Three-dimensional design against fatigue failure and the implementation of a genetic algorithm

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Abstract: - The research presented here utilises a genetic algorithm in a numerical three-dimensional fatigue based optimisation study of a 7050-T7451 aluminium structure. Genetic algorithms have rapidly become popular due to the robustness and the balance between efficiency and effectiveness in many different environments. Genetic algorithm has been utilised in many stress based optimisation applications, however to date, it has not been used in a three dimensional structural fatigue based optimisation study involving short crack lengths in the threshold region. The generalised Frost-Dugdale law was developed to allow for the accurate prediction of fatigue crack growth from short crack lengths. Consequently, design against fatigue failure can include the analysis of near-threshold crack propagation. The structural optimisation procedure proposed integrates geometrical modelling, structural analysis and optimization into one complete and automated computer-aided design process. This paper indicates that the proposed combined procedure provides a more accurate and robust optimised solution. It was discovered that the results resembled the solutions from other optimisation algorithms. As a result, this procedure illustrates a procedure for the design of light weight structures using a fatigue based optimisation in conjunction with a genetic algorithm. Furthermore, the possibility of the application of the generalised Frost-Dugdale model in design optimisation has been demonstrated. This procedure has the potential to be applied to structures with complex structural configurations taking into account crack propagation in the near-threshold.

Key-Words: - Structural optimisation; Genetic algorithm; Finite element method; Short crack growth

1 Introduction

The genetic algorithm concept has been growing since the early 1970s [1]. However only recently has this concept been applied to a variety of disciplines and real-world applications, demonstrating its commercial potential. The application of genetic algorithm to structural optimisation in the engineering industry was first studied by Goldberg [1, 2]. More recently Annicchiarico and Cerrolaza [3-6] developed two- and three- dimensional structural stress based optimisation implementing a genetic algorithm, and modelled the finite element structures using beta-splines. Genetic algorithms have rapidly become popular due to the robustness and the balance between efficiency and effectiveness in many different environments. The application of genetic algorithms to stress based structural and topology optimisation problems is relatively mature. However, the implementation for genetic algorithms to a fatigue based structural optimisation procedure has received little attention and the involvement of initial short cracks within the threshold region has yet to be considered. Therefore an investigation into the development and application of a genetic algorithm into fatigue based structural optimisation procedure involving short cracks within the threshold region is considered in this paper.

Several structural optimisation algorithms in literature have been developed to account for damage tolerance issues and initial cracks, but to date none have analysed the effect of near-threshold crack propagation, i.e. short initial cracks in the low-to-mid ΔK region. This is mainly due to the fact that these structural optimisation algorithms only use Paris like laws which are applicable only in Region II (Paris region). For these reasons the development of an optimisation procedure involving near-threshold crack propagation was investigated.

2 Numerical Formulation

2.1 Crack Growth Model

2.1.1 Paris Law

Fatigue crack growth has traditionally revolved around the belief that the crack growth rate, *da/dN* can be related to the stress intensity factor range, ΔK, and/or the maximum stress intensity factor K_{max}. This correlation was first suggested by Paris et al [6] and resulted in the well known Paris equation:

$$da / dN = C(\Delta K)^m \qquad (1)$$

where C and m are experimentally obtained constants that are considered to be constant for a particular material. This relationship has had a number of modifications to account for various observations, such as R ratio ($R = K_{min} / K_{max}$), K_{max} effects [7, 8] and closure effects [9, 10]. The NASA fatigue crack growth structural analysis program implements Newman's law, which is a closure effect variant of the Paris law. The Paris law, and its variants are only applicable in the Paris region, Region II.

2.1.1 Generalised Frost-Dugdale Law

Recent observation has revealed, for constant amplitude loading, a near log-linear relationship between natural log of the crack length and the fatigue life for crack growth lengths as small as a few microns in the near-threshold region [11-14]. From these observations Barter et al. [11] presented a generalised Frost-Dugdale crack growth law to describe this relationship.

$$\frac{da}{dN} = C(a)^{1-\frac{m}{2}}\left(\Delta K_{eff}\right)^m \qquad (2)$$

where C, a and m are constants, and ΔK_{eff} is the effective stress intensity factor. It has been shown in [11-14] that this relationship holds for the 7050-T7451 aluminium alloy, in which C is 1.78E-10 and m is 3.36. Thus, confirming the implementation of the generalised Frost-Dugdale law for the 7050-T7451 aluminium alloy. The NASA fatigue crack growth structural analysis program [9] has been modified to implement this law.

2.1 3D Structural Analysis

The structural optimisation procedure proposed integrates geometrical modelling, structural analysis and optimization into one complete and automated computer-aided design process, termed an 'Integrated Design Optimisation System'. It determines the shape of the boundary of the three-dimensional structural component under geometric constraints and structural conditions. The same structural analysis, the Finite Element Alternating Method (FEAM) as described in [18, 19] was implemented in this research, to avoid the finite element modelling of a complex three-dimensional cracks. FEAM only requires the location of the crack centres in its analysis. Therefore, 3D semi-elliptical cracks, as specified in [9, 18, 19] were placed all along the design surface of the model allowing for an effective modelling of the stress intensity factor variation around the boundary surface. Using these stress intensity factor solutions the fatigue crack growth structural analysis program then uses the appropriate crack growth law to calculate the fatigue life at the crack locations.

2.2 Genetic Algorithm

Genetic algorithms search from a randomly selected population of design points. Genetic algorithm searches many points simultaneously, searching many optimal peaks in parallel, thus illuminating the probability of finding false local optima. Genetic algorithms converge quickly to the optimum structure with a minimum effort, having to test only a small fraction of the design space to find out either the near optimum or the optimum solution. Genetic algorithm does not calculate the derivatives or use other auxiliary knowledge, but utilises the objective function information. An important aspect of genetic algorithm is the fact that probabilistic transition rules, i.e. probabilistic operators, are implemented to guide the search and not deterministic rules. These differences contribute to a genetic algorithm's robustness and resulting advantage over other more commonly used techniques [1].

Each design parameter is encoded into a particular code, i.e. binary finite-length string, and is termed the genotype. The concatenation of each genotype creates a binary finite-length string representation known as a chromosome. A single chromosome represents the total prescription for the construction of a particular design. The entire parameter sets is termed the population. Therefore, there are two main mechanisms that link genetic algorithm to solving a particular problem. The first mechanism is the method of encoding particular design solutions, represented by design parameters, to the problem on chromosomes and decoding the chromosome within limits of the parameters of the design solution. While the second mechanism is the evaluation of a function that measures the performance of a chromosome in the context of the problem.

The fitness function is the link between the genetic algorithm and the problem to be solved [20]. Since each chromosome represents a parameter or parameter set for a particular design, the objective function of the problem is evaluated to find the response of the design parameters. The fitness function is converted from the objective function and produces a direct indication of performance for each chromosome to solve the optimisation problem subjected to the imposed constraints, which is termed fitness. This allows the population to be ranked according to fitness.

A top-level description of a simple genetic algorithm is as follows. In the initial phase, the population is heterogeneous, that is the randomly-generated chromosomes that are different. This randomly chosen population of chromosomes are copied based on their fitness by the operator known as reproduction. The chromosomes with the higher fitness values have a higher probability of being reproduced. Next the operators, crossover and mutation are implemented, in which bit manipulation, string copying and exchanging of partial strings update the chromosomes in the new improved population. This result in a new generation of chromosomes that are ready to be evaluated, selected and reproduced. This process is repeated, continuously improving the population, until the population becomes homogeneous. The definition of a population becoming homogeneous as defined by [1], is when the variance or standard deviation of the fitness becomes small, or the mean of the fitness approaches the maximum fitness of the population. I.e. when the population has converged, in which the population consists primarily of similar individuals. The next section briefly describes the three genetic algorithm operators.

2.2.1 Genetic Algorithm Operators

The reproduction of a new population begins with the selection procedure. The concept of reproduction is to create another population based on the fitness values of each chromosome. The selection process chooses chromosomes to be paired for reproduction, which will be used by the crossover and mutation operators. Chromosomes with a higher fitness value have a higher probability of being selected and contributing to one or more chromosomes in the next generation. There are a number of algorithmic implementations of this selection operator. The selection operators investigated include deterministic sampling, stochastic sampling with replacement, remainder stochastic sampling methods with and without replacement and stochastic tournament procedure.

The crossover operator exchanges bits between chromosomes selected by the selection operator according to the type of crossover algorithm. The selection operator selects two chromosomes from the population termed the parents from the population; next the crossover operator creates two children from the two parents and begins to create a new population. This process is repeated until the new population is filled. The crossover operator is assigned a probability of being performed with the genetic algorithm. The three common crossover algorithms investigated are single, double and uniform crossover. Single crossover involves the exchange of bits between two chromosomes from a randomly selected point, termed the crossover site, in the chromosome. For example if a chromosome has a length of n, a crossover site position k between 0 and n is randomly chosen to indicate the point after which all the bits from both chromosomes are exchanged, i.e. exchanging all the bits between k+1 and n inclusively. The double crossover operator is similar to the single crossover operator, however there are two exchange points. The bits between the two parent chromosomes are exchanged between the two exchange points. For example if a chromosome has a length of n, crossover site positions of k and m are randomly chosen between 0 and n. Therefore the bits between k+1 and m are exchanged between the two parent chromosomes to produce two new chromosomes. The uniform crossover follows a different principle from that of the single and double crossover. For each bit of the children chromosomes, it is randomly selected which parent contributes its bit value to a child. This leads to the random generation of a binary code template. I.e. a template is randomly created of 1 and 0's. The template indicates which parent will contribute to the first child, leaving the remaining parent bit to contribute to the second parent. The selection and crossover operators give genetic algorithms much of their power.

The mutation operator involves the alteration of a randomly selected bit in a chromosome according to a probability value. The alteration of a randomly selected bit in a chromosome simply involves the exchange of a bit to a randomly selected bit depending on the probability. Therefore the mutation operator goes through each bit in a chromosome, replacing each bit by a randomly selected bit if the probability of exchange is met.

3 Numerical Analysis

A simple benchmark problem of a 'through-hole in a rectangular block under biaxial loading' was considered. It consisted of a 10 mm thick 7050-T7451 Aluminium alloy with a length and height of 200 mm and a centrally located hole with a radius of 10 mm. The block was subjected to a uniform biaxial tensile stress field of 100 MPa × 50 MPa as illustrated in Fig. 17b. Due to the symmetry of the model, only an eighth of the structure was modelled to increase the efficiency of the optimisation procedure (i.e. a quarter of the structure and half its thickness, illustrated by the model in Fig. 1b). The material properties used were Young's modulus = 71.7 GPa and Poisson's ratio = 0.30. The objective of this problem was to obtain the optimal geometry of the centrally located through-hole, therefore the hole boundary was considered the design boundary. The optimisation domain constrained the shape of the hole within the lines KM and LN as illustrated in Fig. 1a.

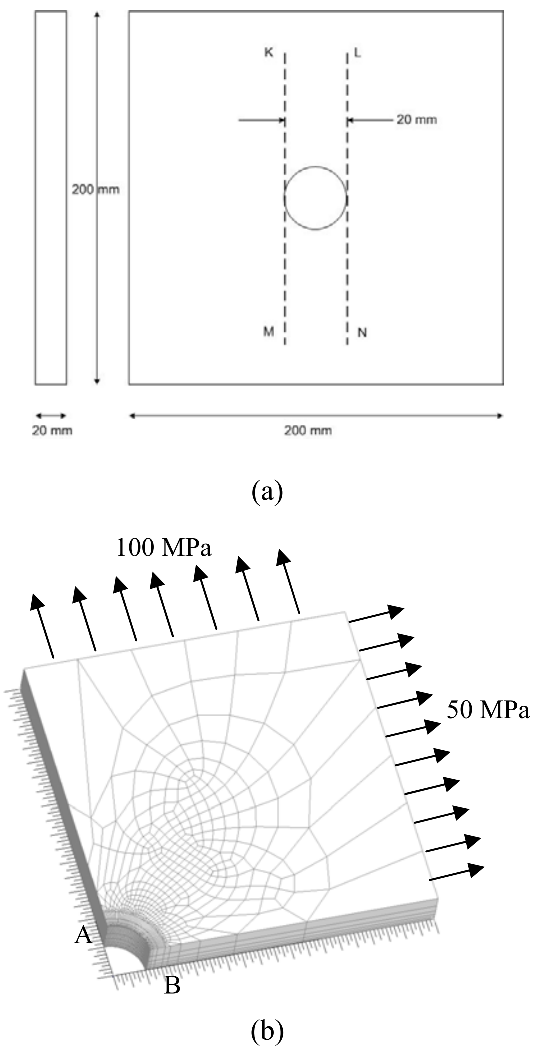

Figure 1. a) Full Schematic of the 7050-T7451 aluminium alloy model. b) Meshed structure of the optimisation problem

This genetic algorithm implementation relies on a simple coding of model design parameters such as the parameters of an equation describing the design boundary. The geometric representation of the design boundary of the centrally located through-hole was chosen so that an effective geometry could be represented by the least possible amount of parameters. In order to maintain the simplicity of the problem and reduce computational efficiency, the number of parameters was kept at a minimum.

The papers [18-22], have shown that the three-dimensional stress optimised solution for the problem of a through-hole in a 3D rectangular block under biaxial loading is approximately a 2:1 ellipse. Therefore, the general polar equation of an ellipse was considered to describe the design boundary of the hole.

$$x = \left(\frac{\cos^p \theta}{a^p} + \frac{\sin^p \theta}{b^p} \right)^{-\frac{1}{p}} \cos\theta \quad (3)$$

$$y = \left(\frac{\cos^p \theta}{a^p} + \frac{\sin^p \theta}{b^p} \right)^{-\frac{1}{p}} \sin\theta \quad (4)$$

It is clear from the equations that there are three variables *a*, *b* and *p*. These represent the design parameters for the optimisation procedure. The parameters *a* and *b* represent the maximum size of the hole in the *x* and *y* directions in the X-Y plane. However, due to a geometric constraint of the problem (as illustrated in Fig. 17a) the hole is constrained in the x-direction by ±10mm. Therefore, parameter *a* was kept at a constant value of 10mm in the symmetrical model. The parameter *b* represented the height of the ellipse and the parameter *p* described the curvature of the hole. This resulted in a two parameter (*b* and *p*) geometric representation of the hole.

The chromosomes represented the parameters of the equation that describes the design boundary. Since two parameters (*b* & *p*) are required for the optimisation, the problem is considered a multi-parameter optimisation. Therefore two chromosomes of length 15 were concatenated to form a chromosome of length 30. Each 15 bit chromosome (l=15) represented a parameter mapped to an un-signed integer ranging between 0 and 2^l $[0,2^l]$, to a specified interval $[S_{min}, S_{max}]$. In this case the first 15 bits of the chromosome represented the *b* parameter ranging from 10 – 30, while the final 15 bits of the chromosome represented the *p* parameter ranging from 2 – 3.

3.1 Stress Optimisation Results

The genetic algorithm developed was initially verified by implementing a stress based optimisation, since the optimal solution was known to be an approximate 2:1 ellipse along the design boundary. The objective function was to minimise the maximum tangential principal stress along the design boundary. Therefore the evaluation function was the maximum tangential principal stress along the design boundary and the fitness function was the normalised value of the evaluation function for each particular population. Eq. (5) describes the fitness function implemented:

$$fitness(i) = 1 - \frac{eval(i)}{eval_{\max}} \tag{5}$$

For i = 1, population size. Where *eval(i)* represents evaluation value for chromosome (parameter set) i and $eval_{max}$ is the maximum evaluation value in the current population.It was found that in this application the genetic parameters and operators indicated in Table 1 was sufficient.

Table 1. Genetic algorithm parameters

Probability of mutation	0.0333
Probability of crossover	0.8
Chromosome length	30
Population size	30
Number of generations	50
Selection Operator	RSSWR
Crossover Operator	Double

Kristensen et. al. [23] calculated that the stress optimised shape of a hole in an infinite plate under a biaxial stress field is an ellipse with an aspect ratio equal to the biaxial stress ratio. Therefore, the total tangential stress = $(1+k)\sigma_1$, where $k = \sigma_2/\sigma_1$. σ_1 represents the stress field in the x-direction and σ_2 represents the stress field in the y-direction. Therefore the two-dimensional stress optimised solution for this case, with σ_1=50MPa and σ_2=100MPa, is a 2:1 ellipse with a constant tangential stress of $\sigma_1 + \sigma_2 = 100 + 50 = 150$MPa along the design hole boundary edge. Table 2 contains the optimal design parameters for the analytical and genetic solution and clearly indicates that the genetic algorithm solution is similar, i.e an approximate 2:1 ellipse. These results are graphically illustrated in Fig. 2, in which the design boundary of the hole in a two-dimensional X-Y plane for the analytical solution and the genetic algorithm solution is shown. Table 2 indicates the maximum tangential principal stress for the 3D analytical and genetic solution and indicates that both produce a similar solution to the 2D analytical solution of 150 MPa. In conclusion, it is clear that the genetic algorithm produced an optimal solution similar to the analytical solutions. These results verify the accuracy of the genetic algorithm in the implementation to finite element engineering structural optimisation problems.

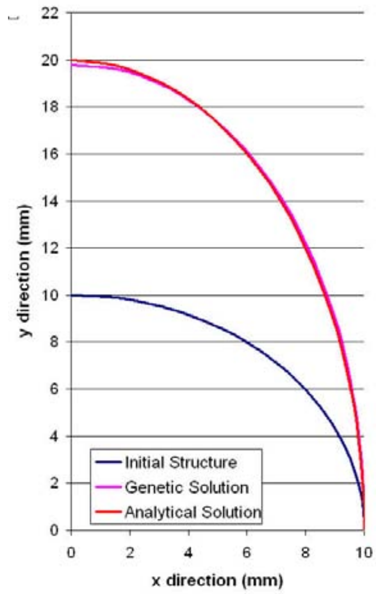

Figure 2. 2-D representation of the analytical and genetic algorithm solution

Table 2. Optimal design parameters and maximum tangential principal stress of the initial structure, analytical and genetic optimised structure

	Initial Structure	3D Analytical Solution	3D Genetic Solution
Hole height (b)	10	20	19.88
Curvature (p)	2	2	2.07
Stress (MPa)	258.13	155.74	156.54

Fig. 3 illustrate the convergence of the design parameters b (hole height) to 19.88 and p (curvature) to 2.05 throughout the generations of the genetic algorithm, respectively. In the same manner, Fig. 4 contains the convergence of maximum tangential principal stress to 156.54 MPa. Each graph contains the maximum, minimum and average values for each generation and indicates the convergence of these values to a near optimum solution as the population becomes homogeneous. The population becomes homogeneous when the standard deviation of the fitness becomes small, or the mean of the fitness approaches the maximum fitness of the population. I.e. when the population has converged, in which the population consists primarily of similar individuals. The graphs indicate a rapid convergence towards the near optimal solution. The rapid convergence is due to the fact that the solution space for this problem is relatively flat with a single peak, therefore once the genetic algorithm locates the peak the convergence occurs quick, thus highlighting the advantage of genetic algorithm. The mutation operator of the genetic algorithm can account for the bumps in the graph. After approximately 40 generations the near optimal solution remains at a steady state.

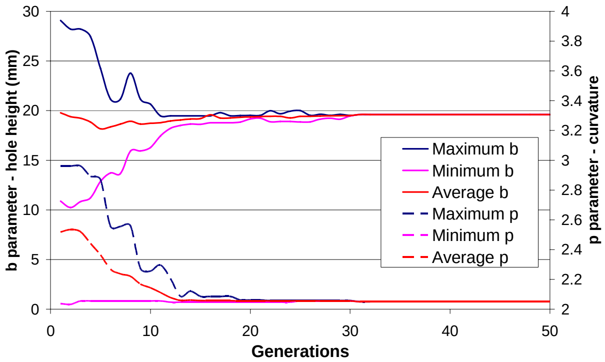

Figure 3. Convergence of design parameter b and p implementing the genetic algorithm

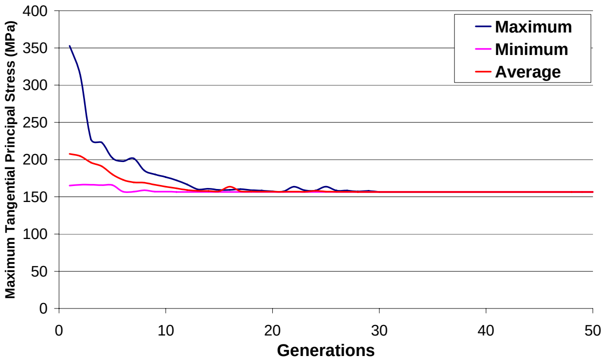

Figure 4. Convergence of the maximum tangential principal stress implementing the genetic algorithm

3.2 Fatigue Optimisation Results

The objective function was to maximise the minimum fatigue life of the cracks along the design boundary. Therefore the fitness function implemented is described by Eq. (23):

$$fitness(i) = 1 - \frac{eval(i)}{eval_{\max}} \quad (6)$$

For i = 1, population size. Where *eval(i)* represents evaluation value for chromosome (parameter set) i and $eval_{max}$ is the maximum evaluation value in the current population. The same genetic algorithm parameters (Table 1) that were used in the stress based optimisation problem (section 6.1.1) was implemented and found to be sufficient.

Molent et al. [12] states that "the multi-scale nature of the Frost-Dugdale hypothesis is evident from the fact that it appears to apply to flaws with sizes that range from 0.01 mm to tens of mm's". Therefore following from Molent's paper, the current research implemented a lower limit of 0.01 mm for the short crack category and an upper limit of 6 mm for the large crack category. The final crack size remained a constant 8 mm in all cases.

Through the implementation of the generalised Frost-Dugdale law, the accurate prediction of the fatigue life is possible for short cracks within the threshold region as the generalised Frost-Dugdale law is applicable in the threshold region, while Newman's law is not. [18, 19] discovered that for the case of a 'through-hole in a plate' problem, when the crack size is short (threshold region) a stress based solution (assuming no cracks) will produce an identical optimised geometry to a fatigue based solution. Ultimately, this solution will prove to be another verification of the accuracy of the genetic algorithm

3.2.1 Short Crack Category

The results indicated in Table 3 illustrate that when the crack was short and within the threshold region the fatigue optimal solution of the design boundary is approximately a 2:1 ellipse, which is similar to the stress based solution. These results are graphically illustrated in Fig. 5. Table 3 indicates the minimum fatigue life for the 3D genetic stress solution and 3D genetic fatigue solution and indicates a 7.07% difference. Table 4 illustrates that the results produced by several different optimisation techniques from [18, 19] produce a similar solution as the genetic algorithm. These results provide further evidence of the accuracy of the genetic algorithm in the application of a fatigue based finite element engineering structural optimisation problems.

Table 3. Comparison of the optimised solution between initial, stress and fatigue optimised structures

	Initial	Stress	G. Frost-Dugdale
b (mm)	10	19.88	21.26
p	2	2.07	2.05
Life (Cycles)	991956	4967890	5345589

Fig. 6 illustrates the convergence of the two design variables, b and p, to a near optimal solution of b = 21.26 mm and p = 2.05. Fig. 7 illustrates the convergence of the fatigue life to the near optimal solution of 5345589 cycles. Each graph contains the maximum, minimum and average values for each generation and indicates the convergence of these values to a near optimum solution as the population becomes homogeneous. The graphs indicate a rapid convergence due to the fact that the solution space for this problem is relatively flat with a single peak. The solution space for this case is graphically illustrated in [18, 19].

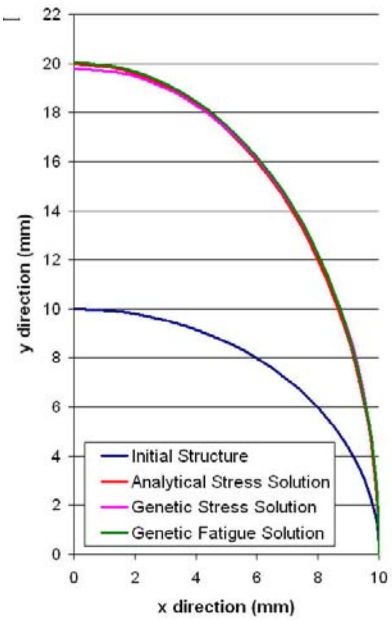

Figure 5. 2-D representation of the initial, stress and fatigue algorithm solution for the short crack category.

Table 4 – Comparison of genetic solution with other optimisation algorithms using the generalised Frost-Dugdale law

	Objective Functions	Design Parameters	
Optimisation Algorithm	Minimum Fatigue Life (Cycles)	b (mm)	p
Gradient-less	3882000	20.65	2.00
Gradient based	5311846	20.71	2.09
Enumeration	4707360	20.70	2.00
Genetic	5345589	21.26	2.05

3.2.2 Large Crack Category

Since the accuracy of the genetic algorithm has been proven to be sufficient for the use in a fatigue based optimisation study, a larger crack comparing two different crack growth laws was investigated. A fatigue based optimisation solution within the common Paris region (i.e. Region II) was

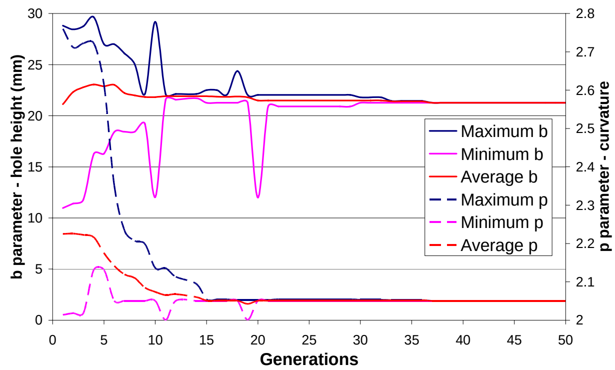

Figure 6. Convergence of design parameter b and p implementing the G. Frost Dugdale

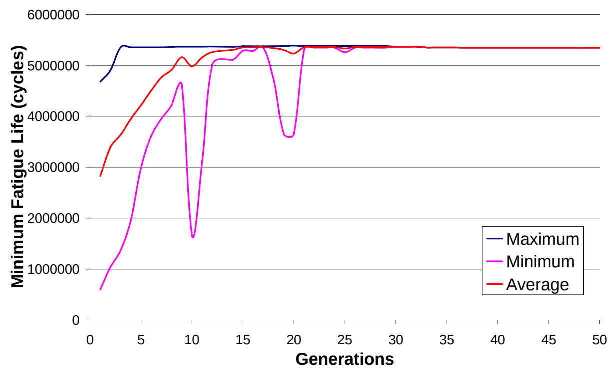

Figure 7. Convergence of the fatigue life implementing G. Frost Dugdale

considered. The common Paris-like law, termed the Newman law was implemented and compared to the solution produced by the generalised Frost-Dugdale law. Table 5 directly compares the optimal results produced by the genetic algorithm between each law and indicates that there is little difference between the solutions. Since Paris like laws have been proven to predict well in Region II of the standard crack growth curve, it can be assumed that the Newman law prediction is accurate. Therefore, the results indicate that when the crack length is large the generalised Frost-Dugdale law produces a similar result to the Newman law. This result is expected as the large crack is within region II of the standard crack growth curve in which the generalised Frost-Dugdale law tends towards the Newman law solution. The near optimal geometry of the design boundary in a 2D X-Y plane for both crack growth laws is graphically illustrated in Fig. 8. Fig. 8 also illustrates the initial and stress optimal solutions and indicates that the fatigue based optimal solutions produces a considerably lighter structure than both the initial and stress based solutions. This fact by itself illustrates the need for fatigue based analysis with an optimisation study. The convergence history for both laws indicates a rapid convergence similar to the stress and short crack results. Table 6 and 7 illustrates that the results produced by several different optimisation techniques [18, 19] and indicate a similar solution as the genetic algorithm. These results provide further evidence of the accuracy of the genetic algorithm in the application of a fatigue based finite element engineering structural optimisation problems.

Table 5 – Comparison between Newman and generalised Frost-Dugdale laws

	Categorised Crack Length		Difference (%)
	Large		
	Newman	G. Frost-Dugdale	
hole height (b)	23.20	23.60	1.72
Curvature (p)	2.08	2.11	1.44
Life (Cycles)	2506	2745	9.54

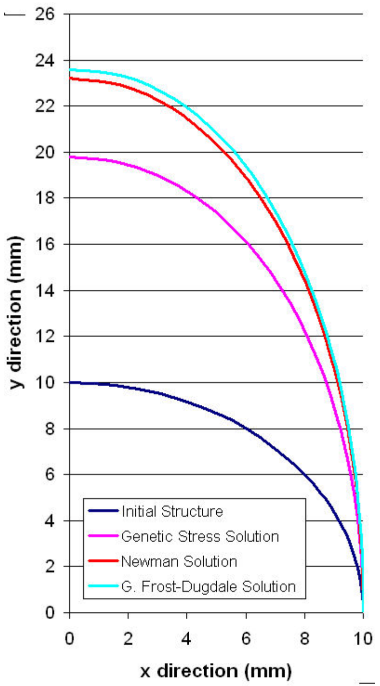

Figure 8. 2-D representation of the Newman and generalised Frost-Dugdale solution

4 Conclusion

The paper has presented and verified a fatigue based optimisation procedure implementing a genetic algorithm developed in FORTRAN for a numerical three-dimensional structural optimisation study of a 7050-T7451 aluminium structure. The 7050-T7451 aluminium structure analysed was the simple benchmark problem of a 'through-hole in a rectangular block under biaxial loading', in which the hole geometry was the design boundary. The genetic algorithm implementing a stress based

Table 6– Comparison of genetic solution with other optimisation algorithms using G. Frost-Dugdale law.

	Objective Functions	Design Parameters	
Optimisation Algorithm	Minimum Fatigue Life (Cycles)	b (mm)	p
Gradient-less	2451	23.90	2.00
Gradient based	2756	23.59	2.11
Genetic	2745	23.60	2.11

Table 7 – Comparison of genetic solution with other optimisation algorithms using the Newman law.

	Objective Functions	Design Parameters	
Optimisation Algorithm	Minimum Fatigue Life (Cycles)	b (mm)	p
Gradient-less	2271	23.78	2.00
Gradient based	2411	22.87	2.09
Genetic	2506	23.20	2.08

solution produced similar results to the analytical optimal stress solution. For the fatigue optimised solution involving short cracks, the genetic solution produced was similar to the stress based solution which was expected. This is due to the fact that if the crack size is very small in comparison to the length scale associated with the cut out, the local rework or the structural detail being optimised, then the stress intensity factors will be directly related to the tangential stress around the cut out. The implementation of the generalised Frost-Dugdale law was required due to the fact that Paris-like laws were not applicable in any other region other than region II. These two solutions provided the necessary validation of the developed genetic algorithm.

A fatigue based optimisation study with large cracks compared solutions between a Paris like law (Newman's Law) and the generalised Frost-Dugdale law. In general it should be stressed that solutions obtained are best thought of as "better", or local optima, rather than true optimal solutions. The results indicated that the optimised geometry and predicted fatigue lives were similar, due to the fact that the crack size was within Region II. As with the other cases convergence to a near optimal solution was quick due to the flat solution space. The results generated from the genetic algorithm for the short and large crack case category produced similar results as the gradient-less, gradient based and enumeration algorithms, thus verifying the solutions. These findings offer the potential for the design of light weight structures, in which a fatigue based optimisation implementing a genetic algorithm provide a more robust methodology. Furthermore, it has the potential to be applied to structures with complex structural configurations with multiple optimum peaks taking into account crack propagation in the near-threshold

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