Mathematical Modeling Pollution From Heavy Traffic in Tbilisi Streets

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Abstract: - Using mathematical simulation, distribution of concentration of harmful substances NO_x at Rustavely Avenue, the crossroad of David Agmashenebeli and King Tamar Avenue, where traffic is congested, and for the whole territory adjoined to the crossroad have been studied. In addition, there have been investigated influences of traffic-lights at streets' intersections on the growth of concentration of harmful substances. Mathematical model of air pollution from traffic is presented. Results of numerical calculations are given.

Key-Words: - Air pollution, mathematical simulation, influences of traffic-lights.

1 Introduction

Considerable technical developments have taken place in recent years with the emergency of advanced transportation systems. Consequently has strengthened reaction on environment. Nowadays the damages caused by air pollution in the world are large. Harmful substances mainly are emitted from power plants and vehicle engines. All over the world the main reason of the atmosphere air pollution is the exhaust of the motor-transport, having about the 75% part of the total pollution. The list of the polluting admixtures, exhausted by the motor-transport is long. It comprises more than 200 substances. The main polluting components exhausted by the motor-transport are: firm particles in the form of dust (mostly soot), carbonic acid (CO), Sulphur dioxide (SO_2), Nitric oxides (NO_x), formaldehyde (CHOH), stannous oxides (Pb_xO_y). Among the most important components of air pollutants are acid constituents, namely Nitric and Sulphur acid are the main contributors to the acidity in rain.

At the present day industrial potential of Georgia is very low, as many existing plants and factories are not functioning. For this reason, exhaust gases represent main air pollutants in the country. Therefore, air pollution is on the high level in Tbilisi, capital of Georgia, with population of 1.5 million. Reasons and results of Tbilisi air pollution is not much different from any urban area in the world. Aerosols are considered as one of the main air pollutants in Tbilisi. As known, air pollution in urban areas due to traffic is much higher in rush-hours. Exhaust gases, which mainly consist in nitrogen oxides, have very adverse effects on human health, besides bad environmental changes. According to official statistical data of Georgia, air pollution in urban areas with heavy traffic is higher than in industrial areas. It is expected, that the continuous economic growth will strengthen the traffic intensity, therefore quality of air will worsen. Unfortunately, for the last 15 years, owing to difficult economic situation and lack of financing, system of meteorological and observation stations have almost been destroyed in Georgia. Now there are functioning only eight meteorological observation laboratories in Tbilisi from 34 existing in 1992. Besides, Tbilisi has such kind rather compound orography, which prevents air ventilation and leads pollutants growing in some districts of Tbilisi having concave shape. Thus, investigation of dispersion of exhaust gases in the main Tbilisi streets, by means of mathematical modelling is very important for human health, environmental management and future economic planning, including revising of street network and traffic management.

Models describing the dispersion and transport of air pollutants in the atmosphere can be distinguished: Plumerise, Gaussian, Semi-empirical, Eulerian, Lagrangian, Chemical and Stochastic models. The model most widely used today for pollutant dispersion within urban street-canyon is empirically derived Street-model [17,18] Similar to the basic equation of the street-models, but with improvement dispersion were described in [24]. These models had been developed by [21]. In 1994, the Revised Version of the Nordic Computational Method was issued [2]. From this model they were able to study parametrisation of dispersion in the case of more general street configuration, frequency of low wind speeds and chemical transformations. For some time, however, investigators have realized non-Gaussian models better describes the dispersion of passive contaminants from surface and elevated releases in the atmospheric boundary layer. [5,9,12, 22} Analyzed the contribution of vehicale emissions. On air-quality improvement. at present widely are used Lagrangian and Eulerian meso-scale meteorological model and chemical transport models for simulation harmful substances transfer and conversion in the atmosphere. The meso-scale meteorological model includes about120 gas phase reactions and 65 separate chemical species. The models simulate gas phase ozone, various peroxides, NOx and NOy compounds, hydroxyl radicals, and hydrocarbons, and is designed for regional-scale analyses [11,13,14,20,26,27] . Also widely are used sulphur transport Eulerian model and atmosphere chemistry box model (STEM). The STEM comprehensive model has been developed to investigate the relationships between the emissions, atmospheric transport, chemical transformation, removal processes, and the resultant distribution of air pollutants and deposition patterns on meso and regional scales[7,8,28]. Also some of the interesting works are concerning to the environmental protection. Namely with the purpose of enhancing fuel economy and reduce the exhaust emissions, a lean burn gasoline engine system is used for NOx emission after treatment. [28] and new town planning for imbalanced development [23].

In Georgia the problem of exhaust gases dispersion in streets canyons are examined at the Institute. of Hydrometeorology [15]. But these investigations carried out using mainly empirical-statistical methods. But these investigations carried out using mainly empirical-statistical methods.

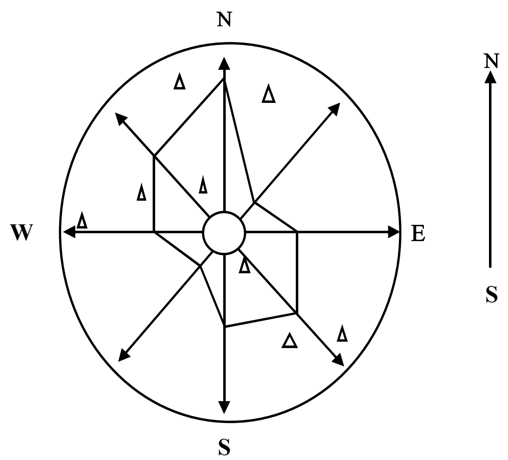
2 Material and Estimation Method

While estimating the atmosphere pollution in Tbilisi, the following criteria of hygiene are used: a single maximum value of the permissible critical concentrations (PCCm) and the average daily, monthly or annual value of the same parameters. (PCCa). The comparisons of single measuring is done from the PCCm and the comparison of the average monthly or annual concentrations from the PCCa. In Tbilisi the condition of the atmosphere air is estimated by the concentration of harmful substances such are: dust, the sulphur oxide, carbon, nitrogen, the nitric acid, dissolved sulphutes, phenol, formaldehyde, manganum oxide and ammonia.

The climate in Tbilisi is mild warm, transitional from the steppe to mild humid subtropical. Winter is mild, not very cold and summer is mild, not very hot. The average annual temperature is 12.7^{0}C, of January – 0.3 ^{0}C and of July – 24.4^0 C. The amount of precipitations in average- 560 mm a year.

For the last ten-year period the observation were carried out in Tbilisi, in 8 posts, dislocated in different districts. The dislocation of the posts (by triangles) and the main directions of the winds (rose of winds) are given on the Fig. 1.

Fig.1



The dislocation of the posts in Tbilisi considering the direction of the wind

Fig.1 shows that there are dislocated only three meteorological observations post in the central (high polluted) part of Tbilisi and North and North-West winds dominate in Tbilisi, also South-East winds often blow as well.

For estimating the current conditions of atmospheric air pollution of Tbilisi the correct analysis of the existing pollution is necessary, which must be based on the analysis of the space-time changes of atmospheric admixture concentrations for different periods and by giving the average space scale.

In regard with this question, to undergo numerous difficulties the notions of the atmosphere pollution index (Ki) and its average total index (K) are accepted: $Ki = q_i/q_{ix}$, where qi and q_{ix} accordingly are the values of the average definite space and time i – all mater concentrations and corresponding permissible critical concentrations: $K=1/n\sum Ki$, (i=1,2,…,n), where n is the number of the admixtures[3,4,10].

With the purpose of imagining level of pollution in the Table 1. are represented average annual values of some polluting admixtures of the atmosphere for the three meteorological observation posts dislocated in the center part of Tbilisi. In the Tab.1 in the second row up numbers denote observation data of concentrations in (mg/m^3) and numbers below – values of PCCa in (mg/m^3).

Table 1. The values of the concentrations of the polluting admixtures in the atmosphere for the three posts dislocated from the North-West to South-East directions in the central part of Tbilisi [15].

Admixture Post numb.	Dust	SO_2	CO	NO_2	NO
6	0.38 0.5	0.03 0.5	4.12 5.0	0.041 0..05	0.03 0.03
2	0.42		3.12	0.051	0.035
29	0.47		5.12	0.047	0.032

It's clear from the Table 1, that the distribution of the concentration of the admixtures on the territory of the city is not equable. Investigations have shown that values of concentrations mainly depends upon the direction of the wind, dislocation of the streets, districts and traffic intensity. Also Table1 shows that average annual concentrations for some harmful substances (NOx, CO) exceed of PCCa.

Tabale. 2. Values of the average total index K of atmosphere pollution according to the observation meteorological posts

No	32	5	28	1	2	6	29	4
K	0,8	0.83	1.13	1.	1.3	1.1	1.3	1.2

Analysis of the data of the Table 2. shows that the distribution of the atmosphere pollution on the territory of the city, that across the dominations direction of the wind from the North-West periphery to the center the index K of the total pollution increases. That is way we investigate air pollution problem in the central part of Tbilisi. As normally average total index K of atmosphere pollution is less than one and an average value of K is 1.08 in Table 2, so level of Tbilisi air pollution is high.
The yearly course of the admixture concentrations of Tbilisi atmosphere pollution also arises interest (Table 3. and Table 4).
The analysis of the data for 1991-1997 shows, that the yearly course of the concentrations isn't characterized by the abrupt of the amplitude. The yearly course of the atmospheric pollution index K are given on the Table 3.

Table 3. Change of values of the average total index K of atmosphere pollution according to the years from 1991 to 1998

Y	91	92	93	94	95	96	97	98
K	1.3	1,1	0.8	0.6	0.5	0.5	0,6	0.6

The Table 3, shows that the change of the atmospheric pollution index K meaning is characterized by the tendency of decreasing during the years, that can be explained by the fact, that the exhaust of admixtures by the industrial works has decreased. It's clear, that it's caused not be the measures, taken for improving the common state, but by the economical and energetic crisis.

Table 4. Change of values of the average total index K of atmosphere pollution according to the years from 1999 to 200

Y	99	00	01	02	03	04	05	06
K	0.7	0.8	0.9	1.05	1.	1.2	1,23	1.3

The Table 4 shows that the change of the atmospheric pollution index K meaning is characterized by the tendency of increasing for the last ten-year period. As for this period many miles and factories were not able to renew functioning but number of motor transport increased about 1.5 times so it is evident that the tendency of increasing of value K is owing to exhaust gases from motor transport in Tbilisi street canyons. That is the result of the current re-organization going on the our country.

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3 Problem Formulation

Amount of vehicle exhaust and content of main harmful substances in gas, such as carbonic acid (CO), nitric acids (NO_x) and hydrocarbons (C_xH_x), depend on the traffic intensity, vehicle type and speed, road width and the number of traffic lines. On the given lengthwise layer in the given time unit, exhaust amount is calculated by the following formula [1,6]:

$$M=a\sum Qi*\Pi i*Ni \,, \qquad (1)$$

where M is an exhaust of the harmful substance (gr/sec. km) per 1 km; $a=a^1\rho$; ρ is average density of fuel (≈0.74 kg/l.); a^1 =1000gr hr/(3600kg hr); a ≈0.2; N is the traffic intensity (number of vehicles, passing the given lengthwise layer in given time unit (hr^{-1})); Q is the amount of the petrol consumed by the vehicle per one km (l/km); Π is un-dimensioned coefficient, expressing the correlation of the harmful exhaust to consumed petrol; n is the number of the cars of different types. If we pick out cars, buses, microbuses and lorries, then from (1) we will have:

$$M=0.2[(Q*\Pi*N)_{cars}+(Q*\Pi*N)_{buses}+(Q*\Pi*N)_{micbuses}+(Q*\Pi*N)_{lorries}],$$

where Q_{cars}=0.11, Q_{buses}=0.35, $Q_{mic\text{-}buses}$=0.18, Q_{buses}=0.31. N_{cars}, N_{buses}, $N_{mic\text{-}buses}$, $N_{lor.}$ are determined by experimental observations. As it is recommended [2], Π_{NOX}=0.04, Π_{CXHX}=0.1, it doesn't depend on the speed and Π_{co} depends on the average speed, namely: $\Pi_{co}(20)$=0.72, $\Pi_{co}(30)$=0.6, $\Pi_{co}(40)$=0.45, $\Pi co(50)$=0.22, $\Pi co(80)$=0.16. CO, NO_x exhaust from the cars with diesel engine is several times less than from the cars with petrol engine, although it's difficult to determine in our conditions. The existence of the crossroads has a definite influence on the concentration of the exhausted substance, that can be represented by the following mathematical dependence:

$$C_{crossroad}=C_o\,(1+N_2/N_1), \quad (2)$$

where C_o is the maximum concentration caused by the exhaust on the observed main arterial road. N_1 and N_2 indicate the intensity of the traffic for the given main and crossroads, respectively.

The fulfillment of the experimental activities, provided by the given method includes the following stages: 1) determining the average typical structure of the motor transport stream; 2) determining the intensity of the roads; 3) determining the average value of the intensity of the traffic by the formula: $N_{aver.}= 1/3\,(N_{morning}+N_{noon}+N_{evening})$.

4 Mathematical Model

The method of researching of atmospheric air pollution by motor transport is based on the well-known and tested different physical and mathematical models [1,9], which approximately represent the dynamics and mechanism of such processes. Namely, this method is based on solving the system of three-dimensional differential equation of non-stationary turbulent boundary layer together with harmful substances transference in the atmosphere which is described by diffusion equation [10,25]. The temperature and wind regime of the lower layer of the atmosphere, where the main mass of the polluting components lies, depends on the processes of synoptic scale (advection, vertical movements) as well as on the boundary layer processes (turbulence, radiation). In our model the influence of the broad-scaled factors, were determined by means of the background meteorological fields (wind velocity, temperature and harmful substances concentrations), is regarded as external parameters. Also there is inflow of the harmful substances in the considered region. The achieved data, such as: background concentrations, geostrophical wind velocity, the vertical distribution of the wind, temperature, the coefficient of the vertical change, are used for investigation of the harmful substances concentrations distribution in the atmosphere lower layer. In this model we assume, that at the initial time the wind velocity is uniform on the horizontal plane (small sixes of the considered area) and it changes on height likewise density and pressure. When we examine the problem of transference and diffusion of adverse substances for the Tbilisi street network, as landscape of Tbilisi is in-homogenous enough, and street network of the city mainly is not arranged in the sane plane so It is necessary to take into consideration relief.. That is way the numerical model is taking into account orography of examined region of Tbilisi [10]. Now let us consider the following specific problem: migration and diffusion of adverse substances ejected from traffic in Tbilisi streets. We consider that pollutant ejected from traffic imagine as linear sources. Let us consider that our linear source of power M, is located on the roof level z=h. According to above harmful substance transfer in the atmosphere together with basic differential equations of mesoscale boundary layer of atmosphere may be written as:

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}+W_s\frac{\partial C}{\partial z}= \frac{\partial}{\partial z}K(z)\frac{\partial C}{\partial z}-\alpha_1 C+Q(x,y,z,t), \quad (3)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+W_s\frac{\partial U}{\partial z}- f(V-\eta V_g)=\frac{\partial}{\partial z}K_m\frac{\partial U}{\partial z}, \quad (4)$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+W_s\frac{\partial V}{\partial z} -f(U-\eta U_g)=\frac{\partial}{\partial z}K_m\frac{\partial v}{\partial z}, \quad (5)$$

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial Ws}{\partial z}=0, \quad (6)$$

$$\frac{\partial T}{\partial t}=\frac{C_p}{C_{1p}}\left(\frac{\partial}{\partial z}K_H\frac{\partial T}{\partial z}+\gamma\alpha\frac{\partial K_H}{\partial z}\right), \quad (7)$$

$$\frac{\partial q}{\partial t}=\frac{\partial}{\partial z}K_q\frac{\partial q}{\partial z}, \quad (8)$$

where C is adverse substance concentration; α is the coefficient that determines the velocity of substance concentration changes during the process of substance decomposition and transformation; $Q(x,y,z)$ are internal sources (in our model Q is used for describing pollutant emitted from vehicles); U, V and Ws are horizontal and vertical axial components of wind velocity along axis Ox, Oy and Oz respectively; U_g, V_g are geostrophical wind velocity components; g is the gravitational acceleration; f is the Coriolis parameter; ρ is density; q is specific humidity; T is temperature; K_m, K_H, K_q are motion, heat and humidity vertical coefficients of turbulence, respectively; $\eta=1/(1-R/H)$; function $R = R(x,y)$ is describes non homogeneity of the earth surface; H is altitude of the atmosphere top level. $W_s=W_p+W_{or}$,

$W_{or}=U\left(\frac{\partial r}{\partial x}+A\frac{\partial^2 r}{\partial y^2}\right)+V\left(\frac{\partial r}{\partial y}+A\frac{\partial^2 r}{\partial x^2}\right)$- terms in W_{or} must to answer the following inequalities [10]:

$$\frac{\partial r}{\partial x} > A\frac{\partial^2 r}{\partial y^2}, \qquad \frac{\partial r}{\partial y} > A\frac{\partial^2 r}{\partial x^2},\ A = (\Delta x + \Delta y)/2$$

,

where $\Delta x and \Delta y$ are grid steps along axis Ox and Oy respectively;

$C^1_p = C_p + 0.622R_1e_sL^2/(P*R_v*T)$,

where R_1=0 if $q<q_s$ and R_1=1 if $q\geq q_s$, q_s is specific humidity of saturation; L is Latent heat of condensation; e_s is elasticity of saturation; R_v is the gas constants. For humidity air C_p is specific heat of dry air at constant pressure; γ_α. Is the gradient of temperature;

$\gamma_\alpha = g/C_p$ if $q<q_s$,

$\gamma_\alpha = g(RT-Lq_s)/RTC_p$ if $q\geq q_s$

Value of the K_m is calculated for the neutral and stability stratification with the following formula [3,10,25],

$$K_m = \chi^2 z^2 r_1 \frac{\partial S}{\partial z} \quad \text{where} \quad S = \sqrt{U^2 + V^2}\ .$$

For un-stably stratification we have

$$K_m = \chi^2 z^2 r_1 \left[\frac{\partial S}{\partial z} + r_2\sqrt{\frac{g}{T}\left|\frac{\partial \theta}{\partial z}\right|}\right]$$

where χ is the Karman constant; r_1 and r_2 are empirical constant; θ is potential temperature.

K_H is defined by the following rellation

$$K_H = K_m/(1+\alpha R_i),$$

where R_i is Richardson's constant . Also we assume that $K_q=K_H$ and α=0.6.

In the near-earth layer vertical thermal streams and momentum are maintained according to altitude, and vertical component K(z) of turbulent stream variation coefficient increases in proportion of altitude .As a result of theoretical studies I. Kibel has received formula for *K(z)*. We are using the analogy formula for *K(z)* [3,10].

$$K(z) = \begin{cases} K_m\,(Z/h_1)^{\,h1-Z}, & \text{when} \quad z < h_1, \\ K_m(1+ (z-h_1)/(H-h_1)), & z \geq h_1, \end{cases}$$

where h_1 = 50 m.

The system of equations (3) - (8) is solved in the rectangular parallelepiped

$G=\{0 \leq x \leq L,\ 0 \leq y \leq M,\ 0 \leq z \leq H\}$,

with boundary surface Γ, with the following initial and boundary conditions:

$$C/_{t=0} = C_0,\quad U/_{t=0} = U_0e^{z/z_1} - U_0,\quad V/_{t=0} = V_0e^{z/z_2} - V_0,$$
$$T/_{t=0} = T_0,\qquad q/_{t=0} = q_0. \qquad (9)$$

On the up level (H) of the Γ, which was on the 700 *MB* surface the boundary conditions are follows:

$$C/_{Z=H} = C_H,\quad T/_{Z=H} = T_H,\quad q/_{Z=H} = q_H,\quad U/_{Z=H} = U_g,\quad V/_{Z=H} = V_g, \qquad (10)$$

where C_H, T_H, q_H, U_g, V_g were obtained from the background forecast.

On the bottom level we have:

$K_m\partial C/\partial z = \beta_1 C$, β_1 = const, Ws = 0, U = V = 0 , T = T_s , $\partial q/\partial z$ = 0 . (11)

when x=0 ; or x=L; then $\partial C/\partial x = 0$; $\partial U/\partial x = 0$; $\partial V/\partial x = 0$; $\partial T/\partial x = 0$;

$\partial q/\partial x = 0$; (12)

when y=0 ; or y=L; then $\partial C/\partial y = 0$; $\partial U/\partial y = 0$; $\partial V/\partial y = 0$; $\partial T/\partial y = 0$; $\partial q/\partial y = 0$; (13)

For harmful substances linear and area sources along the streats canyons we have;

$Q(x,y,z,t) = 0$ when $(x, y,z) \notin \ell_1$,

$Q(x, y, z, t)/_{\ell 1} = \{Q_o$ when $t \leq 3600\}$,

$Q(x, y, z, t)/_{\ell 1} = \{0$ when $t > 3600\}$.

5 Numerical Scheme

The problem (3)-(8) with initial (9) and boundary conditions (10)-(13) is solving numerically by Adams-Betshfort method[3, 10,25].

At the firs step of time is used the Eulierian obviously scheme. For the subsequent time steps is used the following finite-difference scheme:

$C^{n+2}_{i,j,k} = C^{n+1}_{i,j,k} + (3/2)f_C(n) - (1/2)f_C(n-1)$.

For the space approximation is used nine-point finite-difference operator.

Let us denote

$Sr(S,n,i,j,k) \equiv (4S^n_{i,j,k} + 2(S^n_{i-1,j,k} + S^n_{i+1,j,k} + S^n_{i,j-1,k} + S^n_{i,j+1,k}) + S^n_{i-1,j-1,k} + S^n_{i+1,j-1,k} + S^n_{i-1,j+1,k} + S^n_{i+1,j+1,k})$ / 16 ,

$\partial S_x(n,i,j,k) \equiv (S^n_{i+1,j,k} - S^n_{i-1,j,k})\Delta t/4\Delta x + (S^n_{i+1,j+1,k} - S^n_{i-1,j+1,k} + S^n_{i+1,j-1,k} - S^n_{i-1,j-1,k})\Delta t/8\Delta x$

$\partial S_y(n,i,j,k) \equiv (S^n_{i,j+1,k} - S^n_{i,j-1,k})\Delta t/4\Delta x + (S^n_{i+1,j+1,k} - S^n_{i+1,j-1,k} + S^n_{i-1,j+1,k} -$

$S^n_{i-1,j-1,k})\Delta t/8\Delta x$.

$\partial S_z(Ws,n,i,j,k) = (S^n_{i,j,k+1} - S^n_{i,j,k-1})\,\Delta t/\,2\Delta z$
;
where $S = (U,V,C,T,q)^T$ is a matrix column,
S is function of the x, y, z, t when $S_{i,j,k}$ is the function of the $(i\cdot\Delta x,\ j\cdot\Delta y,\ k\cdot\Delta z)$; Δt, Δx, Δy and Δz are temporal and space grid steps.

For example there is writen out the finite-difference scheme for the equation (3), which using the denotations above has the following form:

$C^{n+1}_{i,j,k}$= Sr(C,n,i,j,k) – Sr(U,n,i,j,k) ∂C_x(n,i,j,k) – Sr(V,n,i,j,k) ∂C_y(n,i,j,k)

– Sr(Ws,n,i,j,k) ∂C_z(n,i,j,k) + (Sr(C,n,i,j,k+1) – 2 Sr(C,n,i,j,k) +

Sr(C,n,i,j,k-1)) $KZ_k\Delta t/\Delta Z^2$

(Sr(C,n,i,j,k+1) – Sr(C,n,i,j,k-1)) $(KZ_{k+1} - KZ_{k-1})\,\Delta t/\Delta Z^2 + Q^n_{i,j,k} - \alpha_{1*}$

$C^n_{i,j,k}$.
where $Ws_{i,j,k}$ and $\eta_{i,j}$ were calculated by the following numerical formulas:
$\eta_{i,j} = 1/(1 - r_{i,j} / H)$,
$Ws_{i,j,k} = Wp_{i,j,k} + Wor_{i,j,k}$;
where $r_{i,j}$ is finite-difference analogies of the function r(x,y). Let us denote: **ΔL= Δx= Δy**
Primary values of the grid function r = r(x,y) are taken from the topographic maps (the step on the topographic maps as twice less than ΔL ,). Further these obtained values of the z(x,y) are smoothed with aid of second order derivative and linear interpolation's finite-deference shemes. Specifically for the points I+1/2, I-1/2 of the grid $\overline{\omega}_1$ we have:

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i+1/2,k} = \frac{1}{\Delta L^2}\left(r_{i+1,k} + r_{i,k} - 2\,r_{i+1/2,k}\right)$$

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i-1/2,k} = \frac{1}{\Delta L^2}\left(r_{i,k} + r_{i-1,k} - 2\,r_{i-1/2,k}\right)$$

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i,k} = \frac{1}{2\Delta L^2}\left(r_{i+1,k} + 2\,r_{i,k} + r_{i-1,k} - 2\,r_{i+1/2,k} - 2\,r_{i-1/2,k}\right)$$

from other hand we have:

$$\left(\frac{\partial^2 r}{\partial x^2}\right)_{i,k} = \frac{1}{\Delta L^2}\left(r_{i+1/2,k} + r_{i-1/2,k} - 2\,r_{i,k}\right)$$

consequently we have:

$$r_{i,k} = \left(\frac{2}{3}r_{i+1/2,k} + \frac{2}{3}r_{i-1/2,k} - \frac{1}{6}r_{i+1,k} - \frac{1}{6}r_{i-1,k}\right)$$

Similar formulas we can obtain for the points $\left(i, k+\frac{1}{2}\right)$ and $\left(i, k-\frac{1}{2}\right)$ of the grid. $\overline{\omega}_1$.
Further, for the purpose to remove peaks in the field of the relief, we expand values of the r(x,y) on the row of the Furie and third's of the last terms of the Furie's row are taken out.
The numerical modeling was realized in the domain 1000m*1000m*1000m. The space-temporary numerical grid has consisted 500*500*200 points. The steps of the grid along axes O***x*** and O***y*** were equal to 2m. The step of the grid along axis O***z*** was equal to 5m and on time 0.1s.
In order to compare numerical calculation processes during experiments, we have calculated integral characteristics of prognosis fields at each time step; namely we have calculated tendencies of meteorological elements variation in the area G in average time by the following formula:.

$$\left|C\right| = \sum_{n=1}^{N}\left|C_{i,j,k}^{n+1} - C_{i,j,k}^{n}\right| / N*\Delta t\,,$$

where $C = (U,V,C)^T$ is a matrix column, N is number of grid points.
Results of calculations had shown that $\left|C\right|$ variation had more stable, decreasing character.
It is well known, that if in finite–difference scheme the following quadratic quantities:

$$EK = 1/N\sum_{n=1}^{N}\left|U_n^2 + V_n^2\right| / 2\,;$$

$$\Omega^2 = 1/N\sum_{n=1}^{N}\left(\partial U_n/\partial x - \partial V_n/\partial y\right)^2;$$

are maintained over time period then instability does not occur [10,25]. In each experiment we were examining maintenance of EK and Ω^2 quantities during period of prognosis. Calculation results have shown that relative error of $\mathbf{\Omega}^2$ was not exceed 10^{-4} in average within physical 24 hours. Relative error of the EK. was not exceed 10^{-3}. This fact indicates that numerical scheme and solutions were stable.

6 Results and Discussion

First of all we have investigated air pollution problem of the main avenue of Tbilisi- Rustaveli Avenue by exhaust gases of motor transport. With the purpose to determine the average value of the intensity of the traffic by the formula:
$\boldsymbol{N}_{aver.} = 1/3\ (\boldsymbol{N}_{morning}+\boldsymbol{N}_{noon}+\boldsymbol{N}_{evening})$, we have observed motor transport movement in the Rustaveli Avenue for a week.
. Our observations have shown that in average it was about 3200 motor car per hour during rush hours. Usually there are observed three kinds of motor transports on the Rustaveli Avenue: passenger cars,

microbuses and buses. During our study the transport means have had the following distribution of intensity:

$N_{pass.} = 0,54$; $N_{mic.}=0,26$; $N_{bus.}=0,03$;

By means of obtained data and (1), we have determined masse of detrimental substance M_{nox}=0.936 mg, where M_{Ox} is a corresponding mass fixed in time unit, unit of distance. According to [3,4] and given masse of M_{Ox} we have determined the initial values of the concentrations. By means of the obtained data and mathematical model discussed above we have learnt distribution of the concentrations of the detrimental substance NO_x across the Ruistaveli Avenue. We have assumed that in the initial moment of time motor transport starts movement with the intensity of 3200 car per hour. Traffic keept movement with this intensity during fifty minutes. On the basis of the point of view that at the end of motor transport movement, function of the concentrations reaches its maximal value and in the numerical model the stationary movement after fifty minutes slows down noticeably. Also for the initial period of time (during twenty five minutes) the wind vector was directed from the North-West to South-East across the axis Ox (wind velocity accelerated, in accordance with the height, from 0 to 5 m/s and it reaches the maximal value at the height of Z=600m.). But further, after 25 minutes wind changed direction and air currents were directed from the West to East. This kind of wind vector's alteration is often observed in synoptic practice of Tbilisi. Results of numerical calculations have shown that after one hour of physical times values of concentrations were about 1.3 times more than initial values of concentrations. Some of the results of calculations are presented on the Fig.2.

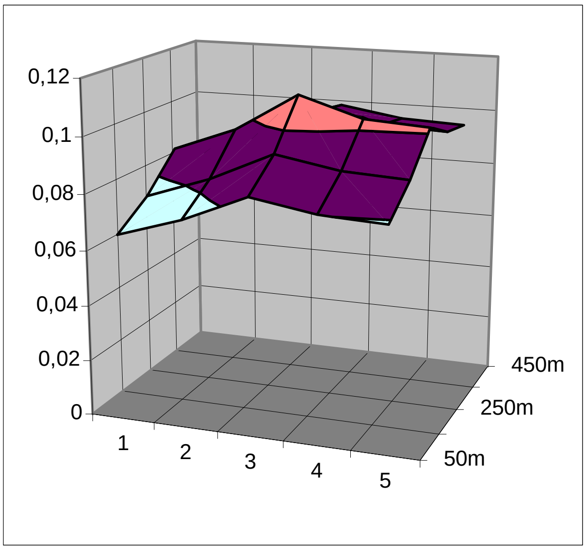


Fig.2

The Fig.2 shows that the maximum concentrations are observed at the central part of the Rustaveli Avenue, that is due to traffic congestion in the central area of the Rustaveli Avenue (lateral arterial streets with intensive traffics are joined the Rustaveli Avenue). Besides probably it is caused by mutation of wind's direction (we have calculated an experiment with constant wind direction during all calculation period and the results of calculations have shown more homogenous concentrations across the Rustaveli Avenue). Also the Fig. 2 shows that there are observed not very high values of concentrations. The point is that the Rustaveli Avenue is directed along the wind direction and the Rustaveli Avenue ventilated well.

Further we have learnt spreading of harmful substances on the intersection of Agmashenebeli and King Tamar Avenues as data of meteorological observations have shown that this place is contaminated high). On Agmashenebeli Avenue the intensity of the traffic was approximately 3000 cars per hour, and on King Tamar Avenue - 3600 cars per hour. Fig.3 illustrates the King Agmashenebeli and King Tamar Avenues allocation and distribution of whole complex of joint fourteen districts distribution. The traffic intensity of the different type cars on Agmashenebeli and King Tamar Avenues have been distributed as follows: Agmashenebeli Avenue -:$N_{pass.} = 0,38$; $N_{mic.}=0,40$; $N_{bus.}=0,05$. King Tamar Avenue - $N_{pass.} = 0,53$; $N_{mic.}=0,38$; $N_{bus.}=0,06$.

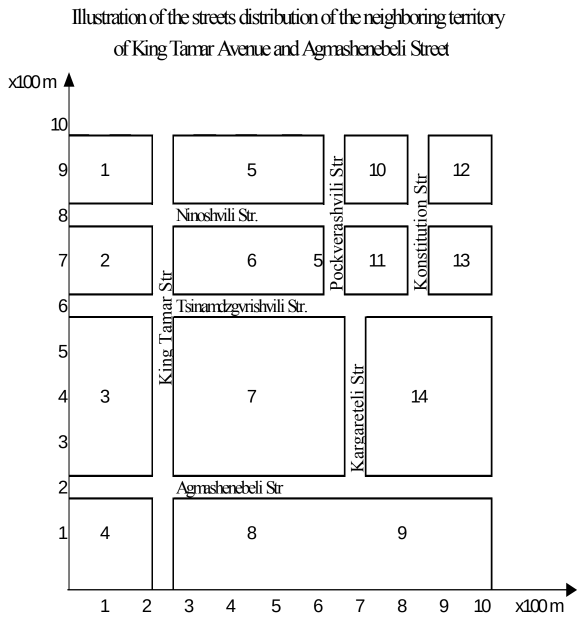
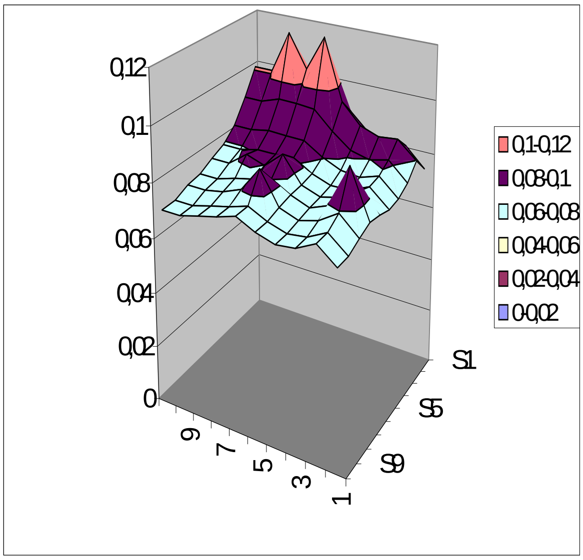


Fig.3

The results of numerical calculations have shown that, how it was expected, the highest concentrations of harmful substances was observed at the streets' intersection, which have exceeded the average daily maximum permissible concentrations approximately 1.5 times – up to 0.12 mg/m^3 (here and further under the harmful substances is meant concentrations of NO_x mg/m^3). With the growing distance from crossroad along the streets, amount of pollution was falling (from 0.09 to 0.4). The concentrations were relatively low at the outskirt territories - from 0.02 mg/m^3 to 0.03 mg/m^3 (Fig.4). High concentrations were observed in the neighborhood of the crossroad, at the south side of King Tamar Avenue and at the east side of Agmashenebeli Avenue. In addition, results of calculations have shown, that after the motor transport was stopped for ten minutes, the concentrations distribution was radically changed. Level of pollution grows from 0.06mg/ m^3 to 0.08 mg/ m^3 from south-east to north-west direction, because transfer of harmful substances was conditioned by the wind direction.



NOx harmful substances concentrations distributio after the motor transport 10 minutes begining at the neighboring territory of the crossing of King Tamar Avenue and Agmashenebeli street

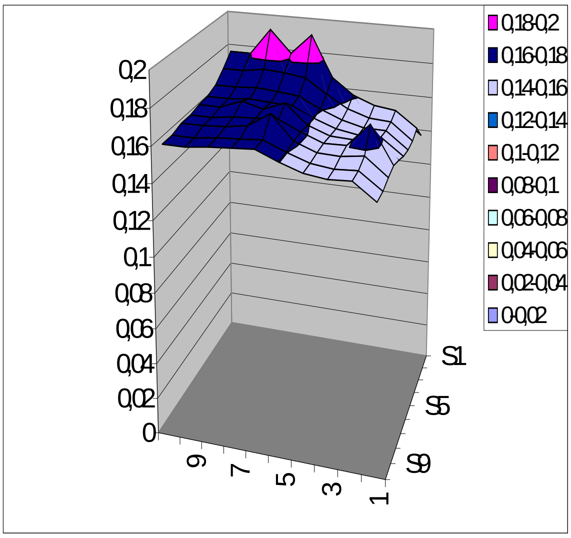
Fig.4

As pollution level radically decreased, we may conclude that during ten minutes the harmful substances were conditionally transferred out of the territory's borders.

We have also studied influence of light-signals in the streets on the distribution of harmful substances. There are light-signals at four angles of King Tamar and Agmashenebeli Avenues crossroad and there working cycle is 35 seconds. During this time, vehicles accumulate at the both sides of the avenues, which consequently leads to the growth of gas masses. The accumulation of cars reaches the average maximum distance of 50 meters away from the intersection. Consequently, in this radius, the growth of unhealthy gas depends on the traffic intensity at the present moment. In our case the concentration grew approximately 2 times (0.19 gr/m^3.).

We have studied the distribution of harmful substances concentration not only for the separate streets, but also for the whole complex of adjoined territories (fourteen districts). We have studied the intensity of transport movement in these streets of Tbilisi, which gave us the following substances: Agmashenebeli Avenue-3000 cars per hour; King Tamar Avenue-3600; Tsinamdzgvrishvili St.-1200; Ninoshvili St.-600; Pockverashvili St.-300; Konstitution St.-1000; Kargareteli St.-400. The intensity of movement among different types of cars are distributed in the following way: Tsinamdzgvrishvili St.-N_{ms}= 0.48; N_{mic}=0.45; N_{bus}=0.07.Konstitution St.-N_{ms}=0.65; N_{mic}=0.30; N_{bus}=0.05. For Agmashenebeli and King Tamar Avenues these significances are given above, for the rest streets the significances are:N_{ms}=0.77; N_{mic}=0.19; N_{bus}=0.04.

It is implied that at the initial time period, the motor transport begins moving with the maximum intensity. Such an intensive movement continuous for an hour, after which the movement radically ceases and after which we observe the distribution of gathered harmful substances concentrations at the streets' adjoined territory.



NOx harmful substances concentrations distributio after the motor transport 1 hour begining at the neighboring territory of the crossing of King Tamar Avenue and Agmashenebeli street

Fig.5

At the initial time period (after twenty minutes) the wind vector's was turned from the South-East to East direction and its speed grows in accordance with height from 0 to 7 m/c, and it reaches the maximum significance at Z=600 meters height. Calculations clearly have shown that the wind vector's alteration influence on harmful substances' concentration distribution. After wind's direction alteration the concentrations distribution monotonously increases from the North-West to the South-East direction at the limits from 0.08 mg/ m^3 to 0.12mg/ m^3, which was caused by motor transport intensity, presence of the light-signals and the wind vector's direction.

7 Conclusions

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Calculations have shown that the wind vector has influence on harmful substances' concentration distribution. The concentration distribution monotonously increases from north-west to south-east direction in Tbilisi. Existence of the light signals made a great effect on the concentration distribution at the territory in the radius of 250-300 m from Agmashenebeli and King Tamar intersection, at the places, where traffic is congested. As the distance from this point grows, concentration gradually decreases. Existence of light-signals also increased the harmful substances concentration approximately 2 times. Taking as a basis this experimental calculations, we conclude that the growth of harmful substances wholly depends on the traffic intensity as well as on the light-signal's placement and working cycle.

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