

Cell-based genetic algorithm and simulated annealing for spatial groundwater allocation

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Abstract: - A genetic algorithm and a simulated annealing approach is presented for the guidance of a cellular automaton toward optimal configurations. The algorithm is applied to a problem of groundwater allocation in a rectangular area consisting of adjacent land blocks and modeled as a cellular automaton. The new algorithm is compared to a more conventional genetic algorithm and its efficiency is clearly demonstrated. Also, comparison is made to a simulated annealing scheme. Finally, the proposed genetic algorithm is combined with simulated annealing to yield a new hybrid. The presented cell – based algorithm is different from related algorithms of the literature, as it relies on local interactions among land blocks. Moreover, it offers a framework for application to more general and detailed problems.

Key-Words: - genetic algorithm, cellular automata, groundwater, optimization, resource allocation, land use.

1 Introduction

Land management presents an interesting class of problems, in which the prevailing characteristic is the spatial distribution of various land uses and the allocation of resources throughout an extensive terrain with a multitude of features and needs.

Areas that need to be managed or reformed may be modeled through two-dimensional cellular automata. These are constructs consisting of adjacent cells and endowed with rules of local interaction. They have been combined with genetic algorithms and simulated annealing for the solution of various simulation and optimization problems.

Genetic algorithms are well-known biologically-inspired meta-heuristics, whose significance and principles are described in textbooks such as [1] and [2]. A plethora of applications can be found in the literature, particularly in relation to special combinatorial optimization problems, such as in [3].

Simulated annealing is based on an analogy to the slow cooling process of a solid. Its theory and applications are given in the text of van Laarhoven and Aarts [4]. A great variety of combinatorial problems has been treated by simulated annealing. A most notable application is the traveling salesman problem [5]. Simulated annealing has also been applied in conjunction with a genetic algorithm, such as in [6] and in [7]. In the latter paper a mixed problem involving both discrete and continuous variables has been presented.

Cellular automata date back to von Neumann. They were developed by Wolfram [8], as a fundamental modeling tool. Many natural, as well as social and economic systems have been simulated by the use of cellular automata [9]. In particular, cellular automata have been used for modeling forest fire propagation [10, 11] and urban development, along with producing possible land-use scenarios [12]. Genetic algorithms and simulated annealing have been employed in combination with cellular automata in order to devise transition rules, such that the automata could perform certain computational tasks [2, 13, 14].

A genetic algorithm was embedded into a cellular automaton as an optimization tool [7]. In the latter reference, as well as in the present work the objective is not to find transition rules, but to guide the system to optimal configurations. This point will be further documented in the discussion section.

The notion of neighborhood plays a significant role in cellular automata for the study of the local interactions and of the mechanisms that underlie the transitions from local to global conditions. The present work utilizes this role in the proposed special genetic algorithm and simulated annealing.

The present paper deals with the optimal allocation of groundwater to the land blocks (or cells) of a rectangular area, which may be viewed as a cellular automaton. The particular characteristics of the problem are examined in relation to the classical allocation problems.

The genetic algorithm presented for this problem guides the evolution of the cellular field toward an optimal configuration or “mosaic” of cells. The algorithm relies heavily on local interaction by forcing each cell to mimic one of its neighboring cells. This device promotes deep exploration but it also favors a gradual exploitation through the implicit diffusion of information due to the overlapping of the neighborhoods. A more conventional genetic algorithm is also employed in order to show the efficiency of the proposed scheme.

A simulated annealing approach is presented for the solution of the same problem. The variations in configuration involved in the annealing algorithm are also based on the use of neighborhood. The annealing approach is compared to the genetic algorithm and, finally, a mixed genetic-annealing algorithm is proposed.

The present work differs from treatments of optimization problems in related subjects in two respects. First, the use of groundwater as the resource to be allocated gives rise to a problem different from the classical allocation problems, due to the spatial nature of the objective function, involving distances from wells and a physical groundwater model. Second, the methodology presented emphasizes a genetic algorithm based on neighborhood concepts, in contrast to related literature, as it will be documented in subsequent sections.

2 Formulation and characterization of the problem.

2.1 Description of the problem

An artificial problem is considered for the purpose of presenting a methodological framework for dealing with land management combined with groundwater allocation. Specifically, a two-dimensional grid is formed, consisting of hypothetical land blocks. The same type of cultivation or, more generally, land use is assumed to apply to all the blocks of the rectangular area.

The resource needed in order to exploit the area is water that has to be transported from sources lying outside the rectangular area. Each one of the land blocks is given the choice of receiving water from only one of the available sources. The latter are extraction wells drawing

from an aquifer of infinite extent but of spatially varying hydraulic conductivity.

The cost involved in acquiring water is divided between pumping and transport from the wells to the individual blocks. The transport cost for each block is taken as proportional to the distance of the block from the respective well. The pumping cost is estimated via a steady-state groundwater phreatic aquifer model. The total cost results from summing over all land blocks and it forms the objective function that needs to be minimized.

The decision variables are the wells assigned to the various blocks. The situation can be depicted with the help of a colored two dimensional rectangle, in which the color of each block signifies the respective well. Fig.1 shows such a rectangle with three wells as water sources. Thus, figuratively, the problem consists in finding the optimum “mosaic” for the two-dimensional grid.

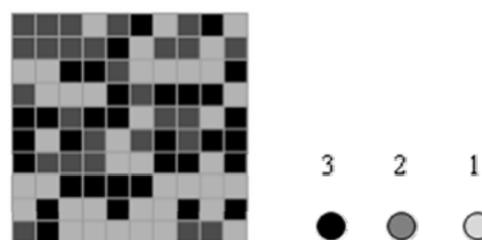


Fig. 1. Two-dimensional mosaic with three wells

2.2 Formulation

Let (i, j) be the coordinates of the center of the typical block with $i=1,2,\dots,a$ and $j=1,2,\dots,b$, where a and b are the lengths of the two sides of the orthogonal grid. The blocks can be numbered consecutively, so that if

$k=1,2,\dots,a \cdot b$, then

$$i = k - a \left[\frac{k}{a} \right], \quad j = \left[\frac{k}{a} \right] + 1 \quad (1)$$

where the brackets denote the integer part of the enclosed number.

Let m be the number of the wells and let the wells be numbered from 1 to m . Also, let

$w_k \in \{1, 2, \dots, m\}$ be the number of the well assigned to block k (Fig.2) with $k = 1, 2, \dots, a \cdot b$, according to the above numbering. Then the transport cost is

$$F_T = \sum_{k=1}^{a \cdot b} \left[(x_k - x_{w_k})^2 + (y_k - y_{w_k})^2 \right]^{1/2} \quad (2)$$

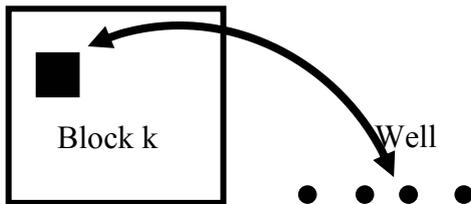


Fig. 2 Connection of block to well

where (x_k, y_k) are the coordinates of block k and (x_{w_k}, y_{w_k}) the coordinates of the respective well.

The pumping cost is expressed as follows:

Let $s_w, w \in \{1, \dots, m\}$ be the number of blocks irrigated from well w . Then the discharge from well w is equal to

$$q_w = \alpha s_w, \quad (3)$$

where α is a quantity representing the water needs of the block. This quantity is assumed to be uniform over the whole area.

The drawdown at each well is given by

$$\Delta h_w = \frac{1}{2\pi b} \sum_{\substack{w'=1 \\ w' \neq w}}^m \frac{q_{w'}}{k_{w'}} \ln \frac{\sqrt{(x_w - x_{w'})^2 + (y_w - y_{w'})^2}}{R} + \frac{q_w}{2\pi b k_w} \ln \frac{r_w}{R} \quad (4)$$

where b is the thickness of the aquifer, assumed constant, R is the influence radius, r_w is the radius of well w and k_w with $w=1, 2, \dots, m$ are the hydraulic conductivities of the areas around each one of the wells.

Finally, from Equations (3) and (4), the total pumping cost is proportional to the quantity

$$F_P = \sum_{w=1}^m q_w \Delta h_w \quad (5)$$

where q_w and Δh_w are given by Equations (4) and (5).

Thus, from Equations (2) and (5), the total cost can be taken to be equal to the sum

$$F = F_T + F_P. \quad (6)$$

2.3 Characterization of the problem

From the above formulation it can be seen that the present problem differs from the classical resource allocation problems, because the cost associated to each cell does not depend only on the quantity of the water to be supplied to the particular block.

It also depends on the position of the block itself. In fact this is true both for the transport cost, which depends on distances, and for the pumping cost, which is determined through the aquifer model with its predominantly spatial character. Moreover, the pumping cost is influenced not only by the well connected to the particular block, but also by the action of the other wells.

The solution methods to be described in the following sections are based on this spatial – cellular character of the problem domain. The present approach is further compared to the treatment of resource allocation problems of the recent literature in the similar context of forest planning.

3 Solution method

3.1 A natural genetic approach

The objective function, Equation (6), to be minimized is a function of the wells assigned to the various blocks. The problem is one of combinatorial optimization with a nonlinear objective function. It concerns a managerial interior arrangement in a field with a cellular character.

Evolutionary algorithms are particularly suited for this kind of resource allocation problem. Indeed, such methods have been applied to resource allocation problems [16, 3], as well as problems involving both water allocation and crop planning [16, 17, 18].

A natural way of encoding the problem in a genetic algorithm framework is to set up the typical chromosome as follows:

$$C_i = \{w_k \mid w_k \in \{1, \dots, m\} \text{ and } k = 1, 2, \dots, a \cdot b\}$$

$$i = 1, 2, \dots, N \tag{7}$$

where N is the population size.

As above, each one of the numbers w_k represents the well assigned to block k, as shown in Fig.

As it is well known (e.g. [1]), the genetic operators of selection, crossover and mutation will be applied to the N chromosomes, resulting in the population of the next generation.

The selection operator will follow the model of the roulette wheel. Crossover will be executed in a two-dimensional fashion, as this fits better the geometry of the problem. Namely, the rectangular area is divided into four parts by means of two random separators, one for each side of the rectangle.

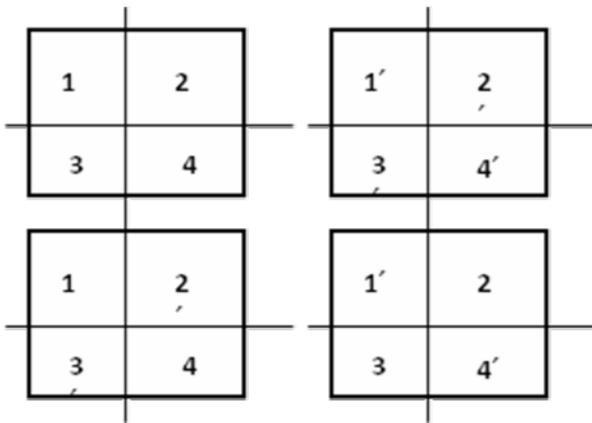


Fig. 3 Parents (above) and offspring (below)

The offspring produced are shown schematically in Fig. 3. Crossover of similar type has been implemented in various problems involving two-dimensional arrangements, such as in [19]. Finally, mutation is executed in the standard fashion.

3.2 An alternative genetic approach

An alternative scheme is presented here, that takes into account the cellular nature of the field to be reformed. The scheme considers the neighborhood of each block. The neighborhood is defined in the sense of von Neumann [9], as the set of “east – west” and “north – south” cells (Fig. 4). Obviously, for the boundary cells the respective neighborhoods contain less than four elements.

Let N_k be the neighborhood of block k and n_k be a selected neighbor of the same block ($n_k \in N_k$).

The typical chromosome will be defined as

$$C_i = \{n_k \mid n_k \in N_k \text{ and } k = 1, 2, \dots, a \cdot b\}$$

$$i = 1, 2, \dots, N \tag{8}$$

where N is the population size.

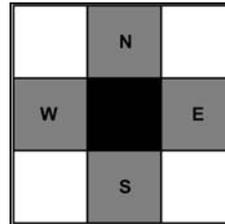


Fig.4 Neighborhood definition

Also, according to this notation, let $w_{n_k} \in \{1, 2, \dots, m\}$ be the number of the well assigned to the neighbor n_k .

The chromosome C_i of Equation (8) can be considered as a replacement rule that dictates to the block k to replace the well w_k assigned to it by the well w_{n_k} of the selected neighboring block n_k (Fig.5).

The appropriate choice of the suitable neighbor n_k will be put forward by the genetic algorithm.

The procedure runs as follows:

- (a) An initial reference configuration or mosaic is formed by randomly assigning a well to each one of the blocks.
- (b) An initial population of chromosomes of the type prescribed by Equation (8) is formed by randomly assigning to each block one of its neighbors.
- (c) The chromosomes of step (b) are evaluated by replacing in the reference mosaic the well w_k of each block with the well v_k of the neighbor n_k , as indicated in the position k of the chromosome, and applying the objective function (6). Thus the chromosomes may be considered as operators transforming the original mosaic.
- (d) The chromosome with the best value of the objective function transforms the original mosaic and a new reference configuration is obtained.
- (e) The evaluations of step (c) are followed by selection, crossover and mutation for all chromosomes. Selection is performed according to the roulette wheel model, crossover is executed by means of one random separator and mutation follows the conventional pattern.

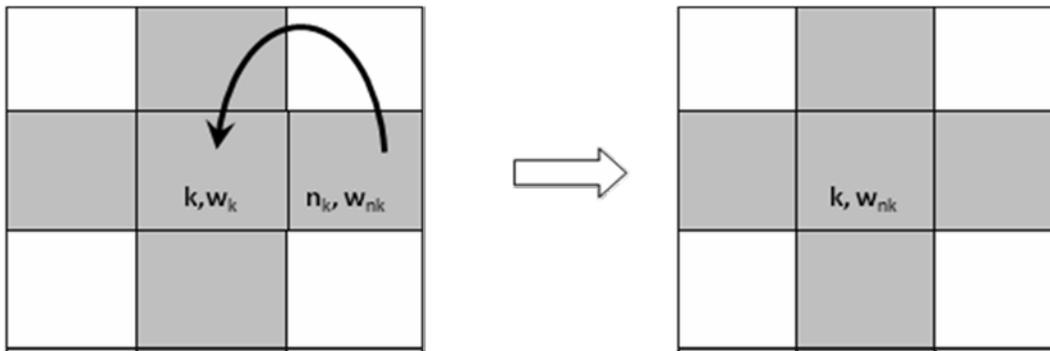


Fig.5. Replacement of well connected to block k with the well connected to block n_k

(f) The resulting renewed population is evaluated according to step (c) with the reference mosaic of step (d).

The algorithm is shown schematically in Fig. 6.

3.3 A simulated annealing approach.

For the simulated annealing procedure, let C be the current configuration of the problem, which has already been represented by the chromosome of Equation (7) and let C_1 be a small variation of C . If $f(C)$ and $f(C_1)$ are the corresponding objective

If $r < e^{-\delta/T}$, then C_1 is accepted.

Otherwise it is rejected and another variation of C is generated, which is also subjected to the above $\delta -$ test.

T is the so called temperature that decreases slowly according to a prescribed “annealing schedule”, which is a basic parameter of the annealing algorithm.

Another parameter is the mode of transition from configuration C to configuration C_1 . One of the ways to effect this transition would be to pick out two cells at random and mutually exchange the

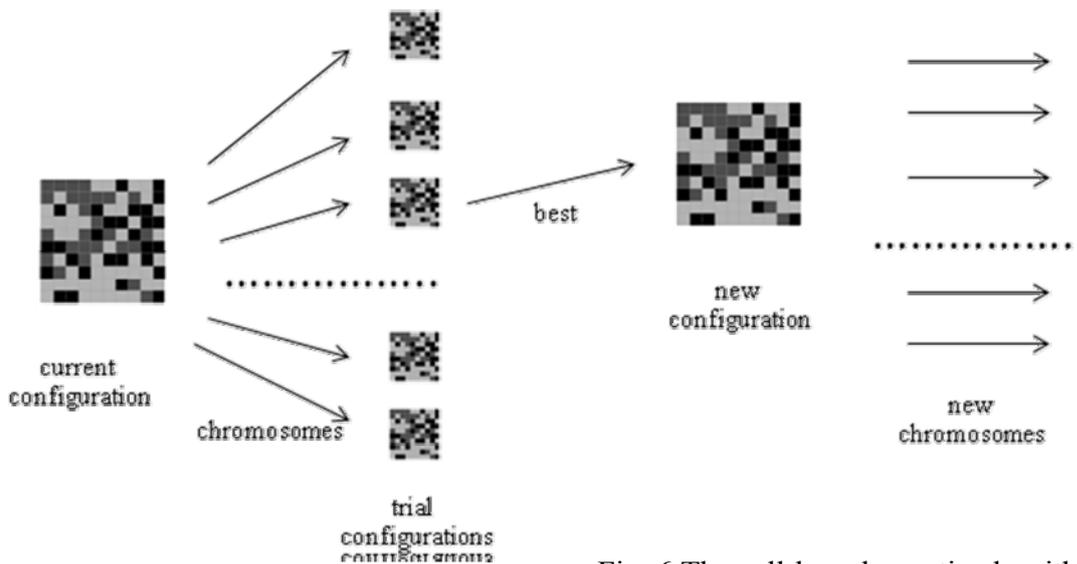


Fig. 6 The cell-based genetic algorithm

function values, then the quantity

$$\delta = f(C_1) - f(C) \text{ is formed.}$$

If $\delta < 0$, then C_1 is accepted as the new current configuration.

Otherwise, a random real number r is generated, that lies between 0 and 1.

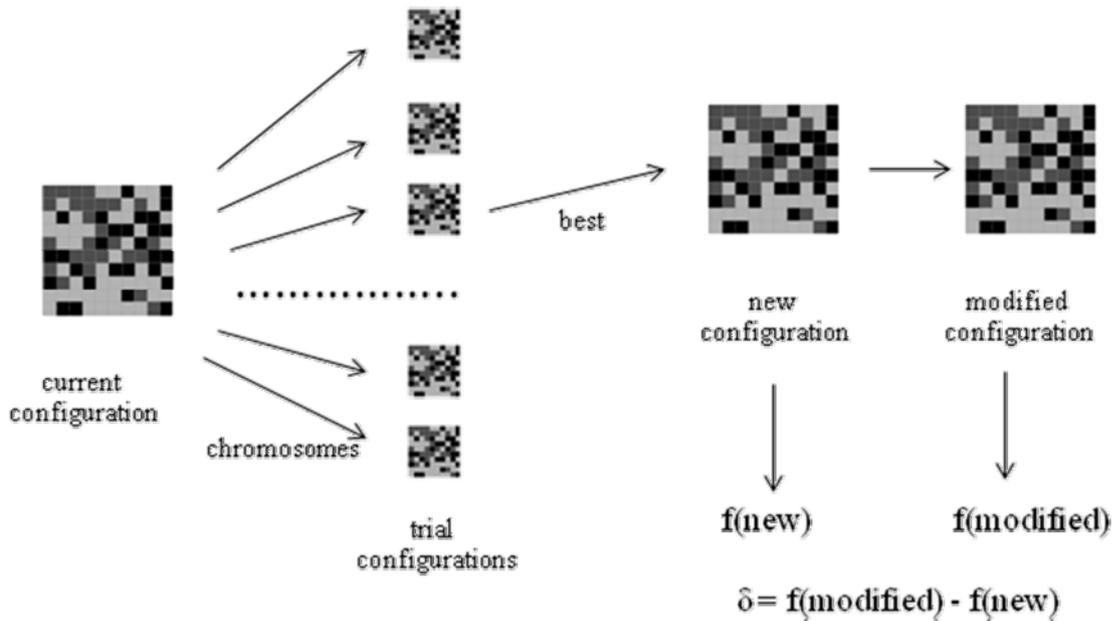
respective wells connected to these cells. This approach is followed in the spatial allocation problem of Aerts et al. [20].

For the present problem this type of transition did not yield good results. Instead, the following scheme was set up in accordance with the cell-based idea of the previous section:

Let p_s be a prescribed probability of change. For each one of the cells, a random number r is generated such that $0 \leq r \leq 1$. If $r < p_s$, the cell will replace the well to which it is connected with the well of a cell selected at random from its

The same type of interaction was tried with cells taken from the rest of the whole configuration and not just from the current cell's neighborhood.

In both cases the update of the cells was done in an asynchronous manner. This means that, if a cell



If $\delta < 0$, then
 new \leftarrow modified
 else if $\delta > 0$, then
 if $\text{random}(0,1) < \exp(-\delta / T)$, then
 new \leftarrow modified,
 else modified is rejected

Fig.7 Schematic representation of the genetic – annealing hybrid

neighborhood, as it was defined in section 3.2 and Fig.4. The replacement is illustrated in Fig. 5. If $r \geq p_s$, then the cell remains unchanged.

The formation of the new configuration is completed, when all cells have been subjected to the above procedure. It must be noted here that the cells are taken in a random order.

By imposing a probability of change, only a fraction of the total number of cells is affected, thus resulting in a small differentiation of the whole configuration.

is differentiated during the formation of the configuration, it uses its most recently acquired state for all subsequent interactions until the formation is completed.

3.4 A hybrid genetic – annealing approach.

The above described cell-based genetic algorithm can be endowed with a simulated annealing ramification if the current configuration that arises after every generation of genetic search, is subjected to annealing processing. Indeed, the configuration, that results after each generation lends itself

naturally to a possible further improvement through annealing.

In the literature genetic – annealing hybrids appear in numerous applications and it may be considered that they fall under two main categories. One broad category concerns using simulated annealing as a genetic operator in the framework of a genetic algorithm. In particular, simulated annealing complements the mutation operator. This basic idea has been applied to discrete optimization problems, such as in [21] and [22], as well as in continuous optimization, such as in [23], [24], [25]. Simulated annealing in combination to both mutation and crossover operators can be found in [26], following the parallel recombinative simulated annealing model of Mahfoud and Goldberg [27]. Another category is characterized by a sequential application of genetic algorithm and simulated annealing [28,29,30]. In these cases the best individual or a number of top individuals obtained by the genetic algorithm are directed to an annealing processing and subsequently inserted back into the population. As already alluded, the present approach

is closer to this latter category, although the current configuration, which is subjected to annealing, is not a population member, but a product of the genetic cycle. The present hybrid genetic – annealing scheme is depicted in Fig. 7.

4 Results

A 10x10 grid was set up with three wells placed in the following positions:

$$x_{w_1} = 20, y_{w_1} = 0$$

$$x_{w_2} = 18, y_{w_2} = 0$$

$$x_{w_3} = 15, y_{w_3} = 0$$

The hydraulic conductivities around the respective wells were

$$k_1=0.05 \times 10^{-3} \text{m/s}, k_2=0.5 \times 10^{-3} \text{m/s}, k_3=1.2 \times 10^{-3} \text{m/s}.$$

The thickness of the aquifer was taken as $b=50\text{m}$, the radius of influence $R=15\text{m}$ and the radii of the wells all equal to $r_w=0.10\text{m}$.

Both the conventional and the cellular-based genetic algorithm were tried with crossover probability $p_c=0.5$, mutation probability $p_m=0.01$ and a population size equal to 70. The cell-based algorithm clearly outperformed the conventional one, yielding an optimal configuration within less than 100 generations. The conventional algorithm

was unable to reach the same result even after 1000 generations.

As shown in Fig. 8, well no 3 is not represented in the configuration. The same is true of the best configuration produced by the conventional algorithm, shown in Fig. 9 and of the configurations produced by the annealing methods, as shown in Figs. 10 and 11 below.

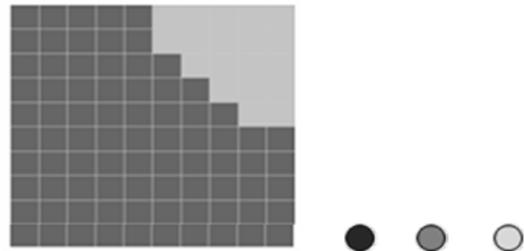


Fig. 8 Proposed cell-based genetic algorithm

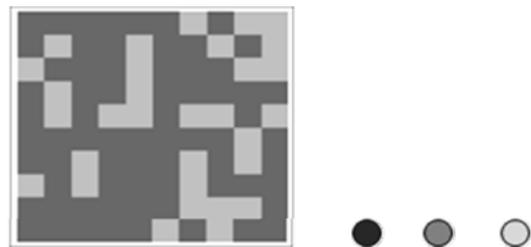


Fig. 9 Conventional land use map type genetic algorithm

Fig. 10 shows the configuration that resulted from the simulated annealing process, in which mimicking a neighbor cell took place, as described in section 3.3.

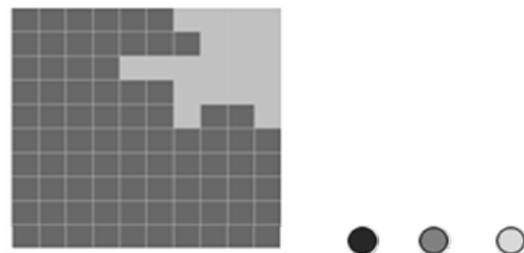


Fig. 10 Simulated annealing with neighbor interaction

Fig. 11 shows the best configuration produced by simulated annealing for the case where the neighborhood is taken equal to the whole rectangular field.

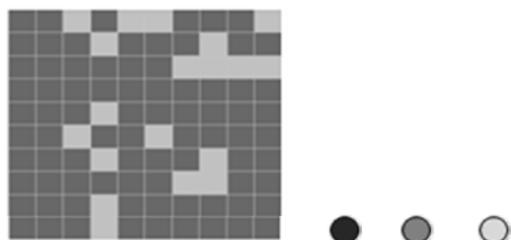


Fig.11 Simulated annealing with the cells mimicking any other randomly chosen cell.

It is noted here that the optimal configuration produced by the cell-based genetic algorithm was not rendered by the conventional approach or by the simulated annealing alone. The same result was only obtained by the hybrid cell-based genetic – annealing algorithm. The result of the hybrid algorithm was again obtained in less than 100 generations, while it took the annealing approaches 900 iterations to yield the configurations shown in Figs. 10 and 11.

A final remark could be made in relation to the better result of Fig. 10 compared to Fig. 11. In the first case the interactions take place within the neighborhood, contrary to what happens in the last figure.

5 Discussion - Conclusions

The optimization procedures presented in the previous sections have the concept of cellular automaton at their base. As it has already become evident from the previous description, cellular automata are characterized by

- (i) A two-dimensional cell lattice structure
- (ii) A set of possible states that the cells may realize
- (iii) A set of rules governing the transition from one state to the next.

In the present approach the cellular modeling has been suggested by the two-dimensional geometry of the land blocks arrangement. The state of each block – cell is identified with the well to which the cell is connected. Thus, in the particular example problem of the previous section the set of possible states contains three members.

The transition rules are not fixed under the methods presented here. In the case of simulated annealing the transition of the cells is done each time by mimicking another cell at random and in the case of the proposed genetic algorithm the changes of state follow varying directions rather than constant rules.

There is an extensive literature related to the evolution of cellular automata through genetic algorithms, with [13] representing a characteristic early example. In this category of problems a genetic algorithm is employed in order to determine the best rules or rule tables that enable a cellular automaton to perform certain given numerical tasks. The chromosomes of such a genetic algorithm represent possible rules that are evaluated in terms of performance and then subjected to genetic operators in order to be improved from generation to generation. The result of this process is an optimal rule that can be used henceforth, so that the cellular automaton can fulfill the desired function.

Clearly, in the present approach no constant rule is sought. The objective is to evolve the cellular automaton toward an optimal configuration and not to find an optimal rule. This point of view is adopted in a number of papers related to land use and forest planning, such as in [31], [32], [33], [34] and [35].

However, in all of these references the problems treated are classical allocation problems and do not carry the particular characteristics of groundwater allocation, as explained in section 2. Also, the objectives of the problems treated therein are different and, therefore, no direct numerical comparison can be made.

Nevertheless, the conceptual similarity allows useful methodological comparisons. Indeed, regarding the specific methodologies, Stewart et al. [34], representing the above group of papers, presented a genetic algorithm whose typical chromosome is a land use map, i.e. it consists of the collection of all cells accompanied by the respective provisional land uses. In contrast, in the present exposition, the typical chromosome does not reflect the current configuration of the cellular automaton, but it points to a possible way toward the upcoming new configuration.

The kind of genetic algorithm of the above references was used here as an example of a natural encoding of the water allocation problem, in order to obtain a standard of comparison for the alternative version. Thus, there is a clear methodological difference in the present work.

Another interesting feature of the above cited references is the consideration of local versus global

objectives that have to be satisfied through the optimization process. For that reason and because of possible arising conflicts, special objective functions were defined in those references. This treatment does not consider objectives of a local nature. However, because the nature of the present typical chromosome is local, any such additional local features could potentially be incorporated into the formulation.

The simulated annealing and genetic algorithm approaches have been compared in the literature to self-organizing algorithms applied to cellular automata in the context of land use planning ([31], [35]). The results obtained favored the use of the self-organizing algorithms. However, the genetic algorithms used in those references were equivalent to land use maps. The alternative algorithm presented here might offer a more efficient version of a genetic algorithm and put the comparison on a different basis. However, this issue and the consideration of local objectives and conditions are possible future directions of research.

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