Generalized Maximum Entropy Estimation of Spatial Panel Interaction Models

ROSA BERNARDINI PAPALIA
Department of Statistics
University of Bologna
Via Belle Arti 41, Bologna
ITALY
rossella.bernardini@unibo.it

Abstract: - Flow data are viewed as cross-classified data, and spatial interaction models are reformulated as log-linear models. According to this view, we introduce a spatial panel data model and we derive a Generalized Maximum Entropy – based estimation formulation. The estimator we propose has the advantage of being consistent with the underlying data generation process and eventually with the restrictions implied by some non sample information or by past empirical evidence by also controlling for collinearity and endogeneity problems.

Key-Words: - Maximum entropy estimation, spatial panel data models, spatial Interaction models.

1 Introduction

The analysis of macro data at the country or industry level in trade has a long tradition in the empirical literature. More specifically, spatial interaction models or gravity models (Linneman, 1966; Ord, 1975) have recently become an useful tool for applied trade analysis. These models relate bilateral trade to the aggregate supply of the exporting country, the aggregate demand of the importing country, transport and transaction costs, and other specific trade factors.

The recent empirical trade literature (Fischer et al., 2006; Cheng et al., 2005; Egger et al., 2003; LeSage et al., 2005) seems to suggest: (i) panel data models which have the advantage to control for heterogeneity among the countries and cross sectional correlation, and (ii) to take into account the potential endogeneity between trade and output. At the same time, stressing the importance of the cross-sectional spatial dependence term that can represent the spatial effect of regional governance on trade flows, an appropriate estimation methodology that allows for such an effect is needed.

In the estimation of gravity models, this paper addresses these issues and also the problem of zero trade flows. Two common approaches to handle the presence of zero trade involve either discarding the zeros from the sample or to add a constant factor to each observation on the dependent variable, introducing a log-transformation of the model with a multiplicative exponential error term and assuming that the pattern observed in the empirical distribution is better represented by lognormal distribution. If the zeros are not randomly distributed, the first strategy induces a selection bias. With reference to the second strategy, the heteroskedasticity inherent in the log-linear formulation of the model can produce both biased and inefficient OLS estimates.

In our approach, trade flows, from incomplete data, are represented by spatial interaction models which were originally developed in geography. Flow data were viewed as cross-classified data, also referred to as contingency tables, and in this perspective spatial interaction models were reformulated as log-linear models. In this view, we proceed by introducing a panel data model specification which recovers information on trade flows from incomplete data and by estimating the spatial econometric flow model by using a Generalized Maximum Entropy estimation approach.

The proposed estimator has the advantage of being consistent with the underlying data generation process and eventually with the restrictions implied by some non sample information or by past empirical evidence while controlling for collinearity and endogeneity problems.
The paper is structured as follows. The next session introduces the theoretical foundations for the gravity model. The third section concentrates on specifications of spatial panel interaction models. Section four formalizes the proposed generalized maximum entropy estimation approach. The forth section shows an empirical application and the last section concludes.

2 Theoretical foundations: literature review

The gravity model was originally founded on Newton's physical theory which states that two bodies attract each other in proportion to their masses and inversely by the square of the distance between them. The application of the gravity model to international trade theory, on the other hand, aims at explaining the bilateral trade flows and patterns between wo economies by regarding each of them as an organic body that attracts each other in proportion to their economic size (GDP) and inversely to their distance.
The basic assumption of the gravity model, therefore, states that the bilateral trade flows are positively related to the product of the two countries' GDPs and negatively related to the distance between them.
The gravity model was first applied to the international trade field by Tinbergen (1962) and Pöynöhen (1963) in the early 1960s. With the increasing importance of geographical factors in international trade theory, the gravity model started to attract a reawakening interest in the 1980s. Works by Krugman and Helpman (1985), Bergstrand (1989), Deardorff (1995) and Evenett and Keller (1998) greatly contributed to the establishment of a theoretical foundation for the gravity model by showing that the gravity equation can be derived from a number of different international trade models.
There are two competing models of international trade that provide theoretical justification for the gravity model. They are the Heckscher-Ohlin (H-O) Model and the Differentiated Products Model.
The standard assumption of the Heckscher-Ohlin model that prices of traded goods are the same in each country has proved to be faulty due to the presence of what trade economists call "border effects." Properly accounting for these border effects requires prices of traded goods to differ among the countries of the world.
Deardorff (1995, 1998) has shown that the gravity model can be derived from several variants of the Heckscher-Ohlin model based on comparative advantage and perfect competition if it is properly considered. He found out that the absence of all barriers to trade in homogeneous products causes producers and consumers to be indifferent to the trading partners, both domestic and foreign, so long as they buy or sell the desired goods. Based on this assumption, he derived expected trade flows that correspond exactly to the simple frictionless gravity equation whenever preferences are identical and homethetic.
While the H-O theory would account for the success of the gravity equation in explaining bilateral trade flows among countries with large factor proportion differences and high shares of inter-industry (so-called 'North-South' trade), the Differentiated Product Model would serve well in explaining the bilateral trade flows among countries with high shares of intra-industry trade (so called 'North-North' trade).
Anderson (1979) employing the product differentiation by country of origin assumption, commonly known as the "Armington assumption" (Armington, 1969). By specifying demand in these terms, Anderson helped to explain the presence of income variables in the gravity model, as well as their multiplicative (or log linear) form. This approach was also adopted by Bergstrand (1985) who more thoroughly specified the supply side of economies. The result was the insight that prices in the form of GDP deflators might be an important additional variable to include in the gravity equations described above. Price effects have also been captured using real exchange rates.
More specifically, Anderson (1979) and Helpman and Krugman (1985) tried to identify the relationship between the bilateral trade flows and the product of two countries' GDPs

by utilizing the Differentiated Products Model. According to Krugman et al., under the imperfect substitute model, where each firm produces a product that is an imperfect substitute for an other product and has monopoly power in its own product, consumers show preference for variety. When the size of the domestic economy (or population) doubles, consumers increase their utility, not in theform of greater quantity but of greater variety. International trade can provide the same effect by increasing consumers' opportunity for even greater variety. Therefore, when two countries have similar technologies and preferences, they will naturally trade more with each other in order to expand the number of choices available for consumption. The correspondence between the gravity equation and the Differentiated Products Model was empirically proven by Helpman (1987) by applying his test on OECD countries' trade data. His results supported the argument that the gravity equation can be applied to the trade flows among industrialized countries where intra-industry trade and monopolistic competition are well developed. In contrast, Hummel & Levinsohn (1995) conducted a similar empirical test with a set of non-OECD countries where monopolistic competition was not so plausible. To their surprise, they proved that the gravity equation is also efficient in explaining the trade flows among developing countries where inter-industry trade is dominant with scarce monopolistic competition. Their findings questioned the uniqueness of the Product Differentiation model in explaining the success of the gravity equation and proved that a variety of other models, including the H-O model, can serve as alternatives.

The monopolistic competition model of new trade theory has been another approach to providing theoretical foundations to the gravity model (Helpman, 1987 and Bergstrand, 1989). The product differentiation by country of origin approach is here replaced by product differentiation among producing firms, and the empirical success of the gravity model is considered to be supportive of the monopolistic competition explanation of intra-industry trade. However, Deardorff (1998) and Feenstra (2003) have cast doubt on this interpretation, noting the compatibility of the gravity equation with some forms of the Heckscher-Ohlin model and, consequently, the need for empirical evidence to distinguish among potential theoretical bases: product differentiation by country of origin; product differentiation by firm; and particular forms of Heckscher-Ohlin-based comparative advantage. In each of these cases, the common denominator is complete specialization by countries in a particular good. Without this feature, bilateral trade tends to become indeterminate.

Alternatively, there are other approaches to gravity-based explanations of bilateral trade that do not depend on compete specialization. This involves accounting for trade frictions in the form of distance-based shipping costs or other trade costs, as well as policy-based trade barriers. Distance costs can also be augmented to account for infrastructure, oil price, and trade composition. The two approaches (complete vs. incomplete specialization) can be empirically distinguished by category of good, namely differentiated vs. homogeneous.

The justification for the gravity equation can be also analysed in the light of a partial equilibrium model of export supply and import demand as developed by Linneman (1966). Based on some simplifying assumptions the gravity equation turns out, as Linneman argues, to be a reduced form of this model. Using a trade share expenditure system Anderson (1979) also derives the gravity model which postulates identical Cobb-Douglas or constant elasticity of substitution (CES) preference functions for all countries as well as weakly separable utility functions between *traded* and *non-traded* goods. The author shows that utility maximization with respect to income constraint gives *traded goods* shares that are functions of *traded goods* prices only. Prices are constant in cross-sections; so using the share relationships along with trade balance/imbalance identity, country j's imports of country i's goods are obtained. Then assuming log linear functions in income and population for traded goods shares, the gravity equation for aggregate imports is obtained.

Further justification for the gravity model approach is based on the Walrasian general equilibrium model, with each country having its own supply and demand functions for all goods. Aggregate income determines the level of demand in the importing country and the level of supply in the exporting country (Oguledo and Macphee 1994). While Anderson's (ibid.) analysis is at the aggregate level, Bergstrand (1985, 1989) develops a microeconomic foundation to the gravity model. He opines that a gravity model is a reduced form equation of a general equilibrium of demand and supply systems. In such a model the equation of trade demand for each country is derived by maximizing a constant elasticity of substitution utility function subject to income constraints in importing countries. On the other hand, the equation of trade supply is derived from the firm's profit maximization procedure in the exporting country, with resource allocation determined by the constant elasticity of transformation. The gravity model of trade flows, proxied by value, is then obtained under market equilibrium conditions, where demand for and supply of trade flows are equal. Bergstrand argues that since the reduced form eliminates all endogenous variables out of the explanatory part of each equation, income and prices can also be used as explanatory variables of bilateral trade. Thus instead of substituting out all endogenous variables, Bergstrand (ibid.) treats income and certain price terms as exogenous and solves the general equilibrium system retaining these variables as explanatory variables. The resulting model is termed a "generalized" gravity equation (Krishnakumar 2002).

Eaton and Kortum (1997) also derive the gravity equation from a Ricardian framework, while Deardorff (1998) derives it from a H-O perspective. Deardorff opines that the H-O model is consistent with the gravity equations. As shown by Evenett and Keller (1998), the standard gravity equation can be obtained from the H-O model with both perfect and imperfect product specialization. Some assumptions different from increasing returns to scale, of course, are required for the empirical success of the model. Economies of scale and technology differences are the explanatory factors of the comparative advantage instead of considering factor endowment as a basis of this advantage as in the H-O model (Krishnakumar 2002).

To test for the relevance of monopolistic competition in international trade Hummels and Levinsohn (1993) use intra-industry trade data. Their results show that much intra-industry trade is specific to country pairings. So their work supports a model of trade with monopolistic competition.

Therefore, the gravity equation can be derived assuming either perfect competition or a monopolistic market structure. Also neither increasing returns nor monopolistic competition is a necessary condition for its use if certain assumptions regarding the structure of both product and factor market hold. Anderson et al. (2003) also derive import gravity equation as a function of income and trade cost. Trade cost is mainly transport cost in this kind of model which is related to distance.

Trade theories just explain why countries trade in different products but do not explain why some countries' trade links are stronger than others and why the level of trade between countries tends to increase or decrease over time. This is the limitation of trade theories in explaining the size of trade flows. Therefore, while traditional trade theories cannot explain the extent of trade, the gravity model is successful in this regard. It allows more factors to be taken into account to explain the extent of trade as an aspect of international trade flows (Paas 2000).

3 Spatial panel interaction model specification

The most common formulation of the models for origin i to destination j flows start by vectorizing the n by n square matrix of interregional flows from each of the n origin regions to each of the n destination regions (with i=1,..n and j=1,..,n). An n^2 –components vector of flows is then obtained by stacking the columns of the flow matrix into a variable vector that we designate as *Y*. The objective of flow models is to explain variation in the magnitude of flows between each origin-destination pair. Following LeSage and Pace (2005), our focus is on a formal methodology for accounting for spatial dependence in the origin-destination flows. The basic idea is that: i) large commodity flows from region O (origin) to region D (destination) might be accompanied by similarly large flows from neighbors to region O to region D; ii) large commodity flows from region O to region D might be accompanied by similarly large flows from region O to neighbors to region D; and iii) large commodity flows from region O to region D might be accompanied by large flows from neighbors to region O to neighbors of region D. In accordance with this: i) is labeled as origin-based dependence, ii) as destination-based dependence and iii) as origin-destination dependence.

Conventional gravity models use explanatory variables containing characteristics of both the origin and destination regions in an attempt to explain variation in the vector Y containing interregional flows. In addition, an intercept term and a n^2 by 1 vector of distances between all origins and destinations are typically used as additional variables.

With reference to a panel data framework, by applying a log-transformation to the standard gravity model, the resulting structural model takes the following log-additive specification:

$$Y = \alpha\iota + X_d\beta_d + X_o\beta_o + D\gamma + \varepsilon \qquad (1)$$

where Y is a vector of dimension n^2T where observations 1 to n reflect flows from origin 1 to all n destinations for all times T.

In (1), the explanatory variable matrices Xd, Xo represent n^2T by K matrices containing destination and origin characteristics respectively and the associated K parameter vectors are βd and βo. The matrix Xd is constructed using characteristics of the destination node for each of the origin-destination (O-D) pair observations, and the matrix Xo is similarly constructed from the origin node in the O-D pairs representing the sample of observations. The vector D denotes the n^2T origin-destination distances and the scalar parameter γ reflects the effect of distance D, α denotes the constant term parameter and ι is a vector of ones of dimension n^2T. Typically these regression models assume $\varepsilon \sim N(0; \sigma^2 I\, n^2{}_T)$. When spatial dependence is introduced, the correlation across cross-sectional units is non-zero, and the pattern of non-zero correlations follows a certain spatial process. With a few exceptions, use of spatial lags typically found in spatial econometric methods have not been used in the spatial interaction models. models where each observation represents a region rather than an origin-destination pair.

The family of models introduced by LeSage and Pace (2005) rely on a spatial autoregression filtering shown in (2) that takes into account origin, destination, and origin-to-destination dependence:

$$\begin{aligned} y &= \alpha\iota + X_d\beta_d + X_o\beta_o + D\gamma \\ &\quad + \rho_1 W_o y + \rho_2 W_d y + \rho_3 W_w y + \varepsilon \end{aligned} \qquad (2)$$

where, $W_o = I_n \otimes W$, where I_n is an identity matrix of dimension n and $\otimes$ denotes the Kronecker product. W represents an nT by nT spatial weight matrix whose diagonal elements are zero. The specification of the spatial weights is typically driven by geographic criteria, such as contiguity (sharing a common border) or distance, including nearest neighbor distance. The matrix W_o captures origin-based spatial dependence of the type labelled 1). Similarly, $W_d = W \otimes I_n$ is used to capture type 2) dependence, or destination-based dependence relations. $W_w = (I_n \otimes W)(W \otimes I_n) = W \otimes W$ reflects type 3) dependence that we referred to as origin-destination based dependence. In addition, ρ_1, ρ_2, ρ_3 are the unknown spatial interaction parameters referred to the W_o, W_d, and W_w spatial weight matrices, respectively.

The model in (2) can give rise to a family of other models by placing various restrictions on the parameters ρ_1, ρ_2, ρ_3. The conventional assumption of a normal distribution for the disturbances in the data generating process (and the implied normal distribution of the origin-destination flow magnitudes) may not be a valid one. It follows that estimation procedures that allow for a fat-tailed error distribution (Gelfand and Smith, 1990, Geweke, 1993) are needed.

4 Generalized Maximum Entropy estimation

In the context of this work, we adapt the approach presented in Bernardini Papalia (2009, a, b) to the spatial interaction model. More specifically, a spatial lag model specification, as specified in equation (2), which includes a spatially lagged dependent variable is here considered:

$$\begin{aligned} y &= \alpha\iota + X_d\beta_d + X_o\beta_o + D\gamma \\ &\quad + \rho_1 W_o y + \rho_2 W_d y + \rho_3 W_w y + \varepsilon \end{aligned} \qquad (3)$$

This model can also be written as:

$$Y = \rho_1 W_o y + \rho_2 W_d y + \rho_3 W_w y + X\beta + \varepsilon; \qquad (4)$$

where $X = \lfloor \iota; X_O; X_d; D \rfloor$, $\beta = \lfloor \alpha'; \beta_O; \beta_d; \gamma \rfloor$, and the other notation is as before.

It is assumed that: spatial effects are not identical across spatial units; differentiated spatial effects within and between spatial units are taken into account.
In empirical applications, it is common practice to derive spatial weights for W_o , W_d and W_w from the location and spatial arrangements of observation by means of a geographic information system. In this case, units are defined 'neighbors' when they are within a given distance of each other, ie $w_{ij}=1$ for $d_{ij} \leq \delta$ and $i \neq j$, where d_{ij} is the distance function chosen, and δ is the critical cut-off value. More specifically, a spatial weights matrix W* is defined as follow:

$$w_{ij}^* = \begin{cases} 0 & \text{if} \quad i = j \\ 1 & \text{if} \quad d_{ij} \leq \delta, i \neq j \\ 0 & \text{if} \quad d_{ij} > \delta, i \neq j \end{cases}. \tag{5}$$

and the elements of the row-standardized spatial weights matrix W (with elements of a row sum to one) result:

$$w_{ij} = \frac{w_{ij}^*}{\sum_{j=1}^{N} w_{ij}^*}, \qquad i, j = 1,.., N. \tag{6}$$

Under the GME framework the objective is to recover simultaneously the unknown parameters, the unknown errors by defining an inverse problem, which is based only on indirect, partial or incomplete information. In this respect, the parameters of the spatial interaction model are estimated with minimal distributional assumptions (Golan et al. 2006; Samilov et al. 2006; Fragoso et al., 2008, Wu et al. 2007).
Each parameter is treated as a discrete random variable with a compact support Z and M possible outcomes, $2 \leq M \leq \infty$. The uncertainty about the outcome of the error process is represented by treating each error as a finite and discrete random variable with J possible outcomes, $2 \leq J \leq \infty$. A set of discrete points, the support space $V=[v_1,v_2,...,v_J]'$ of dimension $J \geq 2$, that are at uniform intervals and symmetric around zero, are chosen and each error term has corresponding unknown weights $r_i=[r_{i1},r_{i2}...,r_{iJ}]'$ that have the properties of probabilities $0 \leq r_{ij} \leq 1$ and $\sum_j r_{ij}=1$. In practice, discrete support spaces for both the parameters and errors are defined according to economic theory or other prior information.
In matrix notation, the unknown parameters and errors are reparameterized as:

$$\rho_1 = \left(Z^{\rho_1} p^{\rho_1}\right), \rho_2 = \left(Z^{\rho_2} p^{\rho_2}\right),$$
$$\rho_3 = \left(Z^{\rho_3} p^{\rho_3}\right), \qquad ,$$
$$\beta = Z^{\beta} p^{\beta}, \varepsilon = Vr$$

yielding the following GME specification:

$$\begin{aligned} Y &= \left(Z^{\rho_1} p^{\rho_1}\right) W_o Y + \left(Z^{\rho_2} p^{\rho_2}\right) W_d Y \\ &+ \left(Z^{\rho_3} p^{\rho_3}\right) W_w Y \\ &\quad + X\left(Z^{\beta} p^{\beta}\right) + (Vr); \end{aligned} \tag{7}$$

where p^{β}=vec(p^{α},$p^{\beta o}$,$p^{\beta d}$,p^{γ}) consists of vectors of weights p^{α},$p^{\beta o}$,$p^{\beta d}$,p^{γ}, each having nonnegative elements summing to unity and r are vectors of proper probability distributions for parameters and errors, respectively.
Given the data consistency (8), the GME objective function H(.), relative to our formulation problem, may be formulated as:

$$H(p^{\beta}, r) = -p^{\beta}{}' \ln p^{\beta} - r' \ln r \tag{8}$$

subject to:
data consistency conditions (7):
(*ii*) adding-up constraints:

$$\begin{aligned} 1' p_k^{\beta} &= 1 \forall k; \\ 1' r_i &= 1 \forall i. \end{aligned} \tag{9}$$

The advantages of the GME estimation approach for spatial panel gravity models are: first, it is possible to obtain consistent estimates of the individual fixed effects when $N \rightarrow \infty$ (the incidental parameter problem); second, this estimation procedure deals with the problem of endogeneity of the spatial lag term as specified in spatial lag interaction models and within a panel data framework.

5 An application to commodity flows between Italy and European countries of the Balkanic area

To illustrate the ideas discussed in Section 2 we produced GME estimates for the spatial lag model in (2) using commodity flows between Italy and European countries of the Balkanic area that covers a total of seven countries (Albania, Bosnia, Erzegovina, Bulgaria, Croazia Macedonia, Romania, Serbia Montenegro) during the years from 1998 to 2004. It is assumed that the volume of exports between Italy and each of the other countries is determined by the following set of explanatory variables (*Xo* and *Xd*): gross domestic product (GDP), openness of treading countries, GDP per capita differential, distance as a proxy of transportation costs, and a set of dummies variables either facilitating or restricting trade between pairs of countries or specific sub-groups of countries that identify some communication or transportation networks. The weight matrix is computed by means of the distance between the capital cities, with a critical cut-off value equal to the first quartile.
We would expect that changes in per capita GDP would exhibit positive signs, leading to higher levels of commodity flows at both the origin and destination regions. The coefficient estimate on distance should be negative indicating a decay of flows with distance.
A negative sign for the spatial autorrelation coefficient indicates negative spatial dependence between flows from an origin-destination pair and flows from neighbors to the origin and neighbors to the destination regions.
Our results (see Table 1) show that Italy's export is positively determined by the size of the economies, and openness of the countries involved. With regards to the country specific effects, we observe that these effects are strongly significant for all countries. It has been found that transportation costs are significant factors in influencing Italy's exports negatively. This implies Italy would be influenced to a grater extent by the border between the Balkanic area and Italy.

TABLE 1 _ GME estimates

Variables	*Coefficient*	*Std. Error*
ln per cap GNP Diff	0.6097	0.0481
ln impj/GDPj	0.0018	0.0863
ln openness	0.0463	0.0987
ln distance	0.6304	0.0890
spatial dep. Var	0.9088	0.0046
2000_dummy	0.0336	0.0024
2001_dummy	0.0077	0.0024
2002_dummy	0.0048	0.0023

Note: Dependent variable, Italian exports to country j (in logs); time effects to control for business cycles are included;

6 Conclusion

Despite numerous applications in empirical trade analysis, there are still open issues related to the estimation of gravity models. First, cross-sectional correlation is present already because of the construction of this kind of models, which involve bilateral trade flows and aggregate national variables. Second, even if few authors estimate dynamic panel data models in order to catch the relevance of persistence in bilateral trade patterns, the introduction of dynamics in a panel data model produces inconsistency of the estimators due to the endogeneity of the lagged dependent variable.
In analyzing trade dynamics throughout spatial interaction models, the contribution of this study comes from the combination of (i) the use of panel data with spatial unobserved heterogeneity which allows estimating elasticity of trade with respect to its determinants, and (ii) an adequate estimation technique which deals with problems of potential endogeneity of the gross domestic product (GDP) variables when bilateral specific effects are accounted for or when a dynamic panel data model specification is assumed.
From a more general point of view, the GME estimation method has advantages in cases involving small samples or ill-posed problems and is computationally efficient and robust for

given support points chosen according to prior information and/or past empirical evidence.
An illustrative application of the spatial GME estimator in the context of the analysis of commodity flows between Italy and European countries of the Balkanic area that covers a total of seven countries (Albania, Bosnia, Erzegovina, Bulgaria, Croazia Macedonia, Romania, Serbia Montenegro) and refers to the period of 1998 to 2004, has been provided. Our results have shown that Italian's trade is determined by the size of the economies, per capita GNP differential of the countries involved, openness of the trading countries and distance between the two countries' capitals (proxy of transportation costs). The role of both the spatial lag dependent variable and the lagged bilateral trade variable seems to be confirmed.
Further empirical investigations could be implemented with the aim of considering alternative formulations for spatial weights, based on different geographic criteria (great circle distance, k nearest neighbors) as well as derived from aggregate trade flows between countries.

Acknowledgements

This paper has been completed within the BLU-ETS project "Blue-Enterprises and Trade Statistics", a small or medium-scale focused research project funded by the Seventh Framework Programme of the European Commission, FP7-COOPERATION-SSH (Cooperation Work Programme: Socio-Economic Sciences and the Humanities).

References:

[1] Anderson, J.E. (1979). A theoretical foundation for the gravity equation. The American Economic Review, 69: 106-16.
[2]Armington, P. (1969). A Theory of Demand for Products Distinguished by Place of Production.'' IMF Staff Papers 16 (3), 159–76.
[3] Anderson, J.E. and Wincoop E. V. (2003). Gravity with gravitas: a solution to the border puzzle. The American Economic Review, 93(1), 170-92, Nashville.
[4] Anselin L. (1988). *Spatial Econometrics: Methods and models*. Dordrecht, the Netherlands, Kluwer.
[5] Anselin L. (2001). Spatial Econometrics. In *A companion to theoretical econometrics*, ed. B.H. Baltagi, 310-330. Malden, Blackwell.
[6] Barry R.P. and Pace R.K. (1999) Monte Carlo estimates of the log determinant of large sparse matrices, *Linear Algebra and Its Applications* , 289, pp. 41-54.
[7] Bell K.P. and Bockstael N.E. (2000) Applying the generalized-moments estimation approach to spatial problems involving microlevel data, *Review of Economics and Statistics*, 82, pp. 72-82.
[8] Bergstrand, J.H. (1985). The gravity equation in international trade: some microeconomic foundations and empirical evidence. *The Review of Economics and Statistics,* 67: 474-81. Harvard University Press.
[9] Bernardini Papalia R. (2009a) Esimating mixed spatial processes in information-theoretic frameworks. In: *Recent advances in mathematics and computers in business and economics*. 10th WSEAS Int. Conference on Mathematics and Computers in Business and Economics (MCBE'09). Prague. March 23-25, 2009. (pp. 235 - 242). ISBN: 978-960-474-063-5. PRAGUE: WSEAS Press (CZECH REPUBLIC
[10] Bernardini Papalia R. (2009b) Estimating spatial heterogeneous panel data: an Information-Theoretic Approach, WSEAS *Transactions on Business and Economics*, 6-4, 167-176.
[11] Deardorff, V. Alan(1995) " Determinants of Bilateral Trade: Does Gravity Work in a Neoclassic World? " *National Bureau of Economic Research,* Working Paper 5377.
[12] Deardorff, A. (1998). Determinants of bilateral trade: does gravity work in a classical world? In The Regionalization of the World Economy, ed. by Jeffrey Frankel. Chicago: University of Chicago Press.
[13] Eaton, J. and Kortum, S. (1997). Technology and bilateral trade', in NBER *Working Paper*, No. 6253, Cambridge, MA: National Bureau of Economic Research.
[14] Elhorst JP (2001) Dynamic Models in Space and Time. Geographical Analysis 33: 119-140.
[15] Evenett, Simon and Wolfgang Keller (1998) "On Theories Explaining the Success of the Gravity Equation" *National*

Bureau of Economic Research, Working Paper 6529.

[16] Fragoso R., De Belem Martins M., Ventura Lucas MR. (2008) Generate disaggregated soil allocation data using a Minimum Cross Entropy Model, WSEAS *Transactions on Envieronment and Development*, 9, 756-566.

[17] Golan A, Judge G, Miller D (1996). Maximum entropy econometrics: robust estimation with limited data. Wiley, New York.

[18] Elhorst JP (2003) Specification and Estimation of Spatial Panel Data Mod*els, International Regional Science Review*, 26:3, 244-268.

[19] Griffith DA (1996) Computational simplifications for Space-Time Forecasting within GIS: The Neighbourhood Spatial Forecasting Models. In Spatial Analysis: Modelling in a GIS Environment, eds, P. Longley and M. Batty. Cambridge, UK: Geo Information International.

[20] Helpman, E. and P. Krugman (1985) "Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition and the International Economy." MIT Press,Cambridge, MA.

[21] Helpman, E. (1987) "Imperfect Competition and International Trade: Evidence from Fourteen Industrial Countries." *Journal of Japanese and International Economy.* MIT Press, Cambridge, MA.

[22] Hepple LW (1978) The Econometric Specification and Estimation of Spatio-Temporal Models. In Time and Regional Dynamics, eds., T. Carlstein, D. Parkes, and N. Thrift. London, Edward Arnold

[23] Honoré B.E. and Hu L. (2004) Estimation of cross sectional and panel data censored regression models with endogeneity, *Journal of Econometrics*, 122, pp. 293-316.

[24]Hsiao C (1989) Modeling Ontario Regional Electricity System Demand Using a Mixed Fixed and Random Coefficient Approach, *Regional Science and Urban Economics*, 19.

[25] Hummels, D. and Levinsohn, J. (1993). Product differentiation as a source of comparative advantage. American Economic Review, Papers and Proceedings, LXXXIII: 445-49.

[26] Hummels, D. and J. Levinsohn (1995) "Monopolistic Competition and International Trade: Reconsidering The Evidence." *Quarterly Journal of Economics.*110(3):799-836.

[27] Karemera, D. et al (1999). A gravity model analysis of the benefits of economic ntegration in the Pacific Rim', *Journal of Economic Integration,* Vol.14, No. 3 (September): 347-67.

[28] Kelejian H.H. and Prucha I.R. (1999) A generalized moments estimator for the autoregressive parameter in a spatial model, *International Economic Review*, 40, pp. 509-533.

[29] Konings J. (2001) The Effects of Foreign Direct Investment on Domestic Firms: Evidence from Firm Level Panel Data in Emerging Economies, *Economics of Transition*, Vol. 9, pp. 619-633.

[30] Krishnakumar, J. (2002). A SUR-EC-AR system gravity model of trade. *Working Paper*, Department of Economics, University of Geneva, Geneva, Switzerland.

[31] Kullback (1959) Information theory and statistics. Wiley, New York.

[32] Lesage J.P., Pace R.K. (2004) *Spatial and Spatiotemporal Econometrics* - Advances in Econometrics, Vol. 18. Elsevier Ltd.

[33] Lesage J.P., Pace R.K. (2005) *Spatial Econometric Modeling of Origin-Destination flows* – Working paper.

[34] Linneman, H. (1966). An econometric study of international trade flows. North Holland, Amsterdam.

[35]Marsh L Mittlehammer RC (2003) Generalized maximum entropy estimation of spatial autoregressive models. Working paper, Kansan State University

[36] Marsh L, Mittlehammer RC, Cardell S (1998) A Structural-Equation GME Estimator. Selected Paper for the AAEA Annual Meeting, 1998, Salt Lake City

[37] Mundlak Y (1978) On the Pooling of Time Series and Cross-Section Data, Econometrica, 36, 69-85.

[38] Oguledo, V.I. and Macphee, C.R. (1994). Gravity models: a reformulation and an application to discriminatory trade arrangements. Applied Economics, 26: 107-120.

[39] Paas, T. (2000). Gravity approach for modelling trade flows between Estonia and the main trading partners', *Working Paper,* No. 721, Tartu University Press, Tartu.

[40] Pesaran MH, Smith R (1995) Estimating long-run relationships from dynamic heterogeneous panels, *Journal of Econometrics*, 68, p. 79-113.
[41] Pöyhönen, Pentti (1963) "A Tentative Model for the Volume of Trade Between Countreis." *Weltwirtschafliches Archive*. Vol. 90, pp. 93-100.
[42] Samilov A. (2006) A development of Entropy Optimization Methods, WSEAS *Transactions on Mathematics*, 5, 568-575.
[43] Smirnov O. and Anselin L. (2001) Fast maximum likelihood estimation of very large spatial autoregressive models: A characteristic polynomial approach, *Computational Statistics & Data Analysis,* 35, pp. 301-319.
[44]Theil H (1971) Principles of Econometrics. John Wiley &Sons: New York.
[45] Tinbergen, Jan. (1962) *Shaping the World Economy – Suggestions for an International Economic Policy*, The Twentieth Century Fund.
[46] Wu Y, Fyfe C. (2007) The On-line Cross Entropy Method for Unsupervised Data Exploration, WSEAS *Transactions on Mathematic*, 6, 865-877.