Long-run structural modelling of the Czech macroeconomy

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Abstract: The aim of this paper is to modify and empirically verify the modelling strategy developed by Garratt, Lee, Pesaran and Shin (2006). The strategy provides a practical approach to incorporating theoretic long-run relationships of a small open economy through a structural vector error correction model (VECM). The basic macroeconomic framework is a core small open economy model consisting of five long-run relationships based on production technology and output determination, arbitrage conditions, long-run solvency requirements and accounting identities and stock-flow relations. This leads to five expected long-run equations: the relative purchasing power parity (PPP), the real money market equilibrium condition (MD), the output gap (OG), the interest rate parity (IRP) and the interest rate relationship – Fisher inflation parity (FIP). We apply this modelling strategy to the Czech economy. The data are quarterly and run from the first quarter 1999 to the fourth quarter of 2010. The domestic variables are real money stock, real gross domestic product, the nominal interest rate, the rate of inflation and the domestic price level. Further endogenous variables are the nominal exchange rate, the foreign price level, foreign real GDP, the foreign interest rate. We are able to identify the long-run structure amongst those variables and to test over-identifying restrictions on the cointegrating vectors. We analyse the consequences of imposing the long-run restrictions for the impulse response function, where we focus on a Czech monetary policy shock. This shock has a significant effect on domestic inflation and real domestic output.

Key-Words: Long-run structural model, Czech economy, Small open economy, Purchasing power parity, Interest rate parity, Output relationship, Money market, Fisher inflation parity, Impulse, Response Functions.

1 Introduction

At present, there are two elementary methodological approaches to the modern macroeconometric modelling of economies – Dynamic Stochastic General Equilibrium models (DSGE) and Cointegrated Vector Autoregression models (CVAR).

DSGE models follow and develop two significant waves of thinking - models of Real Business Cycle (RBC) and new Keynesian models. DSGE models continue in quantitative economics with a strong theoretical background despite the effort to enrich them by standard statistical testing processes. These models are used by creators of macroeconomic policy mainly as a supportive tool for the analysis of the economic policy impacts on the economy in question.

The other option of macroeconometric modelling is a use of vector autoregression (VAR) and mainly structural VAR. Models CVAR follow up modelling of a complex dynamic system the features of which respect real data. CVAR models began to appear at the end of eighties and at the beginning of nineties of the last century and were orientated on the U. S. economy mainly. In the article [14] King and others dealt with the cointegrating relations between consumption, investment, output and nominal interest rates and real monetary balance. Restrictions were the implications of RBC, Fisher equation theory and monetary demand equation.

Cointegrating ideas as introduced by Granger are commonly embodied in empirical macroeconomic modelling through the vector error correction model (VECM). The VECM representation of a dynamic system is obtained as a simple rearrangement of the vector autoregressive (VAR) model advocated by Sims [24], once the variables in the VAR are cointegrated.

Sims had argued that structural identification of the then-existing simultaneous-equation of the macroeconometric models was incredible, and he proposed the alternative strategy of estimating the unrestricted reduced form, treating all variables as endogenous. It has become common practice to treat some variables as weakly exogenous in cointegrated VAR models. Also according to policy analysis VAR models still require identifying assumptions and they can be formulated structural as VAR (SVAR) models. In a similar way the identification of multiple cointegrating relationships by restrictions draw from economic theory, leaving the short-run dynamics and stochastic specification unrestricted, is called "long-run structural modelling" by Pesaran and Shin [18]. This approach is applied in the construction of a small quarterly model of the UK economy by Garratt, Lee, Pesaran and Shin [7].

This paper presents such a study, in the context of a structural vector error correction model (VECM) for the Czech economy with the quarterly data from the first quarter 1999 to the fourth quarter of 2010.

The remainder of the paper is organised as follows. Section 2 briefly summarizes a framework for long-run macromodelling and the econometric formulation of the core model. Section 3 reviews the formalities of the VAR and VECM modelling framework. Section 4 estimates VECM of the Czech economy. First we estimate an unrestricted VAR model, choosing the order of the VAR model and checking its diagnostic properties. Next we carry out cointegration tests, estimate an exactly identified set of long-run relations and consider the imposition of over-identifying restrictions on the cointegrating vectors. We also turn to an analysis of impulse response functions to a Czech monetary policy shock. Section 5 presents a summary of the empirical results and we make concluding remarks and recommendations.

2 The long-run structural modelling strategy

The long-run structural modelling strategy we follow was suggested by Garratt, Lee, Pesaran and Shin [7] who applied it to the UK. Other modifications were made for Germany [23] by Schneider, Chen and Frohn and also, for Switzerland [2] by Assenmacher-Wesche and Pesaran. The first study on the Czech economy was reported in [8].

The basic macroeconomic framework is a core small open economy model consisting of five long-run relationships. This five long-run equilibrium relations are derived from the economic theory and thus having a structural feature. When deriving these relations the authors took into account neoclassical production function, arbitrage conditions, solvency conditions and conditions of portfolio equilibrium. The basic model includes six domestic variables and three foreign variables. Domestic variables comprise: P_t - the domestic producer prices (manufacturing), PR_t - harmonised consumer prices, E_t - bilateral exchange rate CZK/EUR, Y_t - real per capita GDP (EUR), R_t - nominal interest rate as interbank 1Y middle rate, M_t - real per capita domestic monetary aggregate $M2$. Foreign variables for EU25 involve: YS_t - real per capita GDP, RS_t - 1Y interest rate as Euribor 1Y- offered rate, PS_t - producer prices.

Economic theory yields five long-run conditions of equilibrium equation (1) is the log-linear version of relative purchasing power parity (*PPP*), the money market equilibrium condition (*MD*) is equation (2), the output gap relations (*OG*) is represented by equation (3), and follows uncovered interest rate parity (*IRP*) in equation (4) and finally Fisher inflation parity (*FIP*) is described in equation (5).

$$PPP: \quad p_t - ps_t - e_t = b_{10} + b_{11}t + \xi_{1,t+1} \tag{1}$$

$$MD: \quad my_t = b_{20} + b_{21}t + \beta_{22} \cdot r_t + \beta_{24} \cdot y_t + \xi_{2,t+1} \tag{2}$$

$$OG: \quad y_t - ys_t = b_{30} + b_{31}t + \xi_{3,t+1} \tag{3}$$

$$IRP: \quad r_t - rs_t = b_{40} + b_{41}t + \xi_{4,t+1} \tag{4}$$

$$FIP: \quad r_t - dpr_t = b_{50} + b_{51} + \xi_{5,t+1} \tag{5}$$

where

$p_t = \ln(P_t), ps_t = \ln(PS_t), pr_t = \ln(PR_t), dpr = \Delta pr_t,$

$e_t = \ln(E_t), y_t = \ln(Y_t), ys_t = \ln(YS_t), my_t = \ln(M_t / Y_t),$

$r_t = \ln(1+(R_t/100)), rs_t = \ln(1+(RS_t/100)).$

The *PPP, IRP* and *FIP* relations are commonly from the arbitrage conditions. Since 1989 the Czech economy has undergone the transition from the communist regime economic reforms and has been opening and integrating to the European economy. The transformation of the Czech economy should have an impact on productivity and steady state output. Using the economic growth theory (Barro and Sala-i-Martin [4]) we assume that Czech economy has been in a transition state and is converging to the European Union economy and therefore we assume a trend in the output gap relation [4].

As Garratt, Lee, Pesaran and Shin [7] claim, the primary explanation of long-run deviations from relative *PPP* is the Harrod-Balassa-Samuelson (HBS) effect in which the price of basket of traded and non-traded goods rises more rapidly in countries with relatively rapid productivity growth in the traded goods sector. Since we assume that Czech productivity has been generally increasing we capture the HBS effect by assuming a trend in the *PPP* relation (1). We also expect a trend in the money demand relation (2) to capture the possible effect of the changing nature of financial intermediation, and the increasing use of credit cards in settlement of transactions. Finally, successful reforms could decrease the risk premium, and therefore we assume a trend in the *IRP* relation (4).

We do not assume (as Garratt, Lee, Pesaran and Shin [7]) that oil price has a significant impact on the difference between the Czech and foreign long-run economic performance as the Czech economy is a negligible oil producer.

In the spatial panel macroeconomic models a spatial interaction for cross-classified data is a frequent problem. Papalia derives a Generalized Maximum Entropy in his article [17]. The estimator has the advantage of being consistent with the restrictions implied by past empirical evidence by controlling also collinearity and endogeneity problems.

3 Specifying the VARX and VEC model

In this part of the paper we summarise current methodological approaches to the econometric modelling which will be used for the structural macroeconomic modelling in the long-run.

3.1 The unrestricted VARX model

The long-run structural VAR model belongs to the group of VAR (*vector autoregression*) models which are based on generalisation of one-dimensional autoregressive processes. Methodology of the vector autoregression is described in a detail in the book by Helmuth Lütkepohl [15] or Katarina Juselius [13]. A richer structure than for the one-dimensional autoregressive processes is an advantage of VAR models. The structural shape of a general VARX model which can include endogenous and eXogenous variables (indication *X*) has the following form:

$$Az_t = \sum_{i=1}^{p} A_i z_{t-i} + \sum_{j=0}^{s} B_j x_{t-j} + Dd_t + \varepsilon_t, \qquad (6)$$

where $z_t = (z_{1t},\ldots,z_{mt})'$ is the (*m x 1*) vector of endogenous variables, $x_t = (x_{1t},\ldots,x_{kt})'$ is the (*k x 1*) vector of exogenous variables, $d_t = (d_{1t},\ldots,d_{qt})'$ is the (*q x 1*) vector of deterministic variables (e.g. intercept, trend, seasonal variables, dummy variables) and $\varepsilon_t = (\varepsilon_{1t},\ldots,\varepsilon_{mt})'$ is (*m x 1*) vector of serially non-correlated random components independent on x_t with a null medium value and constant positive definite covariance matrix $\Omega = (\omega_{ij})$. A dynamic system (6) is stable for given vector values x_t a d_t, if a root of a determination equation

$$A - A_1\lambda - A_2\lambda^2 - \ldots - A_p\lambda^p = 0 \qquad (7)$$

lies outside of a unit circle in a complex plane. This stability condition ensures an existence of long-term relations between z_t and x_t which can be cointegrating if one or a few exogenous variables are integrated. A prerequisite does not exclude a possibility that endogenous variables can be mutually cointegrated if the model does not include endogenous variables.

It is possible to present the reduced shape of the VARX model following from equation (6) in a such way that a variable z_i would only have a non-delayed shape in *i-th* equation:

$$z_t = \sum_{i=1}^{p} \Phi_i z_{t-i} + \sum_{j=0}^{s} \Psi_j z_{t-j} + \Xi d_t + u_t, \qquad (8)$$

where $\Phi_i = A^{-1}A_i, \Psi = B^{-1}B_i, \Xi = A^{-1}D, u_t = A^{-1}\varepsilon_t$ are i.i.d. (independent and identically distributed) $(0, \Sigma)$, where $\Sigma = A^{-1}\Omega A'^{-1} = (\sigma_{ij})$.

The classic identification problem is how to gain parameters of the structural form $(A, A_1, ..., A_p, B_0, ..., B_s\ D, \Omega)$ based on parameters of a reduced form $(\Phi_1, ..., \Phi_p, \Psi_0, ..., \Psi_s, B_i, i = 0, 1, \ldots, p, \Xi, \Sigma)$. A strict identification of structural parameters requires m^2 of restrictions to be set a priori. Majority of macroeconomic models is non-identified with regard to the number of restrictions. Including the above mentioned restrictions it is convenient to arrange the variables in the VARX model in a way that matrix *A* is a triangle one and matrix Ω was a diagonal one and the structural system will become recursive causal series and it is possible to estimate a model by a least squares method. This pre-requisition is also closely linked to the use of Cholesky matrix decomposition Σ for the identification of response functions.

When we apply the VARX model it is as follows:

- setting the number and list of endogenous variables, exogenous variables and deterministic variables,

- setting rules of delay for endogenous p and exogenous variables s,
- testing of integrating rules of individual variables,
- diagnostic tests of the VARX model.

For this treatment it is necessary to combine properly an economic theory with results of statistical verification.

3.2 The VECM representation

In this part of the paper we will focus on cointegration analysis for VAR(p) model without exogenous variables (e.g. $s=0$ in equation (6)). We will also presume that endogenous variables are integrated $I(1)$. Such VAR(p) model can be further specified by following equation:

$$z_t = \Phi_1 z_{t-1} + \ldots + \Phi_p z_{t-p} + d_0 + d_t t + u_t, \qquad (9)$$

where d_0 and d_1 are m-dimensional vectors of unknown coefficients. An adequate condition of stationarity and linearity of the VAR(p) model process is

$$\left| I_m - \Phi_1\lambda - \Phi_2\lambda^2 - \ldots - \Phi_p\lambda^p \right| = 0, \qquad (10)$$

e.g. all roots of autoregression matrix polynom (10) lie outside or on a unit circle at a complex plane (e.g. all inversion roots of this polynom lie on or inside of the unit circle). Non-stationarity of time series can be excluded by differentiating.

Provided that all endogenous variables are $I(1)$ it is possible to express the VAR(p) model from the equation (9) in a form of the vector error correction model:

$$\Delta z_t = -\Pi z_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + d_0 + d_t t + u_t, \qquad (11)$$

where

$$\Pi = I_m - \sum_{i=1}^{p} \Phi_i, \quad \Gamma_i = -\sum_{j=i+1}^{p} \Phi_j, \quad i = 1, \ldots, p-1. \qquad (12)$$

If endogenous variables are $I(1)$ and they are cointegrated with matrix value Π between $0 < h(\Pi) = r < m$, then $\Pi = \alpha\beta'$, where α and β are the matrix of dimension (m x r) with value r and there are $r < m$ of linear combinations of endogenous variables. Provided a cointegrating relation the equation (11) is expressed as follows:

$$\Delta z_t = -\alpha\beta' z_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + d_0 + d_t t + u_t, \qquad (13)$$

where matrix α contains *adjustment coefficients* (e.g. coefficients of dissimilarity rate of short-term relations and long-term ones, it means how fast a short-term deviation from equilibrium is adjusted or an enforcement of equilibrium stable relation). The elements of matrix β are *coefficients of long-term equilibrium relation* and there is one cointegrating vector corresponding to the each column of this matrix. *Cointegrating equations* (*CE)* are expressed now as equation $\xi_t = \beta' \cdot z_{t-1}$. ξ_t, it is stationary and includes feedback in the system. If there are r of cointegrating relations where $0 < r < m$, then some elements of matrix α must be non-null e.g. Granger causality exists at endogenous variable levels.

An unrestricted estimation Π can be derived from the equation (11). If we use model (11) with restrictions including r cointegrating vectors, we must estimate two matrixes of coefficients α and β. A suitability of matrix Π is reduced later as it includes (m^2-$2mr$) restrictions. If r independent restrictions will be demanded for each cointegrating relation and r^2 of further restrictions then in a restricted model we include $(m^2-2mr)+r^2=(m-r)^2$ restrictions e.g (m^2-$2mr$) restrictions on Π and r^2 strictly identified restrictions.

3.3 Cointegration analysis methods

Cointegration analysis represents a testing of cointegrating relations (CE) of the selected VAR(p) model and setting of statistically convenient number of long-term equilibrium relations r.

Engle-Granger two-phase method is used for a testing of cointegrating relations (see Engle and Granger, [5]) and *Johansen method* published by Johansen [11] and [12] is used. Engle-Granger method can be used in a case of the existence of one cointegrating vector and therefore we will not pay further attention to this approach. Johansen cointegration method is suitable even for the existence of a few cointegrating relations and it is based on the method of maximum plausibility which enables a simultaneous estimation of the investigated system parameters from equation (11) provided the restriction of matrix Π and provided that the residues have a Gaussian distribution. An algorithm includes two basic steps:

In the first phase the rules of integration of the investigated variables are set. We can use three well-known tests:

- *Augmented Dickey-Fuller test* (ADF test) which uses the following hypotheses (14) where ν_t is an independent white interference for detrend time series y_t .

$$\begin{aligned} &H_0: \Delta y_t = \psi y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + \nu_t \ \ for \ \ \psi = 0, \\ &H_A: \Delta y_t = \theta + \vartheta t + \psi y_{t-1} + \nu_t \ \ for \ \psi < 0. \end{aligned} \qquad (14)$$

- *Phillips-Perron test* (PP test) which does not take into account autocorrelation of residues by expansion of autoregression members as ADF test but it corrects the estimated deviations directly.
- *KPSS test* is a test by authors Kwaitkovski, Phillips, Schmidt and Shin from 1992 which reacts on a weak distinguishing ability of ADF tests. In this test the null and alternative hypotheses are proposed totally vice versa comparing to the ADF test.

Provided that it is possible to make all variables stationary in vector z_t after one differentiating then it is possible to write down the model (9) in a form of a vector model with a correction member in the equation (13). In the next phase we determine a proper number of delays in the VAR model, for example based on information criteria with a preference of Schwarz information criterion (SIC).

The main attention is devoted to the matrix Π, which contains information concerning long-term relations. We calculate *m* own numbers and look for maximum of plausibility function provided the existence of *r* cointegrating relations $\left(h(\Pi)=r, \text{where } 0<r<m\right)$, e.g. linear combinations of variables which are stationary. Testing of a null hypothesis with the existence of *r* cointegrating relations:

$$H_0 : h(\Pi) = r \tag{15}$$

is performed by two types of statistics:

- trace statistics (λ_{trace}) and
- maximum eigenvalue statistics ($\lambda_{\max}$)

at the selected level of significance. These statistics can be calculated:

$$\begin{aligned} \lambda_{trace}(r) &= -T\sum_{i=r+1}^{m}\ln(1-\lambda_i), \\ \lambda_{\max}(r,r+1) &= -T\ln(1-\lambda_{r+1}), \end{aligned} \tag{16}$$

where *T* is a number of usable observations and $\lambda_1 > \lambda_2 > \cdots \lambda_m$ are sorted own numbers of matrix Π. Both mentioned statistics have χ^2 of distributions with $(m-r)$ number of the latitude grades.

The alternative hypotheses can be formulated:

$$\begin{aligned} H_{A\_trace} &: h(\Pi) = m, \\ H_{A\_\max} &: h(\Pi) = r+1. \end{aligned} \tag{17}$$

A choice of suitable model form, e.g. inclusion or exclusion of deterministic components (mainly a constant or trend), plays an important role. We estimate the models for *r=0* to *r = m–1*. The choice of the model form is done according to the statistics comparison $\hat{\lambda}_{trace}$ and $\hat{\lambda}_{\max}$ with critical values of MacKinnon-Haug-Michelis or Osterwald-Lenun (see Asteriou and Hall [3] and EViews 7 User's Guide[20], [21]).

A weak exogenity is tested based on a number of given cointegrating relations and hypotheses with restrictions and coefficients of matrix elements α and β are tested in a such way that cointegrating relations for model estimation would be in compliance with economic and statistical theory.

4 Estimation and testing of the VECM of the Czech economy

The VECM for our Czech economy can be written as follows:

$$\Delta z_t = -\alpha\beta' z_{t-1} + \sum_{i=1}^{p-1}\Gamma_i \Delta z_{t-i} + d_0 + d_t t + u_t, \tag{12}$$

where $z_t = (e_t, r_t, dpr_t, y_t, pps_t, my_t, rs_t, ys_t)'$ and $pps_t = ps_t - p_t$.

$\beta' \cdot z_{t-1}$ are the error correction terms; α is a matrix of error-correction coefficients; Γ_i are matrices of short-run coefficients and u_t is a vector of disturbances assumed to be i.i.d.$(0,\Sigma)$, with Σ being a positive definite matrix.

4.1 Data analysis

The data used in the applied study are quarterly, seasonally adjusted series covering the period 1999Q1-2010Q4 for the Czech economy. Databases of Reuters, Eurostat, Czech National Bank and OECD have been used as a data source. Software EViews version 7.1. has been used for following data analysis and modelling.

Plots of the core macroeconomic series (e_t, r_t, dpr_t, y_t, pps_t, my_t, rs_t, ys_t) are provided in Figures 1-4.

Figure 1 presents a logarithm development of domestic (*p*) and foreign (*ps*) industrial producer prices and exchange rate CZK/EUR (*e*). The prices of industrial producers increase in the monitored period and these time series will be non-stationary. On the other hand exchange rate CZK/EUR that expresses a relative competitiveness of the Czech economy in a long-run pursues a trend of strengthening of the Czech economy excluding an establishment of the exchange rate bubble in 2002 as a result of the economic transformation completion and long-term faster productivity growth in comparison with foreign countries. This bubble burst in 2004. The second exchange rate bubble emerged in February 2009 as a result of global financial and economic crisis.

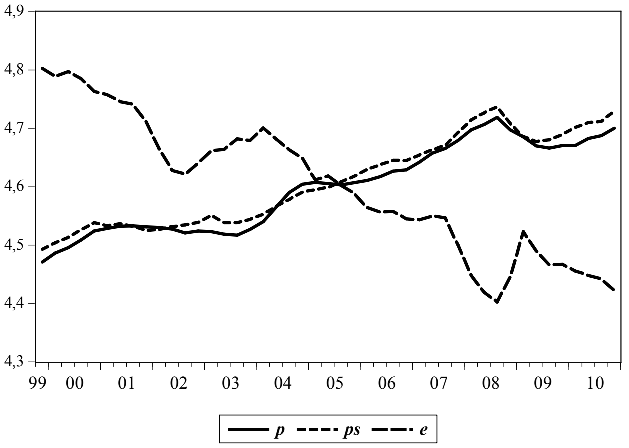

Figure 1 Czech and EU25 producer prices and exchange rate.

Fig. 2 presents Czech money income ratio (*my*) logarithm development and domestic interest rate (*r*) with a scale on left axis side. After 2001 there is a relatively stable period in the development of money income ratio untill 2007 and a sharp growth has been realised.

Figure 2 Czech money income ratio and domestic interest rate.

The logarithm development of real domestic and foreign output per capita (see figure 3) proves an increasing trend that was disrupted by financial crisis in 2008 that passed over to the real economy and secondary impacts on banking sector and a threat of banking sector stability followed. This impact was more intensive in EU25 than in the Czech Republic.

The last figure 4 shows the logarithm development of domestic and foreign interest rate and domestic consumer price inflation. At the beginning of the monitored period domestic interest rate (*r*) was influenced by monetary and political crisis. It is followed by a decrease of domestic and foreign interest rate (*rs*) and relative stabilisation in the long-run. From middle of 2005 to 2008 the interest rate slightly increases again. The growth is lower in the Czech Republic and as a consequence of the global financial crisis a fall appears which is more substantial for the eurozone. An inflation development (*dpr*) shows a slight variability and proves that Czech economy belongs to the low-inflation economies.

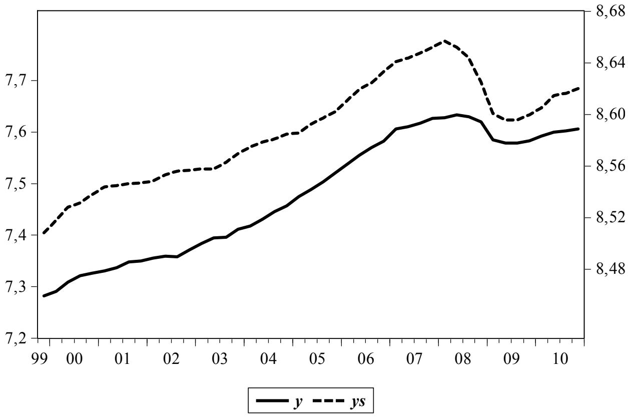

Figure 3 Czech and EU25 output.

Figure 4 Czech and EU25 interest rate and domestic inflation.

In the first part of the data analysis we pre-test the variables for unit roots using ADF, PP and KPSS tests. We can conclude that mainly all the endogenous variables are I(1) at the 5% level of significance.

4.2 Estimation and testing the VAR model

The next step of the modelling sequence is to select the order of the underlying unrestricted VAR in nine endogenous variables. We have applied the Wald test (see Table 1) and also the LR test with SC criterion (see Table 2).

Chi-squared test statistics for lag exclusion:									
Lag	***e***	***r***	***dpr***	***y***	***pps***	***my***	***rs***	***ys***	***joint***
1	$30,1^{***}$	8,7	$24,0^{***}$	$49,1^{***}$	$29,7^{***}$	6,3	$51,1^{***}$	$29,7^{***}$	$\mathbf{581,5}^{***}$
2	$24,2^{***}$	7,1	$17,1^{**}$	$14,0^{*}$	$19,9^{**}$	7,6	$20,1^{**}$	$17,8^{**}$	$\mathbf{399,0}^{***}$
3	13,3	6,9	$26,8^{***}$	4,9	$19, 5^{**}$	10,6	$17,1^{**}$	9,6	$\mathbf{288,2}^{***}$
4	9,3	9,1	$20,2^{***}$	$15,6^{**}$	$18,6^{**}$	8,5	11,9	8,1	$\mathbf{208,4}^{***}$

Table 1 VAR Lag Exclusion Wald Tests
***; **; * indicates significance level (1%; 5%; 10%)

We selected VAR(2) by reason of the length of the time series and also for comparison of our results with other studies and we prefer over-estimation rather than under-estimation and four lags are too many for reason of loss of degrees of freedom.

Lag	**SC**
0	-43,93
1	**-56,71***
2	-55,64
3	-55,49
4	-56,04

Table 2 Schwarz information criterion (SC)
* indicates lag order selected by the criterion

4.3 Estimation of the long-run relations

Next, having established the appropriate order of the VAR model, Johansen's cointegration tests are carried out using the trace and the maximum eigenvalue statistics. According to unit root tests and also graphs we assume an intercept with trend in the cointegrating relationship and a linear data trend. In this case we can indicate five cointegrating equations at the 5% level of significance, especially by the trace statistics (see Table 3).

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized		**Trace**	**0.05**	
No. of CE(s)	**Eigenvalue**	**Statistic**	**Critical Value**	**Prob.****
None *	0,856763	307,7322	187,4701	0,0000
At most 1 *	0,694798	214,4559	150,5585	0,0000
At most 2 *	0,636055	157,4905	117,7082	0,0000
At most 3 *	0,587663	108,9743	88,80380	0,0008
At most 4 *	0,408238	66,45050	63,87610	0,0299
At most 5	0,346219	41,26728	42,91525	0,0724
At most 6	0,225479	20,86808	25,87211	0,1851
At most 7	0,164095	8,603551	12,51798	0,2062
Trace test indicates 5 cointegrating eqn(s) at the 0,05 level				
* denotes rejection of the hypothesis at the 0,05 level				
**MacKinnon-Haug-Michelis (1999) p-values				

Table 3 Johansen's trace statistics for the estimated VAR(2) model
Note: The asterisks indicate the corresponding null hypothesis is rejected at the 5 % significance level.

The trace statistics reject the null hypotheses that the number of cointegrating relations is equal 0, 1, 2, 3 and 4 at the 5 % significance level, but cannot reject the null hypothesis that the number of cointegrating relations is equal 5. With five cointegrating vectors we could impose 25 restrictions to fully identify model – 5 restrictions on each relation.

The next two stages of the modelling sequence are to estimate an exactly indentified set of long-run relations and to test the over-identifying restrictions on the cointegrating vectors.

The matrix β'_{LONG} (13) can be used to impose all the restrictions necessary for the structural long-run relationship but it can be over-identified.

$$\beta'_{LONG} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (13)$$

However, the first step in the estimation is to exactly identify the long-run. We impose *25 exactly* identifying restrictions on cointegrating matrix β'_{EXACT} (14).

$$\beta'_{EXACT} = \begin{pmatrix} \beta_{11} & 0 & \beta_{13} & 0 & -1 & \beta_{16} & 0 & 0 \\ 0 & \beta_{22} & \beta_{23} & \beta_{24} & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & \beta_{35} & \beta_{36} & 0 & \beta_{38} \\ 0 & -1 & \beta_{43} & 0 & 0 & 0 & \beta_{47} & \beta_{48} \\ 0 & \beta_{52} & 1 & 0 & 0 & \beta_{56} & 0 & \beta_{58} \end{pmatrix} \quad (14)$$

The final matrix $\beta'_{FINAL} = \beta'_{LONG}$ is with 38 restrictions. Estimation of the parameters of the model (1)-(5) was carried out using the long-run structural modelling approach and it is based on a modified version of Johansen's estimation of our VEC model.

The estimated long-run relationships for the Czech economy, incorporating all the restrictions suggested by the theory, with the trend restrictions suggested above, and with *t*-statistics in [], are:

$$PPP: p_t - ps_t - e_t = -4,92 + \underset{[18,3]}{0,007}\, t + \xi_{1,t+1} \quad (15)$$

$$MD: my_t = 22,4 - \underset{[-6,56]}{0,023} t - \underset{[-19,7]}{7,02} \cdot r_t + \underset{[28,7]}{3,29} \cdot y_t + \xi_{2,t+1} \quad (16)$$

$$OG: y_t - ys_t = -1,31 + \underset{[8,5]}{0,0048} \cdot t + \xi_{3,t+1} \quad (17)$$

$$IRP: r_t - rs_t = 0,0043 + \underset{[0,021]}{0,000007} \cdot t + \xi_{4,t+1} \quad (18)$$

$$FIP: r_t - dpr_t = 0,064 - \underset{[-5,13]}{0,00085} \cdot t + \xi_{5,t+1} \quad (19)$$

Equation (15) describes PPP relations and does not reject the context for the core model. The convergence of the Czech economy becomes evident by the average quarterly decrease in the real exchange rate by about 0,007%.

Money market equilibrium is presented by equation (16). It turns out that the income elasticity of money demand is positive. We also could not reject the hypothesis of the negative elasticity of real money balances with respect to domestic interest rate. The quarterly long-run rate of decrease in money stock per capita and per capita Czech output is about 0,023% per quarter.

The third long-run output relationship, given by (17), describes the average long-run growth rate in the Czech and EU25 economies. We estimate the increase of the output gap (the Czech and EU25 per capita output ratio) at about 0,005% per quarter.

Next, equation (18) includes the interest rate parity condition. This includes the intercept, which can be interpreted as the deterministic component of the risk premium associated with bond and foreign exchange uncertainties. Its value is estimated at 0,0043 implying a risk premium of approximately 0,43% per annum. It is also noteworthy that co-trending hypothesis cannot be rejected i.e. the coefficient of the deterministic trend in the interest rate parity equation is zero. We cannot conclude there is a decrease in the long-run Czech risk premium (at 5% level of significance). Some articles e.g. Radulescu, Popescu and Matei [22] present an investigation possibility of conceptual aspects of shadow economy by using a real interest rate.

Finally, the fifth equation (19) defines the FIP relationship, where the constant implies the average long-run Czech real interest rate is about 6,6%. The FIP condition contains a deterministic downward trend representing the steady decline in the real interest rate about 0,00085% per quarter.

The interpretation of the short-run evaluation of variables in the model can be investigated in response to deviations from equilibrium and also describing past changes in the variables of the model. The estimated α loading coefficients show that inflation represents a statistically significant factor by the long-run adjustment mechanism.

The results also confirm satisfactory diagnostic statistics of the estimated VECM(1). The assumption of normally distributed errors is tested by the chi-square distributed Jarque-Bera statistics. This assumption is rejected only in the equation of the EU25 output with respect to skewness. The LM tests are employed as a check of the residual serial correlation. We cannot reject the hypothesis of no serial autocorrelation of the first order in all equations at the 5% level of significance. The presence of autoregressive conditional heteroscedasticity is rejected in all equations.

We can also detect a relatively high level of explanatory power for each evaluation – with R^2 in the range < 0,34; 0,75>, averaging 0,62.

The over-identifying restrictions are tested by the log-likelihood ratio statistic which takes the value 61,08. The test statistic is asymptotically distributed as a χ^2 variate with 13 degrees of freedom. We don't make the conclusion directly, because the works by Haug [9] and Abadir et al [1] shown that the asymptotic critical values may not be valid for vector autoregressive models with a relatively large number of variables, unless samples are sufficiently large, which is just our case.

That's why we decided to implement the significance test of the log-likelihood ratio statistics using critical values which are computed by non-parametric bootstrap techniques with 10000 replications. For each replication, an artificial data set of endogenous variables is created by re-sampling with replacement of residuals computed from initial estimation. The test is carried out on each of the replicated data sets and the distribution of the statistics is derived across all replications. This shows that the relevant critical values for the test statistics are 66,45 at the 5 % significance level. We cannot therefore reject the over-identifying restrictions implied by the theory. There are further new methods of Weighted Bootstrap with probability (WPM) investigated in a paper by Norazan, Midi and Imon [16] who empirically try to find out that WPM method is more efficient than the classical bootstrap or diagnostic-before boostrap estimates. Also the authors of another paper Fitrianto and Midi [6] indicate a possibility of applicability of an alternative approach to use a new bootstrap procedure of indirect effect in mediation model which is resistant to outliers. They propose Rescaled Studentized Resiudal Bootstrap using Least Squares (ReSRB). The Monte Carlo simulations showed that the ReSRB is more efficient in the presence of outliers.

4.4 An impulse response analysis

In this part of the paper we focus on the estimates of impulse response function of a monetary policy shock on the selected endogenous variables with regard to a regime of an inflation targeting executed by the Czech National Bank since 1998.

An impulse response function traces the effect of a one-time shock to one of the innovations on current and futures values of the endogenous variables. If the innovations are contemporaneously uncorrelated, interpretation of the impulse response is straightforward. Innovations, however, are usually correlated and we may use Cholesky transformation to orthogonalize the impulses with correction of degrees of freedom. We carry out this impulse response analysis by reordering of the variables in the VAR as follows – *ys, rs, pps, y, my, r, dpr, e.*

The macroeconomic analyses of the effects of these shocks have been of a special interest and help to provide further insights into the short-run dynamic properties of our model. We shall also consider the time profile of the shock effects.

The impulse response functions for the effects of the domestic monetary shock on the selected endogenous variables (*dpr, y* and *rs*) are given in Figure 5 (graphs 1-4). We conducted estimations of the Cholesky impulse responses for twenty quarters ahead. The analysis of four graphs on Figure 5 follows.

The first graph provides an evidence that an idiosyncratic interest rate increase leads to the inflation decrease after about the first quarter up to negative values with a minimum after the second quarter (see the first graph). The increase of domestic interest rate heads also towards the positive real interest differential in the eurozone (see the third graph) and this is followed by a very sharp appreciation of the real monetary exchange rate up to the original level. This fact is demonstrated by the fourth graph on Figure 5. The real interest rate increase and sharp depreciation of the Czech crown results in a gap decrease = gap between domestic and foreign economy output. It is evident from the second graph on the followed figure. The decrease of domestic and foreign demand for Czech products also reduces an external competitiveness of the Czech economy.

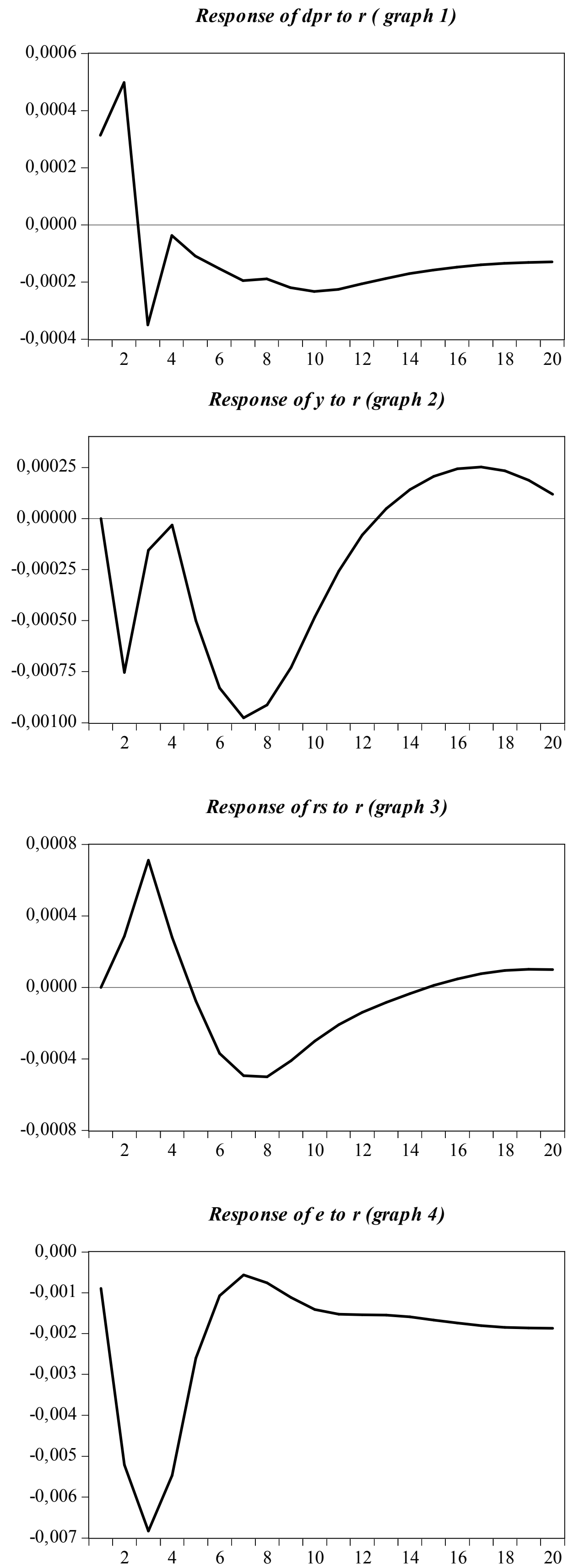

Figure 5 Response to Cholesky one standard deviation innovation.

5 Conclusion

According to the economic growth theory (Barro and Sala-i-Martin [4]) for the convergence of the Czech economy several factors could assist in increasing the steady state output per capita: the transition from the communist regime, the integration to the European economy, the economic reforms and so on. These factors represent the permanent supply economic shocks.

This paper investigated the long-run structural modelling approach for the Czech economy over the period 1999Q1–2010Q4 with eight macroeconomic variables. We estimated the VECM(1) with 13 over-identifying restrictions for the Czech economy.

The convergence of the Czech economy becomes evident:

- by the decline of the average real exchange rate by about 0,007% per quarter,
- by decreasing in the money-income about 0,023% per quarter arising primarily from the technological innovations in financial intermediation,
- by the average decrease in the real interest rate by about 0,00085% per quarter.

We cannot conclude there is a decrease of the long-run Czech risk premium at 5% level of significance.

The estimated long-run relations indicate that:

- We cannot reject the validity of the purchasing power parity relations as the trivariate model.
- For the money market equilibrium condition, we observe that the income elasticity of money demand is positive and we document the negative elasticity of real money balances with respect to domestic interest rate.
- We cannot reject an increasing trend of the output gap the Czech and EU25 economies by about 0,005% per quarter.
- The risk premium represents the deterministic component of the risk premium associated with bonds and foreign exchange uncertainties and its value is estimated at 0,43% per annum.
- Our results of the FIP relationship support the important role played by this relationship. The average long-run Czech real interest rate is about 6,6%.
- The estimated models seem to have reasonable long-run properties. The likelihood ratio test of the 13 over-identifying restrictions can be rejected.

The graphic analysis outcomes of the domestic inflation reaction, Czech economy output reaction and monetary exchange rate CZK/EUR reaction on monetary political shock:

- go along with the conclusion that nominal interest rate growth induces a price level decrease which is followed by a very slow return to the original level,
- documents a domestic production decrease following monetary policy restrictions with a maximum response up to three years,
- they do not confirm an existence of an exchange rate puzzle but in accordance with the conditions of uncovered interest parity the restrictive monetary policy causes a very short depreciation of the Czech crown during the first three quarters and then it sharply returns to the original level.

The current VECM(1) models could be extended into two directions. We can introduce a break variable and also analyse the model dynamics using impulse responses.

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