Application of Seismic Tomography Techniques in Dam Site

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Abstract: - In this paper seismic data obtained from several boreholes located at Nimroud dam site in the east part of Tehran, was used to model 2D tomography of the subsurface model. To increase the accuracy in the modeling procedure , back projection , damped least squares and smoothest function method were used and their results were compared. The back projection method is an approximation weighted solution , while the second method is based on the damped least squares operator. The third method is more stable for estimation of the model parameter than the two previous mentioned methods. Analysis of the model sensitivity showed that , variation of the final residual time with respect to the obtained velocity values diagrams from four boreholes at dam axis differs from the other diagrams. These differences are between – 25 to + 10 percentage greater values. The geological information , sample core from the boreholes as well as the velocity values abnormality obtained at the study area were attributed to a hidden fault at the dam site area.

Key-Words: - Seismic Tomography , East of Tehran , Back Projection method ,Damped least square method , smoothest function ,fault.

1 Introduction

In the past twenty years, Seismic tomography Method has became so widespread among the geologists. Historically speaking the concept of tomography dates to a few decades ago (Radon , 1917 and Denis , 1913). McMechan (1983),Warthington (1984) and Stewart (1985) have applied tomography methods using seismic data in geologic studies[4]. Seismic tomography is a special kinds of inversion problem solution. The function itself is derived , using integrals of a function. A great number of geologic issues are put forth in this way . The ultimate goal of seismic studies is to view the section of underground , which is called tomography in seismic exploration . Tomography is measuring the amount of energy given off in a medium. The features which indicate the energy transmission are amplitude and travel time. In many prospecting such mature sedimentary basins related to the structure of geology, must access a lot of data about boreholes, drilling cores , and at least a model of geology. Regarding big amount of information available , we must apply a method which uses all the data and gives a clear picture of lithologic structure. Using the massive data resulting from field operation in the form of hole– hole and in hole in a tomography design will bear no good results. A

solution with high coefficient and low lithologic image ambiguity in exploration are such shortcomings. In recent years , geologists have successfully used seismic tomography to image velocity variation in earth[4].

2 Forward and inverse modeling

In geophysics one speaks of inversion methods when trying to determine physical and / Or geometrical properties of the subsurface or an object (e.g. drill core) from indirect model is derived from the data(fig. 1).
On the other side there is the method modeling where synthetic data or the theoretical Response from the model are calculated. This is called direct problem in oppositeto the inverse problem(fig. 2). The direct problem has always an unique solution whereas the inverse problem is mostly Characterized by a lack of

unambiguity be cause limiting assumptions have to be made About the model properties –e.g. layered-cake model, constant velocities/conductivities etc. in each layer –and measuring errors or interfering signals (noise) in the data cannot be represented by the model .Often geometrical parameters like depth, layer thickness , lateral extention are explicity added as unknown values

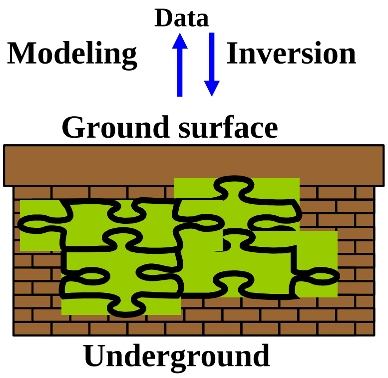

Fig. 1 Fundamental of forward modeling and inversion

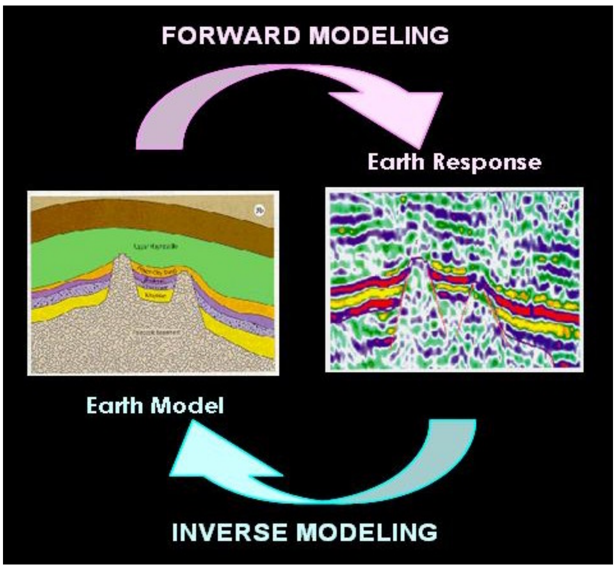

Fig. 2 forward and inversion modeling[1]

There are many different points of view regarding what constitute a "solution" to an inverse problem. If one merely wants to know the numerical values of the model parameters, then the problem becomes one of estimation. Very infrequently, however, is it possible to solve an inverse problem in such a way as to obtain this kind of exact information. Instead, we are often forced to make compromises between what information we desire to get from a solution and the kind of information that in fact is contained in the data. These compromises lead to other kinds of "answers" that are less direct than parameters estimates. Indeed a major part of the practice of inverse theory is to identify what features of a solution are most valuable and then make the compromises that help emphasize these features. we devote a considerable part of this course to this aspect of Inverse Problem.

Relationship of model parameter and observed data:

d=**Gm** (1)

d=observed data vector

G=kernel matrix

m=model parameter vector

m = inverse (**G**) **d** (2)

3 Traveltime Seismic Tomography

Tomography is the process of obtaining a 2 and 3D images of a slice through the interior of a 3-dimensional object by passing rays of some sort through the object and observing how the rays are affected (attenuated or delayed in time) by passage through the material.There are applications of tomographic imaging in many different fields. In medicine, methods include X-ray tomography, ultrasonic tomography, magnetic resonance imaging, and positron emission tomography (PET). In the earth sciences, tomography is widely used In seismic and EM imaging. In cross-well seismic tomography, for instance, acoustic waves are generated within one well, and received by sensors in a second well. In cross well radar tomography, ground penetrating radar waves (at frequencies of hundreds of MHZ) are used. In seismic tomography of the deep earth, seismic waves from naturally occurring earth quakes are sensed at stations on the surface of the earth. Tomography techniques are also used in the analysis of controlled source seismic data generated by reflection of seismic waves from geological structures. Tomography depends on the assumption that energy from the transmitter follows ray paths through the 3-dimensional body to receivers. Under this assumption we can evaluate path integrals along ray paths to determine travel times and attenuations. Of course, computing the travel time or attenuation from the known physical parameters of the 3-dimensional problem is a forward problem. What we want is to determine the physical parameters which result in the observed travel times or attenuations.

In the simplest case, ray paths follow straight lines with essentially no refraction (bending rays) or diffraction (scattering from small features).it is plotted in fig. 3. The problem becomes nonlinear and much more difficult as we consider refraction effects which result in curved ray paths, and especially when we consider diffraction effects. The straight ray tomography problem, however, can be formulated as a linear inverse[1]

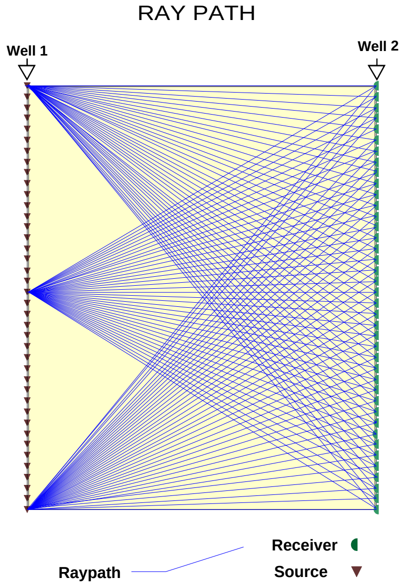

Fig.3 Raypath ,Source and Geophone Location in Wells

3.1 Tomography formulation

The starting point is the traveltime integral of seismic ray from the source to the receiver.

Travel time is given by:

$$t = \int_{\ell} \frac{dt}{dx} dx = \int_{\ell} \frac{1}{v(x)} dx \qquad (3)$$

V(x)=velocity
S(x)=1/v(x)=slowness

We can linearize by making path straight lines. In practice, we evaluate the line integral numerically by discretizing the object into blocks of dimension Δl, and assume that S(x) is constant in each block as is shown in fig. 4 for blocks. This allows us to write the travel time approximation.

Discreet form of equation:

$$\Delta t^{i} = \Sigma \, \Delta l_{j}^{i} \; s_{j} \qquad (4)$$

For example,Side of length 1 and the size of large square in fig.4 is 3*3. Medium is divided and then subdivided to regular cubes in which we assume "slowness" (parameter of the problem) is constant. Transmit the ray along paths and collect temporal data to be used in estimating slowness

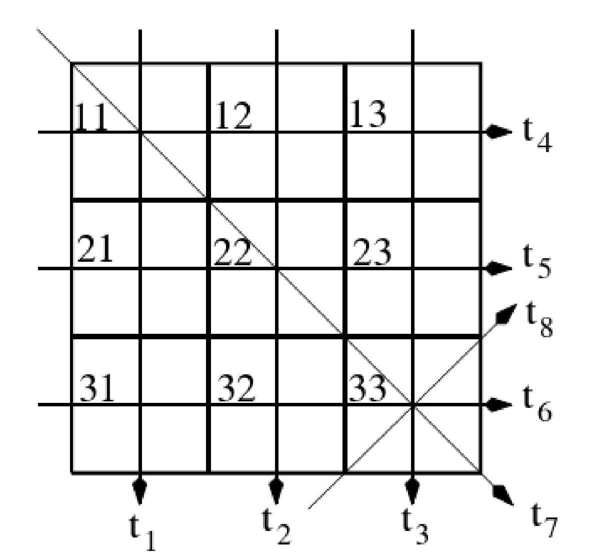

Fig. 4 Discreet section or cellular model and raypaths

G= l ij=distances matrix (kernel)
m= s ij=slowness (model parameter
s ij=slowness (model parameter)
d=ti=ray travel time (observed data)
Equation of (1) Show G, m and d relationships.

Extended form as matrix equation:

$$Gm = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \\ s_{21} \\ s_{22} \\ s_{23} \\ s_{31} \\ s_{32} \\ s_{33} \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{bmatrix} = \mathbf{d}$$

3.2 Tomography Technique

Generally Tomography can be defined as some internal parameters of an object , which are measured from out and clarifies the values of the parameters. It should be pointed out that the parameters in question determine the value of physical property. Then , we initially require data gathering .Then we come up an image of the object using appropriate inversion problem solution. Most of energy transmission is done through seismic source to a geophone. In recording the seismic waves data , travel time and amplitude are measured in the range of 5 to 25KH

These ranges or travel times are considered in the form of projection or sum of values , amplitude attenuation or slowness in the direction the energy passed. The sum or the integral the mentioned values are known as projection or radon transform. Radon transform or projection , are data gathering or direct problem of . Body image or the object under study are produced through radon transform inverse[8].

3.2.1 Back Projection Technique

The Back projection technique is an approximation weighted solution. It is a weighted average from calculated slowness(1/v). The slowness cell is produced by normalizing of sum of transmission raypath in every cell.

Equation systems:

$D1p1=d_1^1\ p1+ d_1^2\ p2 + \ldots + d_1^N\ p^N$

$D1p1=d_2^1\ p1+ d_2^2\ p2 + \ldots + d_2^N\ p^N$

. . . .

. . . .

(6)

. . . .

$D_m p_m=d_m^1\ p^1+ d_m^2\ p^2 + \ldots + d_m^N\ p^N$

Eqation matrix form:

$\mathbf{D}\ \mathbf{P}=\mathbf{A}^T$ (7)

$D_{j} = \Sigma d_j^i$ (8)

p_j=weighted mean of slowness

$p_j = \Sigma d_j^i\ p^i / D_j$ (9)

3.2.2 Damped Least Square

If we demand a solution for **m** from a set of equations **d=Gm** where no unique solution is possible, fundamentally because the data **d** do not provide sufficient constraints to define a unique **m**, then apply some additional constraints on the values that **m** can take. Generalized matrix inversion automatically applies the constraint that squared norm of **m** should be a minimum the additional constraints applied by damped least squares inversion are essentially the same but subtly different.

The standard least square solution to equation (1) *is:*

$$m=[G^T G]^{-1} G^T d \quad (10)$$

The damped least square is equivalent to minimizing the weighted sum of both:

$$|\mathbf{Gm}-\mathbf{d}|^2 + \varepsilon^2|\mathbf{m}|^2$$

$$\mathbf{E}=(\mathbf{Gm}-\mathbf{d})^T(\mathbf{Gm}-\mathbf{d})+ \varepsilon^2\mathbf{m}^T\mathbf{m} \quad (11)$$

$$=\mathbf{m}^T\mathbf{G}^T\mathbf{Gm}- \mathbf{m}^T\mathbf{G}^T\mathbf{d}-\mathbf{d}^T\mathbf{Gm}+\mathbf{d}^T\mathbf{d}+ \mathbf{d}^T\mathbf{d}+ \varepsilon^2\mathbf{m}^T\mathbf{m}$$

Minimize E:

$$\partial \mathbf{E}/\partial \mathbf{m}^T=\mathbf{G}^T\mathbf{Gm}-\mathbf{G}^T\mathbf{d}+ \varepsilon^2\mathbf{m}=0$$

Therefore

$$[G^T G + \varepsilon^2\mathbf{I}]\mathbf{m}=\mathbf{G}^T\mathbf{d}$$

$$\mathbf{m}= [G^T G + \varepsilon^2\mathbf{I}]^{-1}\ \mathbf{G}^T\mathbf{d} \quad (12)$$

3.2.3 Maximum Smooth

The stability in solution produced by a damping factor **I** in formula[5].

The smoothest function in also minimizing the travel time residual and perturbation velocity value in the two neighboring cells.This method longer to calculated to the other methods. MS give better results about the detection of body dimentions.

This method considers the velocity of the neighboring cells prevents over smoothing by making the cells smaller.

The smoothest function is used when the final model gained from other models as initial guess is considered.

Smoother function:

$$[m_{i,j}^{q+1}]' = [m_{i-1,j-1}^{q+1}+m_{i-1,j}^{q+1}+m_{i-1,j+1}^{q+1}+m_{i,j-1}^{q+1}+ m_{i,j}^{q+1}+ m_{i,j+1}^{q+1}1+m_{i+1,j-1}^{q+1}+m_{i+1,j}^{q+1}+ m_{i+1,j+1}^{q+1}]/q$$

i=cell row

j=cell column

q=iteration

′=new amount of velocity

4 Comparision of Techniques

Back projection techniques is the method approximate solution for the body under study in the tomography methods and it is the most anomalous region in the medium detected. The basis of damped least square is minimizing travel time residual from an end and the perturbation of velocity value of the body under question. The findings of this method are better than that of backprojection method , but it takes longer time for computerized calculation . To control the results, it is necessary to minimize the least square solution by damping factor[7]. The basis of deciding the smoothest function is also minimizing the travel time residual and perturbation velocity value in the two neighboring cell in a cellular mode. Minimization is done in conjugate gradient the more modern method of which is least square method. This method lasts longer to calculated compared to the other methods , but gives better results about the detection of body dimension. This method considers the velocity of the neighboring cells is the same and prevents over smoothing by making the cells smaller. This method is used when the final model gained from other models as initial guess is considered. In this study , there are two major steps to come up with a geologic section . First , an appropriate initial model was chosen based on all the geologic and geophysical information. Then , the most optimal method was chosen. Supposition that the initial model is the actual , gained from boreholes and bed lithologic condition in the section. In the second step , from among backprojection least square and the smoothest function , depending on the section condition either least square method or smoothest function led to the final model of velocity–depth optimum. Because both of the methods apply iteration algorithm and minimization of observed and calculated time.

Three tomograms is plotted in fig. 7 that they are related to the pair wells BN 7-5.

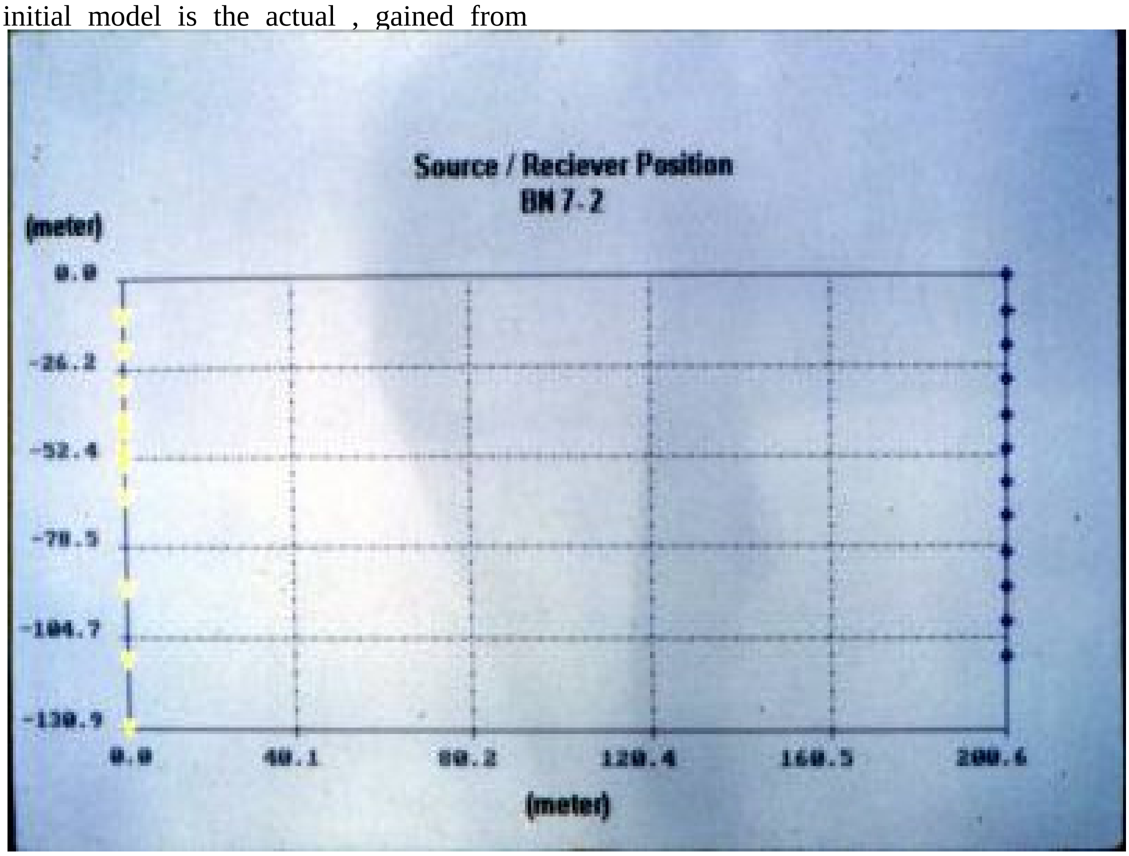

Fig. 5 Source and receiver locations of 7-2 boreholes in dam site

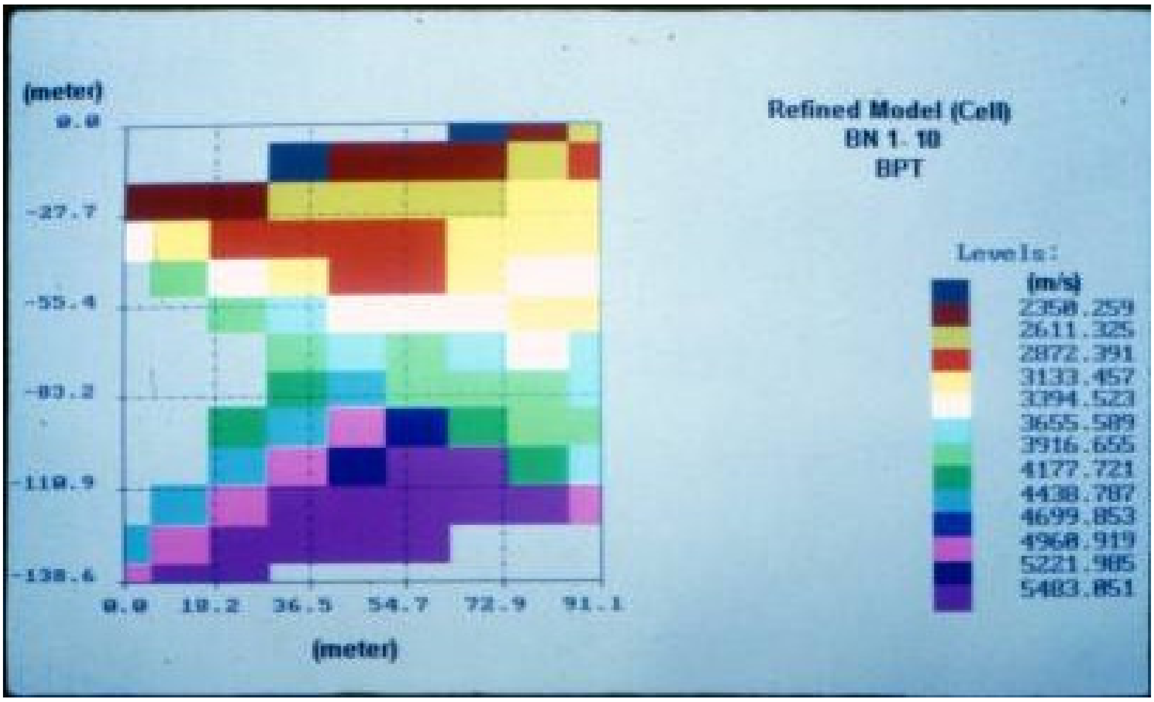

Fig. 6 Inverted Cellular model of 1-10 boreholes section from back projection technique

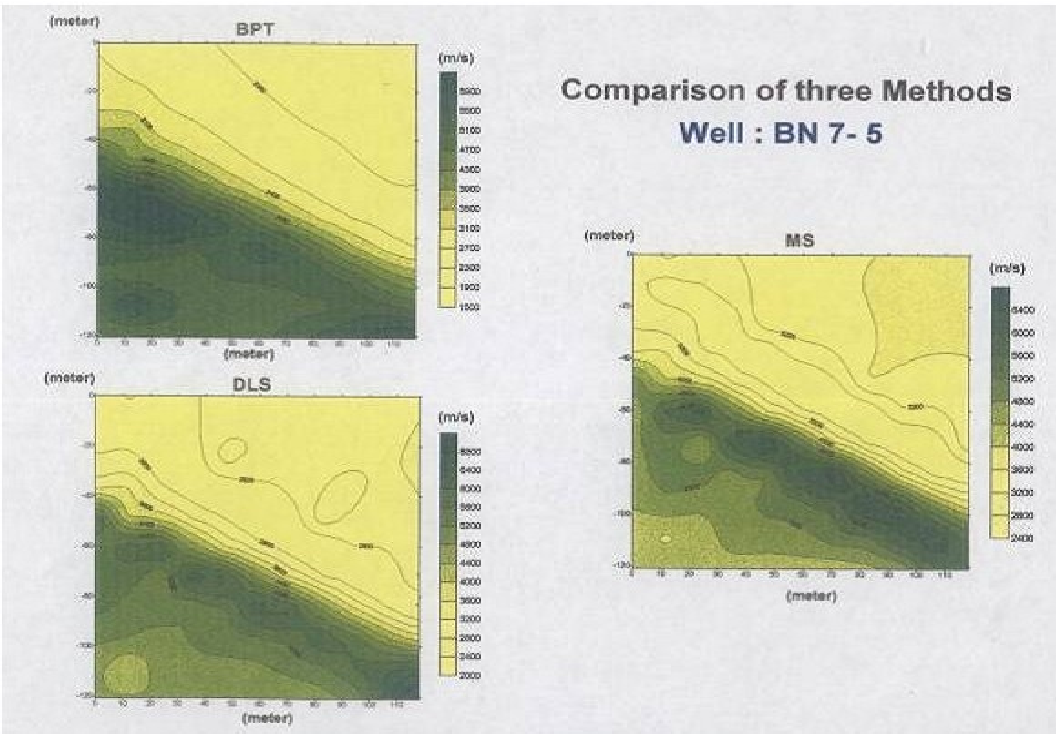

Fig. 7 Tomograms of three methods (backprojection , maximum smoo smooth and damped least square

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5 Sensitivity Analysis

It is necessary one forward model and its response for inversion. if we have geologic model , we can get its geophysical response (gravity ,magnetic or seismic signature).

It must be selected a optimum algorithm or good mathematical technique then, the initial model parameter approach real model. The quality of inversion relater to finding optimum forward model and algorithm .the model sensitivity analysis is proposed for the first step of inverse solution.[ ] For example, It is not happened when we change a parameter 10, 15, 20 percent . therefore, the model is not sensitive with respect to this parameter. one velocity was attributed to initial model and was changed (-30 to +30) percent then the root mean square(RMS) residual times was calculated and have been plotted in fig. 8 (a)-(d),respectively.

Analysis of the model sensitivity showed that , variation of the final root mean square(RMS) residual time with respect to the obtained velocity values diagrams from four pair boreholes at dam axis differs from the other diagrams. These differences are between – 25 to + 10 percentage greater values[9]

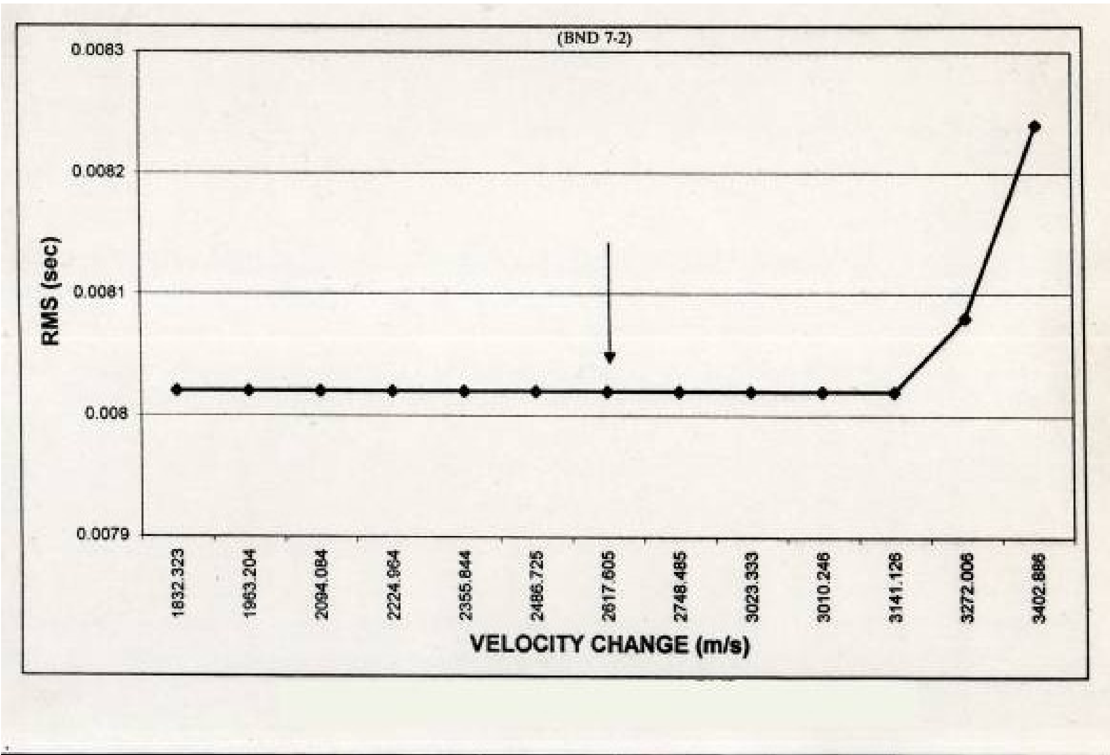

Fig. 7(a) Relationship of root mean square residual time with respect to velocity change for 7-2 boreholes

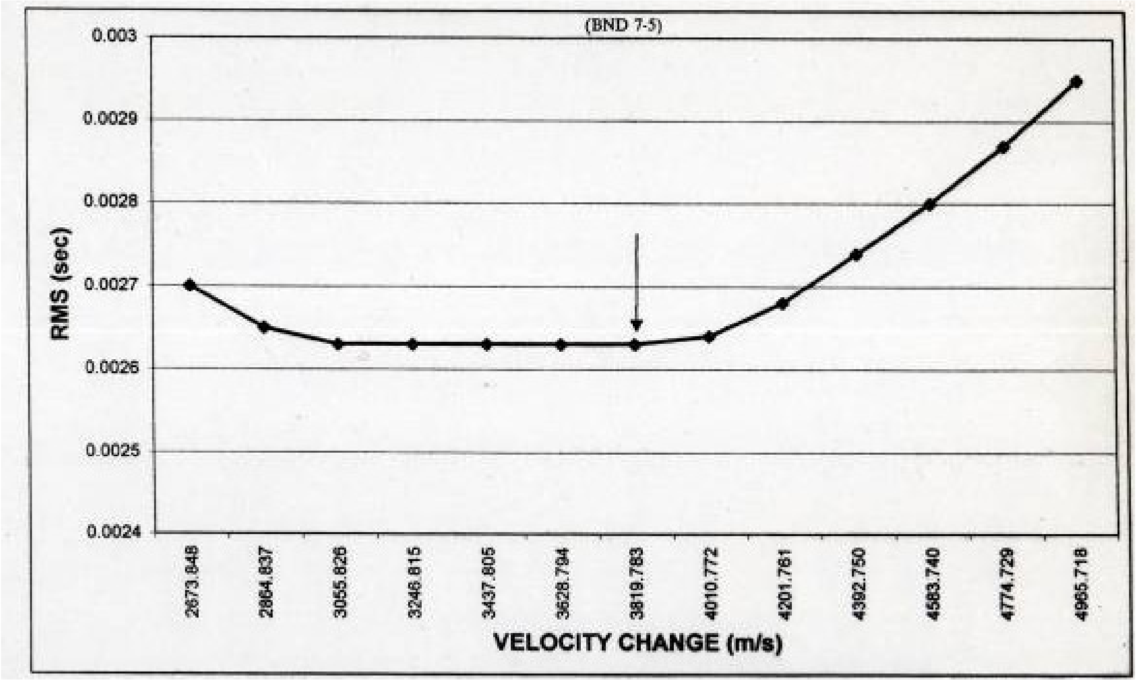

Fig. 7(b) Relationship of root mean square residual time with respect to velocity change for 7-5 boreholes

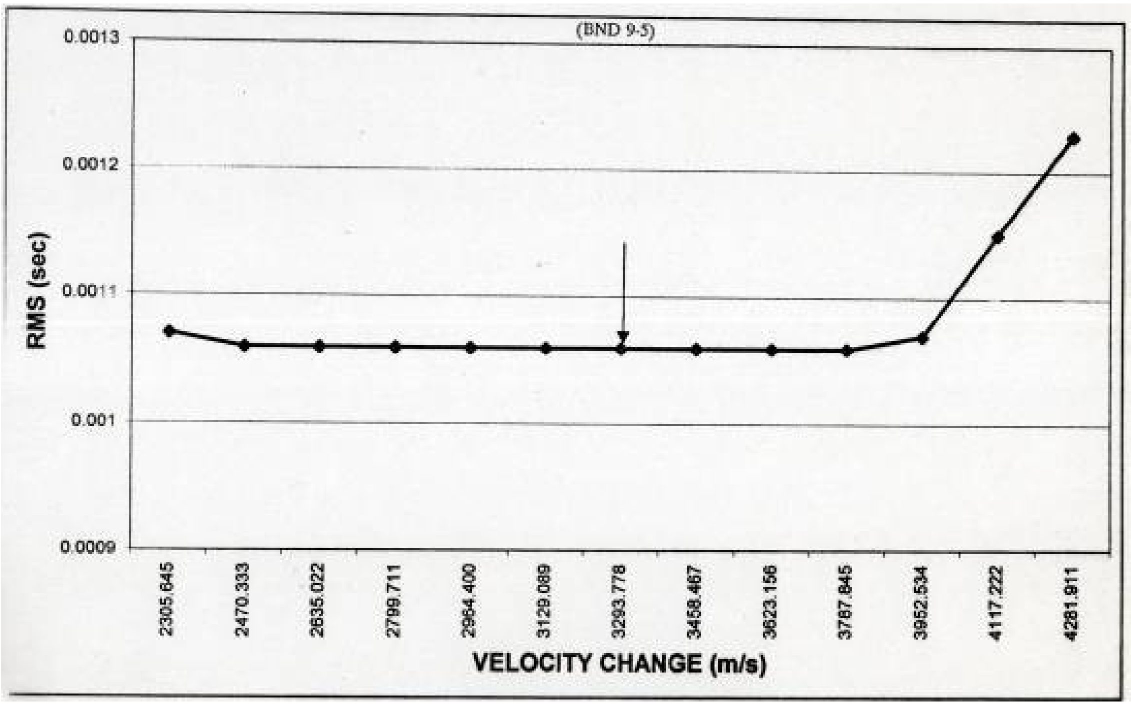

Fig. 7(c) Relationship of root mean square residual time with respect to velocity change for 9-5 boreholes

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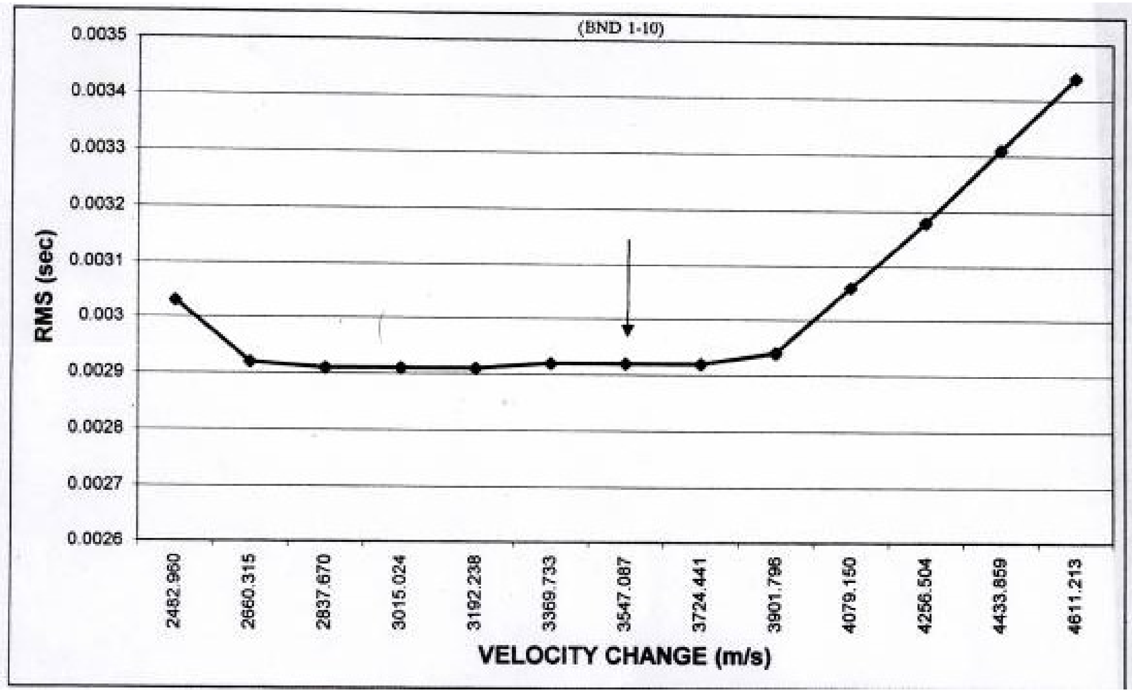

Fig. 7(d) Relationship of root mean square residual time with respect to velocity change for 1-10 boreholes

6 Conclusion

In this research the tomography studies on seismic data from boreholes showed that the change of final residual travel time with respect to velocity change parameter in the curve from the two borehole in the range of dam axis in the range of -25 to + 10 dramatic change in residual times the other three cases. The studies have attributed the problem to the presence and the effect of fault phenomena. Following the geologic studies of the dam site in east Tehran addressing a local underground the anomaly showed by the tomography study of the dimension and extension , is attributed to the existing fault.

Appendix

Review of Linear Algebra

$\mathbf{G}^{-1}$= inverse matrix

$\mathbf{G}^{T}$= transposed matrix

$\mathbf{Gm = b}$

$\mathbf{G^{-1}GM = G^{-1} b}$

$\mathbf{I\ m = G^{-1} b}$

$\mathbf{m = G^{-1} b}$

A matrix is symmetric if:

$\mathbf{G=G^{T}}$

The following statements are true:

1. $\mathbf{A+0 = 0+A = A}$

2. $\mathbf{A+B = B+A}$

3. $\mathbf{(A+B)+C = A+(B+C)}$

4. $\mathbf{A\ (BC) = (AB)\ C}$

5. $\mathbf{A\ (B+C) = AB+AC}$

6. $\mathbf{(A+B)\ C = AC+BC}$

Linear independence:

7 .(s t)$\mathbf{A}$ = s(t $\mathbf{A}$)

8. s ($\mathbf{AB}$) = (s $\mathbf{A}$)$\mathbf{B}$ = $\mathbf{A}$(s $\mathbf{B}$)

9.(s +t)$\mathbf{A}$ = s $\mathbf{A}$+ t $\mathbf{A}$

10. s($\mathbf{A}$+$\mathbf{B}$) = s $\mathbf{A}$ + s $\mathbf{B}$

11.$(\mathbf{A}^{T})^{T} = \mathbf{A}$

12.$(s\mathbf{A})^{T} = s(\mathbf{A}^{T})$

13.$(\mathbf{A}+\mathbf{B})^{T} = \mathbf{A}^{T} + \mathbf{B}^{T}$

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14. $(\mathbf{AB}^{T}) = \mathbf{B}^{T}\mathbf{A}^{T}$

15. $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

16. $(\mathbf{A}^{-1})^{-1} = \mathbf{A}$

17. $(\mathbf{A}^{T})^{-1} = (\mathbf{A}^{-1})^{T}$

18. If **A** and **B** are n by n matrices, and **AB** = I, then $\mathbf{A} = \mathbf{B}^{-1}$

And $\mathbf{B}^{-1} = \mathbf{A}$

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