2}}}(y_j^{1,2})}, \quad (5)$$

$$\dots$$

$$y_{\mu}^{q,1}(B^{h_{q,1}}) = \frac{\sum_{j=1}^q y_j^{q,1} \times \mu_{B^{h_{q,1}}}(y_j^{q,1})}{\sum_{j=1}^q \mu_{B^{h_{q,1}}}(y_j^{q,1})}. \quad (6)$$

The IF – THEN rules $R^{h_{1,1}}, R^{h_{2,1}}, \dots, R^{h_{q,1}}$ of the cascade HFIS, presented in Fig. 4, where q is the number of layers, can be defined as follows:

Layer 1: $FIS_{\mu}^{1,1}$ $R^{h_{1,1}}$: IF x_1^t is $A_1^{h_{1,1}}$ AND x_2^t is $A_2^{h_{1,1}}$
 THEN $y_{\mu}^{1,1}$ is $B^{h_{1,1}}$,
 $FIS_{\mu}^{2,1}$ $R^{h_{2,1}}$: IF x_3^t is $A_3^{h_{2,1}}$ AND x_4^t is $A_4^{h_{2,1}}$
 THEN $y_{\mu}^{2,1}$ is $B^{h_{2,1}}$,
 ...

Layer q : $FIS_{\mu}^{q,1}$ $R^{h_{q,1}}$: IF $y_{\mu}^{q-1,1}$ is $B^{h_{q-1,1}}$ AND x_m^t is $A_m^{h_{q,1}}$ THEN $y_{\mu}^{q,1}$ is $B^{h_{q,1}}$.

Input data of the tree and the cascade HFIS contains parameters $\mathbf{x}^t=(x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t)$, $m=11$. The output contains classifications of objects into classes $\omega_{i,j}^t \in \Omega$. The designs of these models reduce number of p_{FIS} IF – THEN rules. An important part of the models is, besides reducing number of p_{FIS} IF – THEN rules, to reproduce expert decisions in air quality modelling process, in the sense of impeaching relationships of parameters $\mathbf{x}^t=(x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t)$, $m=11$ and their

mutual relations.

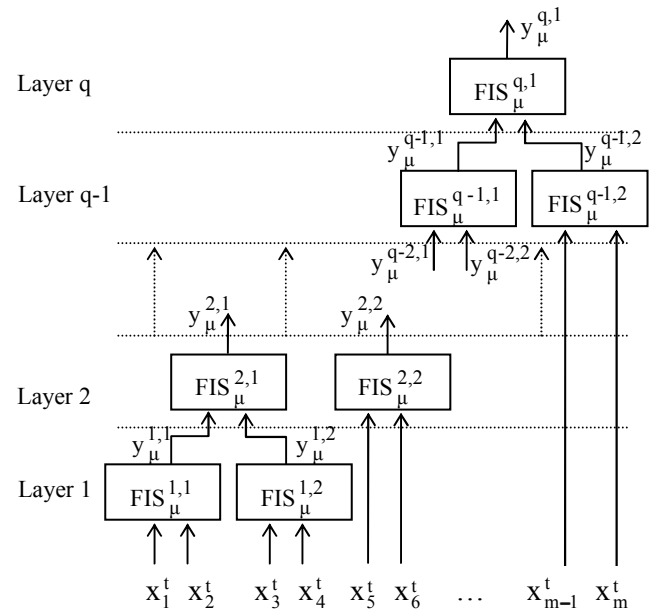


Fig. 3 A tree HFIS

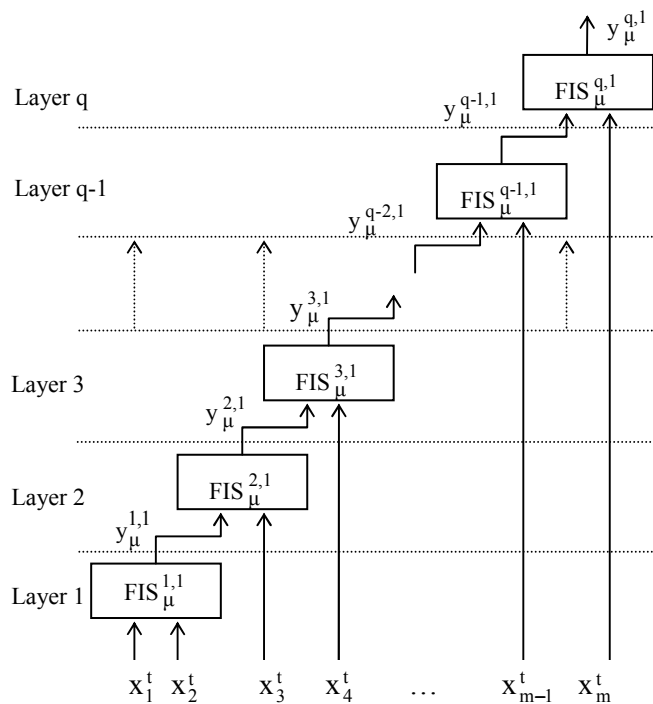


Fig. 4 A cascade HFIS

6 Analysis of the Results

Input parameters x_k^t are represented by two trapezoidal membership functions, while the outputs $y_{\mu}^{1,1}, y_{\mu}^{1,2}, \dots, y_{\mu}^{q,1}$ are represented by three trapezoidal membership functions. Individual membership functions are described by linguistic variables values $low_value_x_k^t$, $high_value_x_k^t$, etc. The design of input and output membership functions for individual subsystems $FIS_{\mu}^{1,1}, FIS_{\mu}^{1,2}, \dots, FIS_{\mu}^{q,1}$ for the tree HFIS is based on the limits set by the government (see Table 2) and on the

recommendations of experts. They are presented in Fig. 5 to Fig. 8.

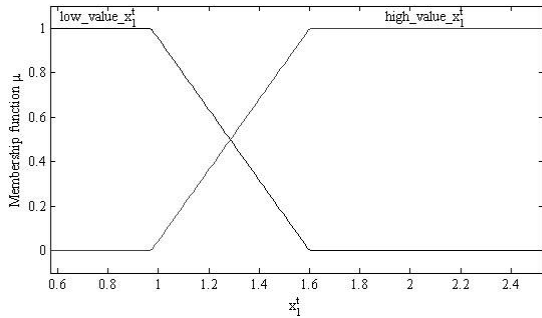


Fig. 5 Input membership functions μ for input parameter x_1^t of subsystem $FIS_{\mu}^{1,1}$

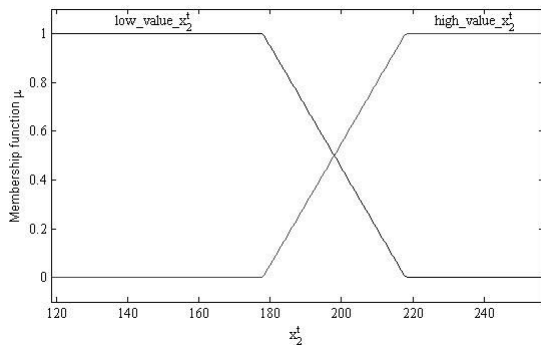


Fig. 6 Input membership functions μ for input parameter x_2^t of subsystem $FIS_{\mu}^{1,1}$

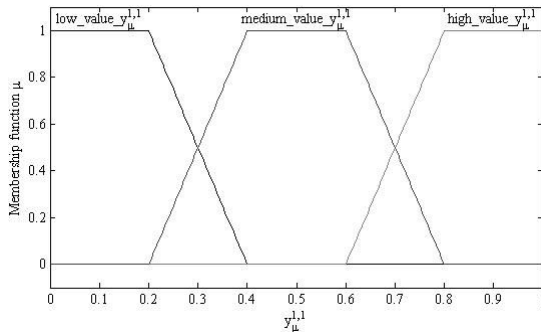


Fig. 7 Output membership functions μ for $y_{\mu}^{1,1}$ of subsystem $FIS_{\mu}^{1,1}$

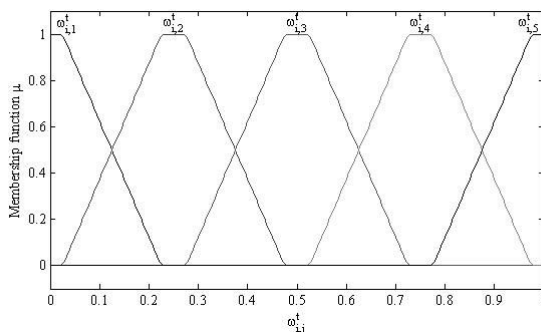


Fig. 8 Output membership functions μ for $y_{\mu}^{6,1}$ of subsystem $FIS_{\mu}^{6,1}$ for the tree HFIS ($FIS_{\mu}^{10,1}$ for the cascade HFIS respectively)

The membership functions designed this way take into account local conditions and, at the same time, a small increase/decrease in pollutant data, near its boundary value, will change the output only slightly.

The relationships among parameters $x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t$, $m=11$, are represented by the base of IF – THEN rules. It represents the knowledge of experts in the area of air quality evaluation. For the tree HFIS, it has following form:

$$\begin{aligned}
 FIS_{\mu}^{1,1} \text{ R}^1: & \text{ IF } x_1^t \text{ is low_value_}x_1^t \text{ AND } x_2^t \text{ is} \\
 & \text{ low_value_}x_2^t \text{ THEN } y_{\mu}^{1,1} \text{ is low_value_}y_{\mu}^{1,1}, \\
 FIS_{\mu}^{1,1} \text{ R}^2: & \text{ IF } x_1^t \text{ is low_value_}x_1^t \text{ AND } x_2^t \text{ is} \\
 & \text{ high_value_}x_2^t \text{ THEN } y_{\mu}^{1,1} \text{ is} \\
 & \text{ medium_value_}y_{\mu}^{1,1}, \\
 & \dots
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 FIS_{\mu}^{1,1} \text{ R}^4: & \text{ IF } x_1^t \text{ is high_value_}x_1^t \text{ AND } x_2^t \text{ is} \\
 & \text{ high_value_}x_2^t \text{ THEN } y_{\mu}^{1,1} \text{ is high_value_}y_{\mu}^{1,1}, \\
 & \dots
 \end{aligned}$$

$$\begin{aligned}
 FIS_{\mu}^{6,1} \text{ R}^1: & \text{ IF } y_{\mu}^{5,1} \text{ is high_value_}y_{\mu}^{5,1} \text{ AND } x_{11}^t \text{ is} \\
 & \text{ low_value_}x_{11}^t \text{ THEN } y_{\mu}^{6,1} \text{ is } \omega_{i,5}^t,
 \end{aligned}$$

$$\begin{aligned}
 FIS_{\mu}^{6,1} \text{ R}^2: & \text{ IF } y_{\mu}^{5,1} \text{ is high_value_}y_{\mu}^{5,1} \text{ AND } x_{11}^t \text{ is} \\
 & \text{ medium_value_}x_{11}^t \text{ THEN } y_{\mu}^{6,1} \text{ is } \omega_{i,5}^t, \\
 & \dots
 \end{aligned} \tag{9}$$

$$\begin{aligned}
 FIS_{\mu}^{6,1} \text{ R}^4: & \text{ IF } y_{\mu}^{5,1} \text{ is low_value_}y_{\mu}^{5,1} \text{ AND } x_{11}^t \text{ is} \\
 & \text{ low_value_}x_{11}^t \text{ THEN } y_{\mu}^{6,1} \text{ is } \omega_{i,1}^t.
 \end{aligned}$$

For the cascade HFIS, it has following form:

$$\begin{aligned}
 FIS_{\mu}^{1,1} \text{ R}^1: & \text{ IF } x_1^t \text{ is low_value_}x_1^t \text{ AND } x_2^t \text{ is} \\
 & \text{ low_value_}x_2^t \text{ THEN } y_{\mu}^{1,1} \text{ is low_value_}y_{\mu}^{1,1},
 \end{aligned}$$

$$\begin{aligned}
 FIS_{\mu}^{1,1} \text{ R}^2: & \text{ IF } x_1^t \text{ is low_value_}x_1^t \text{ AND } x_2^t \text{ is} \\
 & \text{ high_value_}x_2^t \text{ THEN } y_{\mu}^{1,1} \text{ is} \\
 & \text{ medium_value_}y_{\mu}^{1,1}, \\
 & \dots
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 FIS_{\mu}^{1,1} \text{ R}^4: & \text{ IF } x_1^t \text{ is high_value_}x_1^t \text{ AND } x_2^t \text{ is} \\
 & \text{ high_value_}x_2^t \text{ THEN } y_{\mu}^{1,1} \text{ is high_value_}y_{\mu}^{1,1}, \\
 & \dots
 \end{aligned}$$

$$\begin{aligned}
 FIS_{\mu}^{10,1} \text{ R}^1: & \text{ IF } y_{\mu}^{9,1} \text{ is high_value_}y_{\mu}^{9,1} \text{ AND } x_{11}^t \text{ is} \\
 & \text{ low_value_}x_{11}^t \text{ THEN } y_{\mu}^{10,1} \text{ is } \omega_{i,5}^t,
 \end{aligned}$$

$$\begin{aligned}
 FIS_{\mu}^{10,1} \text{ R}^2: & \text{ IF } y_{\mu}^{9,1} \text{ is high_value_}y_{\mu}^{9,1} \text{ AND } x_{11}^t \text{ is} \\
 & \text{ medium_value_}x_{11}^t \text{ THEN } y_{\mu}^{10,1} \text{ is } \omega_{i,5}^t, \\
 & \dots
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 FIS_{\mu}^{10,1} \text{ R}^4: & \text{ IF } y_{\mu}^{9,1} \text{ is low_value_}y_{\mu}^{9,1} \text{ AND } x_{11}^t \text{ is} \\
 & \text{ low_value_}x_{11}^t \text{ THEN } y_{\mu}^{10,1} \text{ is } \omega_{i,1}^t.
 \end{aligned}$$

Each of the subsystems $FIS_{\eta}^{1,1}, FIS_{\eta}^{1,2}, \dots, FIS_{\eta}^{9,1}$ contains a fuzzification process of input variables by membership functions. Further, the inference mechanism of particular subsystems $FIS_{\eta}^{1,1}, FIS_{\eta}^{1,2}, \dots, FIS_{\eta}^{9,1}$ involves also the process of implication (MIN method) and aggregation (MAX method) within IF – THEN rules, and the process of defuzzification by COG method

of obtained outputs to the crisp values. As a result, the outputs can be presented either as crisp values or as membership function values for each class $\omega_{i,j}^t \in \Omega$. The latter approach makes it possible to express, except for the class $\omega_{i,j}^t \in \Omega$, also the membership degree of the i -th district $o_i^t \in O$ in time t to the j -th class $\omega_{i,j}^t \in \Omega$.

Classifications of the i -th district $o_i^t \in O$ in time t to the j -th class $\omega_{i,j}^t \in \Omega$ are displayed in Table 6 in Appendix 2. It shows the comparison of the AQI (classes $\omega_{i,j}^t \in \Omega$) obtained by the tree HFIS with the AQIs of the CNIPH and the Czech Hydro-meteorological Institute. The results show that the classes $\omega_{i,j}^t \in \Omega$ obtained by the tree HFIS make it possible to realize the uncertainty in such a way that the value of membership function $\mu(\omega_{i,j}^t)$ is known for each class $\omega_{i,j}^t \in \Omega$. Three classes $\omega_{i,2}^t, \omega_{i,3}^t$ and $\omega_{i,4}^t$ dominate in the year measurements. Polluted air ($\omega_{i,4}^t$) has been detected in the centre of the town (PP, NR) due to traffic. It is obvious that there is a slight improvement of the air quality in the monitored districts $o_i^t \in O$ during the time $t=2001, 2002, \dots, 2006$.

In addition to yearly values, the monthly values of the AQIs have been also evaluated by the HFISs. Based on the experiments with the data, five classes were observed in the data structure. The classes correspond to those presented in Table 1 with the difference that the class $\omega_{i,6}^t$ is not used. Concerning monthly measurements, the classification of the i -th district $o_i^t \in O$ in time t (months) to the j -th class $\omega_{i,j}^t \in \Omega$ by the HFISs and their frequencies f are shown in Fig. 9 and Fig. 10. The models of the HFISs classify districts $o_i^t \in O$ so that classes $\omega_{i,2}^t, \omega_{i,3}^t, \omega_{i,4}^t$ have highest percentages. That means that areas with slightly polluted air prevail.

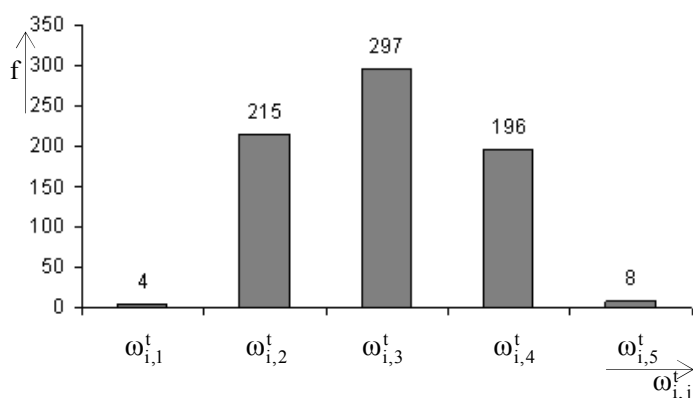


Fig. 9 Classification of the districts $o_i^t \in O$ into classes $\omega_{i,j}^t \in \Omega, j=5$ by the tree HFIS (the frequencies f of the classes)

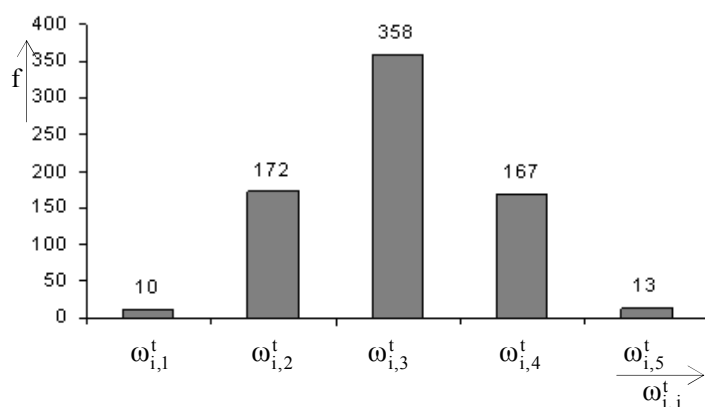


Fig. 10 Classification of the district $o_i^t \in O$ into classes $\omega_{i,j}^t \in \Omega, j=5$ by the cascade HFIS (the frequencies f of the classes)

7 Conclusion

The air quality modelling has been focused on the air quality parameters prediction [25,26] and on the modelling by multi-agents [27,28] or mathematical [29] systems so far while the classification of the districts has been realized either by AQIs [6,7,8] or by neural networks [30,31,32,33,34].

In this paper we define (besides some basic terms from areas of fuzzy sets and fuzzy logic) term linguistic variable and values of linguistic variable related to fuzzy set and set of classification functions. Further, the paper contains possibilities of FIS design and also concept and formalization of the tree and cascade HFISs. The air quality evaluation task is disassembled to elementary tasks, which are trivially solvable. Trivially solvable tasks are represented by individual partial subsystems $FIS_{\mu}^{1,1}, FIS_{\mu}^{1,2}, \dots, FIS_{\mu}^{q,1}, q=6$ for the tree HFIS, while $FIS_{\mu}^{1,1}, FIS_{\mu}^{2,1}, \dots, FIS_{\mu}^{q,1}, q=10$ for the cascade HFIS. Hierarchical fuzzy inference systems designed in this manner are applied in model for air quality classification in the city of Pardubice, the Czech Republic. It is classification of the i -th district $o_i^t \in O$ in time t to the j -th class $\omega_{i,j}^t \in \Omega$ in time t . Designed models with the tree and the cascade HFIS are, considering their efficiency and interpretability, suitable tools for air quality modelling. These models allow processing semantic uncertainty. Therefore, they represent efficient solution based on computational intelligence methods. They also take into account expert's decision-making in air quality modelling, thus take into account relationships among parameters $\mathbf{x}^t = (x_1^t, x_2^t, \dots, x_k^t, \dots, x_m^t)$, $m=11$, and their mutual relations. The results can represent recommendations to Pardubice state administration in the area of air quality progress. The model design was carried out in Matlab Simulink in the MS Windows XP operation system.

In future, it will be useful to propose such a model that

will combine the prediction capabilities of neural networks for individual air pollutants with the evaluation of air quality based on fuzzy logic. This model could be used as an early warning system within the framework of municipal crisis management.

Acknowledgement

This work was supported by the scientific research project of Ministry of Environment, Czech Republic under Grant No: SP/4i2/60/07 with title Indicators for Valuation and Modelling of Interactions among Environment, Economics and Social Relations.

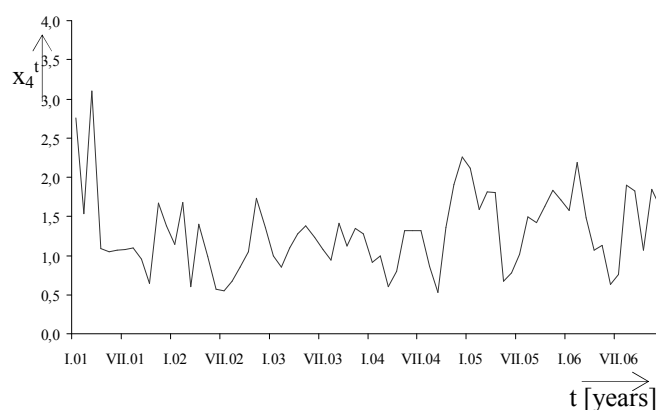
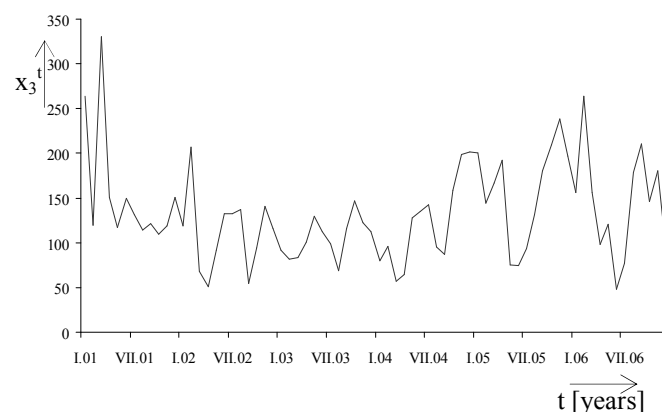
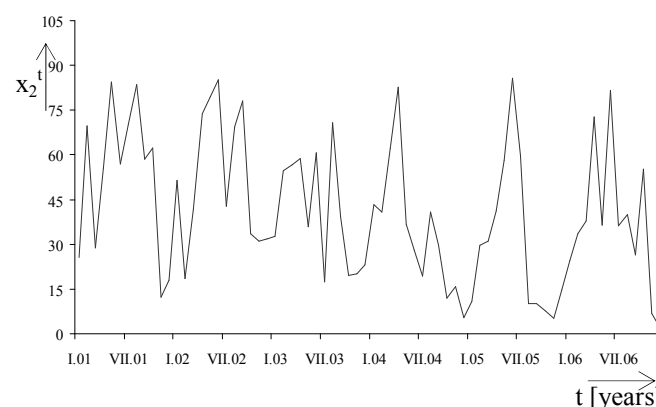
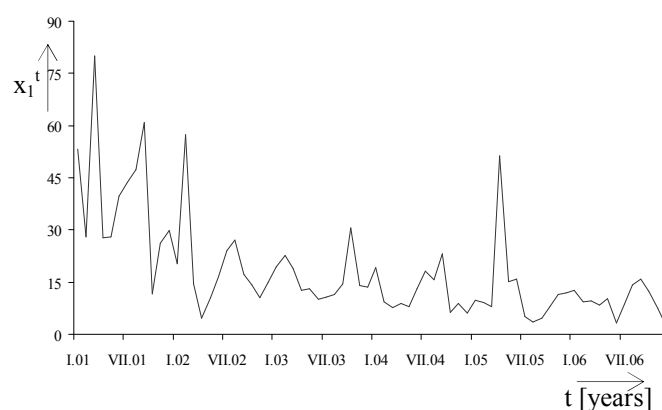
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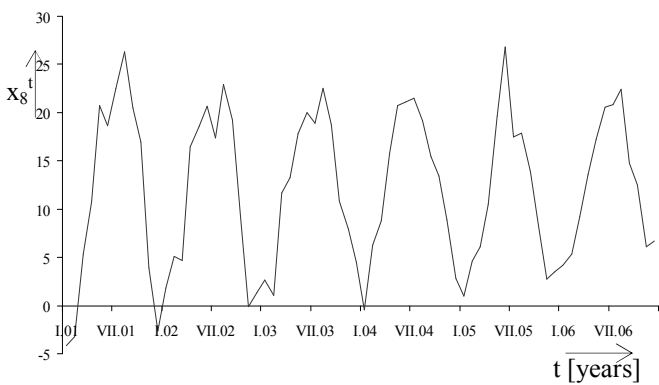
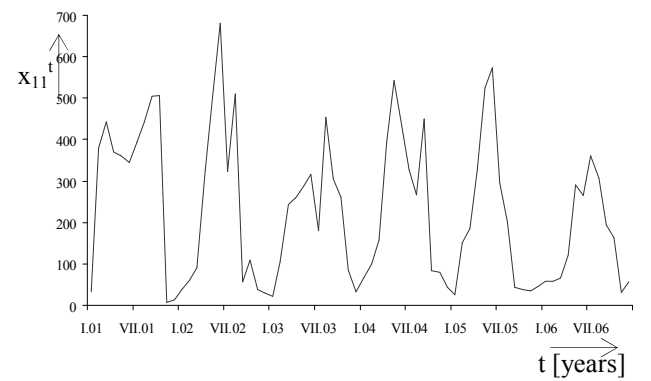
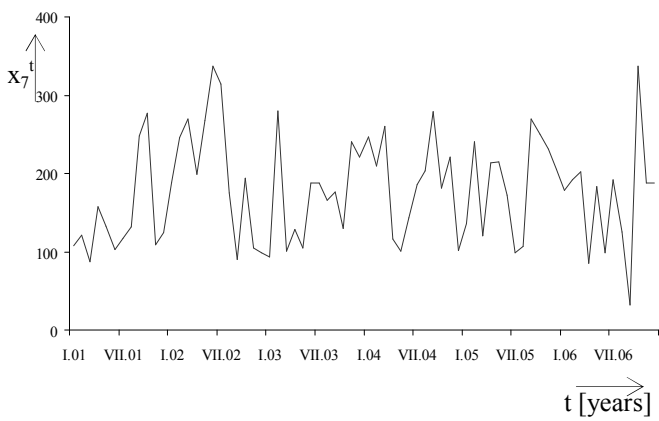
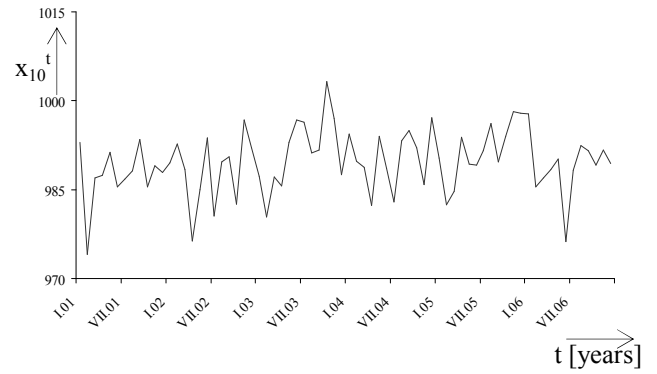
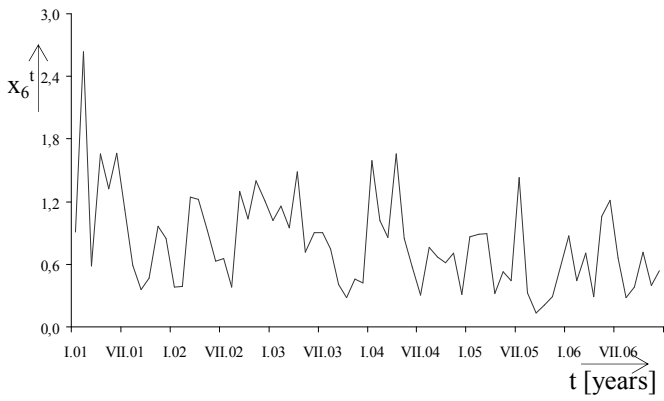
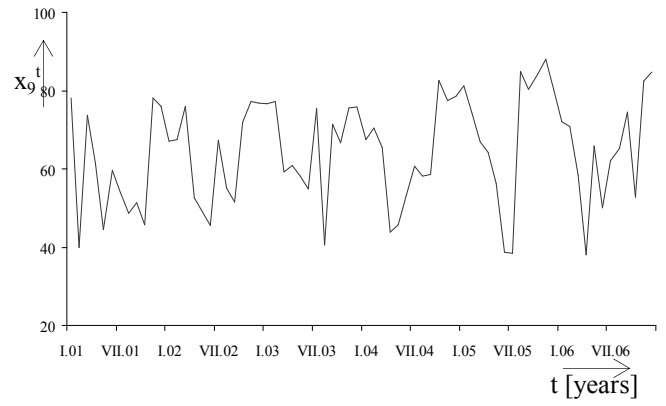
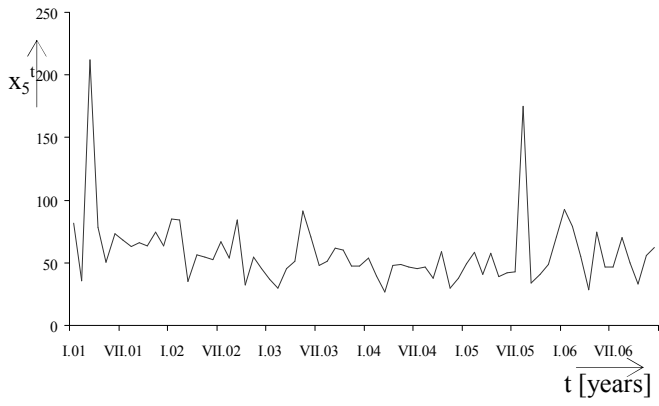
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Appendix 1





Appendix 2

Table 6 Air Quality Indices for districts in years 2001-2006

Classes based on the AQI by the Czech National Institute of Public Health										
year	SR	RY	RO	PP	PO	PA	NR	LB	DU	CI
2001	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,3}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,3}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$
2002	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$
2003	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$
2004	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$
2005	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$
2006	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,2}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$	$\omega_{i,1}^t$
The AQI of the Czech Hydro-meteorological Institute										
year	SR	RY	RO	PP	PO	PA	NR	LB	DU	CI
2001	2.0	2.2	2.0	2.8	2.0	2.2	2.8	2.4	2.2	1.8
2002	2.0	2.0	1.8	2.2	2.0	2.0	2.6	2.0	1.8	2.0
2003	1.6	2.0	1.6	2.2	1.6	2.0	2.6	1.8	1.6	1.6
2004	1.6	1.8	1.6	2.0	1.6	2.0	2.4	1.8	1.6	1.8
2005	1.6	1.8	1.8	2.2	1.8	2.0	2.4	1.8	1.6	1.8
2006	1.6	1.6	1.6	2.2	1.6	1.8	2.2	1.6	1.6	2.0
Classes of the AQI based on the tree HFIS with membership function values										
year	SR	RY	RO	PP	PO	PA	NR	LB	DU	CI
2001	$\omega_{i,3}^t(0.55)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,2}^t(0.80)$	$\omega_{i,4}^t(0.93)$	$\omega_{i,3}^t(0.94)$	$\omega_{i,3}^t(0.91)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,3}^t(0.76)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,3}^t(0.94)$
2002	$\omega_{i,3}^t(0.90)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,2}^t(0.70)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,3}^t(0.95)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,3}^t(0.80)$	$\omega_{i,3}^t(0.95)$	$\omega_{i,3}^t(0.90)$
2003	$\omega_{i,2}^t(0.63)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,2}^t(0.95)$	$\omega_{i,4}^t(0.90)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,3}^t(0.94)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,3}^t(0.59)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,3}^t(0.95)$
2004	$\omega_{i,2}^t(0.63)$	$\omega_{i,3}^t(0.93)$	$\omega_{i,2}^t(0.82)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,4}^t(0.79)$	$\omega_{i,4}^t(0.94)$	$\omega_{i,3}^t(0.84)$	$\omega_{i,3}^t(0.91)$	$\omega_{i,3}^t(0.90)$
2005	$\omega_{i,2}^t(0.90)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,2}^t(0.66)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,3}^t(0.94)$	$\omega_{i,3}^t(0.94)$	$\omega_{i,4}^t(0.92)$	$\omega_{i,3}^t(0.94)$	$\omega_{i,3}^t(0.90)$	$\omega_{i,3}^t(0.91)$
2006	$\omega_{i,2}^t(0.90)$	$\omega_{i,3}^t(0.95)$	$\omega_{i,2}^t(0.80)$	$\omega_{i,4}^t(0.95)$	$\omega_{i,2}^t(0.74)$	$\omega_{i,3}^t(0.57)$	$\omega_{i,4}^t(0.91)$	$\omega_{i,3}^t(0.76)$	$\omega_{i,3}^t(0.92)$	$\omega_{i,3}^t(0.93)$