Manuscript received May 25, 2007; revised July 18, 2007

Mixed H2/H∞ Self-Adaptive Fuzzy Algorithm to Control Satellite Attitude

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Abstract: - The main purpose of this paper is to propose a nonlinear H_∞ and mixed $H2/H_\infty$ self-fuzzy algorithm to solve large-angle robust control problem. The attitude control is one of the studied problems in recent years. Under the space environment, there often exists an unpredictable interference which results some deviations of satellite's attitude. Therefore, it needs to develop a controller still to keep the control ability to the system with respect to the uncertainty of the satellite system and unpredictable environment interference in order to achieve the required control goal and performance specification. In this paper, consider one ROCSAT-3 hardware structure, and use self-fuzzy fuzzy algorithm to perform the attitude control with the time varying and uncertainty parameters. Then, we combine the nonlinear H_∞ and mixed $H2/H_\infty$ control law to reduce the impact of the satellite's attitude system due to external disturbance. Through simulation results, it is shown that the proposed design method can make the ROCSAT-3 system efficiently achieve the desired stability and reject the external interference.

Key-Words: - Nonlinear H_∞ control, Nonlinear mixed H_2/H_∞ control, Self-fuzzy algorithm, Attitude control, Time varying parameter

1 Introduction

The satellite is a nonlinear time varying system that includes the parameter at the time varying uncertainty and be often influenced by the extraneous interference. In recent years, many documents were proposed successively for example, fuzzy control [1, 2], self-adaptive fuzzy control [3], nonlinear H_∞ control [4, 5] and mixed H_2/H_∞ control [6] to control the spacecraft attitude. In this paper, we combined the self-adaptive fuzzy, a nonlinear H_∞ and mixed H_2/H_∞ state feedback control theory, developed a nonlinear H_∞ and mixed H_2/H_∞ self-adaptive fuzzy controller [7, 8], applied to the satellite parameter at the time varying and subjected the influence by the extraneous interference.

At first, the self-adaptive fuzzy control law has been proposed, and held the fuzzy control rule and initial controller parameter setup were sent to the knowledge base. Then, according to the actual response state of the control system, it can be realized by the best adjustment of the self-adaptive parameter, to achieve the desirable satellite attitude using fuzzy reasoning. In fact, the satellite was operated under the complicated space environment, there often exist a lot of external disturbances, such as an interference of atmosphere, the gravity of the earth and magnetic field on the earth, the sun wind, these disturbances result the satellite attitude has some deviation. At this moment, the self-adaptive fuzzy algorithm combined with a nonlinear H_∞ and mixed H_2/H_∞ control theory to enable the system to resist the interference under an uncertain environment. The nonlinear H_∞ and mixed H_2/H_∞ control law is to decide a solution of Hamilton-Jacobi inequality in order to solve the nonlinear H_∞ and mixed H_2/H_∞ controller conveniently.

The main purpose of development nonlinear H_∞ and mixed H_2/H_∞ self-fuzzy algorithm is the satellite system exist the parameter uncertainty, time varying and external interference, in order to solve this problem then the controller was developed. The goal of the controller of the satellite system is still able to make the satellite get back to desired attitude under the worst situation.

2 Mathematics of the Satellite Movement

In the calculation process of the satellite attitude, it needs to use the coordinate transformations, which includes direction cosine matrix, Euler angles, quaternion, etc. Because the direction cosine matrix must deal with more variables and Euler angles has an angular singularity at $\pi/2$, this paper selects the quaternion method to express the attitude and control of ROCSAT-3 due to avoid singularity, simplify the parameter number and perform conveniently numerical operation.

Suppose n is the direction cosine of the Euler axis relative to a reference frame, and the rotated angle is μ, the attitude of satellite then can be expressed by quaternion as follows:

$$\underline{q} = \begin{bmatrix} q1 \\ q2 \\ q3 \end{bmatrix} = n\sin(\frac{\mu}{2}) \text{ and } q_4 = \cos\frac{\mu}{2}$$

The satellite attitude kinematics equation can be expressed the following:

$$\begin{bmatrix} \underline{q} \\ q_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} q_4 I + q^{\times} \\ -\underline{q}^T \end{bmatrix} \underline{\omega} \tag{1}$$

Where I the unit matrix, ω the angle velocity, $q^{\times}$ is the skew-symmetric matrix defined by

$$q^{\times} = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

The satellite attitude dynamics equation expresses as follows:

$$J\dot{\underline{\omega}} = -\underline{\omega}^{\times} J \underline{\omega} + u \tag{2}$$

where J is the moment of inertia matrix, u is the control input, $\omega^{\times}$ is the skew-symmetric matrix defined by

$$\underline{\omega}^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

3 Self-Adaptive Fuzzy Control Design

In the attitude control, due to quaternion value operation convenient, the self-adaptive fuzzy controller design adopt a standard quaternion feedback controller [1, 2, 8, 9]. The controller structure is shown in Fig.1.

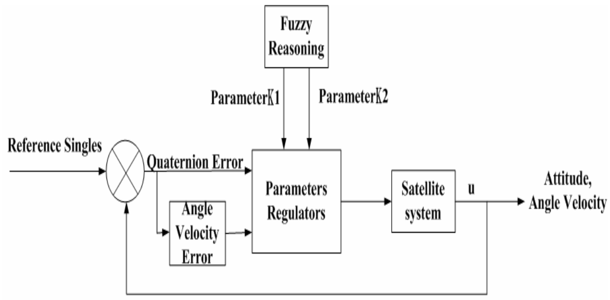

Fig.1 The self-adaptive fuzzy controller structure

The purpose of the controller is to find out the fuzzy relation among k_1, k_2 two parameters and quaternion errors and angular speed errors. Through a continuous examination of quaternion errors and angular speed errors, the on-line parameters should be revised according to fuzzy control principle, in order to meet the different errors with respect to the different demands of the control parameter, and make the plant have a good movement, and static performance.

The quaternion feedback control method is to feedback both attitude and angular velocity. The controller can be defined as follows:

$$u = -k_1 . q_{error} - k_2 . \omega_{error} \tag{3}$$

The main advantage of Eq.(3) is only to relate the state measurement, and need not to know the system parameter clearly. Therefore, it is robust to the error model and the uncertainty and time varying parameter. Where q_{error} and w_{error} express the quaternion error and angular velocity error of the system, respectively [2]. The q_{error} and w_{error} are defined as follows:

$$\cdot\, q_{error} = q_{command}^{-1} . q_{current} = q_c^{-1} . q \tag{4}$$

$$\cdot\, \omega_{error} = \omega_{command} - \omega_{current} \tag{5}$$

4 Nonlinear State Feedback H_∞ Control Theory

Consider the following nonlinear system, suppose all state can be measured:

$$\dot{x} = f(x) + g_1(x) + g_2(x)u \tag{6}$$

$$z = \begin{bmatrix} Q_1(x) \\ u \end{bmatrix} \tag{7}$$

The adopted state feedback control strategy is to assume as follows:

$$u(\underline{q}, q_4, \underline{\omega}) = \delta(x) \tag{8}$$

Where $\delta(x)$ is the undetermined function, the purpose of the nonlinear H_∞ state feedback control is to find a state feedback control law such that the L_2 gain of the closed-loop system from w to z is less than appointed γ.

$$\frac{\|z\|_{L_2}}{\|w\|_{L_2}} \le \gamma\,,\ \forall w \in L_2$$

The purpose of the nonlinear H_∞ state feedback control is, if there exist a $P(x) \ge 0$ and satisfied Hamilton-Jacobi inequality as the following,

$$H_\gamma = P_x^T f + \frac{1}{2} P_x^T \left(\frac{1}{\gamma^2} g_1 g_1^T - g_2 g_2^T \right) P_x + \frac{1}{2} Q_1^T Q_1 < 0 \tag{9}$$

Then there exists a control law $u(\underline{q}, q_4, \underline{\omega})$ such that the nonlinear system is stable and from w to z L_2 gain is less than γ. Suppose if $P(\underline{x}) > 0$ is found, then $u(\underline{q}, q_4, \underline{\omega})$ can be written as follows:

$$u(\underline{q}, q_4, \underline{\omega}) = -g_1^T P_x \tag{10}$$

We can make the nonlinear system equation rearrange as follows:

$$\begin{bmatrix} \dot{\underline{q}} \\ q_4 \\ \dot{\underline{\omega}} \end{bmatrix} = f\left(\underline{q}, q_4, \underline{\omega}\right) + gu + gw \tag{11}$$

$$z = \begin{bmatrix} \sqrt{\rho_1\, \underline{\omega}^T J\, \underline{\omega} + \rho_2 q^2 (q_4)} \\ u \end{bmatrix} \tag{12}$$

Suggest the following candidate function [4, 9, 10] in order to satisfy Eq.(9),

$$P(\underline{x}) = \frac{1}{2} \begin{bmatrix} \underline{q}^T & 1-q_4 & \underline{w}^T \end{bmatrix} \begin{bmatrix} a_1 J & 0 & a_3 J \\ 0 & a_1 & 0 \\ a_3 J & 0 & a_2 J \end{bmatrix} \begin{bmatrix} \underline{q} \\ 1-q_4 \\ \underline{w} \end{bmatrix}$$

$$= a_1 (1-q_4) + \frac{a_2}{2} \underline{w}^T J \underline{w} + a_3 \underline{w}^T J \underline{q} \tag{13}$$

Where a_1 , a_2 and a_3 of Eq.(13) are the undetermined coefficients, and make (11), (12) and (13) substitution Hamilton-Jacobi inequality, can obtain

$$a_2 \ge \sqrt{\frac{\pi^2 \gamma^2 \rho_2}{\gamma^2 - 1}}, a_1 \ge \sqrt{\frac{(3b + \frac{\rho_1}{2})\gamma^2 \|J\|}{\gamma^2 - 1}}$$

If a_1 , a_2 satisfies above Equations, then it can guarantee that $H_\gamma \le 0$, and obtains the control law,

$$u(\underline{q}, q_4, \underline{\omega}) = -(P_x g)^T$$

$$= -a_1 \underline{\omega} - a_2 \underline{q} \tag{14}$$

The nonlinear H_∞ control theory can raise the robustness and stability performance of the satellite system and restrain the influence of the external interference to the system, but it cannot achieve the optimization efficiency. Next section will propose the mixed H_2 / H_∞ control theory to make the overall system be a robust, stable and optimal efficiency.

5 Nonlinear State Feedback Mixed H_2/H_∞ Control Theory

We consider the following nonlinear system equation, suppose all states can be measured:

$$\begin{bmatrix} \dot{\underline{q}} \\ q_4 \\ \dot{\underline{\omega}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}q^{\times}\underline{w}+\frac{1}{2}q_4\underline{w} \\ -\frac{1}{2}\underline{q}^T\underline{w} \\ -J^{-1}\underline{w}^{\times}J\underline{w} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ J^{-1} \end{bmatrix} w + \begin{bmatrix} 0 \\ 0 \\ J^{-1} \end{bmatrix} u \quad (15)$$

$$z = \begin{bmatrix} \sqrt{\rho_1 q^2(q_4)+\rho_2\underline{\omega}^T J\underline{\omega}} \\ \rho_u u \end{bmatrix} \underline{\underline{\Delta}} \begin{bmatrix} Q_1(\underline{x}) \\ \rho_u u \end{bmatrix} \quad (16)$$

Fist, we define two cost functions [10]:

$$J_1(u,w)=\int_0^T\left(\gamma_1^2\|w\|^2-(\|Q_1\|^2+\|\rho_u u\|^2)\right)dt$$

$$J_2(u,w)=\int_0^T(\|Q_1\|^2+\|\rho_u u\|^2-\gamma_2^2)dt$$

The purpose of mixed H_2/H_∞ control is to decide the optimal control law u^* and the worst perturbation w^* and make

$$J_1(u^*,w^*)\le J_1(u^*,w),\forall w$$

$$J_2(u^*,w^*)\le J_2(u,w^*),\forall u$$

Where it should be proved that u^*, w^* can be obtained as,

$$w^*=\frac{1}{\gamma_1^2}g_1^T(\underline{x})T_{\underline{x}} \quad (17)$$

$$u^*=-\frac{1}{\rho_u^2}g_2^T(\underline{x})P_{\underline{x}} \quad (18)$$

Where equations $T_{\underline{x}}<0$, $P_{\underline{x}}>0$ need satisfy a pair Hamilton-Jacobi-Isaacs (HJI) equations [10]

$$T_{\underline{x}}^T(f(\underline{x})+g_1(\underline{x})w^*+g_2(\underline{x})u^*)-\gamma_1^2\left\|w^*(\underline{x})\right\|^2+\left\|Q(\underline{x})\right\|^2+\left\|\rho_u u^*\right\|^2<0$$

$$P_{\underline{x}}^T(f(\underline{x})+g_1(\underline{x})w^*+g_2(\underline{x})u^*)+\left\|Q(\underline{x})\right\|^2+\left\|\rho_u u^*\right\|^2-\frac{\gamma_2^2}{T}<0$$

Consider the following Lyapunov function to solve a pair of HJI equations,

$$T(\underline{x})=-a_1(1-q_4)-\frac{a_2}{2}\underline{w}^T J\underline{w}-a_3\underline{w}^T J\underline{q} \quad (19)$$

$$P(\underline{x})=a_4(1-q_4)+\frac{a_5}{2}\underline{w}^T J\underline{w}+a_6\underline{w}^T J\underline{q} \quad (20)$$

Where a_1 , a_2 , a_3 , a_4 , a_5 and a_6 are the undetermined coefficients at Eq.(19) and Eq.(20), and substitute $T(\underline{x})$, $P(\underline{x})$ into one pair HJI, then obtain

$$a_6\ge\sqrt{\gamma_1^2\pi^2\rho_1},\ a_5\ge\sqrt{(\frac{1}{2}\rho_2-\frac{1}{2}a_6)\gamma_1^2\|J\|}$$

If a_5 , a_6 satisfy above equations, then it can be guaranteed that HJI <0 , and the control law is

$$u^*(\underline{q},\underline{q_4},\omega)=-\frac{1}{\rho_u^2}(a_5\underline{w}+a_6\underline{q}) \quad (21)$$

6 Nonlinear H_∞ and Mixed H_2/H_∞ Self-Fuzzy Control Law

Once satellite system uses only the self-adaptive fuzzy algorithm while it meets an additional interference, simulation results show a relatively serious oscillation. Therefore, we consider select the self-adaptive fuzzy algorithms and combine with a nonlinear H_∞ and mixed H_2/H_∞ control theory, and then obtain a nonlinear H_∞ and mixed H_2/H_∞ self-adaptive fuzzy control law.

This control law is to make use of errors on-line to adjust a_1 and a_2 two parameters. At this moment the control law can still make the satellite attitude maintain balance position when the parameters have a time varying, uncertainty and interfering environment. The interference causes a drift will be rejected. The structure of the overall satellite system is shown in Fig. 2.

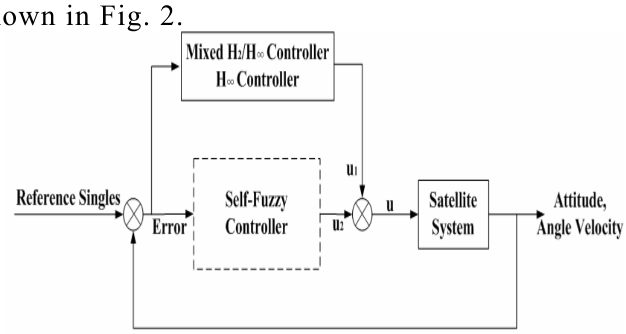

Fig.2 Nonlinear robust self-adaptive fuzzy controller structure

7 Simulation Results

This paper is to propose a control law to emulate ROCSAT-3 attitude measurement successfully. The simulation process includes the time varying inertia parameter and an external interference environment in the space. The initial condition of satellite simulation is to design $q(0)=\begin{bmatrix}0.1705 & 0.5688 & 0.1688 & 0.7868\end{bmatrix}^T$ and $\omega(0)=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$. From simulation results, we can find the nonlinear H_∞ control with the fast response and a good stability to weaken extraneous interference. In order to achieve an accurate satellite attitude control, we developed a nonlinear H_2/H_∞ controller that has combined H_2 optimization performance and H_∞ robust characteristic, better than H_∞ controller had robustness stability to weaken extraneous interference.

Fig.3 to Fig.6 show the response of satellite system quaternion and Fig.7 to Fig.9 show the response of angular velocity. The response of control torque is shown in Fig.10 to Fig.12. From the above simulations, we can understand that the mixed H_2/H_∞ has more high performance attitude control than H_∞ does under the worst condition.

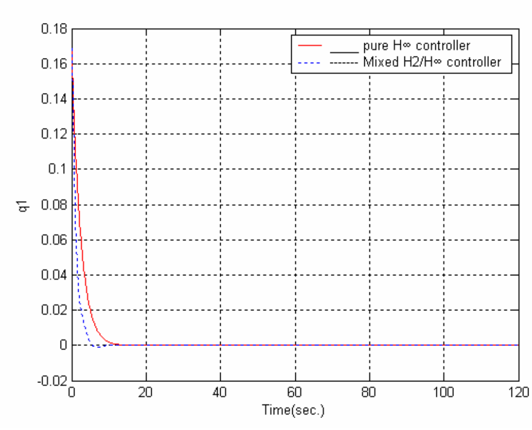

Fig. 3 Time response of quaternion1

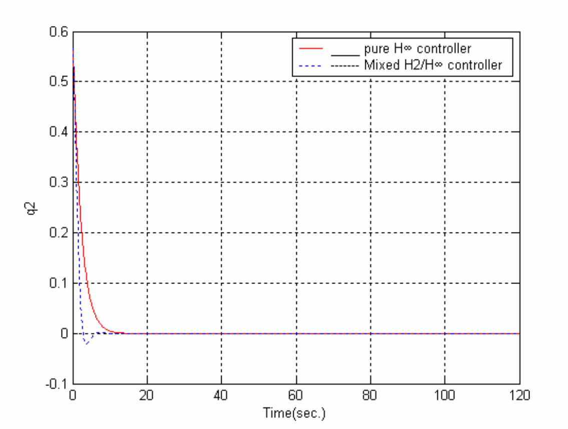

Fig. 4 Time response of quaternion2

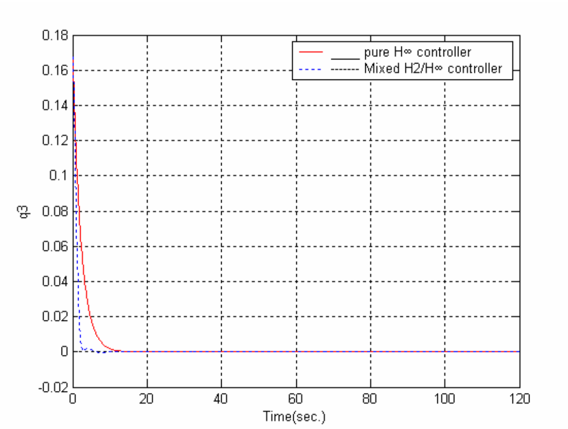

Fig. 5 Time response of quaternion3

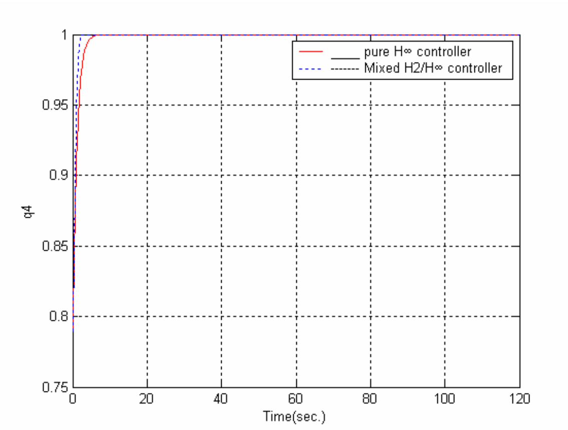

Fig. 6 Time response of quaternion4

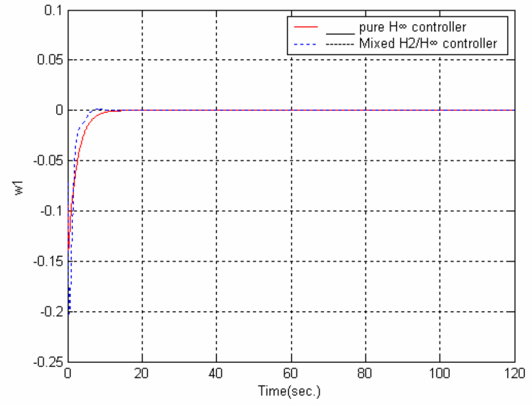

Fig. 7 Time response of angle velocity w1

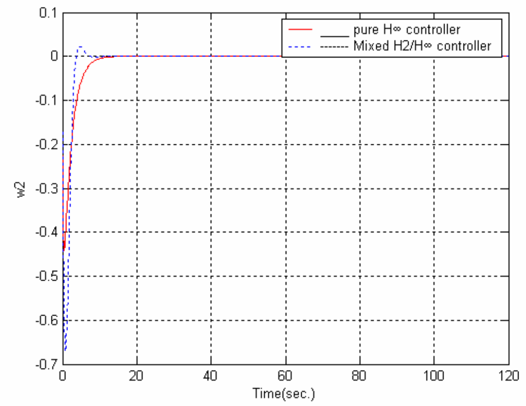

Fig. 8 Time response of angle velocity w2

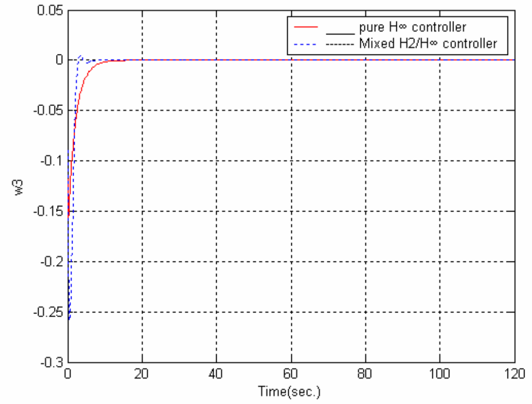

Fig. 9 Time response of angle velocity w3

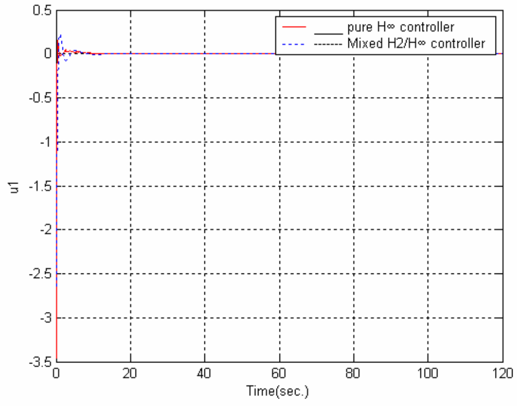

Fig. 10 Time response of control torque u1

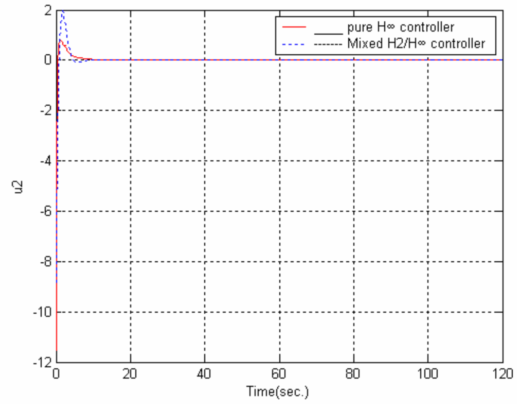

Fig. 11 Time response of control torque u2

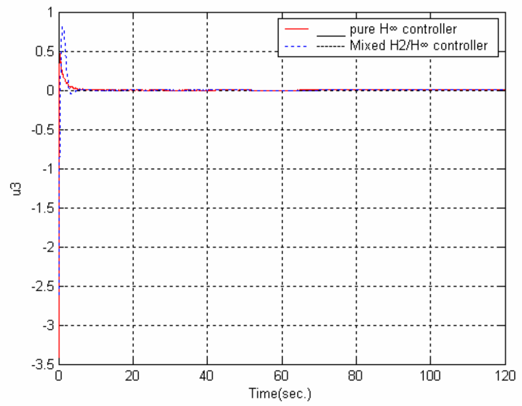

Fig. 12 Time response of control torque u3

8 Conclusions

This paper has succeeded to propose a nonlinear H_∞ and mixed H_2/H_∞ self-adaptive fuzzy algorithm to simulate the attitude response of ROCSAT-3 system. Through the simulation results, it is shown that the satellite's attitude can be back to desired position efficiently and the external perturbation will reduce the impact with respect to output and achieve the purpose of stable attitude.

Hence, the proposed control law can demonstrate the ideal control capability and robustness under an external interference environment to make ROCSAT-3 system have an accurate control attitude during performing the mission.

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