Enhancing Calculus Learning Engineering Students Through Problem-Based Learning

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Abstract. Calculus is most important subject for science and engineering students but phenomenally it is abstract, difficult and highly boring. Underpinning with in constructivism theories, problem-based learning (PBL) is one of student centred and active learning strategies or approach which can be used to improve students' interests and often lead to deep level understand hence acquiring the learning outcomes of a course This approach can be used to improve students' participation, interests and performance in learning of calculus. This study is designed to investigate student's perception about PBL approach in calculus; student's engagement during the acquisition of understanding of calculus and; student's performance as a result of PBL learning experience. A group of 48 Foundation Year engineering students was randomly selected to undergo the PBL approach in Basic Calculus course. A rubric-scoring assessment was used to evaluate students engagement level, a questionnaire with open ended question was used to assess students' perception of PBL learning, and a post-test to measure students' performance after undergoing PBL approach. This pedagogical approach has made the teaching and learning of calculus more interesting and effective. PBL in calculus offer the advantages as it promotes collaborative and distributed learning; development of learners' critical thinking, evaluative and judgment/ decision making skills and enables new and inexperienced learners to access and learn from captured solutions of expert problem solvers. It is also able to develop the potential of individuals to be more creative, critical in their thinking and effective in problem solving.

Key-Words: Problem Based Learning, Calculus, Engagement in learning, Mathematics Education, Performance, Engineering Education

1. Introduction

Calculus is most important subject for science and engineering students in university because its use is widespread in science, engineering, medicine, business, industry, and many other fields to understand and apply the concept rate of change and motion. Knowledge of mathematic (calculus) is necessary for being successful in engineering subject such as mechanics, fluids and thermodynamics. According to McCallum (2000), by end of successful first-year calculus course engineering students should be able to [1]:

(1) Make calculation with agility, accuracy, intelligence and flexibility.
(2) Explain the basic concepts of calculus clearly and reason mathematically with them.
(3) Solve extended problem with good judgment in the choice of tool and in checking answers.
(4) Make connection between different incarnations of the same idea.

(5) Use calculus to model realistic situation from engineering and from the physical, life and social sciences.

However many students cannot achieve a deep understanding and find that calculus is very hard and abstract [2][3]. They frequently questioned and claimed, “What/Why am I going to do with this anyway? It is of no importance”. Many students wonder why they should learn calculus and how calculus would be useful in their future work after graduation.

Generally, traditional approaches is teacher-centered where students are not actively involved in lectures but passively follow their teacher and are isolated from the real world in learning. The task or problems presented during lessons seemed meaningless or do not have any “real” meaning for them [4]. Lecturer tend to teach mechanistically and standard type of problem and solution [5]. Teacher plays a leading role in transferring information. They are the authority, expert, the main source of knowledge, and the focal point of all activity in the class [6]. Thus, teacher centered teaching does not provide an active learning environment for students. Teachers also should not only transmit knowledge but also cultivate some lifelong learning skills in their students [3]. Because the rate of change in knowledge continues to increase, it is a given that students will need to keep learning for much of their careers. Therefore, education system needs to help students develop skills for life-long learning.

Passive learning is relatively easy for both lecturers and students. Students passively involved in class will lack an understanding of the calculus ideas used to solve problems, and will be confused in their future study and work when they encounter problems that are not in textbooks. Students also were forced to learn it to pass the examination rather than understand the subject. A student will have to invest a lot of his or her time if they want to excel in studies. They will miss the chance to mix around with peers to develop their social skills that is bound to be important throughout their lives. Student also may only memorize formulae and procedures that have been taught in the class to find the task has been assigned [7]. Limitations of traditional ways of teaching mathematics students are likely to reproduce the procedures without deep conceptual understanding. When mathematical knowledge or procedural skills are taught before students have conceptualized their meaning, students' creative thinking skills are likely to be stifled by instruction [2][3][7].

Sequentially to motivate students to become independent and active learners many faculties have suggested ways to improve teaching and learning calculus by making better, more active and meaningful strategies in teaching [8]. Active learning will stimulate development of skills in “doing” mathematics rather than “knowing” mathematics because it encourage students to become active, motivated, and independent learners through open communication and collaboration[9]. This type of students is capable of solving problems, building mathematical model, abstracting, inventing and providing [10].

In an effort to improve the teaching and learning in calculus, a reformed curriculum was introduced at some institutions, which makes use of multiple representations of calculus concepts. This new curriculum move from teacher centred teaching to student centred learning that allowed students actively involved in teaching and learning as much as possible. Student centred learning allows students to construct knowledge through active learning. Research shows that student success rate was dramatically improved due to the change of course structure and content delivery [11]. Education of professionals in spatial planning has to prepare students for autonomous decision making and participation in participatory processes [12].

Underpinning with in constructivism theories, problem-based learning (PBL) is one of student centred and active learning strategies or approach which can be used to improve students’ interests and often lead to deep level understand hence acquiring the learning outcomes of a course. [3][13]. Savory and Duffy consider PBL one of the best exemplars of a constructivist learning environment because all eight principles for design of a constructivist learning environment are embodying with PBL. In their view, constructivism can be captured with three primary propositions [13]:

(1) Understanding is constructed individually through our interactions with the environment and we can only test how much our individual understandings are compatible.
(2) Cognitive conflict is the stimulus for learning and determines the organization and nature of what is learned.
(3) Knowledge evolves through social negotiation and through the evaluation of individual understandings.

PBL as an engaging instructional strategy was first popularized by Barrows to enhance the reasoning ability of medical students at McMaster Medical School, Canada in 1969 [14]. Barrows found that medical students more anchor their knowledge by solving clinical problems (real problem) rather than solely around books. Since then, it has expanded worldwide in a variety of disciplines including medical, health, science, business, engineering, nursing, and law [14][15].

This pedagogical approach has given a new dimension to the educational world of mathematics especially calculus. Learning begins with a problem to be solved, and the problem is posed is such a way that students need to gain new knowledge before they can solve the problem. Rather than seeking a single correct answer, students interpret the problem, gather needed information, identify possible solutions, evaluate options, and present conclusions. This learning environment, made the teaching and learning of calculus more interesting and effective. It has potential to develop individuals to be more creative, critical in their thinking and effective in problem solving [16].

2. Problem Based Learning in Calculus

Problem based learning is defined in many ways but the most widely accepted definition of PBL is an instructional strategy whereby student are required to solve the ill structured problem (real world problem) in a collaborative environment by identifying facts related to it. Unlike traditional approach, which is often conducted in lecture format, teaching in PBL normally occurs within small discussion groups of students facilitated by a faculty tutor. The student brainstorms and hypothesizes about the underlying process of the problem and come up with possible solutions [17][18]. Problems were posed to students acting as triggers in this learning environment, thus they became skilled in problem solving, creative thinking, and critical thinking.

In PBL the problem may not be solvable, but nevertheless provides a rich environment for learning. This aim is to learn rather than to solve the problem. Work with real-world and practical problem cooperatively can make them interesting, because these problems may be related to their personal development and hobbies.

Implementation of PBL in calculus class would provide the students more opportunities to think critically, represent their own creative ideas, and communicate with their peers mathematically [18]. A lot of researches show that PBL improved student learning and it widely use in educational settings.

PBL can foster students' intrinsic motivations and develop their self-learning skills because it provides a student-centred and self-oriented learning for students to seek solutions to real world problems proposed that those who undertook PBL are significantly better in the development of critical thinking dispositions than those who took lecture courses [13][19][20].

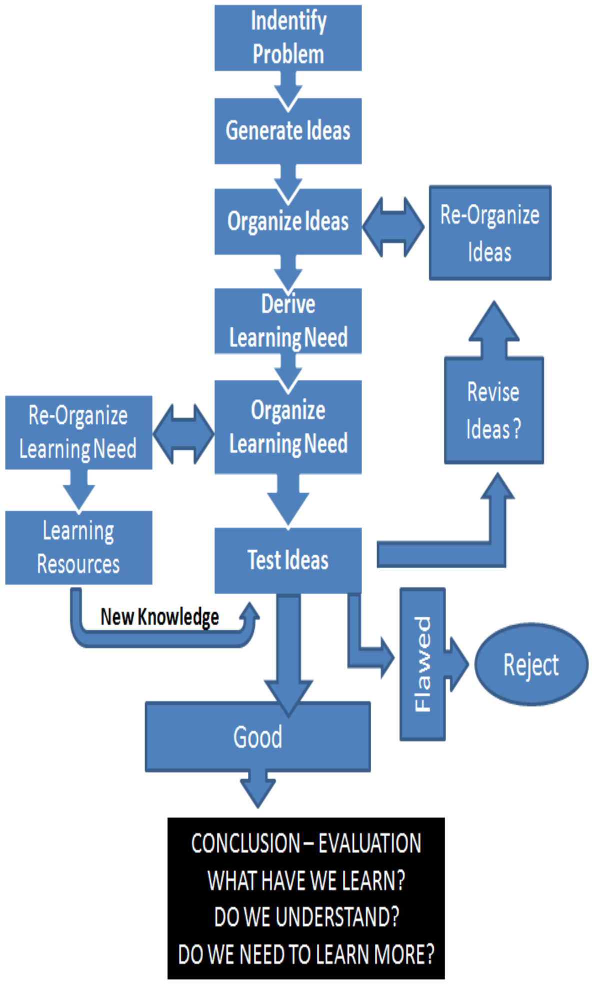


Fig 1: PBL Process

Numerous literatures had indicated that PBL positively influence learning outcomes along with learners' higher order thinking skills such as creative thinking, problem solving, logical thinking and decision making. Elshafei found that students in PBL settings had higher levels of achievement and better solutions in Calculus compared to student in traditional setting [21]. Similar to above studies, several researchers claimed that PBL had a positive influence on problem solving and critical thinking skills [22][23]. However, there are very few studies on the effectiveness of PBL in calculus or mathematics. A study was conducted by Akinoglo and Tandogan to determine the effects of PBL in Introduction of Calculus-Physic on students' academic achievement and concept found that PBL model had positive effect on academic achievement and attitudes [24]. It was also found that the application of PBL model affected students' conceptual development positively.

3. Methodology

3.1 Research Objective

The general aim of this study is to determine the effect of PBL approach in enhancing students' learning experience in calculus among engineering students. Specifically, the objectives of this study are:

(1) To investigate students perception about PBL approach to enhancing learning in calculus;
(2) To investigate students engagement during acquisition of knowledge via PBL;
(3) To determine association between engagement scores and performance scores and;
(4) To compare students performance in calculus after undergoing PBL approach with conventional instructions.

3.2 Research Design and Procedure

This study was conducted using the quasi experimental design which comprised of two groups of students. Participants involved in this study were 48 students Foundation Year students in private university enrolled in Basic Calculus course. The students were from same program of studies and were randomly assigned to control group named as Traditional Group (n=24 student) and treatment group which was named as PBL Group (n=24 student). This design allows for at least eight extraneous variables that may pose threats to the internal validity of the experiment to be controlled[25]. The treatment group underwent learning using PBL approach while the control group underwent learning using a conventional instructional strategy.

The learning sessions were based on the topic of derivative which is part of the Basic Calculus course. The duration of the experiment was two weeks.

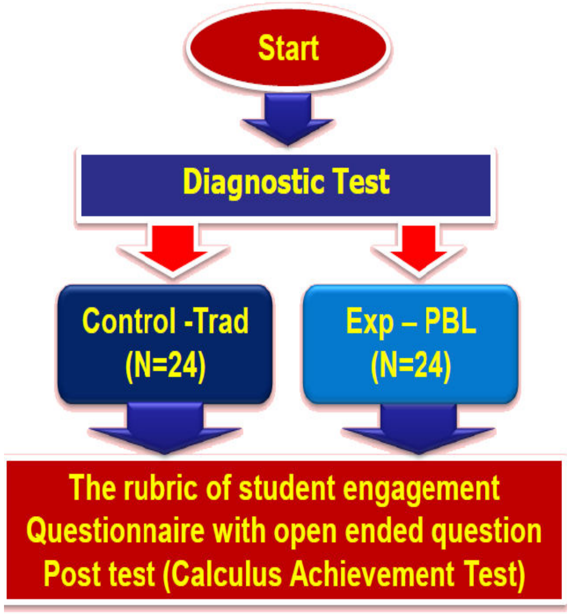


Fig. 1: Flow chart of research process

This study used three phases for the both groups (PBL and Traditional group), namely: 1) Diagnostic Test; 2) Learning Session; and 3) Assessment using a set of Calculus Test and Questionnaire as the posttest. At the phase one, both groups have given diagnostic test to measure level of mathematical competency regarding to calculus. PBL group were given a brief description and training about process of learning in the first class of experimental period. The traditional group (control group), on the other hand, underwent with normal class. During the learning process at the phase 2, PBL group has evaluated their engagement by using Students Engagement Rubric score sheet. Evaluating has done by tutor. During the time when the PBL group underwent the posttest (at Phase 3), the traditional group then was administered the same test. The data were analyzed using independent t-tests

3.3. Materials

The instructional materials for this study consisted of a set of lesson plan on the topic of Derivative in Basic Calculus for first year Engineering Student syllabus, and also a set of module prepared by the researchers. The module which consisted of the content of the lesson was distributed to the students to use as guide throughout the teaching and learning process.

During the first class in second phase (experimental period), the treatment group was given description and training about process of learning in PBL approach In this training, the students were required to familiar with process of learning through PBL. Students in PBL group were divided to five groups. Each group need to nominate a leader.

The conventional group followed teaching and learning session in the traditional teacher-centered approach. During this teaching and learning phase students were given assessment questions to evaluate the extent of short term learning. At the end of the treatment session during the third phase, students were given the calculus achievement test. Examples of some of the problem has used by students in PBL group during their activities as shown in appendix 1.

4 Result

4.1 Students Perception

Based on the questionnaire responses, more than half students (58%) preferred to learn in group and (40%) prefer to learn in pair. These students also preferred to engage in group or pair work (75%) rather than individual work (25%). In terms of study approach, most students preferred case study approach (75%) and project approach (25%). Half of this group preferred their lecturer to direct them on what to learn (50%) and do the classroom teaching (46%). Only small number (21%) is willing to have free hand to decide what they want to learn .This indicates that this group needs the lecturer to assist them to ensure that they were able to participate in the learning activities. In learning process, nearly half of students (48%) preferred to learn through exchanging of ideas with others.

Benefits of this approach perceived by the students included the ability to discuss, query and correct misunderstandings and the development of transferable skills. Students also mentioned that they felt more confident, independent and were therefore more willing to contribute to discussion. Almost all students (84%) stated that the PBL approach requires them to do a lot of self study and do a lot of research.

Seven (7) positive themes about PBL approach were categorized according to the open ended question in questionnaire "What is your comment/view about PBL approach in calculus?". The seven themes were as follows:

(1) PBL enhance working in groups and increase interactivity. *(eg: "We work together to solve the problem. Everybody must play the role to fulfill the requirement and nobody sleeps");*

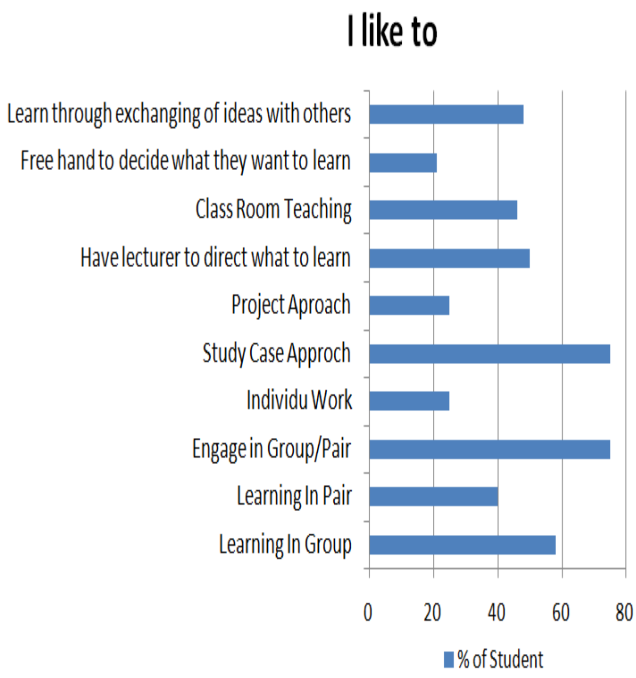


Fig 2: Student Response on Learning Process

(2) PBL encourage search for information. *(eg: "I have the opportunity to state my opinion. I also need to know what I know and identify things that are needed to solve the problem");*

(3) PBL allow applying of interesting or relevant calculus concepts.(*eg: "I will able to relate what I've been learning to the application in daily life. It's more interesti"*);

(4) PBL allows opportunity for hearing different perspectives within group and intergroup *(eg: "Views and solutions form other group helped me better understand to solve the issue has submitted");*

(5) PBL stimulates thinking *(eg : "Stimulate my mind to further outstanding certain topic or ideal and boost our self to adhere independence in finding solution");*

(6) PBL allows flexibility in time *(eg: "It is good approach, so that student will be more independent and unstressed")* and;

(7) PBL allows process of learn to learn *(eg:"I learned to think critically in solving the problem and also learn to cooperate with other").*

Clearly students in PBL approach enjoyed working in groups and recognize the scope for different perspectives within their group. This may suggest that both elements (engagement and thinking opportunity) are beneficial to students in their first semester at university in which they benefited from PBL experiences. Central to the effectiveness of PBL is the ability of students to work together to solve problems. The relevance of applying calculus concepts to an authentic situation or problem is largely attributable to the PBL nature of the exercise. Having flexibility in time and learning activity will open up more opportunities for the student to be independent learners and free to explore deeper learning.

4.2 Engagement

In this study, eleven (11) items were used in the rubric assessment to evaluate level of student engagement. For each item, students were given 3 point if they showed "Exemplary" engagement, 2 point for "Proficient" and 1 point for "Partially Proficient". According to the average rubric engagement score during learning process, 25% of the respondent can be classified as an Exemplary, 58.3% as Proficient and the rest of the students were Proficient Partially as show in Figure 2. Detail percentage frequency student in each engagement categories as in Table 1.

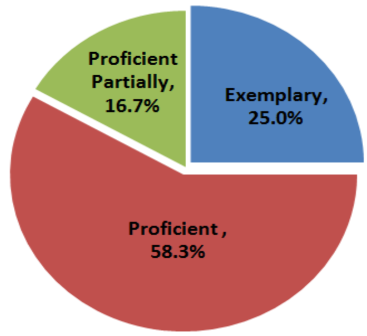


Fig 2 : Average rubric engagement score

Mean average rubric engagement score = 2.08, this indicated that this group maybe classified as Proficient in terms of collaborative and cooperative engagement during their PBL activity in the learning of calculus.

Table 1: Student Engagement

No	Item [Exemplary - 3 point] [Proficient - 2 point] [Proficient Partially - 1 point]	Engagement Categories		
		Exemplary	**Proficient**	**Proficient Partially**
	Focus on the Task and Participation.			
a.	Consistently stays focused on the task given and self directed.	29.2%	37.5%	33.3%
b.	Contributes a lot of effort, encourages and supports the efforts of others in the group.	16.7%	50.0%	33.3%
	Dependability and Shared Responsibility			
c.	Follows through on assigned tasks and does not depend on other to do the work and share responsibility for a task.	16.7%	58.3%	25.0%
	Responsiveness and Listening			
d.	Asks insightful and challenging questions to help understanding and learning.	12.5%	54.2%	33.3%
e.	Demonstrates poise and confidence when answering questions.	25.0%	45.8%	29.2%
f.	Indicates signs of active listening and interest	16.7%	58.3%	25.0%
	Research and Infromation-Sharing			
g.	Routinely gathers research and shares useful ideas when participating in the group discussion.	20.8%	45.8%	33.3%
	Group/Partner Teamwork			
h.	Consistently makes necessary compromises to accomplish a common goal and help keep the group working well together.	33.3%	33.3%	33.3%
1.	Always has a positive attitude about the task(s) and the work of others.	20.8%	50.0%	29.2%
j.	Performed all duties of assigned team role and contributed knowledge, opinions and skills to share with the team.	20.8%	50.0%	29.2%
k.	Shows the high value of acceptance and tolerance in encouraging diversity of ideas in the group.	16.7%	54.2%	29.2%

Table 2 : Comparison of PBL group and TRAD group Performances

Independent Samples T-Test										
Based	Group	N	Mean	Std. Deviation	t	df	**Sig.(2-tailed)**	Mean Diff	Eta	Eta^2
Test1	PBL	24	78.44	13.49	0.05	46	**.960**	0.21	.007	.000
	Control	24	78.23	15.42						

4.3 Performance

To investigate our hypothesis of association between Total Score Engagement Rubric and Calculus Achievement Test, a correlation was computed using Pearson correlation coefficient. Using Cohen's (1988) guidelines, there was a medium positive correlation between engagement scores and performance with an r of 0.45 [$r(22) = 0.45$, $p < 0.05$]. These results indicated that high score in engagement during learning session are associated with high performance in Calculus Achievement Test.

An independent-samples t-test was conducted to compare the performance in Calculus Achievement Test between treatment group (PBL group) and control group (TRAD group), to support the correlation analysis result. There was no significant difference in Calculus test achievement PBL group as table 2 (M= 78.44, SD = 13.49) and TRAD group (M= 78.23, SD = 15.23); $t(46) = 0.05$, $p = 0.960 > 0.05$). The magnitude of the difference in the means was very small (eta = 0.007, eta squared = 0.00).

5. Discussion and Conclusion

Problem-based learning as a constructivist teaching method often plays an important role during teaching and learning process. As a result, PBL can provide meaningful learning experiences hence effective learning. This pedagogical approach hopefully will be able to produce graduate in the 21st century who should be able to transfer the knowledge to solve a real problem, make a judgment individually, have a broad perspective and insights into his/her field, have imaginative and creative powers, communicate and cooperate with others, utilize new technology and ready for lifelong learning. One of the most important skills for today's rapidly changing workforce is the skill in self-reflection. The highly motivated, self-directed learner with the skill in self-reflection will continue to unlearn, learn and relearn.

The study showed that the students have a positive perception towards implement of PBL approach in calculus learning. It also increases their motivation to learn calculus and observe the applicability of calculus in real-life. According to the students, PBL approach is a group based which encourages them to gather information, take active role in lessons, hence increasing success and gaining self confidence amongst them. Learning in group provide students with the opportunity to share their thoughts, check understanding, exchange ideas and communicate with one another. Such interaction among students helps to enhance their motivation in the lesson as they engage in activities that are more interesting and meaningful to them.

The main difference between PBL approach and teacher based method is to solve the problems in scenarios with the group and being forced to do research (learn to learn). PBL approach requires students to critically reflect on authentic problem situations, involving small group learning and self-directed study. The teachers or lecturers empower students to control over their learning process, by valuing their contributions, by encouraging the expression of opinions and by helping them to explore mistakes.

Teacher based approach in calculus are boring, decreased attention, knowledge thought to be gained were easily forgotten. Whilst in PBL approach students were able to undergo searching of information, sharing the knowledge gained in the group and making discussions, increase interactivity and the success motivated the students to improve their achievement and self confidence. PBL developed students to play an effective role during learning, thus as active learners students were lead to better performance by the end of the learning process.

In conclusion, PBL in calculus will offer the advantages as it promotes collaborative and distributed learning; development of learners' critical thinking, evaluative and judgment/ decision making skills and enables new and inexperienced learners to access and learn from captured solutions of expert problem solvers.

Appendix 1: Problem Based Learning

PROBLEM BASED LEARNING

SUBJECT : INTRODUCTION TO CALCULUS AND ANALYTIC GEOMETRY
TOPIC : DERIVATVE
SUBTOPIC: OPTIMIZATION PROBLEM
TIME DURATION: 120 MINUTES

OBJECTIVE TASK:

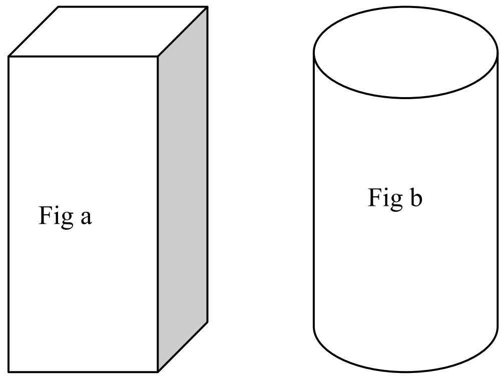
By the end of this task student should be:

a. Simplifying function expression.
b. Finding the domain and range of a function defined by formula.
c. Write formula for function described verbally.
d. Finding critical and inflection point.
e. Indentifying and sketching the graph of the function
f. Applying the first and second derivatives.
g. Solving applied open-interval optimization problem

PROBLEM

CONSTRUCTING A CANDY BOX WITH LID

A candy maker wants to package jelly beans in boxes each having a fixes volume V. Each box is to be an open rectangular box with square base of edge length x (See Fig a). In addition, the box is to have a square lid with a two-inch rim. Thus the box-with-lid actually consists of two open rectangular boxes – the x –by-y box itself with height y ≤ 2 (in.) and the x-by-x-by-2 (which fits the box very snugly). Your job as the firm's design engineer is to determine the dimension x and y that will minimize the cost of the two open boxes that comprise a single candy box with lid. Assume that the box-with-lid is to be using an attractive foil-covered cardboard that cost RM10 per square foot and that its volume is to be V = 400 + 50n cubic inches. (For your personal design, choose an integer n between 1 and 10).



Your next task is to design a cylindrical box-with-lid as indicated in Fig. b. Now the box proper and its lid are both open circular cylinders, but everything else is the same as in the previous problem –two-inch rim RM10 per square foot foil-covered cardboard and volume V = 400 + 50n. What are the dimensions of the box of minimal cost? Which is less expensive to manufacture – the optimal rectangular box with lid or the optimal box with lid in the shape of a cylinder?

HELPING QUESTION

a. What is the information has given?
b. What is variable are involve?
c. What is the main problem we need to solve?
d. What is the function that can be use?
e. What is the domain and range related to the function that we are using?
f. Do we need a diagram to picture our problem?
g. What is mathematical knowledge can be use to solve this problem?

ACTION

a. Discuss with group members how to solve this problem.
b. Your must describe the techniques you used to solve the problem
c. Answer the entire helping question above.
d. State cleanly in several sentences how you solve the problem.
e. Define any variable or function that you are use.
f. Present you solution in the class.

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