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Proposal of Numerically Robust Algorithm for Stochastic Systems Identification

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Abstract: - This work proposes a recursive weighted ELS algorithm for system identification by applying numerically robust orthogonal Householder transformations. The properties of the proposed algorithm show it obtains acceptable results in a noisy environment: fast convergence and asymptotically unbiased estimates. Comparative analysis with others robust methods well known from literature are also presented.

Key-Words: Numerical Method, Modeling, Simulation, Discrete Time Dynamic Systems, Parameter Estimation, Stochastic Systems

1 Introduction

System identification is a general term used to describe mathematical tools and algorithms that build dynamical models from measured data [3][4][14][15]. A dynamical model in this context is a mathematical description of the dynamic behavior of a system or process, as example the movement of a falling body under the influence of gravity and stock markets that react to external influences. Three approaches are common in the data computational modeling field:

- *White-box model*: Based on physical principles, eg., a model for a system from the Newton laws equations; but in many cases such models will be overly complex and possibly even impossible to obtain in reasonable time due to the complex nature of many systems and industrial processes.

- *Black-box model*: No prior knowledge about the dynamic system's behaviour is available, and the problem consists in obtain an adequate mathematical description of the system or industrial process from experimental data.

- *Grey-box model*: Although the peculiarities inside the system are not entirely known, a certain physical model is already available. This model does however still have a number of unknown free parameters which can be estimated using experimental data.

A common approach is therefore to start from measures related to the behavior of the system and external influences as inputs and noises, and try to determine a mathematical relation between them without going into the details of what is actually happening inside the system. To be applicable to real world problems, the parameter estimation must be highly efficient, because the input and output measurements may be contaminated by noise. For low levels of noise the least squares (LS) method, for example, may produce excellent estimates of the consequent parameters. However, with larger levels of noise, some modifications in this method are required to overcome this inconsistency. Generalized least squares (GLS) method, extended least squares (ELS) method, prediction error (PE) method and instrumental variable (IV), are examples of such modifications [9][19][20]. In [10], a robust iterative instrumental variable method with modified residuals is developed for robust identification of systems based on Huber's minimax principle and instrumental variable principle. This algorithm, however, is only used in off-line applications. In [12], a recursive least squares algorithm for fixed order with exponential data weighting, using Givens orthogonal transformations for identification of parameters that vary quickly with time is proposed. A limitation of this proposal is its application just for very low levels of noise environment. In [1], two algorithms for LS system identification via QR decomposition are proposed. This algorithm presents the same limitations of [12]. In [8], a new proposal for robust identification of multivariable nonlinear stochastic systems using fuzzy instrumental variable method [7] is presented.

The proposal of this paper belongs to these contexts. A recursive weighted ELS algorithm based on the numerically robust orthogonal Householder transformations is developed for systems identification in a noise environment, characterizing the black-box model approach. This algorithm, once numerically validated, is used for closed loop identification, according to direct and indirect methods, to provide accurate estimations of the system under feedback control using simple measurements in a noise environment. All experiments are performed in a DC servomotor, of 12 Volts, from the Control Laboratory.

2 Problem Formulation and Algorithm

Consider, for a given dynamic system, the following model structure

$$y_t = b_1 u_{t-1} + \ldots + b_{nb} u_{t-nb} - a_1 y_{t-1} - a_{nc} y_{t-na} + \xi_t$$

where $u(t)$ e $y(t)$ is the input and output of the model, respectively, and ξ_t is a noise signal added to the system. Let:

$$\theta^T = (b_1, \ldots, b_{nb}, a_1, \ldots, a_{na})$$

$$\mathbf{a}_t^T = (u_{t-1}, \ldots, u_{t-nb}, -y_{t-1}, \ldots, -y_{t-na})$$

and in a vetorial form, considering a set of data from the system, gives

$$\mathbf{Y} = \mathbf{A}\theta + \Xi$$

where

$$\mathbf{Y}^T = [\mathbf{y}_1, \ldots, \mathbf{y}_n]$$

$$\mathbf{A}^T = [\mathbf{a}_1, \ldots, \mathbf{a}_p]$$

$$\Xi^T = [\xi_1, \ldots, \xi_n]$$

with p the dimension of the problem, i.e., $na + nb$, and n is the number of sample.

It is intended to obtain a consistent estimation of θ from $\{y_t, u_t\}_1^n$ so that the error between the measured and estimated output of the system be minimum in sense of least squares

$$\min \|\mathbf{A}\theta - \mathbf{b}\|_2^2$$

or in the normal equations form

$$\mathbf{A}^T \mathbf{A}\theta = \mathbf{A}^T \mathbf{Y}$$

.

2.1 Householder Transformation Method

For study on performance of numerical algorithms, the concepts of numerical stability and conditioning are very importants. The former propriety is not based on the computing, and the later one is associated to computing problems and the data to be processed. The numerical errors, in any computational processing, are dependent of the stability of the algorithm and the conditioning of the problem. The proposed algorithm formulation is based on Householder orthogonal matrices for solve the normal equation given in (9) by QR factorization. The use of orthogonal transformations for solve least squares problems are well established by the following: matrices easily invertible, precision and speed of computations perfectly conditioned, simplified error analysis due the use of Euclidian or spectral norms commonly used in such study. The Householder orthogonal matrix is of the following form:

$$\mathbf{H} = \mathbf{I} - 2\frac{\mathbf{v}\,\mathbf{v}^T}{\|\mathbf{v}\|_2^2}$$

where $\mathbf{H} = \mathbf{H}^T$ and $\mathbf{H} = \mathbf{H}^{-1}$. Householder transformations are powerful tools for annul a block of entries in matrices or vectors by selection of Householder vectors **v** as shown in (10) [1][2]. Thus, if **x** is a vector and $\mathbf{e_i}$ is a unity vector with 1 in the **i**-th position, the vector **v** is defined by

$$\mathbf{v} = \mathbf{x} \pm \|\mathbf{x}\|\,\mathbf{e_i}$$

and

$$\mathbf{H}\mathbf{x} = \mp \|\mathbf{x}\|\,\mathbf{e_i}$$

The vectors **x** and **v** are the same dimension and are different only in the **i**-th position. In this analysis, the procedure for explicit form of the Householder matrix is not established which happen in the most cases.

2.2 The QR-ELS Algorithm

In industrial applications of identification algorithms, the model structure can be known but its parameters need to be estimated from measured data of the dynamic system. The variables associated to the dynamical system behaviour can vary with time due the changes of operating conditions. In this case, the off-line methods are insufficient and have a poor performance. The main motivation of this proposal is the development of an algorithm that provides frequently the parameters estimation by adequate processing of its input and output data and can adapt itself to the varying operating conditions of the dynamic system. The problem of interest can be given by

$$\mathbf{A}^T\mathbf{A}\theta = \mathbf{A}^T\mathbf{Y}$$

where $\mathbf{A}_{n\times p}$, $\theta_{p\times 1}$ and $\mathbf{Y}_{n\times 1}$ are, respectively, the matrix of data, the parameter vector and the output vector. The equation in (13), in turn, can be rewrited by

$$\mathbf{A}^T\mathbf{W}\mathbf{A}\theta = \mathbf{A}^T\mathbf{W}\mathbf{Y}$$

where $\mathbf{W}_{n\times n}$ and $\mathbf{W}_n = diag(\lambda^{n-1}, \lambda^{n-2}, ..., 1)$, with $0<\lambda<1$. The scalar λ is the well known *forgetting factor*, and it is used to give more weight for the actual data and less weight for the past data. Developing both side in (14), as **W**, **A** and **Y** are known, results

$$\mathbf{S}\theta = \mathbf{b}$$

where $\mathbf{S}_{p\times p} = \mathbf{A}^T\mathbf{W}\mathbf{A}$ and $\mathbf{b}_{p\times 1} = \mathbf{A}^T\mathbf{W}\mathbf{Y}$. It is important to highlight that the order of the matrix **S** and of the vector **b** are lower than matrix **A** and vector **b**, because p is the number of parameters to be estimated, implying less computational effort and, consequently, more fast for solution of θ. Generically, the matrices **A**, **W** and the vector **b** are given by

$$\mathbf{W} = \begin{bmatrix} \lambda^{n-1} & 0 & . & . & 0 \\ 0 & \lambda^{n-2} & 0 & . & . \\ . & 0 & . & 0 & . \\ . & . & 0 & \lambda & 0 \\ 0 & . & . & 0 & 1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} u_0 & u_{-1} & \cdots & u_{1-nb} & -y_0 & -y_{-1} & \cdots & -y_{1-nc} \\ u_1 & u_0 & \cdots & u_{2-nb} & -y_1 & -y_0 & \cdots & -y_{2-nc} \\ \bullet & \bullet & \cdots & \bullet & \bullet & \bullet & \cdots & \bullet \\ u_{n-1} & u_{n-2} & \cdots & u_{n-nb} & -y_{n-1} & -y_{n-2} & \cdots & -y_{n-na} \end{bmatrix}$$

and

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ y_n \end{bmatrix}$$

Hence, $\mathbf{S}_{p\times p} = \mathbf{A}^T\mathbf{W}\mathbf{A}$ results

$$\mathbf{S} = \begin{bmatrix} \sum_{t=1}^{n} u_{t-1}u_{t-1}\lambda^{n-t} & \cdots & \sum_{t=1}^{n} u_{t-1}u_{t-nb}\lambda^{n-t} & -\sum_{t=1}^{n} u_{t-1}y_{t-2}\lambda^{n-t} & \cdots & -\sum_{t=1}^{n} u_{t-1}y_{t-na}\lambda^{n-t} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \sum_{t=1}^{n} u_{t-nb}u_{t-1}\lambda^{n-t} & \cdots & \sum_{t=1}^{n} u_{t-nb}u_{t-nb}\lambda^{n-t} & -\sum_{t=1}^{n} u_{t-nb}y_{t-2}\lambda^{n-t} & \cdots & -\sum_{t=1}^{n} u_{t-nb}y_{t-na}\lambda^{n-t} \\ -\sum_{t=1}^{n} y_{t-2}u_{t-1}\lambda^{n-t} & \cdots & -\sum_{t=1}^{n} y_{t-2}u_{t-nb}\lambda^{n-t} & \sum_{t=1}^{n} y_{t-2}y_{t-2}\lambda^{n-t} & \cdots & \sum_{t=1}^{n} y_{t-2}y_{t-na}\lambda^{n-t} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ -\sum_{t=1}^{n} y_{t-na}u_{t-1}\lambda^{n-t} & \cdots & -\sum_{t=1}^{n} y_{t-na}u_{t-nb}\lambda^{n-t} & \sum_{t=1}^{n} y_{t-na}y_{t-1}\lambda^{n-t} & \cdots & \sum_{t=1}^{n} y_{t-na}y_{t-na}\lambda^{n-t} \end{bmatrix}$$

and $\mathbf{b}_{p\times 1} = \mathbf{A}^T\mathbf{W}\mathbf{Y}$ is

$$\mathbf{b} = \begin{bmatrix} \sum_{t=1}^{n} u_{t-1}y_t\lambda^{n-t} \\ \vdots \\ \sum_{t=1}^{n} u_{t-nb}y_t\lambda^{n-t} \\ -\sum_{t=1}^{n} y_{t-1}y_t\lambda^{n-t} \\ \vdots \\ -\sum_{t=1}^{n} y_{t-na}y_t\lambda^{n-t} \end{bmatrix}$$

From equations (16) and (17), it can see that the entries of the matrix **A** and of the vector **b** are dependent of actual and immediately past values of input and output from the dynamic system to be identified, according to the dimension of the problem *p*. This imply in generate, directly, i.e., at each sample, the matrix **S** and the vector **b**, without necessity of bench matrices operations as in (14), with advantage the dimension of the problem is lower for QR factorization. So, the problem can be rewrited as to find the solution for

$$\underset{\hat{\theta}}{\text{minimize}} \| \mathbf{S}\theta - \mathbf{b} \|_2^2$$

Applying QR factorization, via Householder orthogonal transformations, gives

$$\underset{\hat{\theta}}{\text{minimize}} \left\| \mathbf{Q}^T \mathbf{S}\theta - \mathbf{Q}^T \mathbf{b} \right\|_2^2$$

and

$$\underset{\hat{\theta}}{\text{minimize}} \| \mathbf{R}\theta - \mathbf{d} \|_2^2$$

where $\mathbf{Q}_{p\times p}$ is an orthogonal matrix, $\mathbf{R}_{p\times p}$ is an upper triangular matrix, and $\mathbf{d}_{p\times 1}$ is a resulting vector. So, the minimizer of (18) can be found by solving $\mathbf{R}\hat{\theta} = \mathbf{d}$ by back-substitution method.

The algorithm receives a set of data for initial estimation and the updating is done by acquisition of input and output measures and using the summation of the matrix **S** and the vector **b**, this is, at *t*-th sampling time, results:

$$\mathbf{S}_{new} = \mathbf{S} + \lambda \begin{bmatrix} u_t u_t & \cdots & u_t u_{t-nb} & -u_t y_{t-1} & \cdots & -u_t y_{t-na} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ u_{t-nb} u_t & \cdots & u_{t-nb} u_{t-nb} & -u_{t-nb} y_{t-1} & \cdots & -u_{t-nb} y_{t-na} \\ -y_{t-1} u_t & \cdots & -y_{t-1} u_{t-nb} & y_{t-1} y_{t-1} & \cdots & y_{t-1} y_{t-na} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ -y_{t-na} u_t & \cdots & -y_{t-na} u_{t-nb} & y_{t-na} y_{t-1} & \cdots & y_{t-na} y_{t-na} \end{bmatrix}$$

and

$$\mathbf{b}_{new} = \mathbf{b} + \lambda \begin{bmatrix} u_t y_t \\ \vdots \\ u_{t-nb} y_t \\ -y_{t-1} y_t \\ \vdots \\ -y_{t-na} y_t \end{bmatrix}$$

The algorithm is as follow:

Step 1: Define the number of input and output pairs from the experimental data for initial estimation;

Step 2: Generate the matrix **S** and the vector **b** from (16) and (17);

Step 3: Apply QR factorization via Householder orthogonal transformation to generate the minimization problem in (20);

Step 4: Solve (20) by back-substitution method;

Step 5: Obtain a new value of input and output pairs from the experimental data;

Step 6: Generate the new matrix **S** and the new vector **b** from (21) and (22), respectively;

Step 7: Go back to **Step 3**.

The LS estimation is biased with the presence of correlated noise on the experimental data. It can be verified from the matricial product involving the matrix **A** and the vector of noise Ξ is different of zero:

$$\hat{\theta} = \theta + \left(\mathbf{X}^T \mathbf{X}\right)^{-1} \mathbf{X}^T \Xi$$

There are several possibility to overcome this problem as outlined in section I of this paper. Particularly, it is of interest the modeling the correlation of the noise and estimate the dynamic system model parameters from this model correlation. It can be performed using the regression model

$$\varepsilon(k) = y(k) - \varphi^T (k-1)\hat{\theta}(k-1)$$

where

$$\hat{\theta} = (a_1 \ldots a_n \; b_1 \ldots b_n \; c_1 \ldots c_n)$$

$$\varphi^T(k-1) = (-y(k-1) \ldots -y(k-n) u(k-1) \ldots u(k-n) \varepsilon(k-1) \ldots \varepsilon(k-n))$$

where the variables $e(k)$ are approximated by the prediction error $\varepsilon(k)$. Thus, the online identification can be done substituting using (25) into the matrix **A** and performing all formulation and steps already outlined.

3 Computational Results

In this section is shown a simulation example to illustrate the main characteristics of the ELS-QR algorithm proposed in this paper, and an experimental application to demonstrate the applicability of the algorithm to open and closed loop identification.

3.2 Simulation Results

In this section, without less of generality, will be presented two simulation results for speed convergence analysis and consistence of the proposed algorithm for open loop identification in a noisy environment.

Example I:

Consider the following linear system described by

$$A(z^{-1})y(t) = B(z^{-1})u(t) + C(z^{-1})e(t)$$

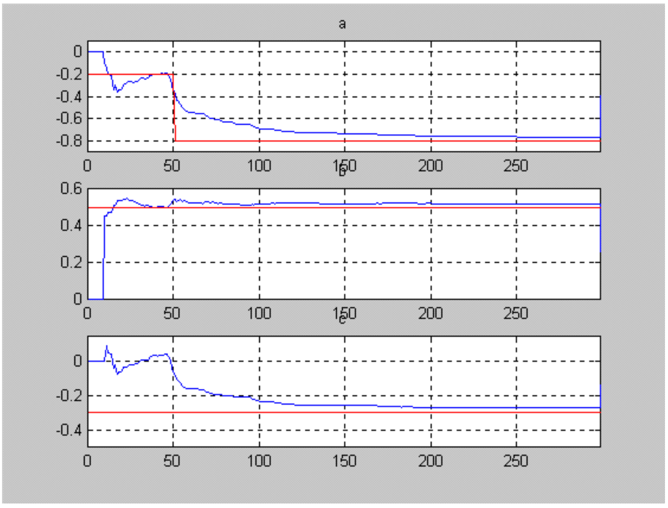

Fig. 2. Performance of the proposed ELS-QR algorithm in a noise environment.

where

$$A(z^{-1}) = 1 - 0.2\,z^{-1}$$

$$B(z^{-1}) = 0.5\,z^{-1}$$

$$C(z^{-1}) = 1 - 0.3\,z^{-1}$$

The input of the system was the well known PRBS (Pseudo-Random Binary Sequence) signal of magnitude one. The perturbation $e(k)$ consists in a white noise signal with mean zero and variance unity. The total of points used in this simulation was *300* and the proposed algorithm was implemented with $\lambda = 0{,}98$, with *10* points for initial estimation. After 50 points of running, the parameter *0,2* in polynomial $A(z^{-1})$ is changed to *0,8*. The Fig. 1 shows the performance of the algorithm LS-QR without influence of noise on the data (red) and with noisy environment (blue). It can be seen that this algorithm track efficiently the time varying parameter of the dynamic system, while in the presence of noise its performance is very poor as expected. The Fig. 2 shows the performance of the proposed ELS-QR algorithm in the same conditions applied to LS-QR. It is observed that the algorithm can track quickly the estimated time varying parameter in a noisy environment. The Table I shows a comparative analysis of the LS-QR and ELS-QR algorithms performances.

TABLE I.
ESTIMATED PARAMETERS FROM LS-QR AND ELS-QR ALGORITHMS

Parameters	True values	LS-QR	LS-QR (with noise)	ELS-QR (with noise)
a	-0,8	-0,7192	-0,2819	-0,7897
b	0,5	0,5305	0,4579	0,5034
c	-0,3			-0,2868

The following norm of the parametric error equation was used for convergence analysis:

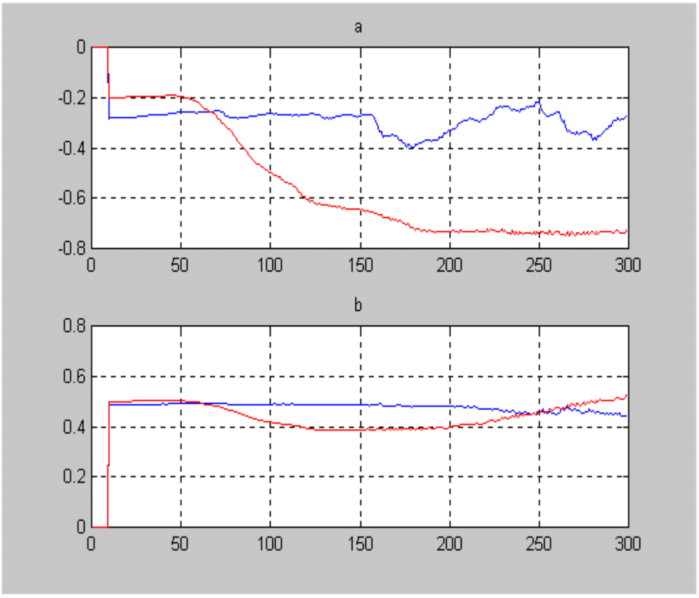

Fig. 1. Parameters estimation by algorithm LS-QR: without noise (red) and with a noisy environment (blue).

$$n(k) = \frac{\| \theta - \hat{\theta} \|_2^2}{\| \theta \|_2^2}$$

where $\| \bullet \|_2^2$ is the Euclidian norm. The Fig. 3 shows the convergence curve of the estimated parameters, according to (30). It is observed fast convergence of the estimated parameters to the nominal ones with minimum error of the norm of *0,0436*.

Example II:

Consider the following second order dynamic systems described by

$$A(z^{-1})y(t) = B(z^{-1})u(t) + C(z^{-1})e(t)$$

with

$$A(z^{-1}) = 1 - 1{,}55\,z^{-1} + 0{,}8\,z^{-2}$$

$$B(z^{-1}) = 1{,}5\,z^{-1} - 0{,}475\,z^{-2}$$

$$C(z^{-1}) = 1 - 0{,}55\,z^{-1} + 0{,}25\,z^{-2}$$

The input of the system was the well known PRBS (Pseudo-Random Binary Sequence) signal of magnitude one. The perturbation $e(k)$ is described by the following equation

$$\wp_\varepsilon = \{F \mid F = (1-\varepsilon)G + \varepsilon H,\ 1 \geq \varepsilon \geq 0\}$$

where

$$G \sim N_1(0, \sigma_1^2 = 0{,}01) \quad ; \quad H \sim N_2(0, \sigma_2^2 = 5{,}0)$$

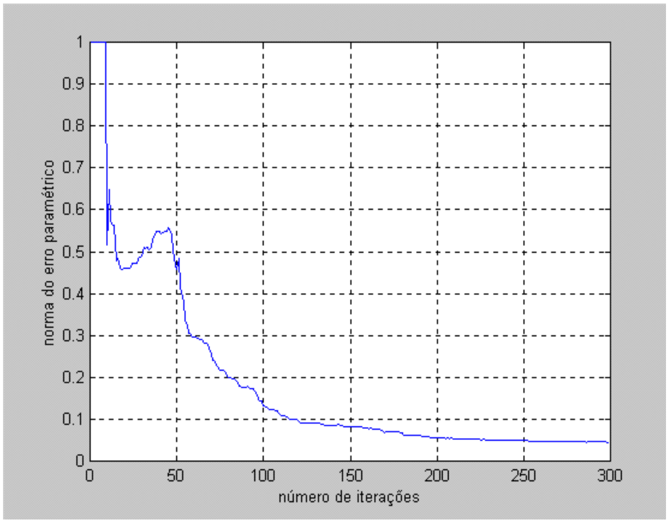

Fig. 3. Convergence property of the algorithm ELS-QR.

Are normal distribution with mean zero and finite variance, and $\varepsilon = 0{,}1$ means the probability of occurrence of outliers on the used data set for model identification. This value imply that *10%* of the data set consists of outliers, which affect considerably the estimation procedure by conventional identification algorithms. So, in this case is treated a robust identification problem. The total of points used in this simulation was *300* and the proposed algorithm was implemented with $\lambda = 0{,}98$, with *10* points for initial estimation. The estimated parameters of the dynamic system in (31)-(34) are listed in Table II, and compared with others robust estimation algorithms RILSMMR and RIIVMMR [10].

TABLE II

PARAMETERS ESTIMATION OBTAINED BY SIMULATION USING LS, RILSMMR, RIIVMM [8] AND ELS-QR

Param.	True	LS	RILSMMR	RIIVMMR	ELS-QR
a_1	1.55	1.373840	1.545272	1.549946	1.54870392
a_0	-0.80	-0.639400	-0.794996	-0.800053	-0.7988945
b_1	1.50	1.452165	1.501897	1.501077	1.50062669
b_0	-0.475	-0.107818	-0.467592	-0.475168	-0.4744999
Norma	0.0	0.187470	0.004380	0.000465	0.00080142

From this result, the robustness of the proposed ELS-QR algorithm is validated, this is, the method can provide robust estimation compared to others robust algorithms in the literature. Figure 4 shows the convergence curve in this application, where the norm of the parametric error decreases very quickly when the number of iterations increases

4 Conclusion

In this paper, an ELS-QR algorithm was proposed for open and closed loop identification in noisy environment. The recursive procedure was based on QR factorization via Householder orthogonal transformations. Simulation results have shown the robustness of the proposed algorithm as compared with others robust methods well known from literature as well as the efficiency to overcome the problem with outliers. Experimental results confirm its application for further adaptive control design, in the sense of the obtained model could represent the plant closed loop behaviour. As future works, the use of this algorithm for TS fuzzy model consequent parameters estimation in noisy environment is addressed. An initial and first proposal in this field can be see in [6][7][8].

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