

## Calculus b-learning with java tools

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*Abstract:* - Nowadays, information society we live in needs wider educational spaces, and internet provides a powerful tool to introduce new educational technologies and to use online applications. Indeed, TICs (technologies for informatics and communication) provide a very positive aid to the learning tasks. Interactive tutorials allow access to a big amount of information in a multi-sequential way, by using specifically designed Java applets suitable for the learning of a specific knowledge. Due to these facts, the use of these tools has many advantages for learning mathematics. In this sense, we present here an interactive tool for differential calculus within a theoretical environment. Moreover, we show how to make a good use of this type of tools, integrating them with traditional teaching methodologies, as well as non-traditional ones, arising in the convergence towards the European Higher Education Area.

*Key-Words:* - Interactive Tutorials, Java Applets, World Wide Web, Calculus, Derivative

### 1 Introduction

In modern society, computers provide access to an increasing number of sources of information. This dynamical and fast development of knowledge implies that higher education needs to train students to manage vast amounts of information. Internet and computer programs offer not only access to information but also give users powerful capabilities for information manipulation and, in general, for their learning task.

This changing society requires an evolution out of the traditional ways of teaching. Moreover, the fast development of knowledge and technology reduces the life span of some concepts, while it increases the necessity of being able to revise and update the formation throughout the professional life. On the other hand, the development of technologies for computers and communication provides a very positive aid to the learning tasks.

One of the most common uses of technology in education appears in the form of learning interactive tools. These are a very good complement to traditional learning materials (notes, books,...). The reading, by exploration or navigation, of a hypertext is multi-sequential and interactive. The reader makes visual sweepings and searches of fragments

of interest. In order to help navigation, it is recommendable to use textual or graphical tools that appear in the screen and that allow the user to identify and to distinguish the contents of the hypertext. Navigation has replaced linear reading, the information is a space to travel, a path to explore.

The work that we present here is the elaboration of an interactive tutorial, including several java applets, to be used by teachers in the classroom lectures and by the students when learning by themselves.

This web tool concerns the notion of derivative of a function in a point, with special attention to Rolle and Lagrange mean value theorems and applications. This is part of the syllabus of the course “Infinitesimal Calculus”, taught in the first semester of Computer Science studies, at the Polytechnic University of Madrid. The tutorial includes definitions, theorems and the most important results, along with some practical applications. We have designed several interactive applets that allow users to experience a high degree of interactivity, offering them the freedom to generate their own examples. These applets have

been made using programming standards for the World Wide Web, like java or javascript.

We have been working, since many years ago (see, for example, [6, 9, 10, 18, 19, 24, 27, 28]), in the development of interactive tools to help teachers to present and display, in an animated and interactive way, many mathematical notions and algorithms in the classroom. These tools can be also used by students, at the web page of the Department of Applied Mathematics, to experiment, in a virtual way, with the contents of the various courses taught by us.

In section 3, we describe in some detail the interactive tutorial for derivation. In sections 4 and 5 we show how to make a good use of this type of tools, as a support to lectures in the classroom, as well as out of it.

For information on calculus we recommend the book [20] by J. Stewart.

## 2 Objectives

The main objective of this work is the development of interactive tools for Calculus (in particular, the current paper focus on the subject “Derivative of a function”) to be used both by teachers in classroom lectures and by students when learning by themselves, using b-learning platforms like moodle.

The use of this kind of tools in the classroom allows to visualize concepts, as well as to show a great number of examples in little time. The saved time can be used to do active learning activities.

We also show in this work how to integrate this java tool with all the non-traditional teaching methodologies arising in the convergence towards the European Higher Education Area.

## 3 Description of the interactive tutorial

This interactive tutorial focuses on practical aspects. Numerous examples are included, mainly in the form of interactive applications for the Web. These applications have been implemented, using Web oriented technologies, as java applets or as dynamical web pages using javascript.

To make the tutorial as friendly and attractive for students as possible, we have given special attention to the following characteristics:

- A graphical interface for the hypertext which can be easily handled by the user. It allows the visualization of the contents and the organization of the information in an immediate way through pull-down menus. One of our goals is that the

different applications which are presented in the tutorial can be easily and quickly found within each section.

- Implementation of didactic applets for the most relevant concepts and applications of differential calculus. These applets are immersed in a theoretical framework. Therefore, as we pretended, the user can interact with the tutorial, so that its use is more attractive and interesting.
- Accessibility from the web page of the department, as additional documentation for the course “Infinitesimal Calculus”. This will be also integrated in a b-learning moodle context.
- Design of the applets using programming standards for the World Wide Web, to avoid incompatibilities.
- Facility to include new functionalities and algorithms in the future, if desired.

Figure 1 shows the page from which the reader can access the different sections that we describe next. It is available at:

<http://www.dma.fi.upm.es/java/calculo/derivacion>

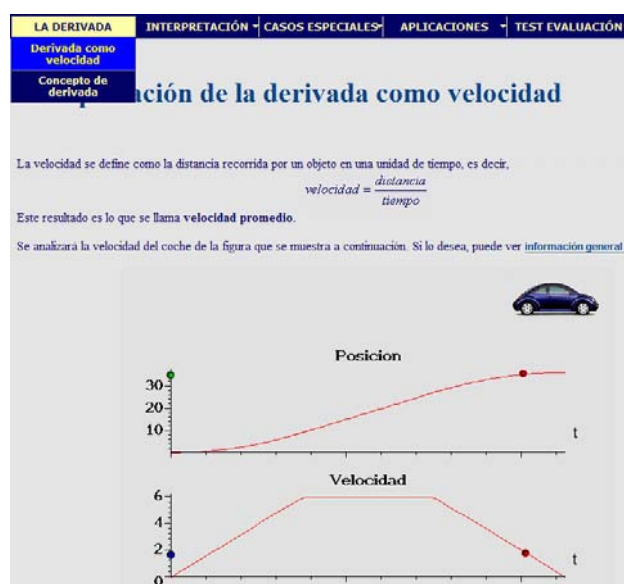


Fig. 1 – Home page of the tutorial

The theoretical part begins with an illustration, made using flash technology, of the derivative as instant speed, followed (in a different page) by the formal definition of derivative of a function in a point, and other theoretical complements that the student will need along the learning of the different parts of calculus that we consider here: side derivatives, higher degree derivatives and the relation between continuity and differentiability of a function.

The geometrical interpretation of the derivative of a function as the slope of the tangent of its graph is the goal of the first applet in the tutorial.

We show the most common particular cases of non-differentiability: non-continuity points, angles, continuity points with infinite oscillation, etc.

We then develop applets to experiment with the main applications of differentiability: Rolle and Lagrange mean value theorems, and the relations between the increase or decrease of a function and the sign of its first derivative. The last page includes some self-evaluation tests.

The different sections that can be accessed using the pull-down menu (see Figure 2) are the following:

- The derivative: The derivative as speed - The notion of derivative.
- Interpretation: Geometrical interpretation - General applet.
- Special cases: Discontinuous function - Angle point - Oscillating non-differentiable function - Oscillating differentiable function.
- Application: Growth and derivative - Rolle theorem - Lagrange mean value theorem - Derivative and growth.
- Evaluation Tests: Test 1 - Test 2 - Test 3 - Test 4 - Test 5.

LA DERIVADA	INTERPRETACIÓN	CASOS ESPECIALES	APLICACIONES	TEST EVALUACIÓN
Derivada como velocidad	Interpretación geométrica	Discontinua	Crecimiento y derivación	Test 1
Concepto de derivada	Applet general	Punto anguloso	Teorema de Rolle	Test 2
		Oscilante no derivable	Teorema del Valor Medio de Lagrange	Test 3
		Oscilante derivable		Test 4
			Derivación y crecimiento	Test 5

Fig. 2 – Expanded pull-down menu

We describe next some of the applets and, in the subsequent sections, we describe the objectives we want students to attain working with them in an adequate way.

### 3.1 The “geometrical interpretation” applet

This application (see figure 3), written in dynamic HTML with javascript, shows that the derivative of a function in a point is the slope of the tangent to the graph of the function at the given point.

Since the goal of this first applet is mainly didactic, the function is fixed. When the user chooses a point (with the mouse or writing its coordinates) the applet displays a sequence of secant lines approaching the tangent line. At the same time, the slopes of these secant lines are computed. The student can see how the values of these slopes approach the value of the derivative of the function.

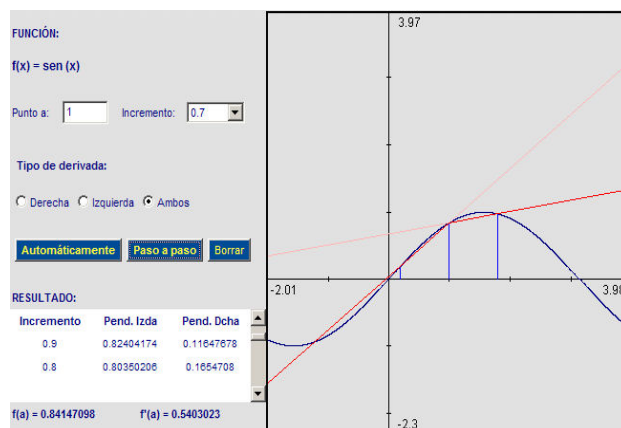


Fig. 3 – The “geometrical interpretation” applet

In addition, the applet gives the user the option to zoom into the point in order to deepen in the relation between the derivative in a point and the tangent line at this point. Moreover, the zoom allows the visualization of the main property of the derivative: the approximation of the function near the point by the tangent line, since when the zoom is deep enough both graphics tend to be undistinguishable.

### 3.2 The general applet

Once the student is familiar with the notion and geometrical interpretation of the derivative, he may further experiment with a more general and versatile applet (see figure 4).

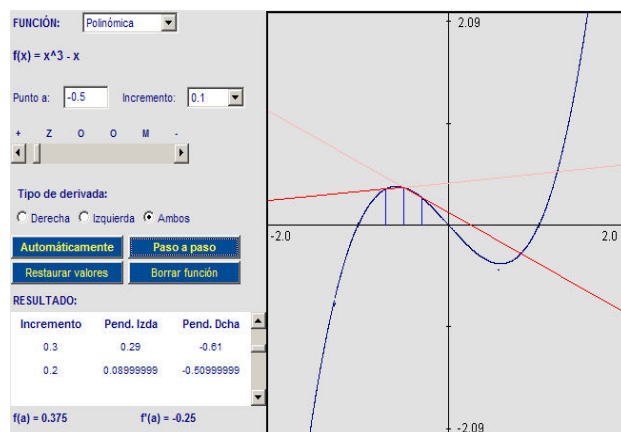


Fig. 4 – The general applet

In this applet, the student can choose the (trigonometric or polynomial) function to differentiate, as well as the point and the approximation rate.

All the applets in our tutorial have the possibility of a step by step visualization, zooming in and out, and moving the graph with the mouse.

### 3.3 Applets for particular cases

When the student has understood the notion of derivative and its geometrical interpretation, he is prepared to face some special cases not included in the family of allowable functions for the above applet (polynomials and trigonometric functions). We have developed several applets which illustrate these special cases.

#### 3.3.1 Discontinuous functions

The first question the student may ask himself is what happens when the function is non-continuous but has a jump discontinuity.

To consider this case, we have included an applet, shown in figure 5, which demonstrates geometrically that at least one of the side derivatives must be infinite. If the step by step button is pressed, the students can deduce this result by themselves, just through the observation of the sequence of secant lines displayed.

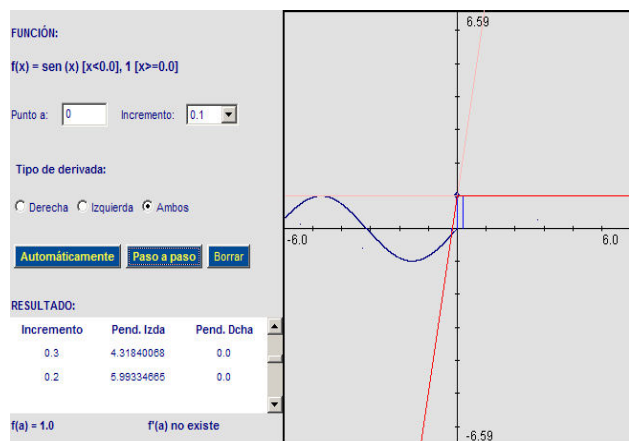


Fig. 5 – Applet for a discontinuous functions

Another question the students frequently ask themselves is if it is possible to find a continuous function with no derivative. The following two applets show different examples to this question.

#### 3.3.2 Continuous non differentiable function

The next applet (see figure 6) illustrates the case of a continuous function with no derivative, and

therefore no tangent, in an angle point with finite (but different) side derivatives.

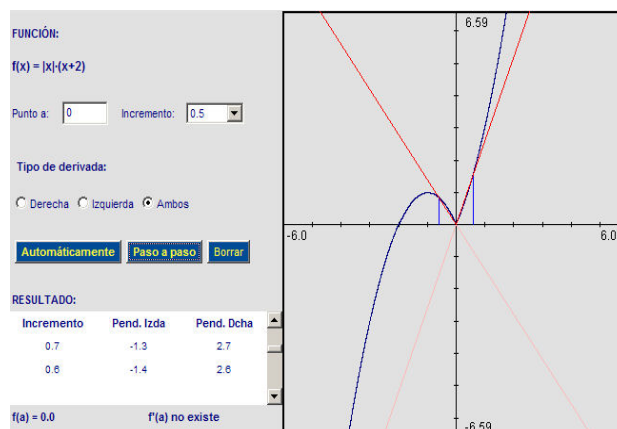


Fig. 6 – Applet for a continuous non differentiable function

The applet shows, in a dynamical and geometrical way, the non-existence of the tangent line in such a point.

#### 3.3.3 Continuous function without side derivatives

Although less frequent than the previous case, there exist functions where the lack of differentiability is due to the non existence of one, or both, of the side derivatives.

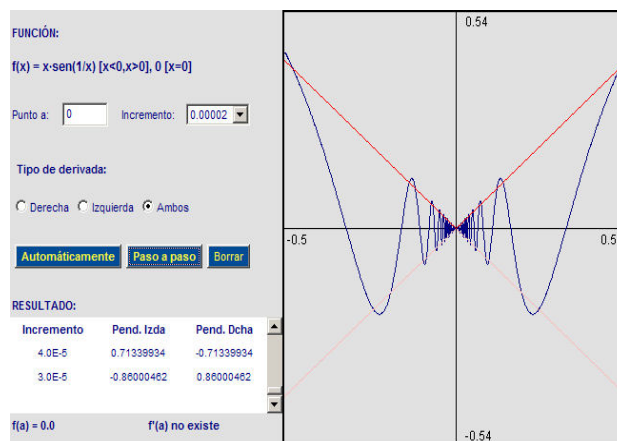


Fig. 7 – Applet for a continuous function without side derivatives

#### 3.3.4 (Non trivially) differentiable function

Finally, we think that it is necessary to make clear to the students that an oscillated function can be a differentiable function. We show an example closely related with the previous one, but in this case the function is differentiable at zero. Although the function  $x^2 \sin(1/x)$  oscillates, it does it between two functions,  $x^2$  and  $-x^2$ , both of them with null



derivative at zero, hence the derivative of the “intermediate” function at zero must also be zero.

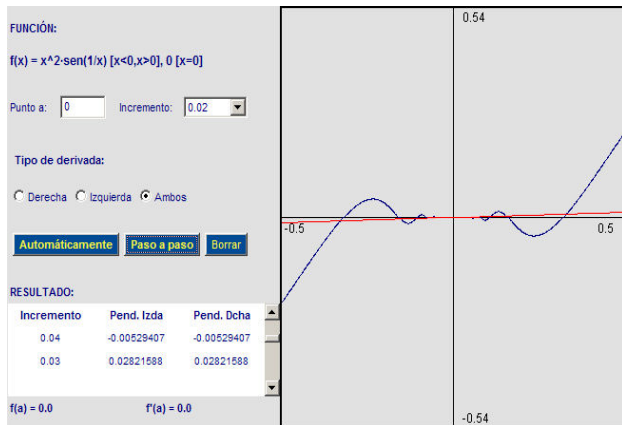


Fig. 8 – Applet for a (non trivially) derivable function

### 3.4 Applications

The last part of the tutorial is devoted to the main applications of differential calculus to the study of functions.

#### 3.4.1 Relative extrema

If a function has a relative extremum point (a maximum or a minimum point) at which the function is differentiable, then its derivative must change its sign when passing through this point. This is the key fact used in the next applet to search for possible local extrema of a function. The geometric idea behind this result is made clear in the applet.

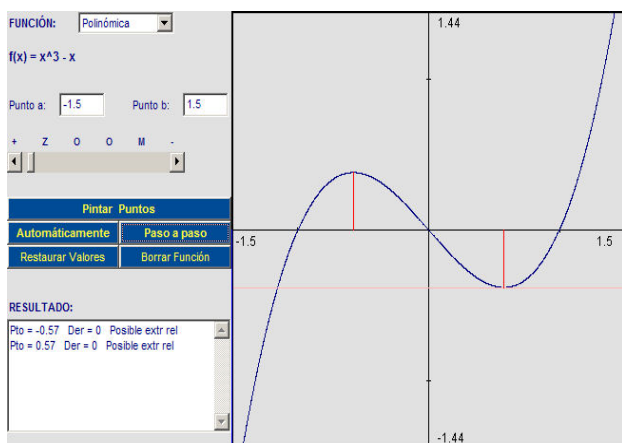


Fig. 9 – Applet to find relative extrema

#### 3.4.2 Rolle Theorem

A consequence of the above characterization of extremum points as points with derivative equal to zero is Rolle Theorem. This fundamental result

states that whenever a differentiable function takes the same value at two points, then the derivative cancels at some intermediate point. Again, we use an applet to explain this result and to search graphically for points at which the derivative vanishes.

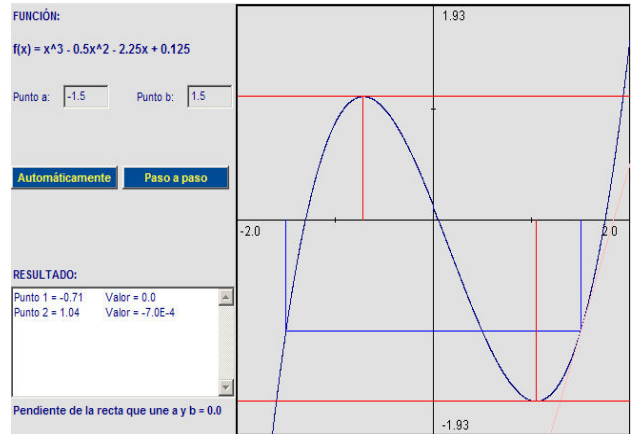


Fig. 10 – Applet for Rolle Theorem

#### 3.4.2 Lagrange mean value theorem

As a consequence of Rolle theorem we obtain one of the most important results in Infinitesimal Calculus: Lagrange mean value theorem. The relation between both results is revealed by a new applet.

This applet searches for the points whose existence is guaranteed by Lagrange mean value, by evaluating the derivative at many points along the interval. When the value of the slope exceeds the searched value, it goes back and makes an oscillating search until it finds the point with the required derivative.

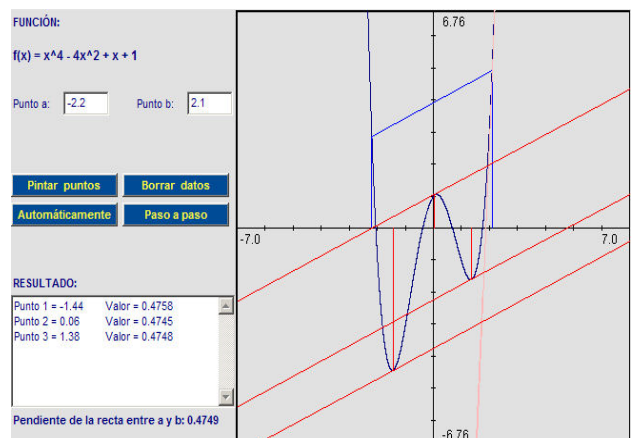


Fig. 11 – Applet for Mean Value Theorem

#### 3.4.3 Increase and decrease of functions

One of the consequences of Lagrange mean value theorem is the existence of necessary and sufficient

conditions, based on the sign of its derivative, for a function to be increasing or decreasing in an interval. This result is used to construct an applet that determines the intervals at which a function is increasing or decreasing.

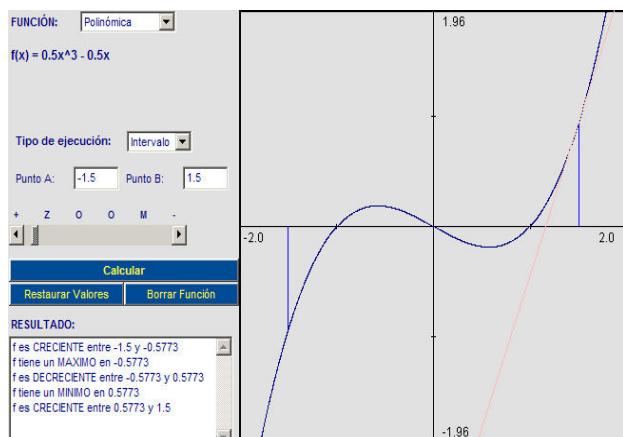


Fig. 12 – Applet to find intervals of increase and decrease

#### 4 Methodology. How to use java tools in and out of the classroom

Nowadays, coinciding with the implantation of the European Higher Education Area, classes tend to be more reduced. This fact, together with the improvement in the means available to teachers, especially those related to technological aspects and to information access, has provided an appropriate framework to design and develop new formative activities. The idea of using applets as a teaching tool, both in face-to-face and online learning, is quite extended. The content of the lecture does not change, but the methods intend to improve students' attitude towards an active learning. In fact, in addition to the classroom, there is now a variety of new learning environments based on the following didactical concepts: teaching fundamental topics, introducing abstract topics with examples and a brief theoretical introduction, using attractive representations with animations to visualize these concepts, allowing a high level of interactivity.

This tutorial on derivation offers a gentle introduction to differentiation theory (including concepts and results such as tangent, Rolle theorem, Mean Value theorem, relative and absolute extrema, intervals of increase and decrease, etc.) which allows the student to analyze the notion of derivative in an experimental way. Relevant parameters of the system can easily be changed, and their impact can be directly analyzed through graphical visualization. The main idea is that

students gain an intuitive understanding of these basic concepts of differential calculus. They can experiment with a range of different examples of functions by changing parameters – like the degree and the coefficients, in the case of polynomials – and observe the resulting graphic behavior. On the other hand, it is interesting for the teachers to give lessons in the classroom using these tools, in combination with other methodologies, to involve the student in a more active learning. Students do not learn much sitting in the classroom listening to lectures, doing stale assignments, memorizing and regurgitating the expected answers. Students need to do what they have learned to do, so that they assimilate this new knowledge. They need to talk about it, write about it, apply it.

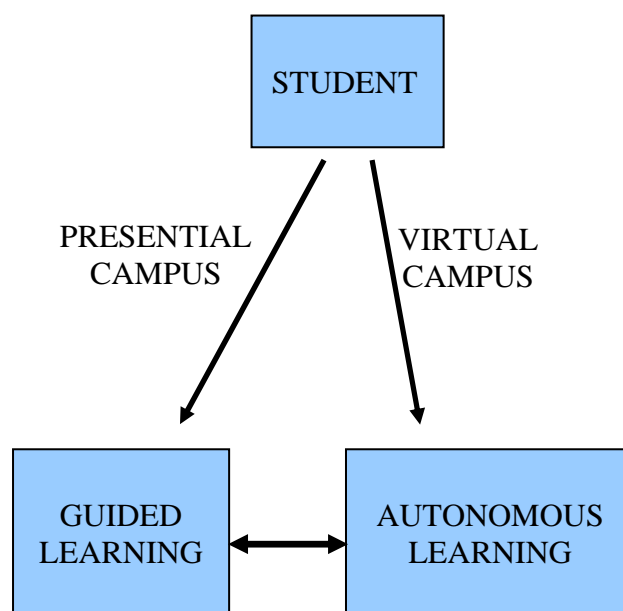


Fig. 13 – Learning methodologies scheme

The above scheme (figure 13) shows how the student's learning is divided within the classroom (directed learning) and outside of it (autonomous learning). The outside of the classroom learning can be conceived as a virtual campus, in which different platforms, like the tutorial and java tools here presented, can be used as a support to teaching. The student has access to these tools in both surroundings. In the classroom, the teacher might train the students in the use of them, but also he may take advantage of the capacities of visualization of these tools in order to experiment, to solve problems or to make any other type of work.

We now describe how to make use of different teaching methodologies (collaborative work, active learning, java tutorials, expositive lectures, etc.) to

give the students the leadership of their own learning.

In order to make works in groups [12], it is advisable to make teams of three students [17], which we will call base groups. It is a good idea to begin each class with meetings of the base groups, in which the homework made previous to the session will be discussed. This meetings help to focus the attention and the interest on the class to come. In addition, it could be necessary to facilitate the work in groups also outside the classroom, although it is also essential to foment the individual work, which is also very necessary.

It is a verified fact that the attention is reduced after 20 minutes, and this is why it is advisable to change the activity after this time lapse. For example, in the expositive sessions, after 20 minutes it would be necessary to stop, so that the students can rest and recover the necessary attention level. At those moments, the incorporation of cooperative tasks is especially interesting. In this way, the exposition can be interrupted without having to change the theme currently being studied. In some cases, depending on the subject to study, it will be necessary to interrupt five minutes just to be able to continue the exposition; whereas in other occasions the work in groups to solve exercises or doubts can be helpful to advance in the learning of the subject.

The next subsections are devoted to show some examples of how to include techniques of cooperative learning in the expositive lectures, and how to use in these sessions complementary technological tools, like the applets which we have presented in this work. We will see how to train the students in the use of these tools in an autonomous way previously to the next teaching session.

#### 4.1 Ideas for a mainly expository session

Some subjects need a long exposition by the teacher. However, as we have mentioned above, after 20 minutes it is necessary to stop the exposition for a while. However, the interruption must be short, useful, and not only it should not contribute to lose the thread of the lecture, but it should help to understand what has been explained so far, and also what it is going to be explained after the interruption. The following is a list of possible activities in this line:

- *To interchange notes:* The students interchange their notes to correct errors (in their own notes and in the notes that its companions have written), to clarify doubts, etc.
- *To decide what is the most important concept and what is the most difficult one:* The base

groups meet to discuss the concepts that have just been explained and to decide what are the most important ideas and what are the most difficult ones.

- *To prepare a question:* The members of the group have to prepare a question on the subject just explained.

#### 4.2 Example for a more complex session

Besides stopping after 20 minutes to make some of the activities previously described, the following works can be incorporated, when the subject is very difficult to understand:

- To make exercises individually, to be corrected later by other members of the group.
- Puzzles [12]. These are a very versatile tool, although its implementation is very time consuming. It consists on dividing the work in three parts. Each member of each base group has to work individually in one of the parts. Next, taking separately the students which have studied a same part, new groups are made to discuss it and clarify doubts. Then, each student has to prepare some notes and examples to explain the part he has studied to the other two members of his base group.
- Other activities of collaborative work may be appropriate at the end of an especially important subject or at the middle of the course. A very interesting activity would be the following: Each group prepares (as homework) an exam of three questions. They pass the exam (with the corresponding solution) to the teacher. At the next teaching session, each group would receive an examination prepared by another group, and its members would have to solve it. Next, the members of each group will correct the exam proposed by them. After the class, the teacher will have to review them, and the evaluation will depend as much on the appropriateness of the proposed exercises, as on the solution of the exam they have received and the evaluation that each group has done.

#### 4.3 The use of tutorials

The current tendencies in educational methodology, recommend teaching sessions to be more active, favoring autonomous learning. Tutorials, as the one presented here (see [22, 23, 24, 26] for other examples), allow, in addition to facilitate the visualization that favors the compression [16], to show numerous examples in less time. This saving

of time allows the implementation of numerous activities like the ones that we have described above.

In this way, the didactical benefits of an interactive tutorial for Infinitesimal Calculus are not only restricted to the fact of being a more or less useful internet tool, but they have also an added value when they are used by teachers as a support in their expositive lectures.

## 5 Teaching activities with the derivative tutorial

We are now going to propose some concrete activities in the line of the ideas in the previous section, showing real examples on the use of our derivative java tutorial in the classroom.

In order to describe a derivation teaching session, making use of the previously described methodologies, we will begin defining some of the formative objectives for this subject, and how they are hoped to be reached.

According to the Taxonomy of Bloom [2], the cognitive levels of the formative objectives are the following ones, where the last three categories will be cognitive abilities of high level.

- *Knowledge*: to learn by heart.
- *Understanding*: to prove that the concepts have been understood.
- *Application*: to apply the new knowledge to solve a problem.
- *Analysis*: to identify the parts which compose a system, to formulate theoretical explanations or mathematical models for observed phenomena.
- *Synthesis*: to create something, to combine elements in an innovating way.
- *Evaluation*: to choose between alternatives and to justify the election made, specifying the criteria used to choose.

Let us see how, with an appropriate plan of activities, and with the support of the derivation tutorial, formative objectives of any level of the previous taxonomy can be obtained.

### 5.1 Formative objectives of derivation

- *Objective 1*: To identify at first glance, in a graph, the points at which there is a derivative and the points at which it does not exist (Knowledge).
- *Objective 2*: To interpret and to identify when the side derivative of a function in a point is infinite. To be able to describe examples. (Understanding).

- *Objective 3*: To solve equations, using Rolle theorem. (Application).
- *Objective 4*: To model mathematically problems of increase and decrease, and problems concerning absolute and relative extrema. (Analysis).
- *Objective 5*: To construct functions, given certain properties about its derivative. (Synthesis).
- *Objective 6*: To design and to solve problems about mathematical modeling of real life phenomena and situations, using the properties of the increase and decrease of function. (Evaluation).

### 5.2 Plan of activities and time

In order to attain the objectives, it is necessary to make a good planning of the activities to be made by the students, as well as of the time necessary to make them. For example, for a course of 5 hours a week, it would be recommendable, in average, to invest 3 hours and a half at home.

In the following, we show an example of a planning of activities.

- *Activities previous to the class* (1h. 10 min. approx.):
  - To study the documentation.
  - To experiment with the derivation tutorial.
  - To prepare a task adapted to each part, for example any of the below proposed exercises to experiment with the applet.
- *Activities in the classroom*:
  - Meeting of base groups: to solve doubts, to compare solutions, to decide common unsolved doubts and to solve a theoretical exercise.
  - Expositive lecture (no more than 20 minutes): explanation at the blackboard and visualization with the applet.
  - Individual work: resolution of exercises.
  - Meeting of base group: resolution of doubts and correction of exercises.
  - Puzzle (1 h. 30 min):
    - Each member of the base group experiments (15 min.) with each of the following three applications of the tutorial (Growth and derivative, Rolle theorem, Mean Value Theorem) and solves exercises adapted to each subject.
    - Meeting of experts (15 min.): Meeting in groups of three students who have worked in the same subject (10 min.), to clarify doubts and to complete the resolution of the problems.



- Individual work (10 min.): To prepare an explanation to the rest of the members of the base group.
- Meeting of base groups (40 min.): Each student explain his part of the work to the rest of the group.
- *Exercises to experiment with the applet:*
  - Geometric visualization of the derivative:
    - choose point=0,
    - choose increment=0.1,
    - choose type of derivative=right,
    - press “step by step”,
    - observe and write down the successive values that the slope of the secant takes,
    - write down for which value the slope is bigger than 0.99,
    - interpret the value of the slope of the secant as the average of growth in the interval,
    - try to relate the derivative to the idea of instantaneous growth.
 (knowledge and compression).
  - Applications, growth and derivative:
    - read the documentation,
    - experiment with the applet until obtaining a polynomial of a certain degree with a given number of relative extremum points and intervals of growth,
    - experiment with the value of the derivative at the relative extremum points,
    - experiment with the zoom lens.
 (synthesis)
  - Applications, Rolle Theorem and Lagrange Mean Value Theorem:
    - read the documentation and experiment with the applet,
    - choose two points and try to find out how many points satisfy the thesis of the theorem,
    - find an example in which the thesis is fulfilled for three points,
    - after understanding both theorems, make exercises of resolution of equations applying these theorems.
 (application, analysis)
  - Test:
    - make the tests at the end of the tutorial and develop similar examples, relating the behavior of the derivative to the increase and decrease of the function,
    - find a real life problem that could be modeled by a single variable function, establishing conditions on the increase and decrease of the solutions of the

- problem in order to find a function that could be a model for it,
  - calculate the relative extrema of the function and interpret the result according to the associated real life problem.
- (evaluation)

## 6 Conclusions and future work

There are many studies comparing the effectiveness of online learning and face-to-face learning. However, researchers have not yet conclusively demonstrated the superiority of any of these two learning strategies. There are a great number of studies that have not found any significant differences between exam results of online students and those of face-to-face students [5, 14, 21]. However, there are cases in which online learning is reported to be more effective than face-to-face learning [13], while some research findings revealed that, in some cases, face-to-face learning is more effective than online learning [3]. These make us think that the best option is to use graphical and interactive tools in both surroundings. On the one hand, these tools help the teacher in the classroom, while on the other hand, the students can work and experiment with them making their own examples, out of the classroom.

Summarizing, the didactical benefits of this interactive tutorial for Infinitesimal Calculus, according to our experience in teaching these mathematical concepts, are:

- It helps the student to understand the concept and the geometrical interpretation of the derivative of a function in a point, as well as the fundamental role Lagrange mean value theorem plays in Calculus.
- It helps the teachers in their lectures by navigating through the examples and the applications implemented along the tutorial.
- It offers the student the opportunity to experiment, increasing interactivity.

In general, interactive tutorials including Java applets are very good aids for learning mathematics, as they improve comprehension, engagement, memorization and the satisfaction of the students, as well as the interest and motivation amongst pupils when the teacher makes use of them.

As a future work, we are currently developing similar tutorials for other parts of Calculus, in an attempt to cover as much as possible of this discipline. We would also like to increase the family of allowable functions to include rational,

exponential and logarithmic functions and basic combinations of them.

Moreover, we are also working in other areas of Mathematics related to Computer Science. For example, we have also made tutorials for Discrete Mathematics, Dynamical Systems, Fractal Geometry and Image Processing.

The use of technology in teaching is a very active field of research. In the last years there have appear a great number of proposals to introduce innovative tools in teaching [7, 8, 11, 15], especially in connection with the World Wide Web [25]. In particular, the subfield of adaptive tests is also being explored [1, 4] and is expanding in connection with the semantic web [29].

As future work we also intend to elaborate a supporting tool for the teaching of mathematics courses. The general plan is divided in several parts which converge in the design and development of an intelligent engine for the generation of adaptive tutorials, including applets and different kinds of tests of an adaptive nature, i.e., than can be dynamically generated according to the knowledge of the students.

## 7 Acknowledgements

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