

A mathematical solution of an industrial problem for Engineering Students

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Abstract: - This paper presents an illustrative industrial example: the liquid volume contained in a horizontal tank is calculated by using integrals and least square method. The equation is used by automated system that can automatically measure the volume of material in horizontal tanks in real time. This system is composed by an industrial PC, where an operator can monitor the variable level through a graphical environment interface developed in Delta V, the interface contains alarms, historical, tendencies about substances weight contained in the tanks. The objective is to provide to mathematics lecturer a practical example in order to motivate engineering students to make mathematics interesting to them and show them why the knowledge of mathematics is essential for their future real word.

Key-Words: - Education, Engineering Mathematics, Automatic Control, Volume Measurement, Industrial Application and Data Acquisition.

1 Introduction

Most engineering educators know that mathematics deficiencies in students are a major impediment to learning. In [1] the importance of explaining to the students why knowledge of mathematics is essential for the future real word is emphasized. Also it is discussed that the objective of teaching mathematics to engineering students is to find the right balance between practical applications of mathematical equations and in-depth understanding.

In [2] said that one of the major causes for the problems that some students have with learning mathematics is attitude towards the subject. How to motivate students in mathematics is a question made by mathematics educators. A modern view of mathematics should be reflected in educational practices [3]. The area of mathematics applications is extensive. Models are used to

- Replace or enhance experiments or laboratory trials.
- Create virtual and/or visualized images of objects and systems
- Optimize certain values of design parameters
- Perform intelligent analyses on measurement data.

- Analyze risk factors and failure mechanisms
The education should bring the flavor of this fascinating art to the classroom.

In [3] consider the goal as a combination of mathematical knowledge with modeling skills, project work and a contact with real world applications. Also, in [3] said that there is a window of opportunity for mutual benefits between academy and industry. This also means a challenge for the university education. In [8] the author suggests that, in engineering problem formulation for mathematics courses, the prime focus should be good engineering problems from which one can extract the opportunity to learn relevant mathematics and useful engineering skills.

Considering the suggestions in [3] and [8], in this paper the solution of a real engineering problem is presented, in order to provide a practical example whose objective is to motivate engineering students to make mathematics interesting to them and show them why knowledge of mathematics is essential for the future real word.

2 Problem Formulation

Nowadays, many industries are using automated systems based on a personal computer, where only one worker can visualize all the process variables used in the production plant, in a form of animated graphics which represent the behavior, in real time, of the processes that are taking place in the plant. This is made in a centralized form, and with great flexibility, making easier the monitoring activities and control of any other elements of input or output of the control loops of the process, constituting with this a Distributed Control System (DCS).

In this work the development of an equation to get the liquid volume or weight of a cylindrical tank with dished only tank heads (See Fig.1) is presented. This equation was used to implement a Distributed Control System Delta V [7] in the Company Sherwin Williams. S. A. de C. V.



Fig. 1 Horizontal cylindrical tanks from the Company Sherwin Williams. S. A. de C. V.

To achieve what was previously exposed, a Delta V [7] Control Distributed System was required. This system has the capacity to control and monitor the levels of all the storage tanks belong to the Company. This system (See Fig. 2) is composed of a work station based on an industrial PC, a graphic interface was built where the process variables might be monitored and controlled. The system also contains data acquisition cards to receive the signals. Also, it contains an Ethernet concentrator may form a communication network with the work station or any other control system. The automation of this measurement system gives the following benefits:

- It gives accuracy in the inventory of raw materials.
- It achieves the requirements of the environmental control authority, as it demands that the tanks will contain no more than 90% of the maximum capacity, to avoid spillage of hazardous chemical substances.
- It has a great advantage over manual form since estimates volumes off-line (with human

assistance) having problems in inaccurate, risk of accidents, inhalation of dangerous chemical substances, because it is necessary to climb the tanks in order to measure the volume.

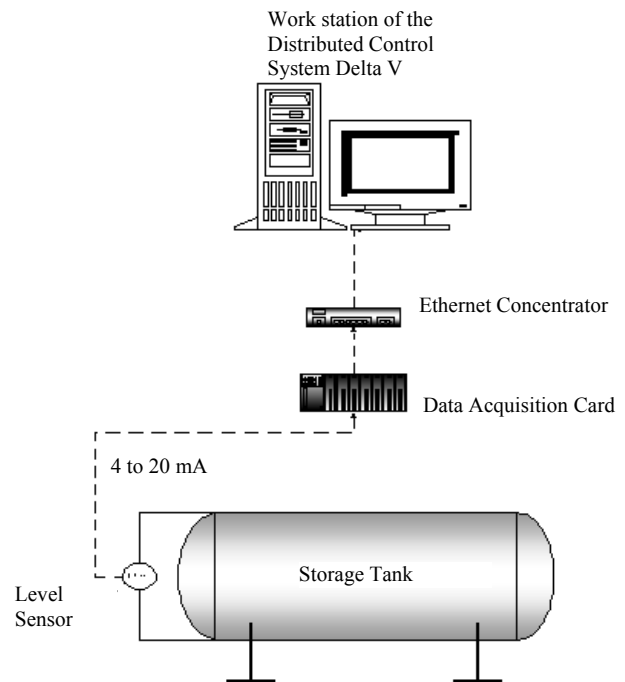


Fig. 2 The volume measurement system for the horizontal tank.

3. Development of the tank volume equation.

The development for obtaining the total volume of the horizontal tank is divided in two parts: Partial volume of the cylindrical section V_{psr} and Partial volume of the dished only tank heads V_{pc} . The total volume V_{pth} is given by

$$V_{pth} = V_{psr} + V_{pc} \quad (1)$$

To obtain the liquid weight contained into the tank, the following equation is used

$$\text{Weight} = V_{pth} * S.W. \quad (2)$$

Where, $S.W$ is the specific gravity of liquid.

3.1 Development of volume equation of the cylindrical section

In order to determine the cylindrical section volume, first the area of the circular section A_{sc} and the liquid level are required, as shown in Fig. 3.

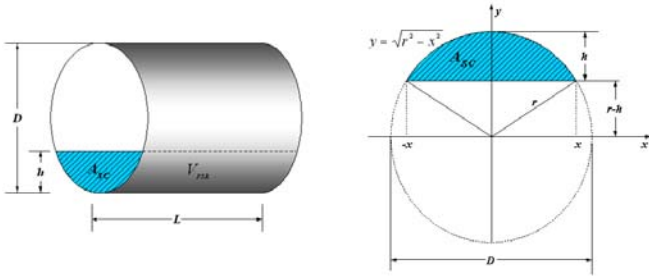


Fig. 3 Cylindrical and circular section of the tank.

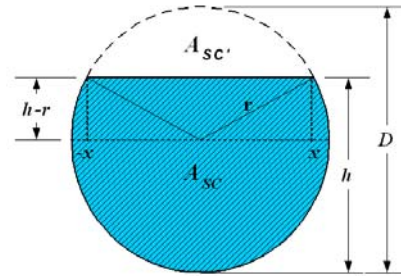


Fig. 4 The circular section of the tank

The following two cases are presented:

- When, the level is equal or less than the half of interior diameter, $h \leq \frac{D}{2}$.
- When, the level is equal or greater than the half of interior diameter, $h \geq \frac{D}{2}$.

Case 1. for $h \leq \frac{D}{2}$

From Fig. 3, the circular section is obtained by integration:

$$A_{sc} = \int_{-x}^x \sqrt{r^2 - x^2} dx - \int_{-x}^x (r-h) dx \quad (3)$$

$$= \frac{x}{2} \sqrt{r^2 - x^2} + \frac{r^2}{2} \text{sen}^{-1} \frac{x}{r} \Big|_{-x}^x - x(r-h) \Big|_{-x}^x \quad (4)$$

$$= x\sqrt{r^2 - x^2} + r^2 \text{sen}^{-1} \left(\frac{x}{r} \right) - 2x(r-h) \quad (5)$$

Making some substitutions, the following equation is obtained

$$A_{sc} = \frac{D^2}{4} \text{sen}^{-1} \left(\frac{2\sqrt{h(D-h)}}{D} \right) - \sqrt{h(D-h)} \left(\frac{D}{2} - h \right) \quad (6)$$

Then, the equation to obtain cylindrical section volume is:

$$V_{psr} = A_{sc}L \quad (7)$$

Case 2. for $h \geq \frac{D}{2}$

From Fig. 4, the circular piece is obtained by

$$A_{sc} = \text{Total Area of the Circle} - A_{sc}' \quad (8)$$

where

$$A_{sc}' = \int_{-x}^x \sqrt{r^2 - x^2} dx - \int_{-x}^x (h-r) dx \quad (9)$$

This equation is the same as Eq. 3 and by substitution in Eq. 5, it is obtained

$$A_{sc} = \frac{D^2}{4} \left[\pi - \text{sen}^{-1} \left(\frac{2\sqrt{h(D-h)}}{D} \right) \right] + \sqrt{h(D-h)} \left(h - \frac{D}{2} \right) \quad (10)$$

Then, the equation to obtain cylindrical section volume for the two cases is:

$$V_{psr} = A_{sc}L \quad (11)$$

3.2 Development of the Partial volume equation of the dished only tank head

The table 1 will be used to obtain the partial volume equation of the dished only tank head. This table published by [4], gives the volume as a fraction of the total head volume. The volume fraction is in function of the liquid level h and the internal diameter D of the head in vertical position, the variable is h/D .

The equation which gives the table is obtained by using least square method [6]. The curve is approximated by the following polynomial

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 \quad (12)$$

No.	h/D	Fración de Volumen	No.	h/D	Fración de Volumen	No.	h/D	Fración de Volumen	No.	h/D	Fración de Volumen
1	0.02	0.0012	14	0.28	0.1913	26	0.52	0.530	39	0.78	0.8761
2	0.04	0.0047	15	0.30	0.216	27	0.54	0.560	40	0.80	0.8960
3	0.06	0.0104	16	0.32	0.242	28	0.56	0.590	41	0.82	0.9145
4	0.08	0.0182	17	0.34	0.268	29	0.58	0.619	42	0.84	0.9314
5	0.10	0.0280	18	0.36	0.295	30	0.60	0.648	43	0.86	0.9467
6	0.12	0.0397	19	0.38	0.323	31	0.62	0.677	44	0.88	0.9603
7	0.14	0.0533	20	0.40	0.352	32	0.64	0.705	45	0.90	0.9720
8	0.16	0.0686	21	0.42	0.381	33	0.66	0.732	46	0.92	0.9818
9	0.18	0.0855	22	0.44	0.410	34	0.68	0.758	47	0.94	0.9896
10	0.20	0.1040	23	0.46	0.440	35	0.70	0.784	48	0.96	0.9953
11	0.22	0.1239	24	0.48	0.470	36	0.72	0.8087	49	0.98	0.9988
12	0.24	0.1451	25	0.50	0.500	37	0.74	0.8324	50	1.00	1.0000
13	0.26	0.1676				38	0.76	0.8549			

Table 1

The parameters of the polynomial are given by

$$\hat{\theta} = (X^T X)^{-1} X^T Y \quad (13)$$

where

$$Y = X \hat{\theta} \quad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{50} \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{50} & x_{50}^2 & x_{50}^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (14)$$

The polynomial obtained is given by

$$y = 0.0001 - 0.0015x + 3.004x^2 - 2.003x^3 \quad (15)$$

It can be approximated by

$$y = 3x^2 - 2x^3 \quad (16)$$

It can be rewritten in terms of the volume fraction h/D , when the tank head is partially loaded.

$$\text{Volume Fraction} = 3\left(\frac{h}{D}\right)^2 - 2\left(\frac{h}{D}\right)^3 \quad (17)$$

This equation gives the Table 1. The Fig. 5 shows the graphic of the Table 1 and the points obtained using Eq. 17.

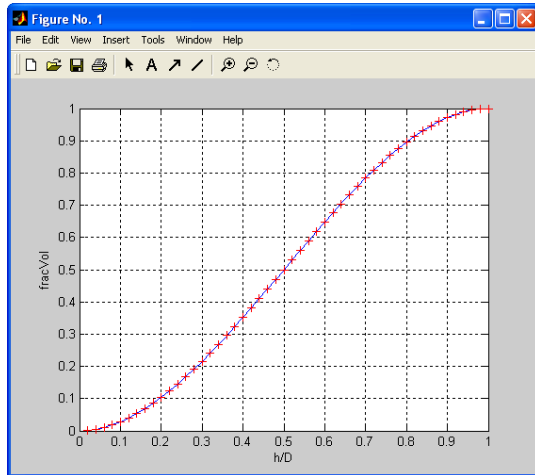


Fig. 5 Graphic of the table 1 and the points obtained by using least square.

The partial volume equation of the hemispherical tank head is obtained by the following equation:

$$\text{Partial Volume of the tank head } (V_{pc}) = \text{Volume Fraction} * \text{Total Volume of the dished only tank head.} \quad (18)$$

The Fig. 6 is used to get the total volume of the dished only tank head, [5] which is given by

$$V_{TC} = \frac{\pi}{6} a \left(\frac{3}{4} D^2 + a^2 \right) \quad (19)$$

Substituting Eqs. 17 and 19 into Eq. 18, the partial volume of the dished only tank head V_{pc} in vertical position is given by

$$V_{PC} = \frac{\pi}{6} a \left(\frac{3}{4} D^2 + a^2 \right) \left(3\left(\frac{h}{D}\right)^2 - 2\left(\frac{h}{D}\right)^3 \right) \quad (20)$$

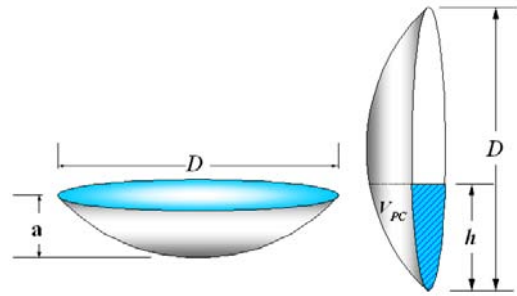


Fig. 6 Dished only tank heads

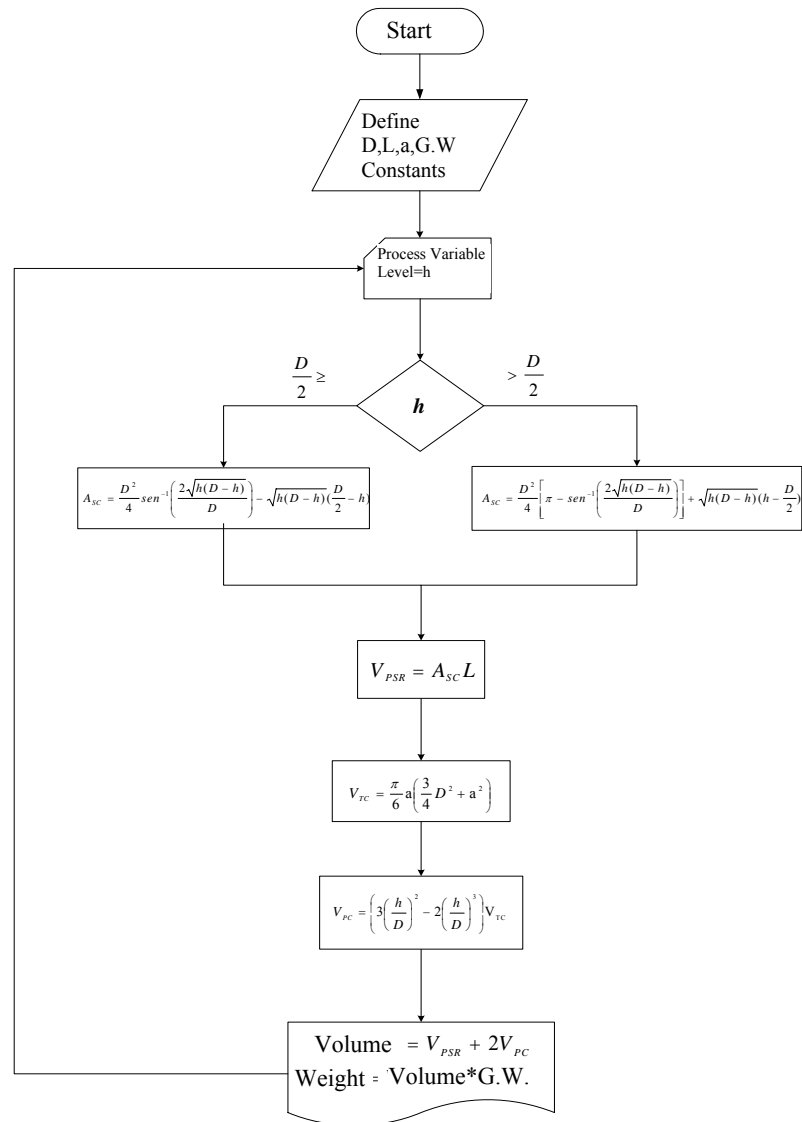


Fig. 7 Algorithm for the volume and weight calculation in partially loaded horizontal tanks.

3.3 Development of the Algorithm for obtaining the volume equation

Figure 7 shows the algorithm used by the program installed in the Distributed Control System Delta V in the Company Sherwin Williams. S. A. de C. V. [7]. This algorithm contains the volume equation obtained in this paper, which has just one variable, the tank level.



Fig. 8 The Operator Interface for the system.

4 Conclusion

This paper presents an illustrative industrial example: the liquid volume contained in a horizontal tank is calculated by using integrals and least square method. The objective of this example is to develop an equation for automated system that can automatically measure the volume of material in horizontal tanks in real time. This system is composed by an industrial PC, where an operator can monitor the variable level through a graphical environment interface developed in Delta V (See Fig. 8), the interface contains alarms, historical, tendencies about substances weight contained in the tanks.

The volume equation is implemented into an algorithm using a Distributed Control System Delta V in the Company Sherwin Williams. S. A. de C. V. The volume equation has just one variable, the tank level; requiring parameters as interior diameter, length of dished only tank heads, which all of them are constant. The operator interface is built, which provides audible and visual alarms to alert the operator of any abnormal conditions.

Considering the suggestions in [3], in this paper the solution of a real engineering problem is presented, in order to provide a practical example whose objective is to motivate engineering students to make mathematics interesting to them and show them why knowledge of mathematics is essential for the future practical word. During presentation of the industrial

problem solution the students seem to be very excited, basically because they can put in practice the learned mathematics in an industrial problem.

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