Optimal minmax analysis of the market equilibrium by generalizing Cobweb and Laffer models

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Abstract: - Equilibrium can be described as an ideal market situation, in which the interests of economic agents are best served, and the resources are allocated and used based on certain criteria and at a normal level of efficiency for every stage. Market equilibrium is derived from a problem specific to non-cooperative game theory with zero-sum and two players. In order to analyze market equilibrium, the main issue is to determine the equilibrium price in different situations: knowing the functions of supply and demand, knowing the elasticity of these functions, taking into account the existence of income tax etc. The results presented in this paper are broad and dwell on the well-known results for Cobweb and Laffer models. An important advantage of these findings is that they are convenient in terms of calculations and they have interesting economic interpretations.

Key-Words: equilibrium price, supply-demand relation, the elasticity of the supply function, the elasticity of the demand function, the equation of price dynamics, the dynamic index of prices.

1 Introduction

Goods market can be defined as the area where, at a certain moment, the consumers' desires - expressing their demand - meet the producers' desires - as shown in their offers. This confrontation yields the price of the analyzed product and accordingly, one can determine the volume of transactions on the market to ensure supply and demand balance.

Such a market, called market with pure and perfect competition corresponds to a theoretical model that reflects an ideal situation, fancied by the neoclassical school representatives such as V. Pareto and Walras L. (Adam Smith also addressed the issue, but he didn't call it the same way). The market model with pure and perfect competition was a theoretical framework for analysis which highlighted the inherent virtues of the "invisible hand" such as the best mechanism for operating and regulating the economy.

Highlighting the assumptions regarding the market with pure and perfect competition is required to establish procedures for determining the equilibrium price and the conditions of its stability. These assumptions are as follows [4]:

- Atomicity of the market - meaning that there is a large number of sellers who are faced with a large number of buyers, but each of them has a minor economic force. Thus, none of the agents is able to influence the state of the market through its decisions and its actions: the level and the dynamics of the equilibrium price, the market demand and the supply industry. In such a market, producers and consumers are "price takers";
- Homogeneity of the products - meaning that all firms produce the same good, with identical characteristics and uses and similar availability of that product; there is no publicity and no differentiation among the products;
- Free entry and exit - meaning that there are no legal, economic, institutional barriers and other measures that restrict either the access on the market of some producers or the exit from the market of others. The only argument that supports such decisions is strictly economic and regards the profitability of related activities;
- Perfect transparency of the market - all economic agents have the same information on market variables: the nature of traded products, their quantity and quality, the level of prices;
- Perfect mobility of production factors - a condition which implies that labor and capital inputs are oriented towards the most effective

destinations.

If one or more of these conditions are not met, the market is characterized by imperfect competition. However, if the above conditions are met, the market, due to its "invisible hand" tends to reach a state of equilibrium. Equilibrium can be described as an ideal situation, in which the interests of economic agents are best served, and the resources are allocated and used based on certain criteria and at a normal level of efficiency for every stage [1].

2 The equilibrium price

2.1 Problem formulation

We shall mark with C and O the demand and the supply functions; obviously $C(p) \geq 0, O(p) \geq 0$, where p represents the prevailing price (hence $p \geq 0$). The demand and supply functions are supposed to be strictly monotonously decreasing and monotonously increasing respectively; by this token these functions verify the following conditions:

$$C(p_1) \prec C(p_2), O(p_1) \succ O(p_2), \forall p_1 \succ p_2, p_1, p_2 \geq 0 \quad (1)$$

Obviously, if C and O are differentiable functions, conditions (1) can be written as follows:

$$C'(p) \prec 0, O'(p) \succ 0, \forall p \geq 0 \quad (2)$$

The price p^*, which verifies the equality $C(p^*) = O(p^*)$ is named the equilibrium price; consequently, the price p^* can be determined as the solution of the following equation:

$$C\ (p) = O\ (p) \quad (3)$$

Generally, it is very difficult to find the solution of equation (3); thereby, most frequently, there are used approximate methods in order to solve this equation. These methods are: the successive approximation method, Newton method, linearizing method etc.

2.2 The determination of the equilibrium price and economic interpretation

A. The direct approach

If we assume that the supply and demand functions are differentiable and we linearize both the left and the right member of the equation (3), considering $\bar{p} \succ 0$ an arbitrary chosen point, the equilibrium point can be determined immediately as follows [12]:

$$p^* = \frac{C(\bar{p}) - O(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} + \bar{p} \quad (4)$$

Remark 1. For the particular case when the demand and the supply functions are linear, namely

$$C(p) = -ap + b, O(p) = cp - d,\ a, b, c, d \succ 0 \quad (5)$$

then the equilibrium price can be determined using the formula:

$$p^* = \frac{b + d}{a + c} \quad (6)$$

Moreover, we can write the following relation:

$$C(p^*) = O(p^*) = \frac{bc - ad}{a + c} \quad (7)$$

If we consider the efficiency function F defined by the relation: $F(x, y) = g(y) - f(x), x, y \geq 0$, where *f* and *g* are known functions, then we can write the following equality:

$$\begin{aligned} &\max_x \min_y (g(y) - f(x)) = \min_y g(y) - \max_x f(x) = \\ &= \min_y \max_x (g(y) - f(x)) \end{aligned} \quad (8)$$

If $x = y = p$, p being the prevailing price on the market, in this particular case we are led to solving the following equation:

$$\min_p g(p) = \max_p f(p) \quad (9)$$

The solution of the above-mentioned equation, p^* represents actually the equilibrium point searched for the case $g \equiv C$ and $f \equiv O$ (figure 1).

B. Determining the equilibrium point starting from the elasticity of supply - demand functions

We shall take into consideration the following elements:

- The minimum prices for which the supply – demand functions are defined are marked with $p_{1,1}$ and $p_{2,1}$ respectively;
- The elasticity of the supply function is e_1 and the elasticity of the demand function is e_{2i}.

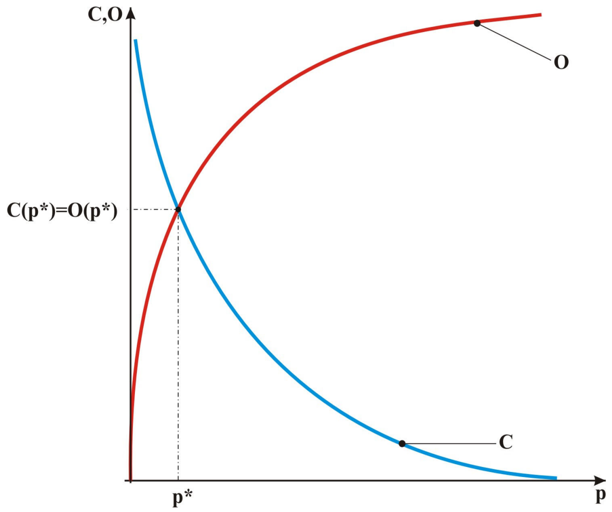

Figure 1. The equilibrium price

Both e_1 and e_{2i} are assumed to be the n degree polynomial in relation to price p, namely:

$$e_1 = a_0 + a_1 p + a_2 p^2 + \cdots + a_n p^n \tag{10}$$

$$e_2 = b_0 + b_1 p + b_2 p^2 + \cdots + b_m p^m \tag{11}$$

We shall run through the following steps:

1. The elasticity for the supply-demand functions can be determined as we can see in the analytical expressions (8) and (9), respectively provided we know the prices $p_{1,1}; p_{1,2}; \ldots; p_{1,n}$ corresponding to each moment $1, 2, \ldots, n$ and also the prices $p_{2,1}; p_{2,2}; \ldots; p_{2,n}$ corresponding to each moment $1, 2, \ldots, m$ taken from statistical data. By using the least squares method, we can calculate the values of coefficients $a_0, a_1, \ldots, a_n$ and $b_0, b_1, \ldots, b_n$; these values represents the solutions of the following algebraic systems:

$$\begin{cases} a_0 \sum_{i=1}^{n} p_{1,i}^0 + a_1 \sum_{i=1}^{n} p_{1,i}^1 + \ldots + a_n \sum_{i=1}^{n} p_{1,i}^n = \sum_{i=1}^{n} O(p_{1,i}) \\ a_0 \sum_{i=1}^{n} p_{1,i}^1 + a_1 \sum_{i=1}^{n} p_{1,i}^2 + \ldots + a_n \sum_{i=1}^{n} p_{1,i}^{n+1} = \sum_{i=1}^{n} p_{1,i} O(p_{1,i}) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_0 \sum_{i=1}^{n} p_{1,i}^n + a_1 \sum_{i=1}^{n} p_{1,i}^{n+1} + \ldots + a_n \sum_{i=1}^{n} p_{1,i}^{2n} = \sum_{i=1}^{n} p_{1,i}^n O(p_{1,i}) \end{cases} \tag{12}$$

$$\begin{cases} b_0 \sum_{i=1}^{m} p_{2,i}^0 + b_1 \sum_{i=1}^{m} p_{2,i}^1 + \ldots + b_m \sum_{i=1}^{m} p_{2,i}^m = \sum_{i=1}^{m} C(p_{2,i}) \\ b_0 \sum_{i=1}^{m} p_{2,i}^1 + b_1 \sum_{i=1}^{m} p_{2,i}^2 + \ldots + b_m \sum_{i=1}^{m} p_{2,i}^{m+1} = \sum_{i=1}^{m} p_{2,i} C(p_{2,i}) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ b_0 \sum_{i=1}^{m} p_{2,i}^m + b_1 \sum_{i=1}^{m} p_{2,i}^{m+1} + \ldots + b_m \sum_{i=1}^{m} p_{2,i}^{2\cdot m} = \sum_{i=1}^{m} p_{2,i}^m C(p_{2,i}) \end{cases} \tag{13}$$

2. Because $O = O(p)$, $C = C(p)$, taking into consideration the elasticity concept we are led to the following finite differential equations [9]:

$$\frac{\Delta O}{O} = e_1 \frac{\Delta p}{p} \quad , \quad \frac{\Delta C}{C} = e_2 \frac{\Delta p}{p} \tag{14}$$

which yields the following first degree differential equations[9]:

$$\frac{dO}{O} = e_1 \frac{dp}{p} \quad , \quad \frac{dC}{C} = e_2 \frac{dp}{p} \tag{15}$$

By integrating these differential equations and taking into account the equalities (10) and (11), we shall get:

$$\begin{aligned} &\int_{p_{1,1}}^{p^*} \frac{dO}{O} = \int_{p_{1,1}}^{p^*} e_1 \frac{dp}{p} \Rightarrow O\left(p^*\right) - O\left(p_{1,1}\right) = \\ &= \int_{p_{1,1}}^{p^*} \frac{a_0 + a_1 p + a_2 p^2 + \cdots + a_n p^n}{p} dp \end{aligned} \tag{16}$$

$$\begin{aligned} &\int_{p_{2,1}}^{p^*} \frac{dC}{C} = \int_{p_{2,1}}^{p^*} e_2 \frac{dp}{p} \Rightarrow C\left(p^*\right) - C\left(p_{1,1}\right) = \\ &= \int_{p_{2,1}}^{p^*} \frac{b_0 + b_1 p + b_2 p^2 + \cdots + b_n p^n}{p} dp \end{aligned} \tag{17}$$

Accordingly, we shall have:

$$O(p^*) = O(p_{1,1}) + a_0 \ln \frac{p^*}{p_{1,1}} + \sum_{i=1}^{n} a_i \frac{\left(p^*\right)^i - \left(p_{1,1}\right)^i}{i} \tag{18}$$

$$C(p^*) = C(p_{2,1}) + b_0 \ln \frac{p^*}{p_{2,1}} + \sum_{i=1}^{m} b_i \frac{\left(p^*\right)^i - \left(p_{2,1}\right)^i}{i} \tag{19}$$

3. Taking into consideration that p^* represents an equilibrium point, it is obvious that

$C(p^*) = O(p^*)$, and consequently, p^* represents the solution of the equation (4).

Generally, it is extremely difficult to solve the above mentioned equation and the equilibrium price p^* can be approximately determined by using specific approximate solving methods of the algebraic equations (successive approximation method, Newton method etc.).

$$O(p_{1,1}) + a_0 \ln\frac{p^*}{p_{1,1}} + \sum_{i=1}^{n} a_i \frac{(p^*)^i - (p_{1,1})^i}{i} = C(p_{2,1}) + b_0 \ln\frac{p^*}{p_{2,1}} + \sum_{i=1}^{m} b_i \frac{(p^*)^i - (p_{2,1})^i}{i} \qquad (4)$$

Particular cases

1) If $n = m$ and we mark $p_0 = p_{1,1} = p_{2,1}$, then the previous equation becomes:

$$(a_0 - b_0)\ln p^* + \sum_{i=1}^{n}\frac{(a_i - b_i)}{i}(p^*)^i = (a_0 - b_0)\ln p_0 + \sum_{i=1}^{n}\frac{(a_i - b_i)}{i}p_0^i + C(p_0) - O(p_0) \qquad (21)$$

2) If $n = m$, $p_0 = p_{1,1} = p_{2,1}$ and the elasticity functions are linear (i.e. $e_1 = a_0 + a_1 p$, $e_2 = b_0 + b_1 p$), the equilibrium price p^* represents the solution of the following equation:

$$(a_0 - b_0)\ln p^* + (a_1 - b_1)p^* = A \qquad (22)$$

where:

$$A = (a_0 - b_0)\ln p_0 + (a_1 - b_1)p_0 + C(p_0) - O(p_0) \qquad (23)$$

3) If the logarithmic function can be written in a linear form, for the point $\bar{p} = 1$, after an immediate calculation, we shall get:

$$p^* = \bar{p} - \frac{O(\bar{p}) - C(\bar{p})}{O'(\bar{p}) - C'(\bar{p})} \qquad (24)$$

This result is concordant with the equality (4).

Economic Interpretation

It is obvious that the equilibrium point p^* represents the solution of the following problem:

$$\max_p O(p) = \min_p C(p) \qquad (25)$$

From equalities

$$O(p) = O(\bar{p}) + (p - \bar{p})O'(p) \qquad (26)$$

$$C(p) = C(\bar{p}) + (p - \bar{p})C'(p) \qquad (27)$$

we can make, immediately, the following deduction:

$$C(p) - O(p) = C(\bar{p}) - O(\bar{p}) + (p - \bar{p})(C'(\bar{p}) - O'(\bar{p})) \qquad (28)$$

Therefore, the area between the graphic representations of the curves $O = O(p)$, $C = C(p)$ and the lines $p = 0, p = p_n$ (marked with A_n) can be determined using the formula (figure 2):

$$A_n = \int_0^{p_n} (C(p) - O(p))dp = p_n(C(\bar{p}) - O(\bar{p}) + (\frac{p_n}{2} - \bar{p})(C'(\bar{p}) - O'(\bar{p}))) \qquad (29)$$

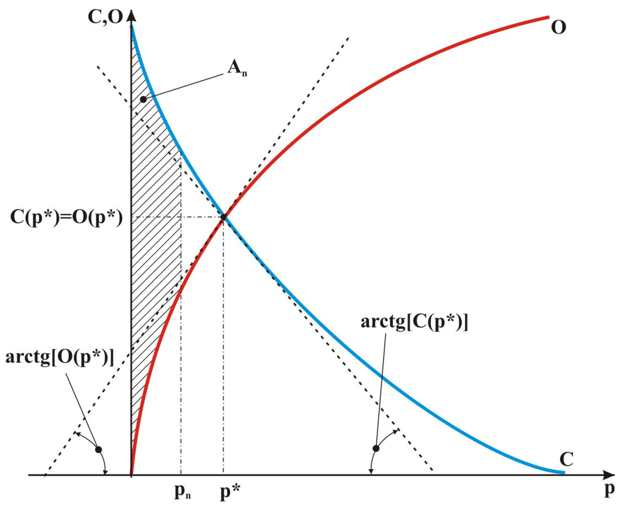

Figure 2. The graphical representation of A_n area

Taking into consideration the requirements $\lim_n p_n = p^*$, $\lim_n A_n = 0$ and the equality:

$$p^* = \bar{p} - \frac{C(\bar{p}) - O(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \qquad (30)$$

after some calculation we can get the following relation:

$$C(\bar{p}) - O(\bar{p}) = \bar{p}(C'(\bar{p}) - O'(\bar{p})) \quad (31)$$

In case $\bar{p} = p^*$ (namely the development of both the demand function and the supply function into Taylor series is made exactly in the equilibrium point), from (7) equality we can obtain:

$$\left|C'(p^*)\right| = O'(p^*) \quad (32)$$

From economic point of view, this equality shows that the marginal values of the demand and of the supply functions have equal absolute values in the equilibrium point p^*.

3 Cobweb model. Determining the equation of price dynamics

Let us assume further on a market of a certain product in a state equilibrium. In order to highlight the dynamic nature of the equilibrium on the competitive market, we shall analyze what would happen if there were changes in the demand, in the supply, or in both cases. If the market re-enters the previous equilibrium state or it develops a new equilibrium point, it will be called *stable equilibrium market*; if the equilibrium of the market breaks because of some disturbance factors and if this imbalance grows more severe in the long run, the market is called *unstable equilibrium market.*

The ability of the competitive market to rebalance itself or not depends on the relationship established between the slope of the demand curve and that of the supply curve, or, in other words, it depends on the relationship between the elasticity of demand in regard to the price and the elasticity of supply in regard to the price. A rebalance is achieved in the long run and it can be detected starting from the model known as the "Cobweb model", due to graphical images generated by the Supply and Demand Functions.

3.1 The case when analytical expressions of supply-demand functions are known

There are practical situations in which the demand is affected by the proposed price at the time it was placed, while the supply is influenced by the prevailing market price from a previous period of time.

For example, in the case of agricultural products, between the intent to provide and the supply itself there is a gap of time (of almost half a year).

Therefore, we shall note p_t, and p_{t-1}, respectively the prices at time t (when the demand was made) and time t-1 (the period of time of the previous offer).

The condition of equilibrium is, in this case, as follows:

$$C(p_t) = O(p_{t-1}) \quad (33)$$

and it will lead to a recurrence relationship between p_t and p_{t-1} (called "the recurring price equation").

The recurring price equation can be determined most comfortable through the linearization of the two members of the equation (30) (developing Mc-Laurin and Taylor series and retaining only the first two terms of the development).

Thus, when developing Taylor series out of the two members of the equation (33) we get:

$$C(\bar{p}) + (p_t - \bar{p})C'(\bar{p}) = O(\bar{p}) + (p_{t-1} - \bar{p})O'(\bar{p}) \quad (34)$$

from where:

$$p_t = \frac{O'(\bar{p})}{C'(\bar{p})} p_{t-1} + \frac{O(\bar{p}) - C(\bar{p})}{C'(\bar{p})} + \bar{p}\left(1 - \frac{O'(p)}{C'(p)}\right) \quad (35)$$

If we note:

$$A = \frac{O'(\bar{p})}{C'(\bar{p})}, B = \frac{O(\bar{p}) - C(\bar{p})}{C'(\bar{p})} + \bar{p}\left(1 - \frac{O'(p)}{C'(p)}\right) \quad (36)$$

$x_t = p_t - p^*$ (i.e. x_t measures the deviation from the prevailing price at time t and the equilibrium price p* given by (3)), then the recurrent relationship (35) becomes:

$$p_t = Ap_{t-1} + B \quad (37)$$

which, after an immediate calculation results in:

$$p_t = A^t(p_0 - p^*) + p^* \quad (38)$$

Practically, the equality (38) reflects the price dynamics (which is why it is called "dynamic pricing equation").

3.2 The case when there are no known analytical expressions of the supply and demand functions

When there are no known analytical expressions of the demand and supply functions but we are aware of the interdependence between the base price p_t and previous prices p_{t-1}, p_{t-2},…,p_{t-k},, the determination of the equation of dynamic pricing is done by going through several stages that involve relatively simple calculations. The interdependence between the basic price and previous prices can be established from statistical data and using well- known approximation methods (interpolation methods, i.e., approximation by polynomials, the method of the least squares etc.).

There are two cases:

Case I. The interdependence between the basic price and previous prices is $f(p_t, p_{t-1}, ..., p_{t-k}) = 0$, f being a known function. This is commonly known as the homogeneous case. Prices take the following form:

$$p_t = pr^t, p_{t-1} = pr^{t-1}, ..., p_{t-k} = pr^{t-k}, p, r > 0 \qquad (39)$$

The functional given interdependence $f(p_t, p_{t-1}, ..., p_{t-k}) = 0$ turns into an equivalent one of the following form: $F(r^t, r^{t-1}, ..., r^{t-k}) = 0$, called a characteristic equation.

Let us note r_1, r_2, …,r_{k+1} the real and non-zero solutions of this last equation. The price at time t has following form:

$$p_t^* = c_1 r_1^t + c_2 r_2^t + c_3 r_3^t + ... + c_k r_k^t \qquad (40)$$

where c_1, c_2, …,c_k are real constants to be determined from the initial conditions, that is to say that at the initial moment and at k-1 previous moments, the prices are known.

Case II. The interdependence between the basic price and previous prices is

$$f(p_t, p_{t-1}, ..., p_{t-k}) = g(t) \qquad (41)$$

where f and g are known functions and g is different from the required function (the homogeneous case).

The price at time t is denoted $\overline{p_t}$ and it has the following form:

$$\overline{p_t} = p_t^* + p_t^0 \qquad (42)$$

where p_t^* is the price given by the homogeneous equation $f(p_t, p_{t-1}, ..., p_{t-k}) = 0$ (which is determined by previous methodology) and p_t^0 is a particular solution of the equation $f(p_t, p_{t-1}, ..., p_{t-k}) = g(t)$; the form of p_t^0 being given by shape of the right member. More precisely if g (t) is polynomial p_t^0 will be polynomial as well; if g (t) is exponential then p_t^0 will be exponential, too etc.

4 Laffer Model

4.1 Theoretical considerations

This model is specific for the case when we take into consideration taxes (in fact, we are dealing with the state's interference upon the market through taxation lever).

Taxes are perceived by the producer as additional costs and, consequently, the producer would try to recover this amount through prices. On the other hand, the increase of prices can determine the decrease of demand and, consequently, the overall recovering of taxes is under risk. Hence, it is possible to identify a phenomenon of taxation handover on the account of the consumer.

If we take no notice of taxation, the equilibrium price p* ca be determined, as usually, by solving the equation: $C(p) = O(p)$. Taking taxation into account, the selling price increases and, accordingly, the equilibrium price varies. Practically, the new equilibrium price p_1* verifies the condition p_1* > p*, and, consequently, $C(p_1^*) < O(p^*) = C(p^*)$.

In other words, the following elements must be analyzed:

• the demand will be reduced due to the increased level of the price paid by the costumer in order to purchase the good;

• the supply will not be increased because the supplier would not get the extra price, this amount being collected by the public budget.

If taxation does not exist, the price corresponding to the above-mentioned supply would be p_2. The difference T = p_1* - p_2 represents, in fact, the tax which corresponds to each physical unit of supply.

We must also show that, under the circumstances of taxation, we can identify two prices used by the producer:

1. the selling price p_c, which is paid by the costumer;
2. the accounting price p_g, namely the reference price determined by the producer in order to analyze his own efficiency.

Practically, p_g represents the remainder of the selling price after paying off taxes to the public budget (figure 3).

Obviously, the following equality is verified:

$$p_c = p_g + T \qquad (43)$$

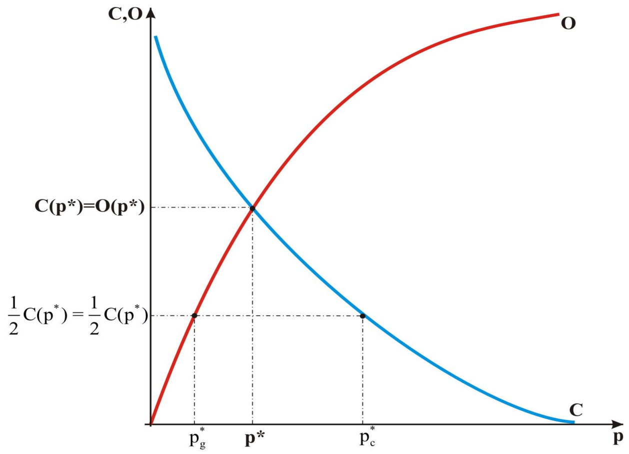

Figure 3. The equilibrium price, the selling price and the accounting price

4.2. Solving the model, economic interpretation

The equilibrium model, under taxation conditions, is based on the equality between the supply and demand, the demand represented by the price p_c while the supply is represented by the price p_g:

$$C(p_c) = O(p_g) \tag{44}$$

that is to say

$$C(p_c) = O(p_c - T) \tag{45}$$

If I_B is the function that describes the budget income (I_B is a function of T, obviously), we are led to solving the following optimization problems:

$$(P) \quad \begin{cases} \max I_B(T) \\ \max\limits_{pc} \min\limits_{T} (C(p_c) - O(p_c - T)) = \\ = \min\limits_{pc} \max\limits_{T} (C(p_c) - O(p_c - T)) \\ T, p_c \succ 0 \end{cases}$$

(P) is an optimal problem in which constraints have a special form (basically a minmax equality). Solving problem (P) involves the following steps:
1) proving that the set of solutions of restrictions:

$$\begin{aligned} &\max_{T} \min_{pc} (C(p_c) - O(p_c - T)) = \\ &= \min_{pc} \max_{T} (C(p_c) - O(p_c - T)) \end{aligned} \tag{46}$$

is not unique and, moreover, it verifies the inequality:

$$p_c - T \prec p^* \prec p_c \tag{47}$$

2) if T^* is the optimal solution of problem (P), a direct relationship between, p^*, $C(p_c)$ and $C(p^*)$ can be determined;
3) from the relationship of the form $C(p_c) = \alpha C(p^*)$ (the parameter α being determined at the previous point), p_c^* and $p_g^* = p_c^* - T^*$ can be immediately calculated.

Building Taylor series from both members of the equation (45) at the point $\overline{p}$ and keeping only the first two terms (basically we would linearize the first two members of (45)), we get:

$$p_c^* = -\frac{C(\overline{p}) - O(\overline{p})}{C'(\overline{p}) - O'(\overline{p})} - T\frac{O'(0)}{C'(0) - O'(0)} + \overline{p} \tag{48}$$

Since, in the absence of taxation, the equilibrium price p* is given by the following relation:

$$p^* = -\frac{C(\overline{p}) - O(\overline{p})}{C'(\overline{p}) - O'(\overline{p})} + \overline{p} \tag{49}$$

from (44) and (45) we get the equality:

$$p_c = p^* + T\frac{O'(\overline{p})}{O'(\overline{p}) - C'(\overline{p})} \tag{50}$$

Since $O'(\overline{p}) \succ 0$, $C(\overline{p}) \prec 0$, it is clear that the equality (50) is followed immediately by the inequality $p_c^* \succ p^*$.

Moreover, equality $p_g^* = p_c^* - T^*$ is equivalent with the equality $p_g^* = p^* + T\frac{C'(\overline{p})}{O'(\overline{p}) - C'(\overline{p})}$ and, consequently $p_g^* \prec p^*$. Therefore, we are led to the following inequalities

$$p_g^* \prec p^* \prec p_c^* \tag{51}$$

which, in economic terms, expresses the relationship between the three equilibrium points. Moreover,

$$C(p^*) = O(p^*) \succ C(p_c) = O(p_g) \tag{52}$$

Remark 2.

In case the demand and the supply functions are linear, we have:

$$C(p) = -ap + b, \quad O(p) = cp - d, a, c > 0 \tag{53}$$

and immediate calculations yield the following relations:

$$\begin{aligned} p_c &= \frac{b+d}{a+c} + \frac{c}{a+c}T > p^* \\ p_g &= \frac{b+d}{a+c} - \frac{a}{a+c}T < p^* \end{aligned} \tag{54}$$

$$\begin{aligned} C(p_c) = O(p_g) = \frac{bc-ad}{a+c} - \frac{ac}{a+c}T < C(p^*) = \\ = O(p^*) = \frac{bc-ad}{a+c} \end{aligned} \tag{55}$$

We shall note I_B the budget revenue function, the analytical expression of which is:

$$I_B(T) = TC(p_c) \tag{56}$$

By linearizing the demand function, we get:

$$\begin{aligned} C(p_c^*) = C(\bar{p}) + (p_c^* - \bar{p})C'(\bar{p}) = \\ = C(\bar{p}) - C'(\bar{p})\left(\frac{C(\bar{p}) - O(\bar{p})}{C'(\bar{p}) - O(\bar{p})} + T\frac{O'(\bar{p})}{C'(\bar{p}) - O(\bar{p})}\right) \end{aligned} \tag{57}$$

As a consequence, the budget revenue function can be determined immediately:

$$\begin{aligned} I_B(T) = T\left[C(\bar{p}) - C'(\bar{p})\frac{C(\bar{p}) - O(\bar{p})}{C'(\bar{p}) - O'(\bar{p})}\right] - \\ -T^2\frac{C'(\bar{p})O'(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \end{aligned} \tag{58}$$

The relations:

$$A = C(\bar{p}) - C'(\bar{p})\frac{C(\bar{p}) - O(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \tag{59}$$

$$B = -\frac{C'(\bar{p})O'(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \tag{60}$$

yield:

$$I_B(T) = AT + BT^2 \tag{61}$$

Remark 3. From calculations we get:

$$A = \frac{C'(\bar{p})O(\bar{p}) - C(\bar{p})O'(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \tag{62}$$

T * that maximizes the size of budgetary revenue is determined as a solution to the following equation:

$$I_B{}'(T) = 0 \tag{63}$$

and taking into account (57), there follows $T^* = -\frac{A}{2B}$, from where, after computation, we can obtain:

$$T^* = \frac{C'(\bar{p})O(\bar{p}) - C(\bar{p})O'(\bar{p})}{2C'(\bar{p})O'(\bar{p})} \tag{64}$$

It is obvious that $T^* \succ 0$; if $p = p^*$; then, after calculations we have:

$$T = \frac{C(p^*)}{2}\left(\frac{1}{O'(p^*)} - \frac{1}{C'(p^*)}\right) \tag{65}$$

Remark 4. On condition: $C(p) = -ap + b, O(p) = cp - d$, the equality (64) becomes:

$$T^* = \frac{bc-ad}{2ac} \tag{66}$$

The determination of the size of T* makes it possible to calculate the equilibrium volume of transactions in terms of taxation,

$$C(p_c) = C(\bar{p}) + (p_c - \bar{p})C'(\bar{p}) \tag{67}$$

i.e., after calculations

$$C(p_c) = \frac{1}{2}\left(\frac{C'(\bar{p})O(\bar{p}) - C(\bar{p})O'(\bar{p})}{C'(\bar{p}) - O'(\bar{p})}\right) \tag{68}$$

as

$$C(p^*) = \frac{C'(\bar{p})O(\bar{p}) - C(\bar{p})O'(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \tag{69}$$

From (64) and (65) we obtain the following equality:

$$C(p_c^*) = \frac{1}{2}C(p^*) \tag{70}$$

and p_c^* yields from the above equality.

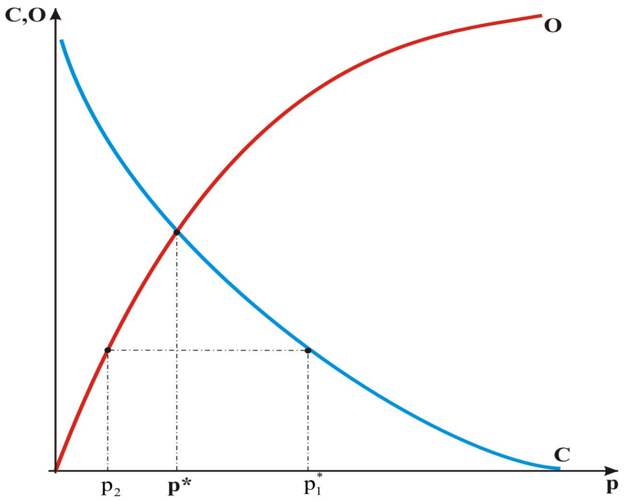

Figure 4. The determination of the equilibrium price within Laffer model

Consequently, the maximum budget revenue is:

$$I_B(T^*) = AT^* + B(T^*)^2 \tag{71}$$

Considering

$$A = \frac{C'(\bar{p})O(\bar{p}) - C(\bar{p})O'(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \tag{72}$$

$$B = -\frac{C'(\bar{p})O'(\bar{p})}{C'(\bar{p}) - O'(\bar{p})} \tag{73}$$

yields the following expression:

$$I_B(T^*) = \frac{\left[C'(\bar{p})O(\bar{p}) - C(\bar{p})O'(\bar{p})\right]^2}{2C'(\bar{p})O'(\bar{p})\left[C'(\bar{p}) - O'(\bar{p})\right]} \tag{74}$$

Remark 5. If $\bar{p} = p^*$, we have:

$$I_B(T^*) = \frac{C(p^*)\left[C'(p^*) - O'(p^*)\right]}{2C'(p^*)O'(p^*)} \tag{75}$$

In a particular case $C(p) = -ap + b, O(p) = cp - d$, the equality (74) becomes:

$$I_B(T^*) = \frac{(ad - bc)^2}{4ac(a + c)} \tag{76}$$

It can be seen immediately that all indicators, both the market and the budget indicators, depend explicitly on the supply and the demand functions.

$$I_B(T^*) = (p_c^* - p_g^*) \cdot \frac{1}{2}C(p^*) = \frac{T^* C(p^*)}{2} \tag{77}$$

Accordingly, for the optimal difference $T^* = p_c^* - p_g^*$ we find that the maximum of the efficiency function I_B equals both the rectangle area $BCp_c^*p_g^*$ and the triangle area $Ap_g^*p_c^*$ (figure 5).

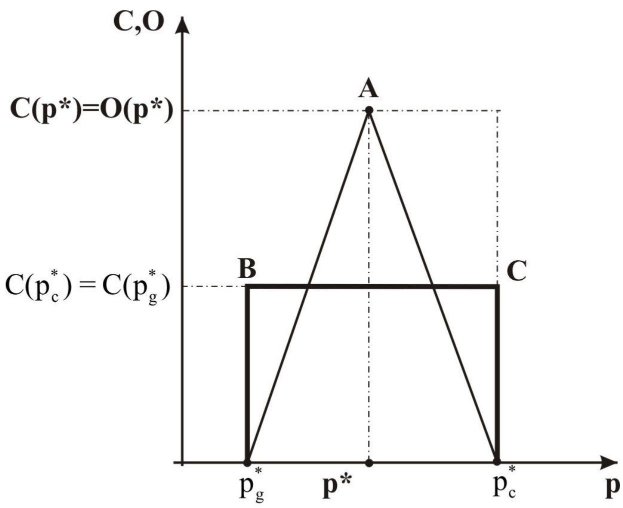

Figure 5. Graphic representation of the maximum efficiency function

Conclusion
The paper contains original elements, which are relatively easy to explain but they have major economic importance:

- The equilibrium price p^* can be determined in two ways: by linearizing the supply and demand functions, starting from the elasticity of supply and demand functions, respectively;
- The Cobweb model presents two types of solutions:
  - the first solution is based on a technique used in solving differential equations. This method is easy to use when there can be established a recurrence relationship between the current price and pprevious prices;
  - a technique based on the least

- squares method. This method is extremely convenient when using electronic computing equipment.

- In the case of Laffer model (an equilibrium model conditioned by the existence of taxation), we can determine:
  - the prevailing market equilibrium price p_c^* which is different from the existing equilibrium price in the absence of taxation;
  - the equilibrium volume of transactions under taxation conditions;
  - a maximum budget revenue.
- The optimal prices p_c^* and p_g^* can be determined by going through certain stages of an optimizing problem with minmax constraints (which accept several solutions).
- Theoretical results are obtained on extremely general conditions and they allow important economic interpretations.

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