Manuscript received May 11, 2007; revised Aug. 15, 2007

A Semismooth Inexact Newton-type Method for the Solution of Optimal Power Flow Problem

XUE LI, YUZENG LI, SHAOHUA ZHANG
Key Laboratory of Power Station Automation Technology Automation Department
Shanghai University
Zhabei District, Shanghai 200072
CHINA
lixue_9808@163.com, yzli@mail.shu.edu.cn, eeshzhan@126.com

Abstract: - The paper presents a semismooth inexact Newton-type method for solving optimal power flow (OPF) problem. By introducing the nonlinear complementarity problem (NCP) function, the Karush-Kuhn-Tucker (KKT) conditions of OPF model are transformed equivalently into a set of semismooth nonlinear algebraic equations. Then the set of semismooth equations can be solved by an improved inexact Levenberg-Marquardt (L-M) algorithm based on the subdifferential. In the algorithm, the positive definitiveness of the iterative coefficient matrix is enhanced by using the L-M parameter, while a reformed nonmonotone line search is used to enforce global convergence of the algorithm. Finally, the feasibility of the proposed method for solving the nondifferentiable problem is verified on Kojima-Shindo problem, and the effectiveness of the proposed method is demonstrated on the IEEE test systems.

Key-Words: - inexact Levenberg-Marquardt algorithm; nonlinear complementarity problem; optimal power flow; power system; subdifferential

1 Introduction

The optimal power flow (OPF) problem has been commonly used as an efficient tool in the power system planning and operating [1]-[3]. Over the years, researchers have examined various algorithmic techniques that seek to speed up the OPF computation. Most of the work done was captured in the 1980s and 1990s [4]-[7], a time when several optimization techniques, such as reduced gradient technique, quadratic programming and Newton methods etc., emerged as the leading nonlinear programming (NLP) algorithms for solving the OPF problem. However, the NLP algorithms are less robust and often experience convergent problems. In recent years, the algorithms based on the interior point method (IPM), especially the primal-dual IPM, have been applied extensively [8]-[11]. The primal-dual IPM has many attractive features, whereas it suffers the drawback of the required positivity of slack variables and their corresponding dual variables at every iteration.

More recently, the nonlinear complementarity method (NCM) is applied to solve the OPF problem [12]-[14]. The method, by introducing the nonlinear complementarity problem (NCP) functions, can make the Karush-Kuhn-Tucker (KKT) conditions of the OPF model transform into a set of nonlinear algebraic equations. The NCM has three appealing features: (i) ease of handling inequality constraints by the NCP functions, (ii) not necessarily identifying the binding constraints and (iii) the iterations are not required to stay in the positive orthant. A damped Newton-type method and a decoupled semismooth Newton method were proposed to solve the transformed nonlinear algebraic equations, respectively [13]-[14]. However, when iterative equations are ill-conditioned, the convergence of two methods can not be guaranteed.

In this paper, the NCM is used to cope with the complementarity conditions of the KKT system. By using a NCP function, the KKT system of the OPF problem is transformed equivalently into a set of the semismooth nonlinear algebraic equations. Then the semismooth equations can be solved by an improved inexact Levenberg-Marquardt (L-M) algorithm based on the subdifferential. The proposed method has the following features:

(i) it can solve the semismooth systems formulated by NCP by using the subdifferential.

(ii) it avoids the emergence of the ill-conditioning equations by using the well-conditioned augmented coefficient matrix.

(iii) it employs a reformed nonmonotone line search to speed up the procedure of obtaining a series of OPF solutions.

Consequently, the proposed method has good convergence performance and computation accuracy, as has been verified by the computational results.

The rest of this paper is organized as follows: Mathematical foundation of the proposed method is described in Section 2. The OPF model is formulated and the NCM is used to transform the KKT conditions into the semismooth equations in Section 3. An improved inexact L-M algorithm is proposed for the solution of the semismooth equations in Section 4. In Section 5, the feasibility of the proposed method for solving the nondifferentiable problem is verified on the Kojima-Shindo problem. Then, the effectiveness of the proposed method is analyzed on the IEEE test systems. Conclusion is given in Section 6.

2 Problem Formulation

2.1 B-Subdifferential

Let $G: R^n \to R^n$ be a locally Lipschitzian function. Hence, G is differentiable almost everywhere [15]. If we indicate by D_G the set where G is differentiable, we can define the B-subdifferential of G at x [16] as

$$\partial_B G(x) = \left\{ \lim_{\substack{x^k \to x \\ x^k \in \boldsymbol{D}_G}} \nabla G(x^k) \right\} \tag{1}$$

Note that the generalized Jacobian of Clarke $\partial_C G(x)$ is just the convex hull of $\partial_B G(x)$ [15].

$$\partial_C G(x) = conv(\partial_B G(x)) \tag{2}$$

where $conv(A)$ denotes the convex hull of the set A.

2.2 Semismooth function

Let $G: R^n \to R^n$ be a locally Lipschitzian function at $x \in R^n$. We say that G is semismooth at x [17], if

$$\lim_{\substack{H \in \partial_B G(x+tv') \\ v' \to v, t \downarrow 0}} \{Hv'\} \tag{3}$$

exists for all $v \in R^n$.

3 Problem Formulation

The OPF problem can be shown as the following nonlinear programming problem

$$\begin{aligned} \min \quad & c(x) \\ s.t. \quad & g(x) = 0 \\ & h(x) \geq 0 \end{aligned} \tag{4}$$

where $x \in R^n$ is the vector of system variables, $c(x): R^n \to R$ is the objective function, $g(x): R^n \to R^m$ represents the nonlinear power flow equations and $h(x): R^n \to R^p$ represents several equipment and system inequality constraints. The explicit OPF model in the paper is expressed subsequently.

In this paper, the objective function of the OPF problem is considered as the fuel cost minimization.

$$\min \quad \sum_{i \in S_G} (a_{2i} P_{Gi}^2 + a_{1i} P_{Gi} + a_{0i}) \tag{5}$$

where S_G is the set of power generation; P_{Gi} is the active generation output; a_{2i} , a_{1i} and a_{0i} are the generation cost coefficients.

The equality and inequality constraints that include flows, real generation, reactive generation, transformer taps, voltage and current of a *N*-bus power system are considered as follows.

$$\begin{aligned} & \left.\begin{aligned} & P_{Gi} - P_{Di} - P_i(e, f, t) = 0 \\ & Q_{Gi} - Q_{Di} - Q_i(e, f, t) = 0 \end{aligned}\right\}, \quad i = 1, \cdots, N, i \neq slack \\ & P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \ i \in S_G \\ & Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \ i \in S_G \\ & t_{ij}^{\min} \leq t_{ij} \leq t_{ij}^{\max}, \ (i, j) \in S_T \\ & (V_i^2)^{\min} \leq (e_i^2 + f_i^2) \leq (V_i^2)^{\max}, \ i = 1, \cdots, N \\ & I_{ij}^2 \leq (I_{ij}^2)^{\max}, \quad (i, j) \in S_L \end{aligned} \tag{6}$$

where

P_{Di} : real power load at load bus *i*;
Q_{Di} : reactive power load at load bus *i*;
Q_{Gi} : reactive generation of generator *i*;
P_i : real power injection at bus *i*;
Q_i : reactive power injection at bus *i*;
e_i : real part of nodal voltage at bus *i*;
f_i : imaginary part of nodal voltage at bus *i*;
t_{ij} : transformer ratio of transformer branch *ij*;
V_i : voltage at bus *i*;
I_{ij} : current at line *ij*;
S_T : set of transformer branches;
S_L : set of transmission line *L*.

The superscript “*min*” and “*max*” stand for the lower and upper bounds of a constraint, respectively.

The Lagrange function $L(\cdot)$ for (4) can be written as follows:

$$L(w) = c(x) - \lambda^T g(x) - \mu^T h(x) \tag{7}$$

where λ and μ are the vectors of Lagrange multipliers about the equality constraints and inequality constraints, respectively. $w=[x^T,\lambda^T,\mu^T]^T$ is a vector of primal and dual variables.

The KKT first-order conditions of optimality for (4) can be written as the following equations and complementarity conditions:

$$\nabla_x L(w)=0 \tag{8}$$

$$g(x)=0 \tag{9}$$

$$h(x)\ge 0,\mu\ge 0,\mu_i h_i=0,(i=1,\cdots,p) \tag{10}$$

where $\nabla_x L(w)=\nabla c(x)-J^T\lambda-K^T\mu$, $\nabla c(x)$ is the gradient of primal objective function at the point x, J and K are the Jacobian matrices of $g(x)$ and $h(x)$ about x^T, respectively.

To deal with a set of complementarity conditions in (10), the NCM is employed by means of a function $\varphi: R^2\to R$, called the NCP function, which is introduced as follows:

$$\varphi(a,b)=\sqrt{a^2+b^2}-a-b \tag{11}$$

The NCP function (11) satisfies the basic property:

$$\varphi(a,b)=0\Leftrightarrow a\ge 0,b\ge 0,ab=0 \tag{12}$$

Using (11), the complementarity conditions (10) can be expressed as the following set of semismooth nonlinear equations:

$$\varphi_i\equiv\varphi(\mu_i,h_i(x))=0 \quad , \quad i=1,\cdots,p \tag{13}$$

Next, using (13), equations (8)-(10) can be equivalently reformulated as the nonlinear system:

$$F(w)=\begin{bmatrix}\nabla_x L(w)\\ g(x)\\ \phi(x,\mu)\end{bmatrix}=0 \tag{14}$$

where $\phi(x,\mu)=(\varphi_{1,}\cdots,\varphi_p)^T$ is a semismooth system, and the nonsmooth points exist if $i\in\{i:\mu_i=h_i(x)=0\}$.

4 The Solution Algorithm

An improved inexact L-M algorithm is proposed to solve (14). The primal inexact L-M algorithm is based on the recently developed theory for solving semismooth systems formulated by NCP. Due to the semismooth function in (14), the notion of subdifferential is introduced in this algorithm to determine the search direction. Only the approximate solution of a linear system is required at every iteration of the algorithm, which rendered the algorithm quite applicable to the large-scale cases.

The computational procedure of the primal algorithm can be summarized in the following steps. Note that the symbol $\|\cdot\|$ indicates the Euclidean vector norm or its associated matrix norm.

Step 0) Initialization: Set k=0, ρ >0, q>2, $\alpha\in(0,0.5)$, $\varepsilon\ge 0$, and choose a starting point w^0.

Step 1) Stopping criterion: The natural merit function is defined as:

$$\psi(w)=\frac{1}{2}F(w)^T F(w) \tag{15}$$

If $\|\nabla\psi(w^k)\|\le\varepsilon$, stop; otherwise, go to step 2).

Step 2) Search direction calculation: Select a B-subdifferential element $H^k\in\partial_B F(w^k)$ and then find a solution Δw^k of the system

$$((H^k)^T H^k+\sigma^k I)\Delta w^k=-(H^k)^T F(w^k)+r^k \tag{16}$$

It can be simplified as

$$(W^k+\sigma^k I)\Delta w^k=-Z^k+r^k \tag{17}$$

where $\sigma^k\ge 0$ is the L-M parameter and r^k is the residul vector. Set $\Delta w^k=-\nabla\psi(w^k)$ if the following condition (18) is not satisfied

$$\nabla\psi(w^k)\Delta w^k\le-\rho\|\Delta w^k\|^q \tag{18}$$

Step 3) Armijo line search: Find the smallest $i^k\in\{0,1,2,\cdots\}$ such that

$$\psi(w^k+2^{-i^k}\Delta w^k)\le\psi(w^k)+\alpha 2^{-i^k}\nabla\psi(w^k)^T\Delta w^k \tag{19}$$

Step 4) Variables update: The variable w^{k+1} is updated as follows:

$$\begin{aligned}w^{k+1}&:=w^k+2^{-i^k}\Delta w^k\\ k&:=k+1\end{aligned} \tag{20}$$

then go to step 1).

4.1 Specifying a B-subdifferential element

The matrix H in (16) is any element among B-subdifferential of F at w [17]. Let $\beta:=\{i:\mu_i=0=h_i(x)\}$ be the index set. Then, the matrix H is defined by

$$H=\begin{bmatrix}\hat{H} & -J^T & -K^T\\ J & 0 & 0\\ (\partial_1\phi)\cdot K & 0 & \partial_2\phi\end{bmatrix} \tag{21}$$

where $\hat{H}=\frac{\partial(\nabla c(x))}{\partial x^T}-\frac{\partial(J^T\lambda)}{\partial x^T}-\frac{\partial(K^T\mu)}{\partial x^T}$,

$\partial_1\phi(x,\mu)=diag(A_i(x,\mu))$, $\partial_2\phi(x,\mu)=diag(B_i(x,\mu))$, $\partial_1\phi$ and $\partial_2\phi$ are $p\times p$ diagonal matrices whose ith diagonal elements are given, respectively, by

$$A_i(x,\mu)=\begin{cases}\dfrac{h_i(x)}{\|(\mu_i,h_i(x))\|}-1, & if \quad i\notin\beta\\ \xi_i-1, & if \quad i\in\beta\end{cases}$$
$$B_i(x,\mu)=\begin{cases}\dfrac{\mu_i}{\|(\mu_i,h_i(x))\|}-1, & if \quad i\notin\beta\\ \eta_i-1, & if \quad i\in\beta\end{cases}$$
$$(i=1,\cdots,p) \quad (22)$$

where ξ_i and η_i which are the subdifferential parameters satisfy $(\xi_i,\eta_i)\in R^2$ and $\|(\xi_i,\eta_i)\|\le 1$ $(i=1,\cdots,p)$.

4.2 Improvement of positive definitiveness of the coefficient Matrix W

Note that the equation (17) is always solvable. In fact, If $\sigma^k=0$, $(W^k+\sigma^k I)$ reduces to W^k, which is guaranteed to be only positive semidefinite. In this case, provided that H^k is non-singular, the equations (14) is equivalent to the generalized Newton equation $H^k\Delta w^k=-F(w^k)$ and is solvable; when H^k is singular, the equations (14) becomes ill-conditioned. If $\sigma^k>0$, then $(W^k+\sigma^k I)$ is always positive definite and surely solvable. Therefore, the positive definitiveness of the matrix $(W^k+\sigma^k I)$ in (14) can be improved by adjusting the value of L-M parameter σ.

4.3 Improved nonmonotone line search

The Armijo line search in (19) is used to enforce global convergence of the algorithm. However, the line search can lead to very small step sizes, in turn this can bring very slow convergence and even numerical failure of the algorithm. To circumvent the problem, a generalization of nonmonotone Armijo line search is proposed to substitute the line search (19) by the following expression.

Find the smallest $i^k\in\{0,1,2,\cdots\}$ such that

$$\psi(w^k+2^{-i^k}\Delta w^k)\le\Omega+\alpha 2^{-i^k}\nabla\psi(w^k)^T\Delta w^k$$
$$\Omega=\max_{0\le j\le s(k)}\psi(w^{k-j}) \quad (23)$$

where $0\le s(\tau)\le\min[s(\tau-1)+1,10]$, $\tau\ge 1$, $s(0)=0$.

If $s(\tau)=0$, the above nonmonotone line search reduces to the Armijo line search. The nonmonotone line search has proved very useful, allowing a considerable saving both in the number of line search and in the number of function evaluations [18].

5 Numerical Examples

5.1 Feasibility for solving the nondifferentiable problem

To verify the feasibility of the proposed approach for the solution of the nondifferentiable problem, the Kojima-Shindo problem, which is the standard test problem of NCP, is used to illustrate the process.

The function of the Kojima-Shindo problem is defined by

$$F(x)=\begin{bmatrix}3x_1^2+2x_1x_2+2x_2^2+x_3+3x_4-6\\ 2x_1^2+x_1+x_2^2+10x_3+2x_4-2\\ 3x_1^2+x_1x_2+2x_2^2+2x_3+9x_4-9\\ x_1^2+3x_2^2+2x_3+3x_4-3\end{bmatrix} \quad (24)$$

According to the introduced NCM, the NCP is formulated as

$$x_i\ge 0,\ F_i(x)\ge 0,\ x_iF_i(x)=0 \ \text{(i=1,2,3,4)} \quad (25)$$

The associated NCP has two solutions:

$x^1=(1.2247,0,0,0.5)$, $F(x^1)=(0,3.2247,0,0)$;

$x^2=(1,0,3,0)$, $F(x^2)=(0,31,0,4)$.

The solution x^1 is a degenerate solution because of $x_3^1=F_3(x^1)=0$.

Equations (25) are transformed equivalently into the semismooth equations as.

$$\begin{bmatrix}\varphi(x_1,F_1(x))\\ \varphi(x_2,F_2(x))\\ \varphi(x_3,F_3(x))\\ \varphi(x_4,F_4(x))\end{bmatrix}=0 \quad (26)$$

The equations are solved by the improved inexact L-M algorithm. In order to demonstrate the performance of the proposed method, the comparisons with the successive quadratic programming (SQP) algorithm for the nonsmooth equations [19] is used. The results are shown in Table 1.

Table 1 Comparison of results

Method	Starting Point	Iteration	Solution	F vectors at the solution
Proposed method	(0,0,0,0)	6	x^1	$F(x^1)$
SQP	(0,0,0,0)	7	x^1	$F(x^1)$
Proposed method	(1,1,1,1)	6	x^2	$F(x^2)$
SQP	(1,1,1,1)	7	x^1	$F(x^2)$

The solutions obtained from the two methods are same when the starting point are (0,0,0,0) and (1,1,1,1), respectively. However, the proposed method has less computational expense than the SQP algorithm.

It is worth pointing out that the proposed method can solve the semismooth equations (26) when the nondifferential case, i.e., $x_i^{(k)} = F_i(x^{(k)}) = 0$ (k is the iteration number) occurs. The case is shown in Fig.1. The iterative process is given when the starting point is (1,0,1,0).

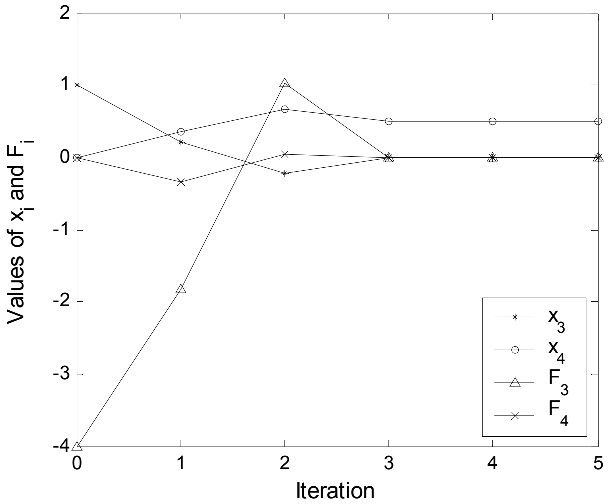

Fig.1 Iteration process

From Fig.1, the expression $x_4^{(0)} = F_4(x^{(0)}) = 0$ and $x_3^{(5)} = F_3(x^{(5)}) = 0$ are satisfied, respectively, namely, $(x_4^{(0)}, F_4(x^{(0)}))$ and $(x_3^{(5)}, F_3(x^{(5)}))$ are the nonsmooth points. Under the circumstances the conventional solution method can not be used to solve the (26) since the nonsmooth points exist. However, the proposed method can do it. It can be seen from Fig.1 that the iteration process can pursue at the nonsmooth points. The results from Fig.1 also indicate that the proposed method can solve the nondifferentiable problem.

5.2 Numerical examples on IEEE test systems

A computer program was implemented in MATLAB to solve the OPF problem. The proposed method was tested on three systems, i.e., the IEEE 9, 30, 118-bus test systems. The parameters ρ, q, α, ξ_i and η_i were given as 10^{-8}, 2.1, 10^{-4}, 0.05, 0.05, respectively. The values of L-M parameter σ and Lagrange multipliers λ_i and μ_i are shown in Table 2. Nx, Nl and Nu denote the number of the system original variables, the number of Lagrange multipliers for the equality constraints and inequality constraints, respectively. The convergence criterion is $\varepsilon \le 10^{-6}$. ε aims at any below and k is the iterative number.

(i) $\left\|\nabla \psi(w^k)\right\| / \left\|\nabla \psi(w^0)\right\|$;

(ii) $\left\|r^k\right\| < (0.1/(k+1))\left\|\nabla \psi(w^k)\right\|$;

(iii) the maximum value of nodal unbalance power.

In the tests, the contributing convergence criterion, which has been verified by the computer program, is shown as

$$\varepsilon = \left\|\nabla \psi(w^k)\right\| / \left\|\nabla \psi(w^0)\right\| \quad (27)$$

Table 2 Information of test systems and initial values

Test Systems	Nx	Nl	Nu	σ	λ_i	μ_i
IEEE-9	21	16	39	0.0000	10	10
IEEE-30	73	58	133	0.0000	50	10
IEEE-118	350	234	649	0.0001	50	10

5.2.1 Convergence and performance

The SQP algorithm in MATPOWER program exploited by Cornell University is used to compare with the proposed method on the same test systems. The tests are conducted on the Intel CPU 1.60 GHz with 2 Gbytes RAM, running the Miscrosoft Windows server 2003 operating system. Table 3 lists the performance comparison of two methods in computing the formulated OPF.

Table 3 Comparison of computational performances

Test Systems	Iterative Numbers		Execution Time(s)	
	Proposed Method	SQP	Proposed Method	SQP
IEEE-9	10	15	0.625	1.34
IEEE-30	11	28	2.125	3.06
IEEE-118	13	-	121.593	-

It can be seen from Table 3 that the proposed method has less both the iterative numbers and execution time than the SQP algorithm. With the system scale increasing, the iterative numbers of the proposed method do not vary too much whereas that of the SQP algorithm increases drastically. Furthermore, the SQP algorithm tested on the IEEE 118-bus system fails to solve the OPF problem which provides the quadratic cost function however the proposed method can do it. The results from Table 3 also indicate that the proposed method shows better performances.

The convergence characteristics for the IEEE 30-bus system are shown in Fig.2. The results from two line search techniques, i.e., the Armijo line search in (19) and the improved nonmonotone line search in (23), are compared in Fig.2. The former fails to converge even when the iterative number is large than one hundred, which is caused by the small step sizes in the process of the Armijo line search.

However, the latter can bring the satisfying results with the eleven iterations. The results from Fig.2 illustrate that the improved nonmonotone line search is superior to the Armijo line search.

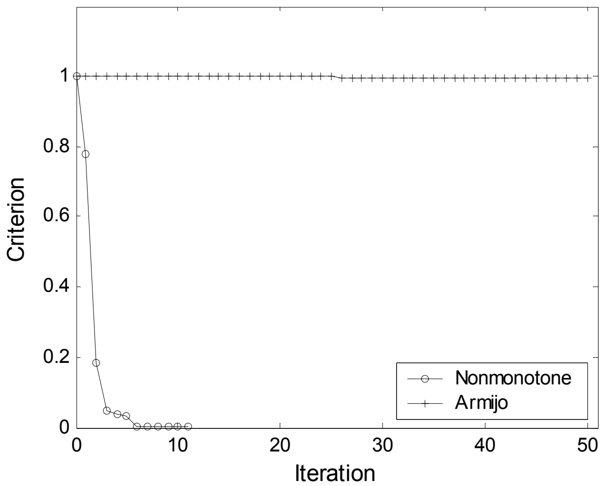

Fig.2 Comparison of convergence characteristic with the Armijo and nonmonotone line search for IEEE 30-bus system (50 iterations are given due to the limited place)

The convergence characteristics for the IEEE 118-bus system are shown in Fig.3. With the system scale expanding, numerical ill-conditioning frequently occurs owing to the non-positive definitiveness of the coefficient matrix. in (17). Thus, there are more uncertainties in convergence for the IEEE 118-bus system than IEEE 9 and 30-bus systems. However, in the proposed method, the case can be improved by the L-M parameter σ in (17). In Table 2, the value of parameter σ is 0.0001 for IEEE 118-bus system, which can improve the positive definitiveness of the matrix W. It can be shown from Fig.3 that the iterative process takes on the oscillation and experiences the convergent problem in the case of $\sigma = 0.0000$ even when the iterative number is large than one hundred, whereas the iterative process presents to be convergent efficiently in the case of $\sigma = 0.0001$.

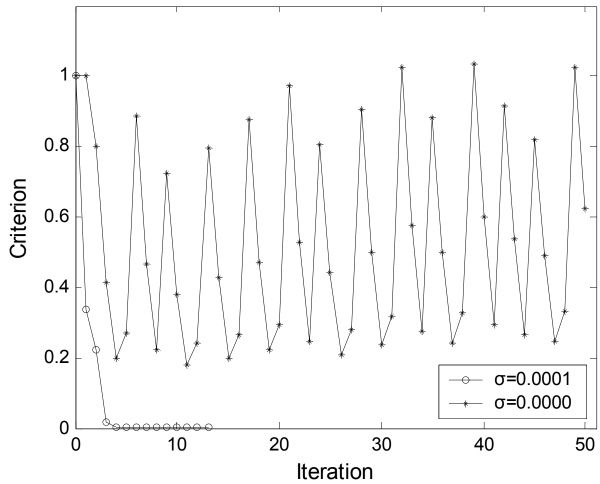

Fig.3 Comparison of convergence characteristic with $\sigma = 0.0000$ and $\sigma = 0.0001$ for IEEE 118-bus system (50 iterations are given due to the limited place)

5.2.2 Accuracy
The Lagrange multipliers of the equality constraints of OPF problem correspond to the shadow prices of nodal power injections and have the same economical significance with the spot prices. Fig.4 shows the results of the spot prices. It is obtained by the proposed method and the SQP algorithm for IEEE 30-bus system. The results of two methods agree so well, which shows the effectiveness of the proposed method.

5.2.3 Some details in Numerical Simulations
(1) Equations (17) are solved by the Gaussian elimination method, which can ensure the convergence of the proposed algorithm. In addition, factorization techniques are used in the process of Gaussian elimination and sparsity techniques are employed to store the nodal admittance matrix,

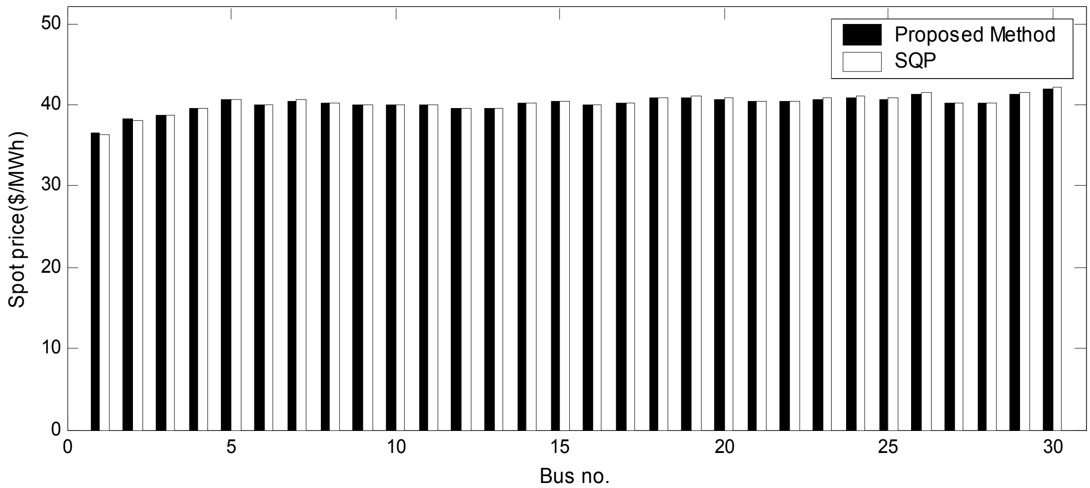

Fig.4 Comparison of active spot prices

which saves the computational time and improves the computational efficiency.

(2) A rearranged variables sequence about the primal and dual variables, similar to the variables sequence of Newton OPF, is used. After rearrangement, a diagonal sub-matrix of the matrix H in (21), in which each diagonal block is constructed by (4×4) small block elements, is obtained and has similar frame with the nodal admittance matrix. Thus, the diagonal sub-matrix can be stored by the sparsity techniques. The rearrangement sequence can reduce the fill-in elements and save the memory. Using the above involved techniques, the proposed method can be extended to the real size power networks.

6 Conclusion

This paper proposed a semismooth inexact Newton-type method for the solution of the OPF problem. After introducing the NCP function, the KKT conditions of OPF model are transformed equivalently into a set of semismooth nonlinear algebraic equations. Then the set of semismooth equations can be solved by an improved inexact Levenberg-Marquardt (L-M) algorithm based on the subdifferential. Numerical studies showed that the proposed method is reliable and better than some existing ones.

In spite of the fact that the nonsmooth cases were not encountered on the IEEE test systems, the subdifferential idea presented in this paper can be extended to cope with other nondifferential problems in the electricity market, for example the nondifferential piecewise cost function, the real energy, reactive energy and voltage support traded in discrete bids and offers in deregulated markets.

References:

[1] H. A. Shayanfar, A. Kazemi, and J. Aqhaei, Effective Location of Facts Devices for Optimal Power Flow in Deregulated Systems, *WSEAS Tran. Circuits and Systems*, Vol.4, No.1, 2005, pp. 32-37.

[2] C. Wang, C. Jiang, A New Arithmetic of Optimal Power Flow Problem Aiming at Inequality Constraints, *WSEAS Tran. Circuits and Systems*, Vol.4, No.8, 2005, pp. 985-991.

[3] A. Kazemi, H. A. Shayanfar, et al, A New Method for Optimal Reactive Power Pricing Considering Power Losses and Voltage Profile, *WSEAS Tran. Circuits and Systems*, Vol.4, No.9, 2005, pp. 1166-1171.

[4] H. W. Dommel, and W. F. Tinney, Optimal Power Flow Solutions, *IEEE Trans. Power App. Syst.*, Vol.PAS-87, No.10, 1968, pp. 1866-1876.

[5] D. I. Sun, et al, Optimal Power Flow by Newton Approach, *IEEE Trans. Power App. Syst.*, Vol.PAS-103, No.10, 1984, pp. 2684-2880.

[6] R. C. Burchett, H. H. Happ, D. R. Vierath, Quadratically Convergent Optimal Power Flow, *IEEE Trans. Power App. Syst.*, Vol.PAS-103, No.11, 1984, pp. 3267-3275.

[7] J. L. Carpentier, Optimal Power Flows: uses, methods and developments. *Proc. IFAC Conf.*, 1985, pp. 11-21.

[8] R. A. Jabr, A. H. Coonick, and B. J. Cory, A Primal-dual Iinterior Point Method for Optimal Power Flow Dispatching, *IEEE Trans. on Power Syst.*, Vol.17, No.3, 2002, pp. 654-662.

[9] I. M. Nejdawi, K. A. Clements, et al, Nonlinear Optimal Power Flow with Intertemporal Constraints. *IEEE Power Engineering Review*, Vol.20, No.5, 2000, pp. 74-75.

[10] H Wang, C. E. Murillo-Sanchez, et al, On Computational Issues of Market-based Optimal Power Flow, IEEE Trans. on Power Syst., Vol.22, No.3, 2007, pp. 1185-1193.
[11] K. C. Almeida, R. Salgado, Optimal Power Flow Solutions under Variable Load Conditions, *IEEE Trans. on Power Syst.*, Vol.15, No.4, 2000, pp. 1204-1211.
[12] H. Yang, Y. Zhang, and X. Tong. Smooth Nonlinear Complementarity Function Based Dynamic Modelling of Power Market. *Electrical and Electronics Engineering. 2006 3rd International Conference*. 2006.
[13] G. L. Torres, V. H. Quintana, Optimal Power Flow by a Nonlinear Complementarity Method, *IEEE Trans. Power Syst.*, Vol.15, No.3, 2000, pp. 1028-1033.
[14] X. Tong, Y. Zhang, F. Wu. A Decoupled Semismooth Newton Method for Optimal Power Flow. *Power Engineering Society General Meeting*, 2006, pp. 1-6.
[15] F. H. Clarke, *Optimization and nonsmooth analysis*, Wiley, New York, 1983.
[16] L. Qi, A Convergence Analysis of Some Algorithms for Solving Nonsmooth Equations, *Mathematics of Operations research*, Vol.18, No.1, 1993, pp. 227-244.
[17] T. De Luca, F. Faccinei, C. Kanzow, A Semismooth Equation Approach to the Solution of Nonlinear Complementarity Problems, *Mathematical Programming*, Vol.75, No.3, 1996, pp. 407-439.
[18] L. Grippo, F. Lampariello, S. Lucidi. A Nonmonotone Line Search Technique for Newton's Method, *SIAM Journal on Numerical Analysis*, Vol.23, No.4, 1986, pp. 707-716.
[19] J. S. Pang and S. A. Gabriel, NE/SQP: A Robust Algorithm for the Nonlinear Complementarity Problem, *Mathematical Programming*, Vol.60, No.1-3, 1993, pp. 295-337.