On Reducing the Number of Existing Cycles in Connected Graphs Obtained from Comparison Matrices

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Abstract: Both classical Analytic Hierarchy Process and Analytic Network Process appear to be somewhat laborious for application in projects with restricted budgets. Their binary and ternary versions seem to be quite sufficient for a reasonably small amount of alternatives and criteria. Occurrences of 'ties' in a ternary AHP is considered in particular in this work. Correlations between the number of 'ties' and the total number of cycles in a connected graph associated with a comparison matrix are discussed.

Key–Words: imprecision, uncertainty, consistency, multi-criteria decision-making, ternary AHP

1 Introduction

Solving complex decision problems where multicriteria comparisons are involved is often effected by imprecision, uncertainty, and indetermination. In [20] such occurrences are related to difficulties of determination, a relatively arbitrary choice between options and values' changes due to time constrains, respectively. A potential choice is evaluated according to seven criteria originating from predetermined objectives and constraints. Indifference and preference thresholds for each criterion are determined based on the relative importance of that criterion. The authors developed also a method to appraise the impact of imperfect information on ranking.

Decision makers are expected to be absolutely consistent or at least nearly consistent. This implies a form of consistency measurement. A variety of methods for measuring both inconsistencies and inconsistencies' reduction have been developed. The Analytic Hierarchy Process (AHP), [21] and its variant Modified Eigenvector Method belong to the group of eigenvector methods while the Least squares method, Weighted least squares method, Logarithmic least squares method, and Chi-square method belong to the group of extremal methods.

AHP facilitates development of a hierarchical structure of a complex evaluation problem. This way subjective judgement errors can be avoided and an increase of the likelihood for obtaining reliable results can be achieved. Inconsistency in AHP is measured by comparing a consistency index obtained from a real data with indexes obtained from randomly generated data. It is criticized for not being able to reflect on different decision situations and at the same time not being able to interpret inconsistency, [9]. The singular value decomposition is measuring inconsistency by applying weights and taking the Frobenius norm for determining differences between the original data and the obtained one, [6]. Several theoretical and numerical properties of the inconsistencies of pairwise comparison matrices are obtained by statistical analysis. The inconsistency of asymmetry is addressed while showing a different type of inconsistency.

Preserving consistency in a grading process is a challenge for most decision makers. Cycles in a graph associated with an AHP related comparison matrix imply inconsistency, [21] and [23]. Inconsistencies can be measured by the number of cycles in an associated graph, [16]. We propose an approach to reduce the amount of work related to locating and subsequently correcting inconsistencies by looking at the possible maximum length of cycles in a connected graph associated with a comparison matrix. Further on we perform a sensitivity analysis that illustrates how a change of a particular criterion can effect the final outcome.

The rest of the paper is organized as follows. Related work and supporting theory may be found in Section 2. The decision process is presented in Section 3. Conclusions and future work can be found in Section 4.

2 Related Work

'Situations that call for an ordinal scale are generally those involving subjective, none measurable qualities, as in sensory testing, personnel rating, or the study of preference and choice behaviour, for example. 'Ordinal, "pick-the-winner" type data also occur naturally in sporting contexts or when attempting to elicit preferences from subjects who are incapable of quantifying their judgements (animals in food testing, say) or for whom the task of comparing alternatives on a ratio scale would be too tedious or time-consuming', [7].

AHP [21] employs paired comparisons in order to obtain ratio scales. Both actual measurements and subjective opinions can be used in the process. A decision committee makes pairwise comparison of independent alternatives with respect to each criterion and among the involved criteria. The elements $a_{ij}, i, j = 1, 2, ..., n$ in the obtained matrices satisfy the conditions $a_{ij} > 0, a_{ij} = a_{ji}^{-1}, a_{ii} = 1, i, j = 1, 2, ..., n$. The AHP steps can be summarized as follows:

1. Development of a decision hierarchy with an objective, alternatives and criteria. The number of levels in an AHP hierarchy can vary greatly according to the need of a particular decision situation. Alternatives carry information of either a quantitative nature or a qualitative nature. For n alternatives only $\frac{n(n-1)}{2}$ paired comparisons need to be elicited since reciprocal response data is assumed.

2. The relative importance of criteria and preferences among the alternatives is stated based on pairwise comparisons. The standard rating system employs 9-point scale where equal importance is denoted by 1 and extreme importance is denoted by 9. The upper bound is a result of research in psychology indicating humans' inability to consistently repeat their expressed gradations of preference finer than seven plus or minus two.

3. A priority weight vector for the criteria is obtained via a synthesis process based on the preference scores [1], [22].

4. Calculation of the final weight vector representing the priority ordering of the alternatives.

Existence of a weight vector in a pair-wise comparison matrix is proven by the Perron-Frobenius Theorem 1, [13].

Theorem 1 *Let $A = (a_{ij})$ be a real $n \times n$ matrix with non-negative a_{ij} entries and irreducible. Then the following statements hold:*

- *there is a real eigenvalue r of A such that any other eigenvalue λ satisfies $|\lambda| \leq r$. This property may also be stated more concisely by saying that the spectral radius of A is an eigenvalue.*

- *there is a left (respectively right) eigenvector associated with r having non-negative entries.*

- *one has the eigenvalue estimate*

$$min_i \sum_j a_{ij} \leq r \leq max_i \sum_j a_{ij}$$

Table 1: Random indexces for matrices of size n

n	1	2	3	4	5
RI	0.00	0.00	0.58	0.90	1.12

Table 2: Random indexces for matrices of size n

n	6	7	8	9	10
RI	1.24	1.32	1.41	1.45	1.49

Decision makers' judgements are consistent if $a_{ij}a_{jk} = a_{ik}, i, j, k = 1, 2, ..., n$. In this content consistency means that if a basic amount of row data is available than all other data can be logically deduced from it. Application of eigenvector vectors leads to a very useful consistency measure called consistency index CI, [21]. The CI of a comparison matrix is defined as $CI = \frac{\lambda_{max}-n}{n-1}$ where n is the order of the comparison matrix and λ_{max} is its maximum eigen value. CI measures the transitivity of a preference that is a part of the pairwise comparisons.

A random index RI is the mean CI value of random generated matrices of size n, [21], see Table 1 and Table 2.A consistency ratio CR [21] is defined as $CR = \frac{CI}{RI}$ and is a measure of comparison between a given matrix and a random generated matrix in terms of consistency indexes. The upper bound for an acceptable CR is 0.1. A revision of judgements is required if larger values are obtained.

Decision makers find AHP to be a very useful tool. At the same time, an increase of the number of alternatives and criteria results in a larger amount of pairwise comparisons. The latter is time consuming and thus increases the loads of the decision makers. Binary and ternary AHP have been proposed for solving problems that do not require a large scale of values representing the intensities of judgements, [12] and [27].

2.1 Rank Reversal

Addition or deletion of alternatives can lead to possible rank reversal [24], [25], and [28]. A change of local priorities can cause rank reversal before and after an alternative is added or deleted, [29]. In order to avoid rank reversal the authors suggest an approach where the local priorities should be kept unchanged.

2.2 Associated Graphs

A connected graph is a graph such that there exists a path between all pairs of vertices, [11].

A subgraph of a graph G is a graph whose vertex set is a subset of that of G, and whose adjacency relation is a subset of that of G restricted to this subset.

The number of cycles of length 3 in a given complete directed graph can be calculated as in Theorem 2 and Theorem 3, [17].

Theorem 2 *Given a comparison matrix A in a binary AHP, the trace $tr(V)$ of the third power of the vertex matrix V, $tr(V)$ corresponding to A is three times the number $\mathcal{N}$ of cyclic graphs of length 3, i.e.*

$$\frac{tr(V^3)}{3} = \mathcal{N}$$

Let $S = v^2 \star V^t$ be a matrix where operation $'\star'$ is defined as an element-wise multiplication and V^t is the transposed of matrix V.

An arc of a graph is an ordered pair of adjacent vertices, [8].

Theorem 3 *[17] The number of cycles that includes arc (i, j) in the corresponding graph is the element $s_{ij} \in S$.*

An algorithm for discovering elements of cycles in incomplete directed graphs is presented in [18]. The algorithm finds cycles of even and odd length. In binary AHP, the algorithm measures consistency for incomplete comparison cases and indicates misjudgements. An incomplete comparison case contains unknown pairwise comparisons, As an example see Fig. 1 where two couples of elements (M, Q) and (N, P) are not compared. The authors also observe that in such cases it is more common to have cycles of length 4 rather than of length 3. They refer to the fact that taking a couple of not compared elements and applying any of the two possible ranking between them will lead to a cycle of length 3. Therefore, in the process of evaluating consistency in the presence of incomplete comparison one should consider all cycles of lengths up to n.

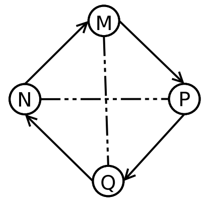

Figure 1: Unknown pairwise comparisons for two couples of elements (M, Q) and (N, P)

Some interesting applications of AHP can be found in [2], [5], [4], and [19]. In [5] the authors concentrate on analyzing and improving the process and evaluating the equipment manufacturers in semiconductor industry applying fuzzy analytic hierarchy process. Critical factors regarding wafer supplier selection are first chosen along with evaluation criteria. The fuzzy analytic hierarchy process is further involved in calculating criteria's weights. A fuzzy multi-criteria model of wafer supplier selection is build afterwards.

A part of the research community, working on decision making, focuses on 'naturalistic decision making', which is related to establishing a meaningful process of planning, perception, comprehension, and forecasting, [26]. The naturalistic decision making is connected to the image theory and recognition-primed decision theory. The image theory investigates how humans' knowledge effects decision making. Some of the types of images in image theory are value image - includes morals, values, beliefs and ethics of an individual; trajectory image - individual's agenda of future goals; and strategic image - individual's plans for achieving those goals. The recognition-primed decision theory involves mainly decision making in high-stakes, time-pressured situations.

3 The Decision Process

Let's first have a look at the comparison outcome at the end of the 1 to 9 scale. If a comparison between alternative β_1 and alternative γ_1 is ranked as 8 then the comparison between alternative γ_1 and alternative β_1 is defined as $\frac{1}{8}$. If a comparison between alternative β_2 and alternative γ_2 is ranked as 9 then the comparison between alternative γ_2 and alternative β_2 is defined as $\frac{1}{9}$. The difference between the two cases with alternatives β_1 and γ_1 and β_2 and γ_2 based on classical AHP is $\frac{1}{8} - \frac{1}{9} = \frac{1}{72}$.

Similar considerations for the ternary AHP again the end of the scale follow. If a comparison between alternative ϕ_1 and alternative ψ_1 is ranked as 2 then the comparison between alternative ψ_1 and alternative

Table 3: Pairwise comparison of the criteria

	C1	C2	C3	C4	C5	C6	C7
C1	1	θ	1	$\frac{1}{\theta}$	$\frac{1}{\theta}$	$\frac{1}{\theta}$	$\frac{1}{\theta}$
C2	$\frac{1}{\theta}$	1	1	θ	$\frac{1}{\theta}$	1	θ
C3	1	1	1	$\frac{1}{\theta}$	1	$\frac{1}{\theta}$	$\frac{1}{\theta}$
C4	θ	$\frac{1}{\theta}$	θ	1	θ	$\frac{1}{\theta}$	1
C5	θ	θ	1	$\frac{1}{\theta}$	1	θ	θ
C6	θ	1	θ	θ	$\frac{1}{\theta}$	1	1
C7	θ	$\frac{1}{\theta}$	θ	1	$\frac{1}{\theta}$	1	1

ϕ_1 is defined as $\frac{1}{2}$. If a comparison between alternative ϕ_2 and alternative ψ_2 is ranked as 3 then the comparison between alternative ψ_2 and alternative ϕ_2 is defined as $\frac{1}{3}$. The difference between the two cases with alternatives ϕ_1 and ψ_1 and ϕ_2 and ψ_2 is based on the ternary AHP is $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

These calculations show that application of the classical AHP requires much more precision from the experts compare to the ternary AHP. The latter one however runs out of different comparison options very fast and is not applicable for a larger number of criteria or alternatives. The ternary AHP is suitable for small cases that should be solved quickly and at low cost.

3.1 Cycles in an Associated Graph

A cycle in graph theory is often understood as a closed path, with no other repeated vertices or edges other than the starting and ending vertices, [3].

Previous research relating a comparison matrix and the corresponding graph was mainly focused on directed graphs (see f.ex. [17] and [27]). In this work we consider the number of cycles of maximum length in a connected graph, that will not violate the accepted CR bound. Our findings are summarized in Proposition 4.

Proposition 4 *Let $\mathcal{G}$ be a connected graph with no multiple edges some of which are undirected. The maximum length of a cycle in $\mathcal{G}$ is $n - l$ if there are l disjoint couples of consecutive nodes, among the graph's n nodes, where the two nodes in each of the l disjoint couples are connected by an undirected edge.*

The connected graph $\mathcal{G}_7$ in Fig. 2 corresponds to a case with seven criteria and four alternatives. The graph has a cycle of length 5, which happens to be the cycle of maximum length in $\mathcal{G}_7$, (see Fig. 3 and Fig. 4). For simplicity we label the nodes in $\mathcal{G}_7$ by C1, ..., C7. There are no cycles of length 7 or 6 since comparisons among four criteria, forming two couples of criteria with consecutive numbers, i.e. (C2, C3) and (C6, C7), result in two 'ties'. In other words the criteria in both couples are found to have equivalent influence on the selection process. The 'ties' (C2, C3) and (C6, C7) limit the search for cycles of length 5. Introducing another cycle of length 5 immediately leads to a $CR > 0.1$, which is unacceptable, [21]. 'Ties' between two criteria are graphically represented by a dash line, Fig. 2.

Examples of cycles of length 4 and 3 are shown in Fig. 5, Fig. 6, and Fig. 7. The cycle of length 4 in Fig. 5 contains nodes C1, C2, C4, and C5. The cycle of length 3 in Fig. 7 contains nodes C1, C2, and C4. The cycle of length 3 in Fig. 6 contains nodes C1, C2, and C7.

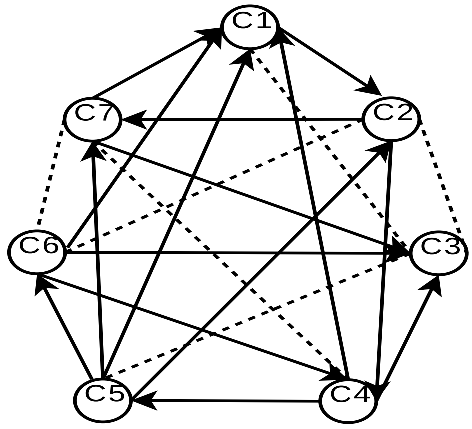

Figure 2: Connected graph associated with the comparison matrix in Table 3

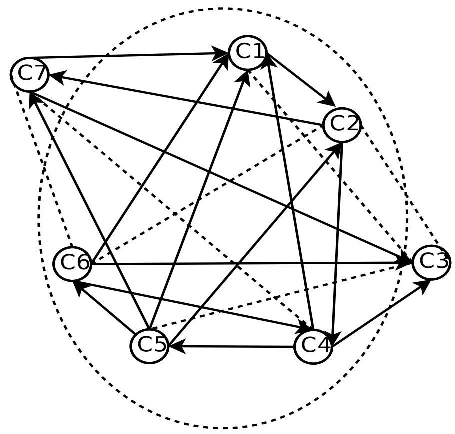

Figure 3: A cycle connecting nodes C1, C2, C4, C5, C6

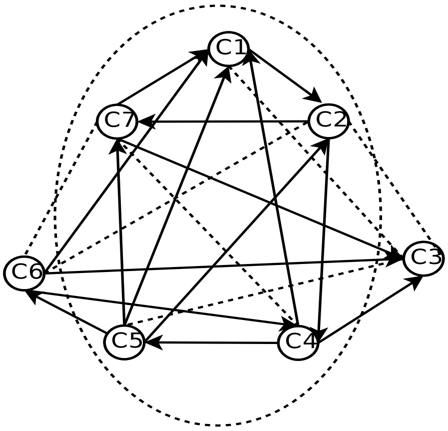

Figure 4: A cycle connecting nodes C1, C2, C4, C5, C7

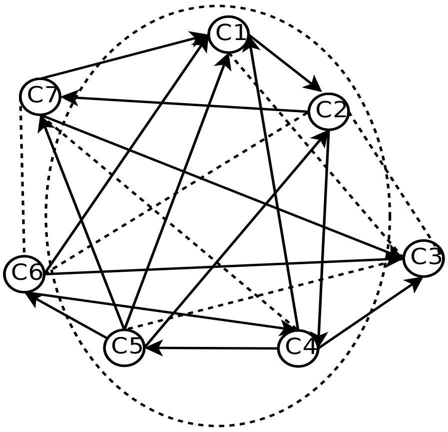

Figure 5: Connected graph with a cycle of length 4

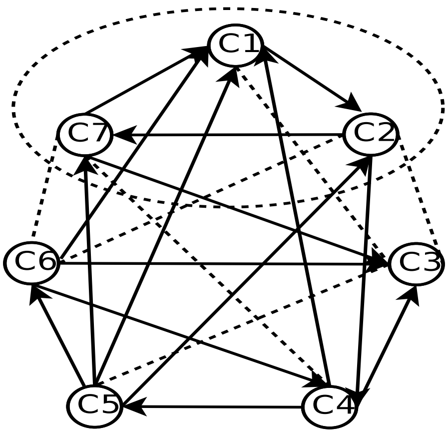

Figure 6: Connected graph with a cycle of length 3, nodes C1, C2, C7

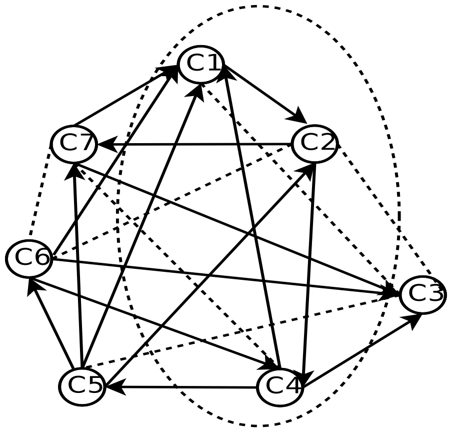

Figure 7: Connected graph with a cycle of length 3, nodes C1, C2, C4

Application of Proposition 4 can reduce the amount of work related to locating and correcting inconsistencies. This can be achieved by working with a connected subgraph with a set of nodes that has only one of the two nodes from each consecutive couple of nodes in the original graph. Proposition 4 can be applied whenever there are 'ties' by performing permutations in criteria numbering that will result in consecutive nodes connected by an undirected edge. Permutations in criteria numbering does not cause rang reversal, [22].

Inconsistencies in a comparison matrix can be represented by the number of directed cycles in a complete directed graph associated with a comparison matrix. Misjudgments are further on assumed to generate cycles in the graph, [17]. Theorems 2 and 3 address the issue of calculating the number of cycles in a complete directed graph.

Binary and ternary AHP lend themselves very well to cases with 'ties' between two criteria. The number of 'ties' can be easily obtained from the comparison matrix. Proposition 5 illustrates how the number of 'ties' in a comparison matrix effects the number of possible cycles of length 3 in the complete directed graph associated with that matrix.

Proposition 5 *Let α be the number of criteria in decision making process and β be the number of couples of criteria involved in 'ties'. The maximum number $\mathcal{M}$ of possible cycles among triplets of nodes in the complete directed graph is*

$$\mathcal{M} = \left(\begin{array}{c} \beta \\ \alpha \end{array} \right) - \beta(\alpha - 2)$$

Example 6 *The connected graph in Fig. 2 has seven nodes which implies possible cycles among 35 triplets*

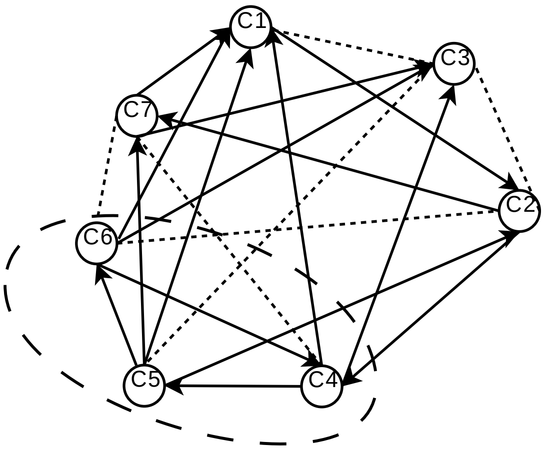

Figure 8: Connected graph where criteria C4, C5, and C6 are involved in a cycle

of nodes. The five 'ties' reduce the number of possible cycles of length 3 to be considered.

A cluster or component in the ANP is a collection of elements whose function derives from the synergy of their interaction and hence has a higher order function not found in any single element, [23]. Application of Proposition 5 in decision making where clusters are involved is particularly useful. It helps not only to reduce the number of criteria triplets to be considered but simplifies listing of all criteria triples that ought to be considered. This means that we can limit our investigations to triplets criteria C_i, C_j, C_k where none of the couples $(i, j), (i, k), (j, k)$ belongs to set of couples of criteria involved in ties. Appearance of cycles in clusters is easily detected and it can be given priority in the process of improving the consistency ratio. If criteria C4, C5, and C6 involved in a cycle, Fig. 8 form also a cluster then this cycle should be given priority. Another example with criteria C2, C4, and C5 involved in a cycle is shown in Fig. 9.

3.2 Pairwise Comparison of Alternatives With Respect to Each Criterion

The global priority is graphically illustrated in Fig. 10. It indicates that alternative A2 is on the top of the ranking list with respect to both distributive mode and ideal mode.

Stability of priority ranking is often tested applying sensitivity analysis. It facilitates the process of eliminating alternatives, enhancing a group decision process, or in providing information as to the robustness of a decision. If changes within criteria importance are likely to happen it is also recommended to calculate the degrees of sensitivity of the involved alternatives.

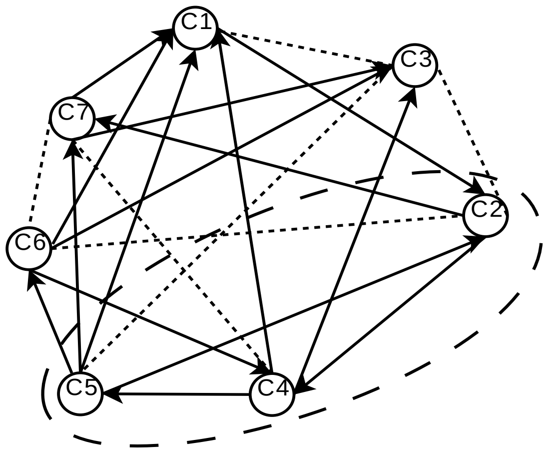

Figure 9: Connected graph where criteria C2, C4, and C5 are involved in a cycle

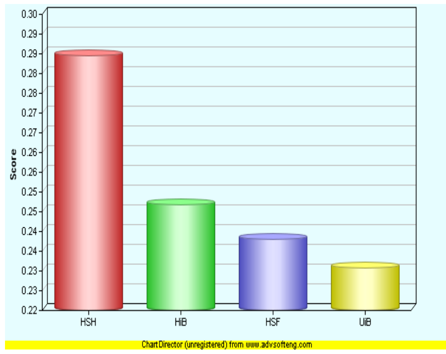

Figure 10: Global priority

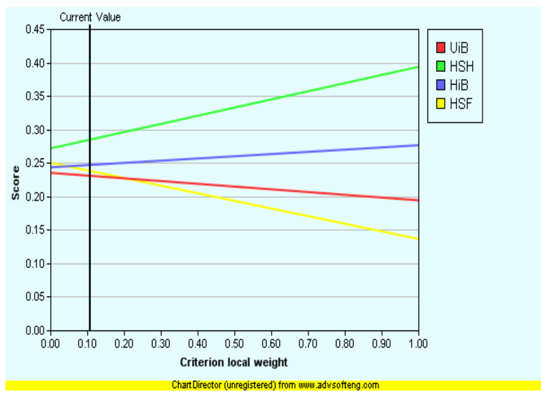

Figure 11: Sensitivity analysis of the alternatives with respect to criterion C1

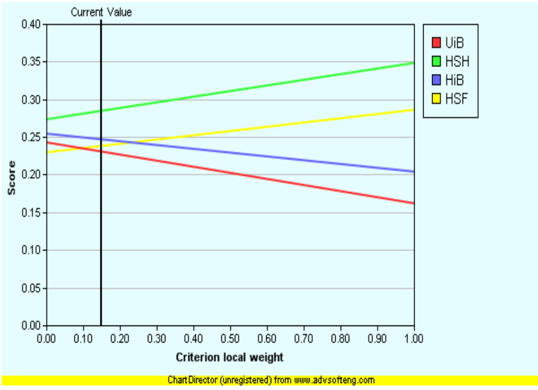

Figure 12: Sensitivity analysis of the alternatives with respect to criterion C2

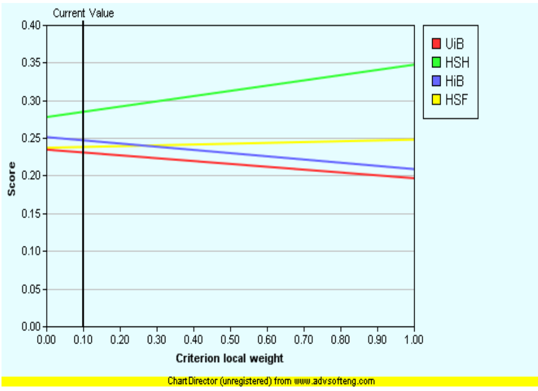

Figure 13: Sensitivity analysis of the alternatives with respect to criterion C3

Alternative A2 dominates with respect to criterion C1, Fig. 11. In the interval (0, 0.35) alternative A1 is least preferable.

Alternative A2 dominates with respect to criterion C2, Fig. 12. For $x > 0.35$ alternative A1 is least preferable.

Alternative A2 dominates with respect to criterion C3, Fig. 13 and alternative A1 is least preferable.

Alternative A2 dominates with respect to criterion C4, Fig. 14 in the interval (0, 0.35), $x > 0.35$ alternative A1 is most preferable.

Alternative A2 dominates with respect to criterion C5, Fig. 15 and alternative A1 is least preferable for $x > 0.15$.

Alternative A2 dominates with respect to criterion C4, Fig. 14 in the interval (0, 0.35), $x > 0.35$ alternative A4 is most preferable.

The importance of the criterion C7 can be seen on Fig. 17. The current priority for the criterion is 0.14 (the vertical line). The intersections of the vertical line with the lines of the alternatives illustrate the alternatives' priorities. A2 is the preferred alternative in the interval (0, 0.35), but if the cost criterion be-

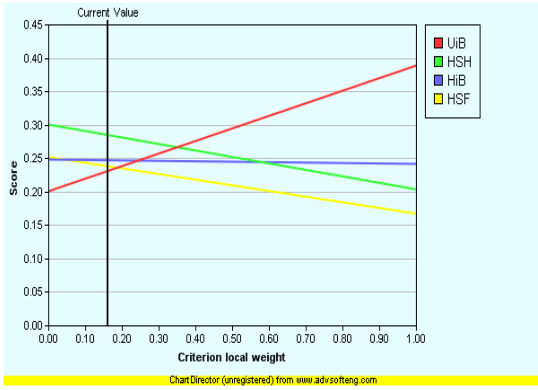

Figure 14: Sensitivity analysis of the alternatives with respect to criterion C4

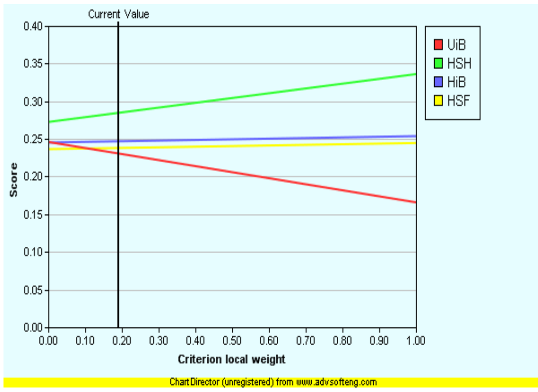

Figure 15: Sensitivity analysis of the alternatives with respect to criterion C5

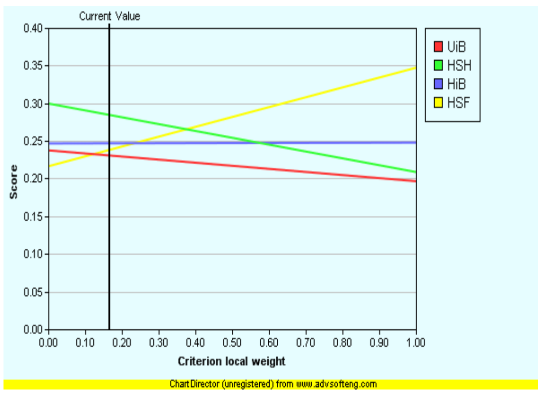

Figure 16: Sensitivity analysis of the alternatives with respect to criterion C6

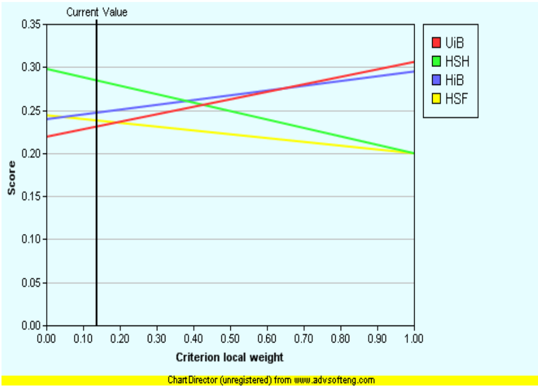

Figure 17: Sensitivity analysis of the alternatives with respect to criterion C7

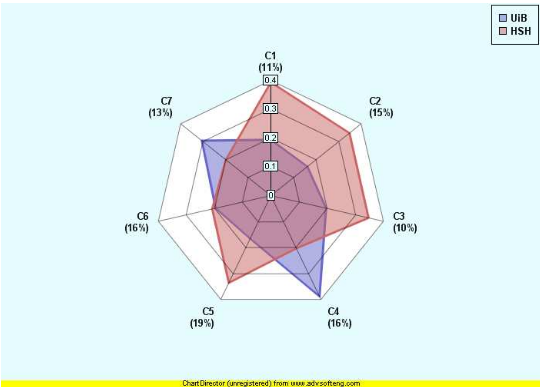

Figure 18: Head-to-head sensitivity analysis of the alternatives A1 and A2

comes more important then A1 will be the preferred alternative.

A head-to-head sensitivity analysis between two alternatives shows the relative magnitude of the alternatives compare with respect to the involved criteria.

Fig. 18 indicates dominance of alternative A2 over alternative A1 with respect to criteria C1, C2, C3 and C5.

Fig. 19 indicates dominance of alternative A2 over alternative A3 with respect to criteria C1, C2, C3 and C5.

Fig. 20 indicates dominance of alternative A2 over alternative A4 with respect to criteria C1, C2, C3 and C5.

Fig. 21 indicates dominance of alternative A3 over alternative A1 with respect to criteria C1, C2, C5 and C6.

Fig. 22 indicates dominance of alternative A4 over alternative A1 with respect to criteria C2, C3, C5 and C6.

Fig. 23 indicates dominance of alternative A3 over alternative A4 with respect to criteria C2, C3, and C5.

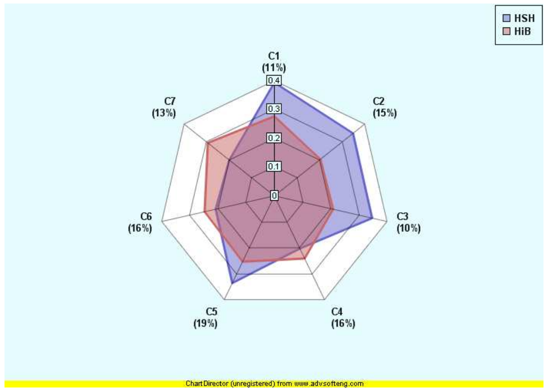

Figure 19: Head-to-head sensitivity analysis of the alternatives A2 and A3

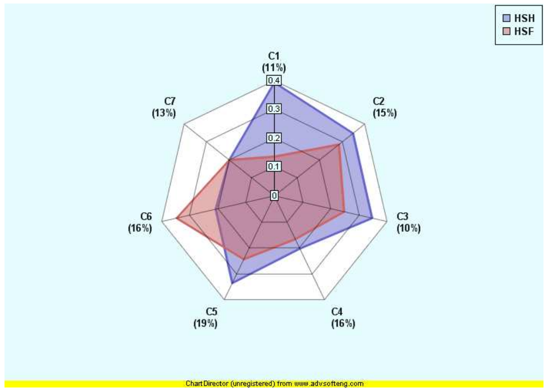

Figure 20: Head-to-head sensitivity analysis of the alternatives A2 and A4

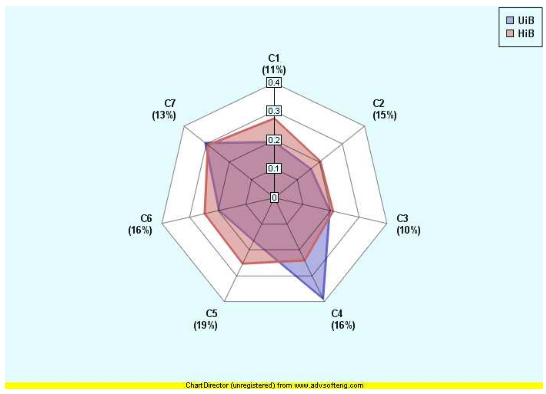

Figure 21: Head-to-head sensitivity analysis of the alternatives A1 and A3

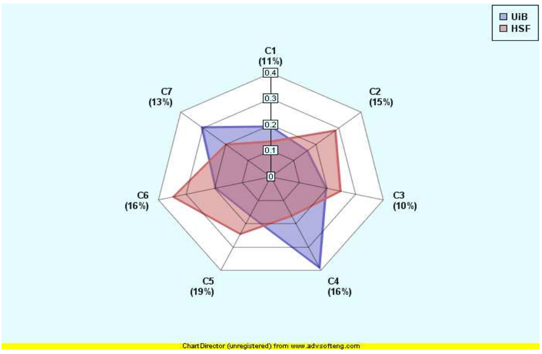

Figure 22: Head-to-head sensitivity analysis of the alternatives A1 and A4

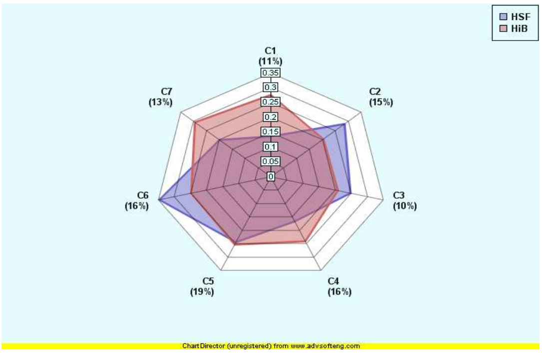

Figure 23: Head-to-head sensitivity analysis of the alternatives A3 and A4

Sensitivity and head-to-head analysis, performed for all criteria and alternatives, are obtained via the AHPproject - Free Web-Based Decision Support Tool [14].

4 Conclusion

In this work we have been considering the effect of 'ties' on the possible number of cycles and the level of significance of appearance of cycles in clusters. Theoretically obtained results can be applied on real data. The outcome is quite encouraging for continuing with further investigations related to graphs and cycles. We hope that in due time this approach will be often used by private and public sector decision-makers.

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