The Computer in the Mathematics Teaching

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Abstract - The difficulty in learning mathematics is often caused by the prior fear of mathematics. Besides the necessity of changing the methods of teaching and teachers training, computer may be a tool that can be used to avoid this fact and to make learning funnier. The use of Information and Communication Technologies (ICT) in schools is still unusual. Several factors contribute to that: the lack of training of teachers, the lack of material resources and the excessive number of students per class. Computers should be used as tools to assist students in the exploration and discovery of concepts, to make the transition from concrete experiences to abstract mathematical ideas, in practice routines, and in the process of troubleshooting, but only as auxiliary and not as the goal of education itself. The use of computer in solving a problem requires that users of these tools have knowledge of errors theory; otherwise they can get wrong values that they do not know how to explain. This work aims to reflect on the occurrence of some of the most common mistakes when using these tools.

Key-words: - Mathematics Education, Self Regulation, Educational Technologies, Numerical Analysis.

1 Introduction

Mathematics at an undergraduate level is frequently presented to the students in quite a traditional way. When implementing the Bologna education reform in Portuguese universities, the number of contact hours of the courses decreased, therefore increasing the need of a more self-responsible learning by the student. This means that the student has to work by himself on a regular basis (in Irene Brito, Jorge Figueiredo, Maria Flores, Ana Jesus, Gaspar Machado, Teresa Malheiro, Paulo Pereira, Rui M. S. Pereira, Estelita Vaz [8]). In the early years of teaching the same situation is verified, aggravated by the fact that students do not like Mathematics and find it difficult to learn. It is in this context that the Information and Communication Technology (ICT) are vital because they can make learning funnier.

Learning and teaching are two mutually dependent elements of a teaching process. From traditional up to the present forms of teaching where ICT technology plays an important role, the focus on the process of teaching has changed. Since teaching has its definite goals it should also be specially tailored to meet the needs of a student. Different forms of teaching have taken into consideration the above mentioned facts in a different way. The tendency of individual approach to a student is constant, but the real level of individualization depends on actual situation and specific circumstances in which the teaching process is carried out. The application of ICT in teaching, more precisely in learning, has considerably changed the above mentioned circumstances (in Ivan Pogarcic, Tatjana Sepic, and Sanja Raspor [9]). Learning should be seen as an active, cognitive, constructive, significant, mediated and self-regulated process (in Beltran [3]). To make it possible the computers are essential.

The use of ICT in Secondary Education schools is still uncommon. Several factors contribute to this fact such as the lack of teacher training, lack of material resources and an excessive number of students per class. However computers can be used as tools to assist students in their exploration and discovery of concepts, in the transition from concrete experiences to abstract mathematical ideas, and in the process of solving problems, but only as an aid and not as purpose of education. The student has to be called upon to use computers in his self-study, but he must have knowledge of the errors

theory, otherwise he takes the wrong lessons from the results. The use of computers in the education is a fact that we cannot escape from, but some factors must be kept in mind. On the one hand the technologies are a factor to motivate the students and facilitate learning; on the other they may lead students to an error if they aren't properly used.
Extensive discussions and debates about the advantages of using technology to create a shared space among learning participants have been presented in studies in the field of e-learning. One of the approaches in using or adopting technology for learning is through the use of online discussion forums. Therefore, it is essential to consider how online discussion forums may promote knowledge constructions in students. Online discussion forum is also a form of learning through networking which provides opportunities for students to seek, obtain, and share information. Therefore, students' participation and interaction in the forum can provide some insight into how they learn about a course in a virtual environment (in Nor Fariza Mohd, Norizan Razak and Jamaluddin Aziz [11]). But for this sharing conducing to an effective learning it is necessary to take into account how they use ICT otherwise the shares may induce students into error. This paper attempts to reflect on some of the most common errors in the use of ICT in the learning of mathematics.

2 Mathematical Expressions

Suppose we have the expression

$$z = \frac{(x-y)^2 + 2xy - y^2}{x^2} \tag{1}$$

If we use the addition and the multiplication properties we get:

$$z = \frac{(x-y)^2 + 2xy - y^2}{x^2} =$$

$$= \frac{x^2 - 2xy + y^2 + 2xy - y^2}{x^2} =$$

$$= \frac{x^2}{x^2} = 1$$

ie

$$z = \frac{(x-y)^2 + 2xy - y^2}{x^2} = 1 \tag{2}$$

However, if we calculate the z value giving values to variables x and y, we obtain a result completely wrong. Why?

It is important that students realize what is happening to avoid this error. For this, consider that we use the computer to do the calculations and assign x and y values with very different magnitudes. Consider the presented below (see Table 1 and Table 2)

x	y	z
10^{-1}	1500	1.0
10^{-3}	1500	1.0002
10^{-4}	1500	0.9965
10^{-5}	1500	2.0489
10^{-6}	1500	-491.7383
10^{-7}	1500	-4.2170×10^{4}

Table 1: fix the value of y to vary the value of x

x	y	z
10^{-1}	15×10^{2}	1.0
10^{-1}	15×10^{4}	0.9998
10^{-1}	15×10^{6}	3.1250
10^{-1}	15×10^{8}	2.56×10^{4}

Table 2: fix the value of x to vary the value of y

As it can be seen, in some situations the z value is completely wrong

$$z = -4.2170\times10^{4}$$

or

$$z = 2.56\times10^{4}$$

Why? What is happening?
What is happening is that in terms of absolute values assigned to x and to y stay too far, then the $(x - y)^2$ value approaches to the y^2 value, and so we have a problem of subtractive cancellation. The subtractive cancellation arises when we want to calculate the difference between two quantities very close to each other. The result of this operation leads to a loss of significant digits in the result which leads to large errors in future uses of this value.

3 2nd Degree Equations

Suppose we want to calculate the roots of the equation

$$x^2 + bx + c = 0 \tag{3}$$

with

$$\begin{cases} b = -0.9432 \times 10 \\ c = 0.1000 \times 10^{-3} \end{cases}$$

If we use the formula to calculate the zeros with 4 decimal places we have:

$$\begin{cases} x_1 = 0.9431 \times 10 \\ x_2 = 0.5 \times 10^{-4} \end{cases} \tag{4}$$

The exact amounts with 4 decimal places are:

$$\begin{cases} x_1 = 0.9431 \times 10 \\ x_2 = 0.1060 \times 10^{-4} \end{cases} \tag{5}$$

Why the smaller root in (4) has a large relative error?

In this case we have again a problem of subtractive cancellation. By using the formula as if we have a equal to 1 we have

$$x = \frac{-b \pm \sqrt{b^2 - 4c}}{2} \tag{6}$$

b and c are orders of magnitude too different, so the square root of the ratio of discriminant has a value very close to the value of b, as seen in (7):

$$\begin{cases} b^2 - 4c \cong b^2 \\ x \cong \dfrac{-b \pm b}{2} \end{cases} \tag{7}$$

So the biggest root is closer to b value and the smallest is close to zero because it is the difference between two quantities very close to each other. This fact leads to a loss of significant digits in the result which will reflect in the later calculations.

To resolve this situation, we use the fact of knowing that the product of the roots is equal to ac, and so $a = 1$ we have (see eq. 8).

$$x_1 x_2 = c \tag{8}$$

Thus, we calculate the root of largest absolute value of the formula and the other using this property. We obtain then the values

$$\begin{cases} x_1 = 0.9431 \times 10 \\ x_2 = \dfrac{c}{x_1} = 0.1060 \times 10^{-4} \end{cases} \tag{9}$$

4 Systems

Suppose you want to find the solution of the system

$$\begin{cases} -1.4x + 2y = 1 \\ x - 1.4y + z = 1 \\ 2y - 1.4z = 1 \end{cases} \tag{10}$$

Using the Gauss method we have

$$\left[\begin{array}{ccc|c} -1.4 & 2 & 0 & 1 \\ 1 & -1.4 & 1 & 1 \\ 0 & 2 & -1.4 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} -1.4 & 2 & 0 & 1 \\ 0 & 0.02 & 1 & 1.7 \\ 0 & 2 & -1.4 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} -1.4 & 2 & 0 & 1 \\ 0 & 0.02 & 1 & 1.7 \\ 0 & 0 & -101.4 & -169 \end{array}\right]$$

ie

$$\begin{cases} x = \dfrac{1-2y}{-1.4} = 1.43 \\ y = \dfrac{1.7-z}{0.02} = 1.5 \\ z = \dfrac{-169}{-101.4} = 1.67 \end{cases} \quad (11)$$

If we make the check we have

$$\begin{cases} -1.4x + 2y = -1.4\times 1.43 + 2\times 1.5 = 0.9980 \\ x - 1.4y + z = 1.43 - 1.4\times 1.5 + 1.67 = 0.85 \\ 2y - 1.4z = 2\times 1.5 - 1.4\times 1.67 = 0.6620 \end{cases} \quad (12)$$

So we can verify in (12) that the values for x, y and z given by (11) are not a good solution. Because so we can see in (12), (11) don't verify (10).

However if we solve the same system by the same method, but by another process, i.e., changing first the order of the equations we have

$$\left[\begin{array}{ccc|c} -1.4 & 2 & 0 & 1 \\ 1 & -1.4 & 1 & 1 \\ 0 & 2 & -1.4 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} -1.4 & 2 & 0 & 1 \\ 0 & 0.02 & 1 & 1.7 \\ 0 & 2 & -1.4 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} -1.4 & 2 & 0 & 1 \\ 0 & 2 & -1.4 & 1 \\ 0 & 0.02 & 1 & 1.7 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} -1.4 & 2 & 0 & 1 \\ 0 & 2 & -1.4 & 1 \\ 0 & 0 & -0.986 & -1.69 \end{array}\right]$$

ie

$$\begin{cases} x = \dfrac{1-2y}{-1.4} = 1.71 \\ y = \dfrac{1+1.4z}{2} = 1.70 \\ z = \dfrac{-1.69}{-0.986} = 1.71 \end{cases} \quad (13)$$

If we check the solution (13), we have

$$\begin{cases} -1.4x + 2y = -1.4\times 1.71 + 2\times 1.70 = 1.006 \\ x - 1.4y + z = 1.71 - 1.4\times 1.70 + 1.71 = 1.04 \\ 2y - 1.4z = 2\times 1.70 - 1.4\times 1.71 = 1.006 \end{cases} \quad (14)$$

So we work with 2 decimal places, (13) is a good solution for (10) (see eq.14)

In a system, the exchange of the order of the equations does not change its solution. What will be the reason for this difference in results?

The difference is explained easily if we note that in the method of Gauss, the error in the solution of the system is proportional to the ratio between the elements and the "pivot". And in the first case we had an error proportional to

$$\max\left\{\frac{1}{1.4},\quad \frac{2}{0.02}\right\}=$$

$$=\max\{0.71,\quad 100\}=$$

$$=100 \qquad (15)$$

And in the second had an error proportional to

$$\max\left\{\frac{1}{1.4},\quad \frac{0.02}{2}\right\}=$$

$$=\max\{0.71,\quad 0.01\}$$

$$=0.71 \qquad (16)$$

If we instead of the Gaussian method we had used the replacement method, we could have the same problem but we were doing it the same way.

Suppose you want to find the solution of the system

$$\begin{cases} 0.0003x+1.246y=1.249 \\ 0.4370x-2..402y=1.968 \end{cases} \qquad (17)$$

Using the replacement method:

$$\begin{cases} 0.0003x+1.246y=1.249 \\ 0.4370x-2..402y=1.968 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\dfrac{1.249-1.246y}{0.0003} \\ 0.4370\left(\dfrac{1.249-1.246y}{0.0003}\right)-2..402y=1.968 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\dfrac{1.249-1.246y}{0.0003} \\ -1817y=-1818 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=5.8467 \\ y=1.001 \end{cases} \qquad (18)$$

If we check the solution (17), we have

$$\begin{cases} 0.0003(5.8467)+1.246(1.001)=1.2647861 \\ 0.4370(5.8467)-2..402(1.001)=0.1506059 \end{cases} \qquad (19)$$

Ie, so we can see in (19), (18) is not a good solution of (17).

However, if we reorganize the equations in the system (17) we have

$$\begin{cases} 0.4370x-2..402y=1.968 \\ 0.0003x+1.246y=1.249 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\dfrac{1.968+2..402y}{0.4370} \\ 0.0003\left(\dfrac{1.968+2..402y}{0.4370}\right)+1.246y=1.249 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\dfrac{1.968+2..402y}{0.4370} \\ 1.248y=1.248 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=10 \\ y=1 \end{cases} \qquad (20)$$

If we check the solution (20), we have

$$\begin{cases} 0.4370\times 10 - 2..402\times 1 = 1.968 \\ \\ 0.0003\times 10 + 1.246\times 1 = 1.249 \end{cases} \quad (21)$$

i.e., so we can expect, (20) is a very good solution for the system (17).

The methods that we have used, are direct methods, they compute the system solution in a finite number of steps. These methods would give the precise answer if they were performed in infinite precision arithmetic. In practice, finite precision is not used and the result is an approximation of the true solution. In contrast to the direct methods, the iterative methods starting from an initial guess and form successive approximations that converge to the exact solution only in the limit. A convergence criterion is specified in order to decide when a sufficiently accurate solution has been found.

If we use an iterative method to solve the system, we must have the same care. The iterative methods calculate an approximate result, which is so close to the exact result as the number of iterations performed.

Representing by x the exact solution, these methods start from an initial solution, $x^{(0)}$, and through operations with matrices, construct a sequence of successive approximations

$$x^{(1)}, x^{(2)}, \cdots, x^{(n)}, \cdots$$

such that

$$x = \lim_{k\to\infty} x^{(k)}$$

In fact, we not can perform an infinite number of iterations so we need to introduce a criterion for stopping the iterative process that ends when the solution is good enough. The are a lot of stopping criterion, we can use the following criterion, stop the process when:

$$\max_{1\le i\le n}\left| x_i^{(k+1)} - x_i^{(k)} \right| < \varepsilon \quad (22)$$

where ε is the desired tolerance.

If the exact solution is very close to zero, this criterion can stop the iterative process before the solution is good enough. In this case it is better to use a relative criterion.

In this case we can then use the following criterion: stop the iterative process when

$$M_R^{(k+1)} < \varepsilon$$

where

$$M_R^{(k+1)} = \begin{cases} \underset{1\le i\le n}{Máx.} \left| \dfrac{x_i^{(k+1)} - x_i^{(k)}}{x_i^{k+1}} \right| & se \quad x_i^{(k+1)} \ne 0 \\ 0 & se \quad x_i^{(k+1)} = x_i^{(k)} = 0 \\ 1 & \text{se} \quad \begin{cases} x_i^{(k+1)} = 0 \\ x_i^{(k)} \ne 0 \end{cases} \end{cases} \quad (23)$$

The convergence to the exact solution is not guaranteed for any system, so we need be careful with the convergence of these methods. We need to alert the students that there are certain conditions that must be met by a system of linear equations to ensure convergence of the method.

Consider the system

$$\begin{cases} 3x + 5y = 2 \\ 8x - 2y = 4 \end{cases}.$$

Solve it by ensuring that the solution has a limit of error of 0,005 (ε ≤ 0.005).

First we need reorganize the system, we have

$$\begin{cases} 8x - 2y = 4 \\ 3x + 5y = 2 \end{cases}$$

So we have

$$\begin{cases} x = \dfrac{4+2y}{8} \\ y = \dfrac{2-5y}{3} \end{cases} \quad (24)$$

There are a lot of different iterative methods for solving linear systems. In the first years, usually we teach the Jacobi and the Gauss-Sidel methods.

If we use the Jacobi method (methods of simultaneous displacements) and the Gauss-Sidel method (method of successive displacements), for the criterion defined by the equation (22) we have

It.	x_1	x_2	$\max_{1\le i\le n}\left\|x_i^{(k+1)}-x_i^{(k)}\right\|<\varepsilon$
	0.000000	0.000000	
1ª	0.500000	0.100000	0.5 > 0.005
2ª	0.525000	0.085000	0.025 > 0.005
3ª	0.521250	0.087250	0.00375 < 0.005

Table 3 – Gauss-Sidel Method

It.	x_1	x_2	$\max_{1\le i\le n}\left\|x_i^{(k+1)}-x_i^{(k)}\right\|<\varepsilon$
	0.000000	0.000000	
1ª	0.500000	0.400000	0.5 > 0.005
2ª	0.600000	0.100000	0.3 > 0.005
3ª	0.525000	0.040000	0.075 > 0.005
4ª	0.510000	0.085000	0.045 > 0.005
5ª	0.521250	0.094000	0.01125 > 0.005
6ª	0.523500	0.087250	0.00675 > 0.005
7ª	0.521813	0.085900	0.001687 < 0.005

Table 4 – Jacobi Method

And for the criterion defined by the equations (23) we have only 2 more iterations for the Jacobi method and 1 more for the Gauss-Siudel, but a much better solution

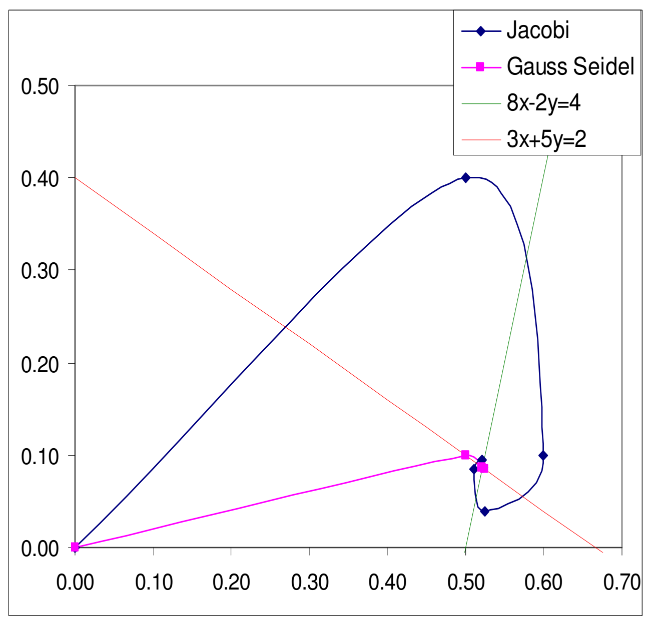
It.	x_1	x_2	$\max_{1\le i\le n}\left\|x_i^{(k+1)}-x_i^{(k)}\right\|<\varepsilon$
	0.000000	0.000000	
1ª	0.500000	0.100000	1.000 > 0.005
2ª	0.525000	0.085000	0.176 > 0.005
3ª	0.521250	0.087250	0.026 > 0.005
4ª	0.521813	0.086913	0.004 < 0.005

Table 5 – Gauss-Sidel Method

It.	x_1	x_2	$\max_{1\le i\le n}\left\|x_i^{(k+1)}-x_i^{(k)}\right\|<\varepsilon$
	0.000000	0.000000	
1ª	0.500000	0.400000	1.000 > 0.005
2ª	0.600000	0.100000	3.000 > 0.005
3ª	0.525000	0.040000	1.500 > 0.005
4ª	0.510000	0.085000	0.529 > 0.005
5ª	0.521250	0.094000	0.096 > 0.005
6ª	0.523500	0.087250	0.077 > 0.005
7ª	0.521813	0.085900	0.016 > 0.005
8ª	0.521475	0.086913	0.012 > 0.005
9ª	0.521728	0.087115	0.002 < 0.005

Table 6 – Jacobi Method

We can see the exact solution in the graphic resolution



Graphic 1– System solution by iterative methods

5 Exponential

Another common mistake is the problem of overflowing, under flowing or the errors that arise when we work with numbers below the unit rounding error of the machine (a condition that occurs when a calculation produces a result that is greater than what a given register can represent or is smaller in magnitude- that is, it is closer to zero-than the smallest value representable as a normal floating point number in the target datatype)

As we know

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \quad (25)$$

If we use a calculator or a computer to estimate the value of

$$\left(1+\frac{1}{n}\right)^n \quad (25)$$

we can expect that the higher the value of n, the value of (26) is closer than the exponential (25).

However we have table 7:

n	$\left(1+\frac{1}{n}\right)^n$
2^{10}	2.716955729
2^{20}	2.718280532
2^{30}	2.718281808
2^{40}	2.718281828
2^{50}	2.718281828
2^{60}	1

Table 7: exponential values

What's happening?

What is happening is that for $n = 2^{10}$, $n = 2^{20}$ and $n = 2^{40}$, $\frac{1}{n}$ is greater than the zero of the machine while for $n = 2^{60}$, $\frac{1}{n}$ is already lower than this value and is therefore considered to be zero (see Table 8).

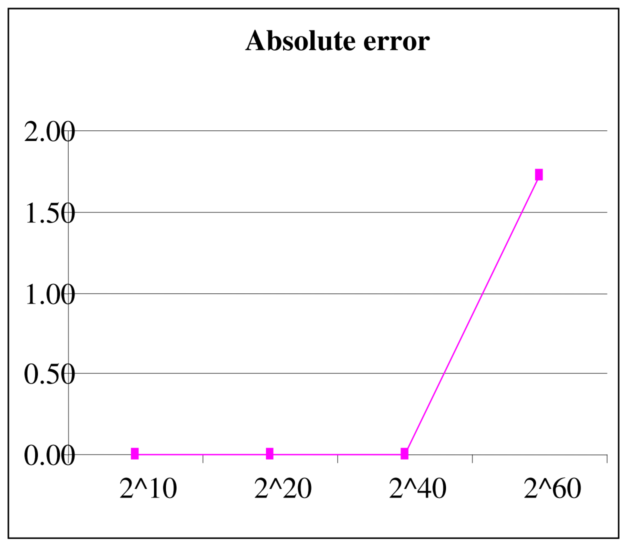
So for $n = 2^{60}$ we has

$$\left(1+\frac{1}{2^{60}}\right)^{2^{60}} \cong (1+0)^{2^{60}} = 1 \quad (27)$$

n	$\frac{1}{n}$
2^{10}	0.000976563
2^{20}	9.53674E-07
2^{30}	9.31323E-10
2^{40}	9.09495E-13
2^{50}	8.88178E-16
2^{60}	8.67362E-19 $\approx$ 0

Table 8 – 1/n values for different values of n

If we calculate the absolute error we have the graphic 2:



Graphic 2: Absolute error

6 Conclusion

We live in a time of fast and big changes, accelerated communication and information exchange, which is inevitably reflected in the educational system and so often called "the era of informatics and communication". Accelerated changes demand quick response. The E-learning has become an imperative for further survival on the market. The e-learning system offers numerous advantages but also some disadvantages (in Vladimir Simovic [12]).

I anticipate that there will be an expanded use of new methods to facilitate teaching and learning, taking advantage of the many new methods being proposed or tried to improve educational activities and student learning. The use of computers in the mathematics teaching can be important because it helps students learning math in a funny way, helping to combat the "myth" that exists towards the learning of mathematics (in Marc Rosen [10]).

In the current paradigm of education the contact hours between the student and teacher are decreased, so the student need more time for self study. The computer can be an essential tool because it facilitates the autonomous learning and makes the contact between the student and the teacher and other students easier. So, answering to questions and solving problems become easier too. The use of several existing software can help students to discover the concepts and can help on the transition from concrete experiences to abstract ideas and also in the process of troubleshooting. However, it is essential to alert students to the need they use these tools correctly and to the potential occurrence of some problems. For this, it is necessary to teach them not only how to use the various existing software, but also the basic knowledge of errors theory.
The study of errors forms an important part of numerical analysis. There are several ways in which error can be introduced in the solution of the problem. It is important alert the students to the most common errors and problems and teach them how to avoid them. Otherwise, there is a risk that students do not correctly use ICT and to remove the wrong conclusions from the results they achieve with them, thus not contributing the ICT for an effective learning of students

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