

Modelling and Simulation of Step-Up and Step-Down Transformers

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Abstract: The current and power (active and reactive parts) at the terminals of the step-down transformer are positive if the transit is in line busbar to charge. The study of the dynamic behavior of the system developed in the later, will use numerical simulations and the calculation of eigen values.

Key-Words: Step-up and step-down transformers, Power model and current.

1 Introduction

The initial state of the system will be a steady one. For this, we must give the value of state variables of system and also the interface variables between different blocks of the model. The value of some of these variables can be chosen arbitrarily (within the limits of validity of models), the value of others may be inferred from the first through the use of relationships between variables in steady state. The definition of an initial state is to choose a set of variables of the system so we can assign a value independently and that the set of all other variables in the system can be inferred. We will describe later in this section the overall system initialization, that is to say the choice of independent variables and calculating the interface variable models of areas and lines. Initialization within each component models will be presented at the same time as the description thereof. In steady state, the frequency is the same everywhere in the system. We assume here that the initial state, this frequency is the nominal frequency. We take as a basic option that the active production and voltage generators are given, and also the active and reactive power of the charges for voltage and nominal frequency. In this context, in general, the active power balance is not checked. Therefore, the active production of one of the generators will not be fixed (node beam). In this node, a reference phase is chosen. Compared to the conventional approach to charge flow, it is necessary to make the following remarks [1],[3],[7]

1) The voltage generator is set to stator terminals of the alternator. We assume here that the voltage is fixed to the bus bar high voltage area. If the step-up transformer of the production group is an ideal one,

there is a relationship of proportionality between the two. If the model of transformer includes internal impedance, the voltage across the alternator stator is calculated from other variables, including voltage busbar HT, as will be described later in the paragraph on the initialization of variables of the step-up transformer (Fig. 1).



Fig. 1: Power three-phase oil transformers.

2) For the initial situation, the voltage across the charge is not necessarily the nominal voltage. If the models are including a sensitivity of active and reactive power charges in tension, it must be taken into considerations in order to determine the powers and the initial tensions. If the step-down transformer is an ideal transformer, the voltage across the charge is proportional to the voltage node HT, supposedly given in the initial conditions. Therefore the active and reactive power consumed can be calculated. The initialization of variables relating to charges and step-down transformers will be developed in Section 2.

3) What has been said about the tension can also be about the active power of group generator. We assume that the net active power produced is known, it is said the power delivered to the busbar HT.

2 Basic Model of a Transformer without Losses

As part of the study presented here, we can reasonably suppose that the magnetizing current of the transformers and the losses by Joule effect in the windings have a negligible influence [11], [16]. Therefore, the model of the transformer is reduced to a series reactance X and an ideal transformer of ratio $1:r$. The scheme and the notations used are given in Fig. 2. The amplitude of the input voltage is U_{in} and its phase is θ_{in} , whose time derivative is proportional to frequency f_{in} .

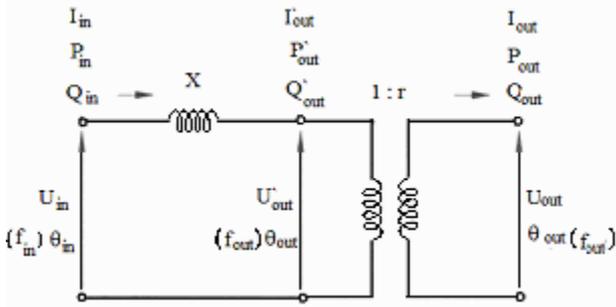


Fig. 2: Schematic of the transformer.

The active and reactive powers to the entry are P_{in} and Q_{in} , the input current being I_{in} . The quantities corresponding to the output are designated by the “out” suffix. As the transformer is without active losses active, we have: $P_{in}=P_{out}$ and $I_{P,in}=I_{P,out}$. Note also that the ideal transformer preserve active and reactive powers and phases, and therefore the frequencies. The dephasing of voltages between input and output is given by [12]:

$$\theta = \theta_{in} - \theta_{out} \quad (1)$$

Therefore, the relationship between frequencies is:

$$2\pi(f_{in} - f_{out}) = \frac{d}{dt} \theta \quad (2)$$

2.1 The Basic Equations of the Current Model of the Transformers

From the diagram in Fig. 3, we can establish the following relations:

$$\bar{I}_{in} = \bar{I}'_{out} = r \cdot \bar{I}_{out} \quad (3)$$

$$\bar{I}'_{out} = I'_{P,out} \cdot \bar{I}'_{P,out} + I'_{Q,out} \cdot \bar{I}'_{Q,out} = r \cdot \bar{I}_{out} \quad (4)$$

where $\bar{I}'_{P,out}$ denotes the vector of unit length oriented along U'_{out} , and $\bar{I}'_{Q,out}$ denotes the vector

of unit length quadrature backwards relatively to U'_{out} . Similarly, we have:

$$\bar{I}_{in} = I_{P,in} \cdot \bar{I}_{P,in} + I_{Q,in} \cdot \bar{I}_{Q,in} \quad (5)$$

where $\bar{I}_{P,in}$ and $\bar{I}_{Q,in}$ are vectors of unit length oriented respectively in phase and quadrature backwards relatively to U_{in} .

The relationship between the two axis systems is:

$$\begin{pmatrix} \bar{I}_{P,out} \\ \bar{I}_{Q,out} \end{pmatrix} = \begin{pmatrix} \cos \theta + \sin \theta \\ -\sin \theta \cos \theta \end{pmatrix} \begin{pmatrix} \bar{I}_{P,in} \\ \bar{I}_{Q,in} \end{pmatrix} \quad (6)$$

Relations (3), (4), (5) and (6) provide

$$\begin{aligned} \bar{I}_{P,in} &= I'_{P,out} \cdot \cos \theta - I'_{Q,out} \cdot \sin \theta \\ \bar{I}_{Q,in} &= I'_{P,out} \cdot \sin \theta + I'_{Q,out} \cdot \cos \theta \end{aligned} \quad (7)$$

From the diagram in Fig. 3, the angle θ is given by

$$\sin \theta = \frac{X \cdot I'_{P,out}}{U_{in}} \quad (8)$$

The internal voltage of the transformer U'_{out} also follows the Fig. 3 by the relationship

$$U'_{out} = U_{in} \cos \theta - X \cdot I'_{Q,out} \quad (9)$$

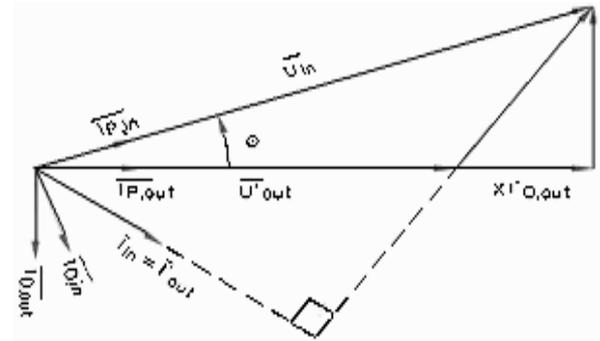


Fig. 3: The phase diagram of the currents and voltages.

The relationship between the internal voltage and the voltage at the output of the transformer is:

$$U'_{out} = r U_{out} \quad (10)$$

Under (2), the frequency deviations are related by:

$$\Delta f_{out} = \Delta f_{in} - \frac{1}{2\pi} \frac{d\theta}{dt} \quad (11)$$

Table 1 below gives the input and output variables of the current models of the step-up and step-down transformers.

Tab. 1: Variable input and output of current models of the transformer.

Current models	Input variables	Output variables	Access
Step-up transformer	U_{alt}, f_{alt} I_{Pnet}, I_{Qnet}	U_{HT} I_{Palt}, I_{Qalt}	in: alt out: HT
Step-down transformer	U_{HT}, f_{HT} I_{Pch}, I_{Qch}	U_{ch}, f_{ch} I_{PchHT}, I_{QchHT}	in: HT out: ch

2.2 The Basic Equations of the Power Model of the Transformers

A certain analogy exists between the model of the transformer and the one with line without losses. The relations developed above can be repeated here, but taking into considerations the sign conventions that are different.

$$P_{out} = P_{in} = \frac{U_{in}U'_{out}}{X} \sin\theta \quad (12)$$

$$Q_{out} = \frac{U_{in}U'_{out}}{X} \cos\theta - \frac{U'^2_{out}}{X} \quad (13)$$

$$Q_{in} = -\frac{U_{in}U'_{out}}{X} \cos\theta + \frac{U'^2_{in}}{X} \quad (14)$$

$$U_{out} = rU'_{out} \quad (15)$$

The input and output variable of power models of the transformers are listed in Tab. 2.

Tab. 2: Variable input and output of models in power transformer.

Models	Input variables	Output variables	Access
Step-up transformer	U_{alt}, f_{alt} P_{net}, Q_{net}	U_{HT}, f_{HT} P_{alt}, Q_{alt}	in: alt Out: HT
Step-down transformer	U_{HT}, f_{HT} P_{ch}, Q_{ch}	U_{ch}, f_{ch} P_{chHT}, Q_{chHT}	in: HT out: ch

Figures 2 and 3 show that there is a similarity between the two models (step-up and step-down transformers) regarding to input and output variables. The input variables are the voltage U_{in} and the frequency f_{in} to an access, the active current $I_{P,out}$, and the reactive one $I_{Q,out}$, out to the other access or active power P_{out} and reactive power Q_{out} out to another access. The output variables are: the voltage U_{out} and the frequency f_{out} at the second access, the active current $I_{P,in}$ and the reactive one $I_{Q,in}$ at the first access, or active power P_{in} and reactive power Q_{in} at the first access [13]. The following developments aim to make expressions of output variables based on input variables for the power model. From (12), we get (16)

$$P_{in} = P_{out} \quad (16)$$

From (12) and (13), we have respectively:

$$P^2_{out} = \frac{U^2_{in}U'^2_{out}}{X^2} (\sin\theta)^2 \quad (17)$$

$$\left(Q_{out} + \frac{U'^2_{out}}{X}\right)^2 = \frac{U^2_{in}U'^2_{out}}{X^2} (\cos\theta)^2 \quad (18)$$

the sum of (17) and (18) provides:

$$\frac{U'^4_{out}}{X^2} + \frac{U'^2_{out}}{X} \left(2Q_{out} - \frac{U'^2_{in}}{X}\right) + (P^2_{out} + Q^2_{out}) = 0 \quad (19)$$

Solving this quadratic equation in U'^2_{out} gives

$$U'^2_{out} = \frac{1}{2} \left(U'^2_{in} - XQ_{out} + \sqrt{U'^2_{in} - 4XU'^2_{in}Q_{out} - 4X^2P^2_{out}} \right) \quad (20)$$

Having determined U'_{out} by (20), we obtain U_{out} by (15). The phase shift θ is obtained by relations (12) and (13), which can be rewritten:

$$\sin\theta = \frac{P_{out}X}{U_{in}U'_{out}} \quad (21)$$

$$\cos\theta = \frac{Q_{out}X + U'^2_{out}}{U_{in}U'_{out}} \quad (22)$$

Hence, we get:

$$\text{tg}\theta = \frac{P_{out}}{Q_{out} + \frac{U'^2_{out}}{X}} \quad (23)$$

Under (2), the frequency f_{out} is given by:

$$f_{out} = f_{in} - \frac{1}{2\pi} \frac{d}{dt} \theta \quad (24)$$

Finally, using (13) and (14), Q_{in} is given by

$$Q_{in} = Q_{out} + \frac{U'^2_{in} - U'^2_{out}}{X} \quad (25)$$

3 Block Diagrams of Models of the Step-Up and Step-Down Transformers

The block diagram of the current model of the transformer is shown in Fig. 4 where the numbers of the equations used are indicated in parentheses. The block diagram of the power model is in Fig. 5.

3.1 The Current Models of the Transformers

The calculation of the derivative of θ uses a

differentiator filtered to reduce the high frequency of the output signal and thus prevent numerical oscillations. The time constant T_δ is about 0.005s. The filtered differentiator can also introduce the initial value of phase shift depending on other initialization values of the model. The input quantities of step-up transformer (I_{Pnet} , I_{Qnet}) and the output voltage U_{HT} will be expressed in physical quantities. The internal quantities of the model of step-up transformer (U_{HT} , I_{Pnet} , I_{Qnet}), the output quantities (I_{Palt} , I_{Qalt}) and the input (U_{alt}) are expressed in per unit. For the model of step-up transformer, the transformation ratio of tensions r_U is therefore the nominal secondary voltage $U_{Te,nom}$ of the transformer (in kV), the nominal primary voltage worth 1 pu. This ratio r_U provides the U_{HT} voltage in kV according to the internal voltage U_{HT} in per unit. In practice, $U_{Te,nom}$ is slightly higher than the rated voltage $U_{HT,nom}$ of the busbars. For the step-up transformer the unity change factor of the currents r_I is introduced to pass input quantities, expressed in physical units, to internal quantities expressed in per unit. This ratio r_I is $U_{Te,nom}/S_{nom}$. We assume that the nominal power of the generator is equal to the one of the step-up transformer: $S_{alt,nom}=S_{Te,nom}=S_{nom}$. The input quantities of the step-down transformer (I_{Pch} , I_{Qch} , U_{HT} , etc.) are expressed in physical quantities. The quantities of the internal model of the step-down transformer (I_{Pch} , I_{Qch} , U_{HT} , etc.) and the quantities of output (I_{PchHT} , I_{QchHT} , U_{ch}) are also expressed in physical quantities. For the model of step-down transformer, the transformation ratio r_U is therefore r_a in physical quantities; this produces the voltage U_{ch} in kV according to the internal voltage U_{ch} in kV. The ratio r_I is also r_a in this case.

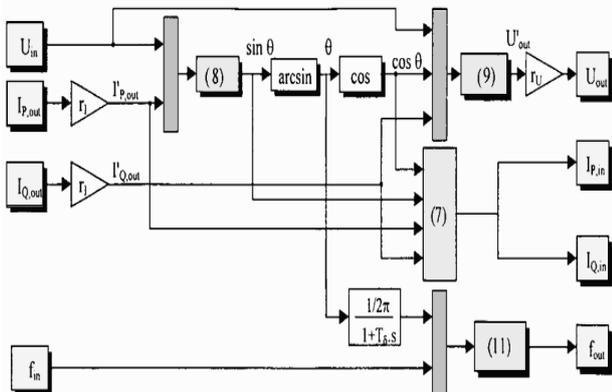


Fig. 4: Current model of the transformer.

3.2 Power Models of the Transformers

The remark on the derivative of θ expressed in paragraph 3.1 also applies to power model of the transformers. For the model of step-up transformer,

by analogy with what has previously been expressed for the input quantities, the output and the internal quantities of the current model, the transformation ratio of tensions is $U_{Te,nom}$. The factor of units change between the input powers, expressed in physical quantities and the internal and output variables expressed in per unit, is $r_P=1/S_{nom}$. For the step-down transformer, the transformation ratio of tensions is simply a report of physical quantities (nominal ratio of transformer), the ratio of power transforming r_P is 1 [1], [14].

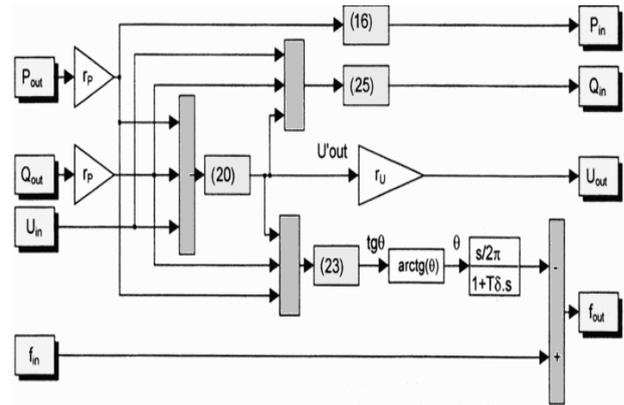


Fig. 5: Power model of the transformer.

Table 3 gives the sizes (r_U , r_I and r_P) used for the models of step-up and step down transformers.

Tab. 3: Size used for models transformers.

	Step-up transformer	Step-down transformer
R_U	$U_{Te,nom}$	r_a
R_I	$U_{Te,nom} / S_{nom}$	r_a
R_P	$1 / S_{nom}$	1

3.3 Initialization of Variables of the Step-Up Transformers

The step-up transformer parameters are: reactance X_{Te} , the transformation ratio transformation $r_U=U_{Te,nom}$, the apparent nominal power $S_{nTe}=S_{nom}$. The input and output variables are shown in Tables 1 and 2 depending on the model used. Assume a priori that the voltage U_{HT} of the high voltage node is fixed. The overall initialization of the system that was developed in paragraph 1 allows determining the production and expenses reported to the high voltage node, that is to say P_{net} and Q_{net} for the power models and the active and reactive currents I_{Pnet} and I_{Qnet} for the current models. By referring to the notation of Section 2 and part of Tab. 1 relatively to the step-up transformer, we find that at this stage of initialization for the current models, we know the quantities $U_{out} \equiv U_{HT}$, $I_{P,out} \equiv I_{Pnet}$ and $I_{Q,out} \equiv I_{Qnet}$, and we must determine $U_{in} \equiv U_{alt}$,

$I_{P,in} \equiv I_{P,alt}$, $I_{Q,in} \equiv I_{Q,alt}$ and the angle θ , noted θ_e in the case of step-up transformer [5].

From (10), Fig. 4 and Tables 1 and 3, we get

$$U_{HT} = \frac{U}{r_U}, \quad I'_{P,net} = r_I I_{P,net} \quad \text{and} \quad I'_{Q,net} = r_I I_{Q,net}.$$

From (8) and (9) we can then determine $U_{in} \equiv U_{alt}$ and the internal angle θ_e of the transformer by:

$$U_{alt} = \sqrt{(U'_{HT} + X_{Te} I'_{Q,net})^2 + (X_{Te} I'_{P,net})^2}$$

$$\theta = \arcsin\left(\frac{X_{Te} I'_{P,net}}{U_{alt}}\right)$$

The currents $I_{P,in} \equiv I_{P,alt}$ and $I_{Q,in} \equiv I_{Q,alt}$ are obtained by relations (7). Similarly, referring to the portion of Tab. 2 relatively to the step-up transformer, it appears that, for power models, we know the quantities $U_{out} \equiv U_{HT}$, $P_{out} \equiv P_{net}$ and $Q_{out} \equiv Q_{net}$, and we must determine $U_{in} \equiv U_{alt}$, $P_{in} \equiv P_{alt}$, $Q_{in} \equiv P_{alt}$ and the angle θ_e . Relation (16) determines P_{alt} ($P_{in} \equiv P_{out}$), and we get $U'_{HT} \equiv U_{out}$ from (15) and the angle θ_e is obtained from equation (23). Knowing P_{alt} , U'_{HT} and θ_e we determine $U_{alt} \equiv U_{in}$ from equation (13). Hence the expression (14) calculates Q_{in} , that is to say the reactive production Q_{alt} of the alternator [3], [11].

3.4 Initialization of Variables of Charges and Step-Down Transformers

Regarding the initialization of variable charges and step-down transformers, different approaches are possible depending on the perspective adopted and the quantities that are initially binded. For each approach considered here, the nominal voltage charge $U_{ch,nom}$ is assumed known. Tables 4 (current models) and 5 (power models) below summarize the four approaches we have considered (for current and power models) based on data and quantities to be determined. The fact that there are several approaches shows that in the model structures, there are several degrees of freedom. For the approaches 1 and 4, the values of currents or power charges are given, while for approaches 2 and 3 they are the nominal quantities (at nominal voltage) that are given. The approaches 1 and 4 are therefore used in calculating charge flow. For approach 1, the quantities are given to busbar HT, while for the approach 4, they are the terminals of the charge. In this study, for calculations, we have used the approach 4, which is closest to the methods used in practice for charge flow calculations [4].

Tab. 4: Approximations of the models.

	Given	Quantities to
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		determine
Approximation 1	$U_{HT}, I_{PchHT}, I_{QchHT}$	$U_{ch}, I_{Pch}, I_{Qch}, I_{Pch,nom}, I_{Qch,nom}, \theta_a$
Approximation 2	$U_{ch}, I_{Pch,nom}, I_{Qch,nom}$	$U_{HT}, I_{Pch}, I_{Qch}, I_{PchHT}, I_{QchHT}, \theta_a$
Approximation 3	$U_{HT}, I_{Pch,nom}, I_{Qch,nom}$	$U_{ch}, I_{Pch}, I_{Qch}, I_{PchHT}, I_{QchHT}, \theta_a$
Approximation 4	U_{HT}, I_{Pch}, I_{Qch}	$U_{ch}, I_{Pch,nom}, I_{Qch,nom}, I_{PchHT}, I_{QchHT}, \theta_a$

The step-down transformer parameters are: the reactance X_{Ta} , the transformation ratio r_a , the apparant power $S_{nTa} = S_{nom}$.

Tab. 5: Approximations of the models.

	Given	Quantities to determine
Approximation 1	$U_{HT}, P_{chHT}, I_{QchHT}$	$U_{ch}, P_{ch}, Q_{ch}, P_{ch,nom}, Q_{ch,nom}, \theta_a$
Approximation 2	$P_{ch}, P_{ch,nom}, Q_{ch,nom}$	$U_{HT}, P_{ch}, Q_{ch}, P_{chHT}, Q_{chHT}, \theta_a$
Approximation 3	$U_{HT}, P_{ch,nom}, Q_{ch,nom}$	$U_{ch}, P_{ch}, Q_{ch}, P_{chHT}, Q_{chHT}, \theta_a$
Approximation 4	U_{HT}, P_{ch}, Q_{ch}	$U_{ch}, P_{ch,nom}, Q_{ch,nom}, P_{chHT}, Q_{chHT}, \theta_a$

3.4.1 Current models

Ist approche: Starting from the high voltage node (HV), we give a priori the voltage U_{HT} and the active current I_{PchHT} and reactive current I_{QchHT} . Using the notation of Table 1, we therefore know U_{HT} , I_{PchHT} and I_{QchHT} . The problem is to determine U_{ch} , I_{Qch} and the angle θ , noted θ_a in the case of step-down transformer. The sum of the equations squares from the (7) allows to write:

$$I_{PchHT}^2 + I_{QchHT}^2 = I'_{Pch}{}^2 + I'_{Qch}{}^2 \quad (26)$$

Similarly, the sum of the equations squares from the relations (8) and (9) gives:

$$U_{HT}^2 = (U'_{ch} + X_{Ta} I'_{Qch})^2 + (X_{Ta} I'_{Pch})^2 \quad (27)$$

From these last two relations, (26) and (27), we get I'_{Pch} and I'_{Qch} and we calculate the currents I_{Pch} and I_{Qch} dividing these values by r_a . The angle θ_a is given by (8) and the voltage U_{ch} is derived from (9) and (10). We have also obtained the active and reactive current, I_{Pch} and I_{Qch} , of the charge and its operating voltage U_{ch} . With the characteristic equations of the charges written here, given that the initial frequency is the nominal frequency f_0 :

$$I_{Pch} = I_{Pch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\alpha \quad (28)$$

$$I_{Qch} = I_{Qch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\beta \quad (29)$$

we calculate the values $I_{Pch,nom}$ and $I_{Qch,nom}$ of the charge at the nominal voltage

2nd approche: starting from the charge, we are given a priori the quantities $I_{Pch,nom}$, $I_{Qch,nom}$ and U_{ch} of the charge. In this case, the active and reactive currents I_{Pch} and I_{Qch} of the load are calculated using relations (28) and (29). We know then $U_{out} \equiv U_{ch}$, $I_{P,out} \equiv I_{Pch}$ and $I_{Q,out} \equiv I_{Qch}$, and we determine $U_{in} \equiv U_{HT}$, θ_a , $I_{P,in} \equiv I_{PchHT}$ and $I_{Q,in} \equiv I_{QchHT}$ in the same manner as the description of the current models. This gives U_{HT} , I_{PchHT} and I_{QchHT} .

3rd approche: we are given a priori the nominal values $I_{Pch,nom}$, $I_{Qch,nom}$ of the load, and we assume the tension U_{HT} of the node HT as known (achieved in practice by setting production groups). This approach requires the simultaneous initialization of the step-down transformer and the charge. It remains to be determined U_{ch} , θ_a , I_{PchHT} , I_{QchHT} , I_{Pch} and I_{Qch} . Using the notation of Tab. 1, relations (10), (28) and (29) are written [4]:

$$U_{in} \equiv U_{HT}$$

$$I_{Pch} = I_{Pch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\alpha \quad (30)$$

$$I_{Qch} = I_{Qch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\beta \quad (31)$$

$$I_{Qch} = I_{Qch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\beta \quad (32)$$

$$U_{ch} = r_a U'_{ch}$$

Substituting the previous expressions of $I_{P,out} \equiv I_{Pch}$ and $I_{Q,out} \equiv I_{Qch}$ in equation (27) yields:

$$U_{ch}^2 + U_{ch} \cdot 2 \cdot X_{Ta} \cdot r_a \cdot I_{Qch,nom} \left(\frac{r_a U_{ch}}{U_{ch,nom}} \right)^\beta + X_{Ta} \cdot r_a \cdot \left[I_{Pch,nom}^2 \left(\frac{r_a U_{ch}}{U_{ch,nom}} \right)^{2\alpha} + I_{Qch,nom}^2 \left(\frac{r_a U_{ch}}{U_{ch,nom}} \right)^{2\beta} \right] - U_{HT}^2 = 0 \quad (32)$$

We obtain an equation with one unknown ($U'_{ch} \equiv U'_{out}$) whose exponents for various terms in the equation are: 2, $1+\beta$, 2α , 2β and 0. The equation (32) has no general analytical solution. Carrying out an iterative calculation, it is possible to solve it. However, this equation is readily soluble in many practical cases:

For $\alpha=0$ (charge at active power constant). If $\beta=0$, equation (32) becomes a quadratic equation in U'_{out} , which is easily soluble. If $\beta=1$, equation (32) is also a quadratic equation in U'_{out} , so readily soluble. If $\beta=2$, there is a fourth degree equation whose analytical solution is expressed not so simple. If $\beta=3$, the equation is of degree six.

For $\alpha=1$ or $\alpha=2$ (charge at constant active current (=1) or constant impedance ($\alpha=2$)). If $\beta=0$, the equation (32) is a quadratic equation in U'_{ch} for a charge at constant active current (=1). By cons, for a charge at constant impedance (=2), the equation (32) is an equation of fourth degree in U'_{ch} whose analytical solution is expressed not so simple. If $\beta=1$, it remains either a quadratic equation in U'_{ch} ($\alpha=1$), or a quadratic equation in U'^2_{ch} ($\alpha=2$) which are both readily soluble. If $\beta=2$, we get an equation of fourth degree in U'_{ch} whose analytical solution is expressed not so simple. If $\beta=3$, we can be reducing to a cubic equation in U'^2_{ch} . Table 6 summarizes the situations where the resolution of equation (32) is easy (sign *).

Tab. 6: The situations where the resolution of equation (32) is easy.

$\alpha \backslash \beta$	0	1	2
0	*	*	
1	*	*	*
2			

Having determined $U'_{ch} \equiv U'_{out}$, we obtain the output voltage $U_{out} \equiv U_{ch}$ by (15). The characteristic equations of charges (28) and (29) allow to calculate $I_{P,out} \equiv I_{Pch}$ and $I_{Q,out} \equiv I_{Qch}$. Hence, we determine successively the internal angle θ_a of the step-down transformer by (8), and the current $I_{P,in} \equiv I_{PchHT}$ and $I_{Q,in} \equiv I_{QchHT}$ by (7).

4th approche: we start from the charge, for which we fix the active and reactive currents I_{Pch} and I_{Qch} and the nominal voltage $U_{ch,nom}$ and we assume as known the tension U_{HT} of the node HT. As the third approach, it also requires the initialization of both the step-down transformer and charge. We know here $U_{in} \equiv U_{HT}$ and it remains to determine U_{ch} , θ_a , I_{PchHT} , I_{QchHT} , $I_{Pch,nom}$ and $I_{Qch,nom}$. From relation (27), taking into consideration the relation (10) and the fact that the currents I'_{Pch} and I'_{Qch} respectively represent the products of I_{Pch} and I_{Qch} and with the transformation ratio r_a , we get:

$$U_{HT}^2 = \left(\frac{U_{ch}}{r_a} + X_{Ta} r_a I_{Qch} \right)^2 + (X_{Ta} r_a I_{Pch})^2 \quad (33)$$

This quadratic equation in U_{ch}^2 provides the tension of the charges $U_{ch} \equiv U_{out}$. The currents I'_{Pch} and I'_{Qch} are obtained from I_{Pch} , I_{Qch} and r_a . The angle θ_a is given by (8), the currents $I_{PchHT} \equiv I_{P,in}$ and $I_{QchHT} \equiv I_{Q,in}$ are given by relations (7). We finally calculate the currents $I_{Pch,nom}$ and $I_{Qch,nom}$ by relations (28) and (29).

3.4.2 Power Models

1st approche: starting from the high voltage node (HT), we are given a priori tension U_{HT} and the active and reactive powers P_{chHT} and Q_{chHT} . Using the notation of Tab. 2, we therefore know $U_{in} \equiv U_{HT}$, $P_{in} \equiv P_{PchHT}$ and $Q_{in} \equiv Q_{chHT}$. We have to determine $U_{out} \equiv U_{ch}$, $P_{out} \equiv P_{ch}$ and $Q_{out} \equiv Q_{ch}$ and the internal angle θ_a . Relation (12) allows us to calculate the power $P_{ch} \equiv P_{out} \equiv P_{in} \equiv P_{PchHT}$. By making the sum of the squares of relations (12) and (14) we obtain:

$$P_{chHT}^2 + \left(Q_{chHT} - \frac{U_{HT}^2}{X_{Ta}} \right)^2 = \frac{U_{HT}^2 \cdot U_{ch}^2}{X_{Ta}^2}$$

This allows us to calculate $U'_{ch} \equiv U'_{out}$. The relation (15) provides then $U_{ch} \equiv U_{out}$. Relation (12) gives the internal angle θ_a and the reactive power $Q_{ch} \equiv Q_{out}$ is determined from the relation (13). This resulted in the active and reactive powers P_{ch} and Q_{ch} of the charge and also its operating voltage U_{ch} . With the charges characteristics equations (20) and (21) that are written here, given that the initial frequency is the nominal frequency f_0 :

$$P_{ch} = P_{ch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\alpha \quad (34)$$

$$Q_{ch} = Q_{ch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\beta \quad (35)$$

We calculate the values $P_{ch,nom}$ and $Q_{ch,nom}$ of the charge at the nominal voltage.

2nd approche: we start from the charge and we are given a priori the quantities $P_{ch,nom}$, $Q_{ch,nom}$ and U_{ch} of the charge. In this case, the active and reactive powers of the charge P_{ch} and Q_{ch} are calculated using relations (34) and (35). We have to determine $U_{out} \equiv U_{ch}$, $P_{out} \equiv P_{ch}$ and $Q_{out} \equiv Q_{ch}$ we get $U_{in} \equiv U_{HT}$, $P_{in} \equiv P_{PchHT}$ in the same is calculated the same way as the description on the power models (section 3.2).

3rd approche: we are given a priori the nominal values $P_{ch,nom}$ and $Q_{ch,nom}$ of the charge and we assume the tension U_{HT} of the node HT as known

(achieved in practice by setting production groups). This approach requires the simultaneous initialization of the step-down transformer and the charge. It remains to be determined U_{ch} , θ_a , P_{chHT} , Q_{chHT} , P_{ch} and Q_{ch} . Using the notation of Tab. 1, relations (15), (34) and (35) are written:

$$P_{ch} = P_{ch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\alpha \quad (36)$$

$$Q_{ch} = Q_{ch,nom} \left(\frac{U_{ch}}{U_{ch,nom}} \right)^\beta \quad (37)$$

$$U_{ch} = r_a U'_{ch}$$

Given $U_{in} \equiv U_{HT}$, and replacing the previous expressions of $P_{out} \equiv P_{ch}$ and $Q_{out} \equiv Q_{ch}$ in equation (19) we have:

$$\frac{U'_{ch}{}^4}{X_{Ta}^2} + \frac{U'_{ch}{}^2}{X_{Ta}} \left[2 \cdot Q_{ch,nom} \cdot \left(\frac{r_a \cdot U'_{ch}}{U_{ch,nom}} \right)^\beta - \frac{U_{HT}^2}{X_{Ta}} \right] + P_{ch,nom}^2 \cdot \left(\frac{r_a \cdot U'_{ch}}{U_{ch,nom}} \right)^{2\alpha} + Q_{ch,0}^2 \cdot \left(\frac{r_a \cdot U'_{ch}}{U_{ch,nom}} \right)^{2\beta} = 0 \quad (38)$$

We obtain an equation with one unknown ($U'_{out} \equiv U'_{ch}$) whose exponents for various terms in the equation are: 4, $2+\beta$, 2, 2 and 2β . Equation (38) has no general analytical solution. In carrying out an iterative calculation, it is possible to solve it. However, this equation is readily soluble in many practical cases [5]:

For $\alpha=0$ (charge at constant active power). If $\beta=0$, equation (38) becomes a quadratic equation in $U'_{ch}{}^2$ which is easily soluble. If $\beta=1$, we have a fourth degree equation whose analytical solution is expressed not so simple. If $\beta=2$, equation (38) is also a quadratic equation in $U'_{ch}{}^2$ therefore easily soluble. If $\beta=3$, we are dealing with an equation of six degree.

For $\alpha=1$ or $\alpha=2$ (charge at constant active current ($\alpha=1$) or constant impedance ($\alpha=2$)). If $\beta=0$, the equation (38) is a quadratic equation in $U'_{ch}{}^2$. If $\beta=1$, we divide equation (38) by $U'_{ch}{}^2$ and we get a quadratic equation in U'_{ch} . If $\beta=2$, we divide equation (38) by $U'_{ch}{}^2$ and obtain an equation of first degree in $U'_{ch}{}^2$ that solves easily. If $\beta=3$, it can be reduced to a fourth degree equation whose analytical solution is expressed not so simple. Table 7 below summarizes the situations where the resolution of equation (38) is easy (sign *).

Tab.7: The situations where the resolution of equation (38) is easy.

$\alpha \backslash \beta$	0	1	2
0	*	*	*
1		*	*
2	*	*	*

Having determined U'_{out} we obtain the output voltage $U_{out} \equiv U_{ch}$ by (15). The characteristic equations of charges (36) and (37) allow the calculation of $P_{out} \equiv P_{ch}$ and $Q_{out} \equiv Q_{ch}$. Hence, we successively determine the angle θ_a by (23), $Q_{ch} \equiv Q_{chHT}$ by (14) P_{in} and $P_{in} \equiv P_{chHT} \equiv P_{out}$ by (12).

4th approche: we start from the charge, for which we fixe the active and reactive powers P_{ch} and Q_{ch} (assuming the voltage $U_{ch,nom}$ known), and we assume the tension U_{HT} of the node HT as known. Knowing $P_{out} \equiv P_{ch}$ and $Q_{out} \equiv Q_{ch}$ and also $U_{in} \equiv U_{HT}$ the relation (20) determines $U'_{out} \equiv U'_{ch}$ and the relation (15) gives $U_{out} \equiv U_{ch}$. The angle θ_a of the step-down transformer is given by (23), $Q_{chHT} \equiv Q_{in}$ is calculated from (25) and $P_{chHT} \equiv P_{in} \equiv P_{out}$ is given by (12). Finally, the relations (36) and (37) allow the calculation of $P_{ch,nom}$ and $Q_{ch,nom}$.

4 Ideal Transformers

The ideal transformer shown schematically in Figures 6 and 7 corresponds to that of paragraph 2 where the reactance X is zero. This model retains the active and reactive powers, and also the frequency and phase. The voltage output (U_{out}) is equal to the input voltage (U_{in}) multiplied by the transformation ratio (r). The ratio of output current and input current is the inverse transformation ratio ($1/r$).

4.1 Current Model of the Ideal Transformer

The input quantities of the current model of the ideal step-up transformer are the active and reactive currents I_{Pnet} (kA) and I_{Qnet} (kA), the voltage U_{alt} (p.u.) and the frequency deviation Δf_{alt} (Hz). To express the output quantities, active and reactive current I_{Pnet} (kA) and I_{Qnet} (kA), in quantities per unit, we must introduce the conversion factor $r_I = U_{Te,nom} / S_{nom}$ between currents. The relationship between the voltages is $r_U = U_{Te,nom}$. The current model of ideal step-down transformer has for the inputs the active and reactive current réactif I_{Pch} (kA) and I_{Qch} (kA), voltage U_{HT} (kV) and frequency deviation Δf_{HT} (Hz). The output quantities are $I_{PchHT} = r_a \times I_{Pch}$, $I_{Qch} = r_a \times I_{Qch}$, $U_{ch} = r_a \times U_{HT}$ and $\Delta f_{ch} = \Delta f_{HT}$. For step-down transformer, we have

$r_I = r_a$ and $r_U = r_a$. Fig. 6 and Tab. 3 summarize these results [6].

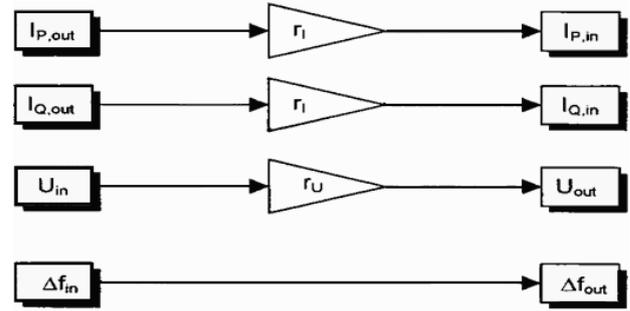


Fig. 6: Current model of the ideal transformer.

4.2 Power Model of the Ideal Transformer

The power model of the step-up transformer has for inputs the active and reactive powers P_{net} (MW) and Q_{net} (Mvar), the voltage U_{alt} (p.u.) and the frequency deviation Δf_{alt} (Hz). For obtaining the active and reactive powers P_{alt} and Q_{alt} in quantities per unit to the exit, we must introduce the conversion factor $r_p = 1/S_{nom}$. The relationship between the voltages is $r_U = U_{Te,nom}$. As the power model of ideal step-down transformer has entries for the active and reactive power P_{ch} (MW) et Q_{ch} (Mvar), U_{HT} voltage (kV) and frequency deviation Δf_{HT} (Hz), the output quantities are $P_{chHT} = P_{ch}$, $Q_{chHT} = Q_{ch}$, $U_{ch} = r_a \times U_{HT}$ and $\Delta f_{ch} = \Delta f_{HT}$. For the step-down transformer, we have $r_U = r_a$ and $r_p = 1$. Fig. 7 and Tab. 3 summarize these results.

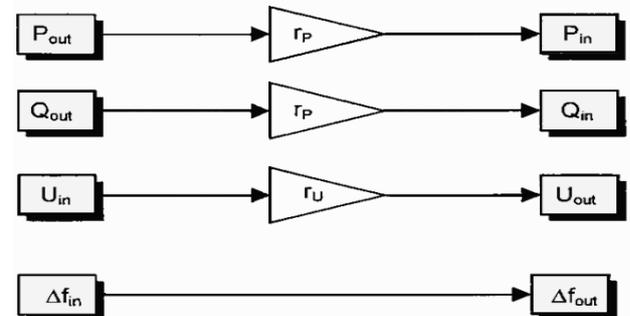


Fig. 7: Power model of the ideal transformer.

5 Simulations

This demonstration illustrates the use of the linear transformer to simulate a three-winding distribution transformer rated 75 kVA - 14400/120/120 V (Fig. 8). The transformer primary is connected to a high voltage source (14,400 V rms). Two identical inductive loads (20 kW -10 kvar) are connected to the two secondaries. A third capacitive load (30 kW -20 kvar) is fed at 240 V. Initially, the circuit breaker in series with Load 2 is closed, so that the system is balanced [17]. Open the powergui block

to obtain the initial voltage and current phasors in steady state [8], [10].

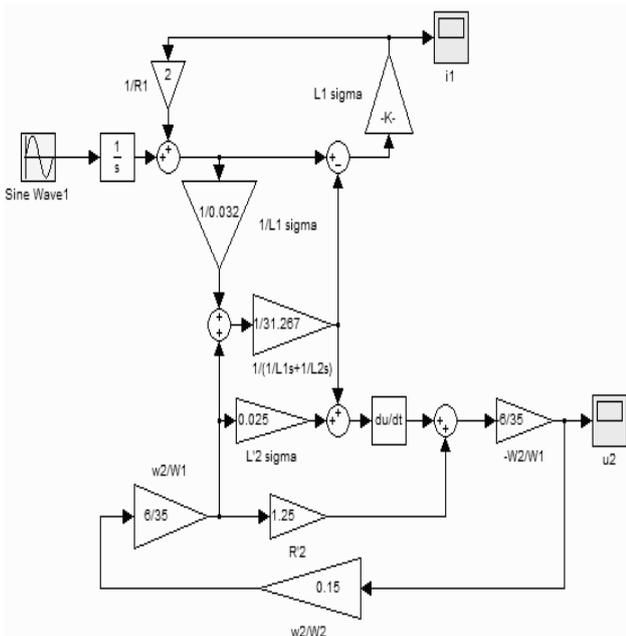


Fig. 8: Transformer model in Simulink.

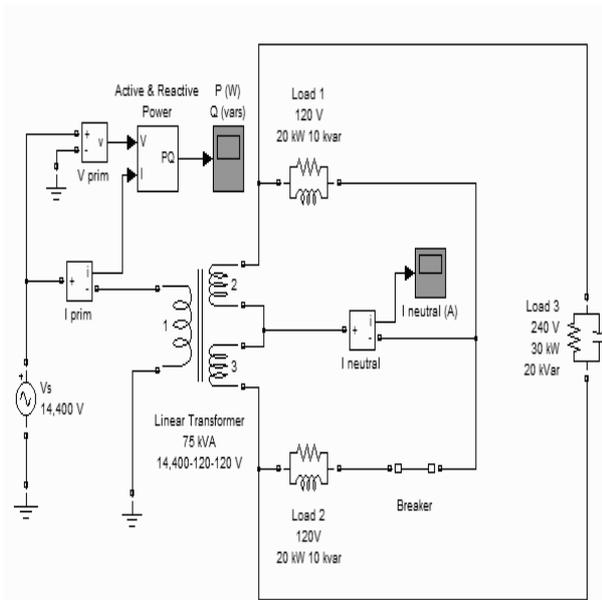


Fig. 9: Linear transformer by Simulink.

As loads are balanced the neutral current is practically zero. Furthermore, as the inductive reactive power of Load 1 and Load 2 (2×10 kvar) is compensated by the capacitive reactive power of Load 3 (20 kvar), the primary current is almost in phase with voltage. The small phase shift (-2.8 deg) is due to the reactive power associated with transformer reactive losses. Open the two scopes and start the simulation. The following observations can be made: when the circuit breaker opens, a current starts to flow in the neutral as a result of the load unbalance. The active power computed from

the primary voltage and current is measured by a Simulink block which can be found in the Extras/Measurement library. When the breaker opens, the active power decreases from 70 kW to 50 kW.

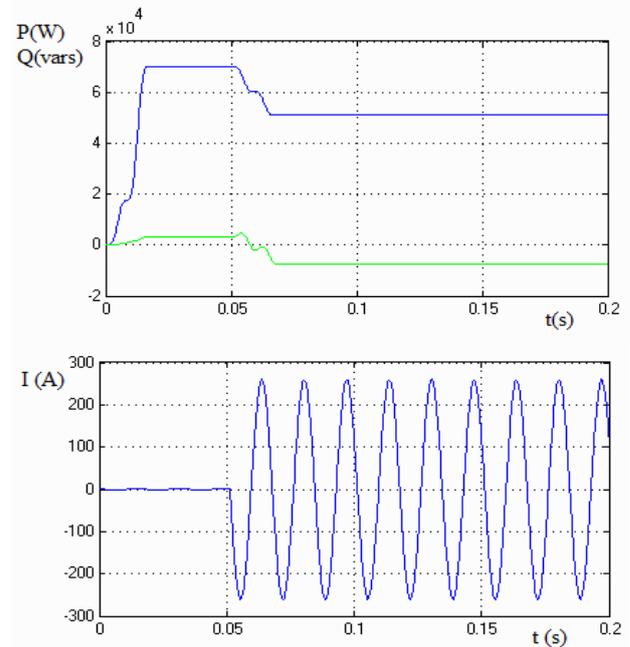


Fig. 10: Parameters transformer.

This demonstration (Fig.11) illustrates measurement distortion due to saturation of a current transformer (CT). A current transformer (CT) is used to measure current in a shunt inductor connected on a 120 kV network [9]. The CT is rated 2000 A / 5 A, 5 VA. The primary winding which consists of a single turn passing through the CT toroidal core is connected in series with the shunt inductor rated 69.3 Mvar, 69.3 kV ($120\text{kV}/\sqrt{3}$), 1 kA rms. The secondary winding consisting of $1 \times 2000/5=400$ turns is short circuited through a 1 ohm load resistance.

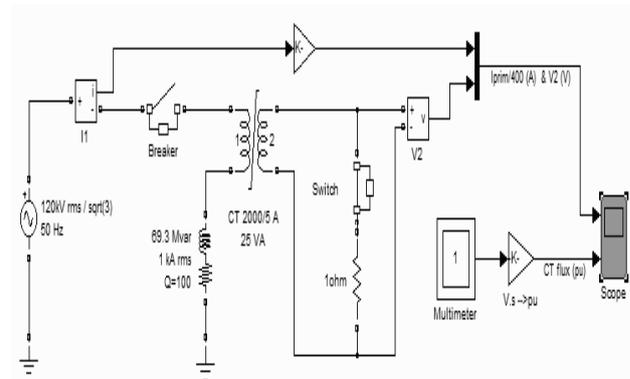


Fig. 11: Current Transformer Saturation by Simulink; in order to observe CT saturation, change the Breaker closing time to $t=1/50$ s (1 cycle).

A voltage sensor connected at the secondary reads a voltage which should be proportional to the primary current. In steady state, the current flowing in the secondary is $1000 \times 5 / 2000 = 2.5$ A (2.5 Vrms or 3.54 Vpeak read by the voltage measurement block V2). Open the CT dialog box and observe how the CT parameters are specified. The CT is assumed to saturate at 10 pu and a simple 2 segment saturation characteristic is used. The primary current reflected on the secondary and the voltages developed across the 1 ohm resistance are sent to trace 1 of the Scope block. The CT flux, measured by the Multimeter block is converted in pu and sent to trace 2. The switch connected in series with the CT secondary is normally closed. This switch will be used later to illustrate overvoltages produced when CT secondary is left open.

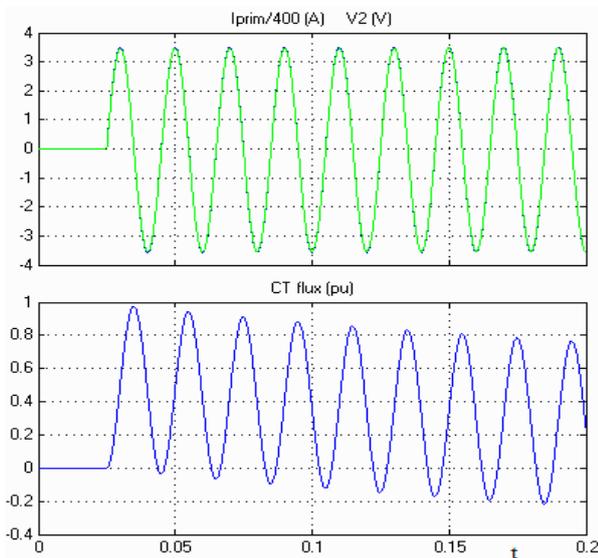


Fig. 12: Parameters transformer
 $(1 \text{ pu flux} = 0.0125 \text{ V} \cdot \sqrt{2}) / (2 \cdot \pi \cdot 50) = 5.63 \text{ e-}5 \text{ V} \cdot \text{s})$

Normal operation. In this test, the breaker is closed at a peak of source voltage ($t=1.25$ cycle). This switching produces no current asymmetry. Start the simulation and observe the CT primary current and secondary voltage (first trace of Scope block). As expected the CT current and voltage are sinusoidal and the measurement error due to CT resistance and leakage reactances is not significant. The flux contains a DC component but it stays lower than the 10 pu saturation value [15].

CT saturation due to current asymmetry. Now, change the breaker closing time in order to close at a voltage zero crossing (Use $t=1/50$ s). This switching instant will now produce full current asymmetry in the shunt reactor. Restart the simulation. Observe that for the first 3 cycles, the flux stays lower than the saturation knee point (10 pu). The CT voltage output V2 then follows the

primary current. However, after 3 cycles, the flux asymmetry produced by the primary current causes CT saturation, thus producing large distortion of CT secondary voltage.

Overvoltage due to CT secondary opening. Reprogram the primary breaker closing time at $t=1.25/50$ s (no flux asymmetry) and change the secondary switch opening time to $t=0.1$ s. Restart the simulation and observe the large overvoltage produced when the CT secondary is opened. The flux has a square wavelike shape chopped at +10 and -10 pu. Large $d\phi/dt$ produced at flux inversion generates high voltage spikes (250 V). This demonstration illustrates simulation of hysteresis in a saturable transformer (Fig. 13). One phase of a three-phase transformer is connected on a 500 kV, 5000 MVA network. The transformer is rated 500 kV/230 kV, 450 MVA (150 MVA per phase). The flux-current saturation characteristic of the transformer is modelled with the hysteresis or with a simple piecewise nonlinear characteristic. A Three-Phase Programmable Voltage Source is used to vary the internal voltage of the equivalent 500 kV network. During the first 3 cycles source voltage is programmed at 0.8 pu. Then, at $t=3$ cycles (0.05 s) voltage is increased by 37.5% (up to 1.10 pu). In order to illustrate remanent flux and inrush current at transformer energization, the circuit breaker which is initially closed is first opened at $t=6$ cycles (0.1 s), then it is reclosed at $t=9$ cycles (0.15 s). The Initial flux ψ_0 (ϕ_0) in the transformer is set at zero and source phase angle is adjusted at 90 degrees so that flux remains symmetrical around zero when simulation is started.

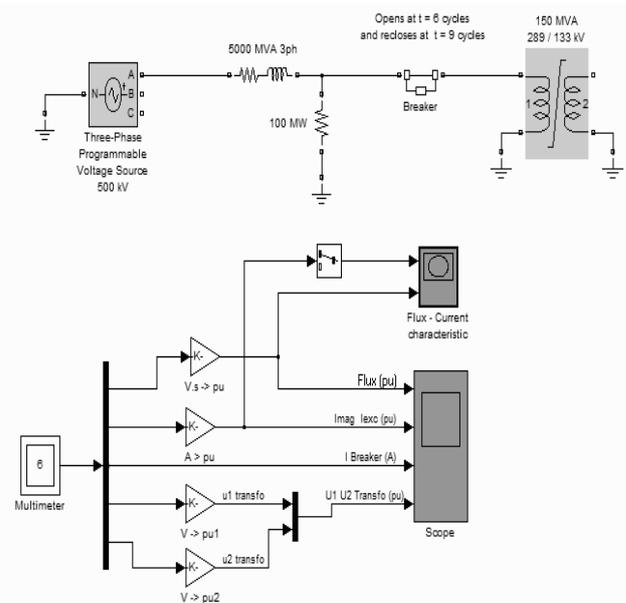


Fig. 13: Saturable transformer with hysteresis by Simulink.

A Multimeter block and a Scope block are used to monitor waveforms of flux, magnetization current (not including the eddy currents which are modelled by the R_m resistance), excitation current (including eddy current modeled by R_m), voltages and current flowing into primary winding. A X-Y Graph block is used to monitor the transformer operating point moving on the flux-current characteristic (Fig. 14).

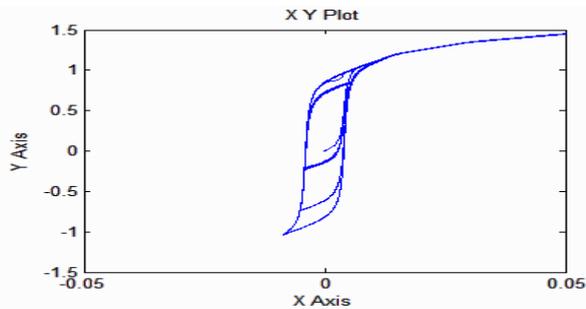


Fig. 14: Flux-current characteristic.

A) *Simulation of saturation with hysteresis.* The saturation characteristics consist of two regions:

- The main hysteresis loop: In this region there are 2 different flux values for a single current value.
- The saturated region defined by a simple line segment starting from the maximum point (I_s , F_s) of the main loop (Fig. 14).

The hysteresis loop is defined by the following 3 points marked by red crosses on the main loop:

[$I=0$; Remanent flux ($F_r=0.85$ pu)], [Coercive Current($I_c=0.004$ pu); $F=0$], [Saturation current ($I_s = 0.015$ pu); Saturation flux ($F_s=1.2$ pu)]

plus the slope dF/dI at coercive current ($F=0$).

Using the 'Zoom around hysteresis' checkbox and 'Display' button you can view the whole characteristic or zoom on the hysteresis. Start the simulation and observe the following phenomena on the two scope blocks:

a) From 0 to 0.05 s: voltage and flux peak values are at 0.8 pu. Notice typical square wave of magnetization current. As no remanent flux was specified, magnetization current and flux are symmetrical. Flux travels on inner loops (inside the main loop).

b) From 0.05 to 0.1 s: voltage is 1.1 pu. Flux now reaches approximately +1.1pu. A slight flux asymmetry is produced at voltage change and the flux which varies between +1.14 pu and -1.05 pu now travels on the main loop. Current pulses appear on the magnetization current (Imag), indicating beginning of saturation.

c) From 0.1 to 0.15 s: at first zero crossing after the breaker opening order, the current is

interrupted, and a flux of 0.84 pu stays trapped in the transformer core.

d) From 0.15 to 2 s: the breaker is reclosed at $t=9$ cycles, at a zero crossing of source voltage, producing an additional flux offset of approximately 1 pu. The peak flux now reaches 1.85 pu, driving the transformer into the saturated region. Peak excitation current now reaches 0.81 pu.

B) *Simulation of saturation with a piecewise nonlinear characteristic.* Open the transformer menu and deselect 'Simulate hysteresis'. The saturation will now be simulated by a piecewise nonlinear single-valued characteristic defined by 7 points. Current/Flux pairs (in p.u.) are: [0 0; 0.0 0.85; 0.015 1.2; 0.03 1.35; 0.06 1.5; 0.09 1.56; 0.12 1.572]. The saturated region is the same, but the hysteresis loop is not simulated. Note that this single valued characteristics still allows specification of a remanent flux (keep $\phi_0=0$ flux as with hysteresis saturation model).

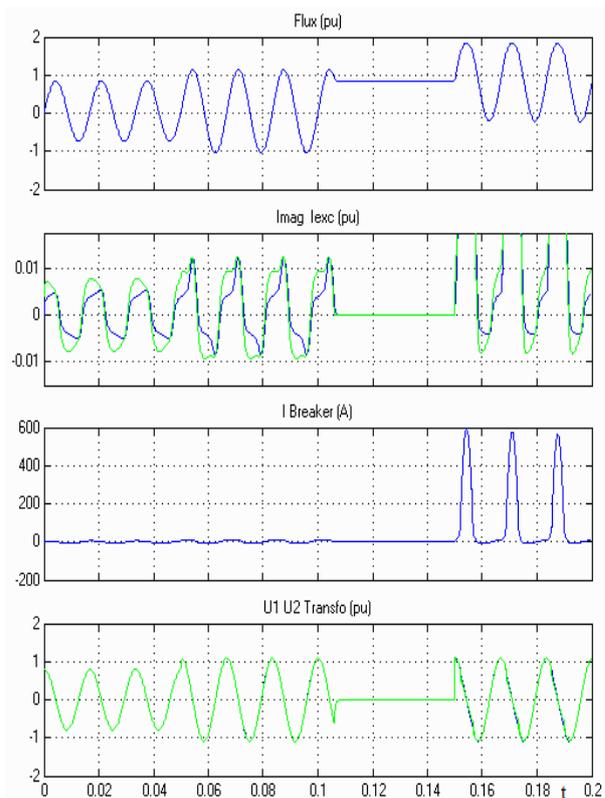


Fig. 15: Parameters transformer.

6 Conclusions

Theoretical results were compared with data from transformer manufacturers and the good agreement between both validates theoretical results.

If the model of the step-up transformer considers the active losses in the transformer, it will calculate

the gross active power from the power net and other quantities. With the assumptions described above, the quantities set a priori to describe the initial situation of the system are: the frequency (nominal value), the voltage at each busbar HT, the production and consumption of each zone (except for the balance node where only the active power consumption is set) and consumption of reactive charges.

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