Optimization Framework for Road Network Directed by Unblocked Reliability for Given Network Topology and Inelastic Demand with Stochastic User Equilibrium

SHUJUN HOU*, NAOKI MARUYAMA, MASAFUMI HIROTA and SEIZO KATO
Graduate School of Engineering
Mie University
1577 Kurimamachiya-cho, Tsu, 514-8507
JAPAN
*Corresponding author: shujunhou@yahoo.com

Abstract: - To provide more reliable service for drivers, it is necessary to establish an optimization framework in which a reliability index is adopted as a performance index to evaluate road networks. First, a modified four-level model of unblocked reliability (link, path, Origin-Destination pair and entire road network) is proposed, in which the unblocked reliabilities of the Origin-Destination pair and the entire road network are formulated by the law of total probability when the unblocked reliability of path is a conditional probability. Then, a bi-level program is established based on the new model of unblocked reliability since planners design the road network and drivers respond the change in the road network. The objective function of the upper-level program is the maximized balance between the unblocked reliability of the entire road network and the road network expansion ratio which implies cost of improving the road network. The program equivalent of Stochastic User Equilibrium is adopted as the lower-level model so as to achieve consistency in route choices and to model the congestion effect in the road network. Next, a set of link capacity expansions is determined as a planning scheme by solving the bi-level program. This planning scheme not only improves the link of lower reliability but also takes into account the performance of the Origin-Destination pair and the entire road network. The proposed optimization framework is capable of improving the road network to its highest possible reliability level with a minimum scale of road network expansion.

Key-Words: - Road Network Planning, Bi-level Program, Unblocked Reliability, Stochastic User Equilibrium, Logit loading model

1 Introduction

This paper studies an optimization method to improve a road network, in which the unblocked reliability is regarded as the performance index. From the viewpoint of the planner, the highest performance and the lowest cost are desired. With the increasing demand for better and more reliable service, the road network system has incorporated reliability analysis as an integral part of its planning, design and operation. Therefore, a reliability index of road network system performance is needed for road network planning. The problem of optimizing a road network for inelastic traffic demand and a given network topology is analyzed.

There are several reliability concepts in road network systems, such as the connectivity reliability [1], travel time reliability [2], capacity reliability [3], and unblocked reliability. Unblocked reliability is the probability of the road unit or system being able to maintain an unblocked state at the peak hour at which the highest traffic volumes are observed for one day [4]. A four-level model of road network unblocked reliability, comprising link, path, Origin-Destination (OD) pair, and entire road network, to assess the operation performance of the road network [5], in which the OD pair unblocked reliability is regard as parallel combined structures of paths. In graph theory, a network is comprised of links and nodes. All links referred to in this paper are directed, for example, link 1-2 and link 2-1 are different. A path is a sequence of nodes connected by directed links so that movement is feasible from the first node to the last node in the sequence. The OD pair represents the source and objective of a trip. In this paper, the four-level model is modified by the viewpoints of conditional probability and total probability.

Planners and managers in the upper level part determine the parameters of a road network, and drivers in the lower level part make choices with regard to the route of their travel in response to the change in the road network. Corresponding to this condition, a bi-level program model is used to solve the problem of road network optimization. A bi-level program has a hierarchical structure in which upper-level and lower-level decision makers select their strategies so as to optimize their objective functions, respectively. The objective function of the upper-level program is the maximized balance between the unblocked reliability of the entire road network and the road network expansion ratio. The entire road network unblocked reliability is formulated by the law of total probability when the unblocked reliability of a path is the conditional probability of route choice.

The upper-level decision maker should consider how the lower-level decision maker would react to a given upper-level decision, although he can not intervene in the lower-level decision of the decision maker. The equivalent program of Stochastic User Equilibrium (SUE) is adopted as a lower-level model so as to achieve consistency in route choices and to model the congestion effect in the network. The SUE is a state in which no driver can improve his/her perceived travel time by unilaterally changing routes and the perceived travel time has random error [6]. The lower-level program is solved by the Method of Successive Average [7] in conjunction with the logit loading model [8] based on a simple path enumeration. A path with no repeated nodes is called a simple path. The Hooke-Jeeves (H-J) [9] algorithm is applied to solve the proposed bi-level program. Then, the proposed optimization framework is tested by a local road network, which has 7 nodes and 18 links.

Nomenclature

A set of links in network
C capacity, pcu/h
d control factor of road network expansion scale
E expectation value
F objective function of upper-level program
f path flow, pcu/h
G number of links in road network
H function of unreliable probability
I set of all origin nodes
J set of all destination nodes
L length of a link, km
M set of paths used
n number of traffic zones
p route choice proportion
q traffic demand between Origin-Destination pair, pcu/h
R unblocked reliability index
s initial step length of exploratory move for link capacity expansion in Hook-Jeeves algorithm, pcu/h
T sum of travel time of each link on studied path, h
t travel time of a link, h
u importance factor of an Origin-Destination pair
V traffic volume, pcu/h
v step length of pattern move in Hook-Jeeves algorithm
X auxiliary link flow, pcu/h
x link flow, pcu/h
w number of simple paths between Origin-Destination pair
y link capacity expansion, pcu/h
Z objective function of lower-level program
α proportion to minimize objective function of a mathematical program
β direction factor in Hook-Jeeves algorithm, $\beta = 1$ for positive capacity expansion, and $\beta = -1$ for negative capacity expansion
δ indicator variable, has a value of 1 if link a is on path m from i to j; 0 otherwise
ε a small positive number as index of convergence test in Method of Successive Average algorithm, pcu/h
η convergence step length of exploratory move for link capacity expansion in Hook-Jeeves algorithm, pcu/h
θ dispersion parameter
λ conversion factor for physical dimension, h/(km·pcu)
μ expansion direction
ϕ reduction factor of step length of exploratory move in Hook-Jeeves algorithm
ω variable of integration in objective function of Stochastic User Equilibrium problem

Superscripts

e free-flow state
k counter for Method of Successive Average algorithm
h counter for pattern move in Hook-Jeeves algorithm
m used path between Origin-Destination pair, $m \in M$

Subscripts

a link in network, $a \in A$
g counter for exploratory move in Hook-Jeeves algorithm
i origin node, $i \in I$
j destination node, $j \in J$

2 Model of Unblocked Reliability

2.1 Link Unblocked Reliability

The value of the link unblocked reliability equals the unblocked trips divided by the total observation trips at a link at the peak hour. A curve that expresses the relationship between reliability and congestion can be obtained by means of a traffic survey. The unreliable probability of link a, $H(x_a/C_a)$, is defined as the following function [10]:

$$H(x_a/C_a)=\begin{cases} x_a/C_a, & 0 \le x_a/C_a \le 1 \\ 1, & x_a/C_a > 1 \end{cases} \quad (1)$$

Then, the link unblocked reliability, R_a, is defined by the following equation:

$$R_a = 1 - H(x_a/C_a) \quad (2)$$

2.2 Path Unblocked Reliability

The path unblocked reliability, R_{ij}^m, on path m connecting origin i and destination j, can be calculated similar to the link unblocked reliability as follows [11]:

$$R_{ij}^m = \begin{cases} 1 - V/C, & 0 \le V/C \le 1 \\ 0, & V/C > 1 \end{cases} \quad (3)$$

where V/C is the nominal degree of congestion on path.

2.3 Origin-Destination Pair Unblocked Reliability

The physical meaning of the OD pair unblocked reliability is the ratio of the unblocked trip time between an OD pair to the traffic demand. From the perspective of probability theory, the unblocked reliability of a path is the conditional probability that a driver maintains an unblocked state on his path when the total drivers choose the route by a rule in the road network. In this paper the rule is SUE. If a path-based assignment of SUE (lower-level program) is completed, a route choice pattern is obtained.

By the law of total probability, when a sample space is limited in an OD pair, the OD pair unblocked reliability, R_{ij}, is expressed as follows:

$$R_{ij} = \sum_{m \in M} R_{ij}^m \cdot p_{ij}^m \quad (4)$$

where p_{ij}^m is the loading proportion on the path m between the OD pair ij. The contribution of each path to the OD pair unblocked reliability is given by Eq. (4).

2.4 Unblocked Reliability of Entire Road Network

In a real network the importance of the OD pair should be considered. The importance factor of an OD pair, u_{ij}, that is the ratio of an OD demand to total demand, can be calculated by the following function:

$$u_{ij} = \frac{q_{ij}}{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne i}}^{n} q_{ij}} \quad (5)$$

where q_{ij} is the traffic demand between the OD pair ij and n is the number of traffic zones. For the entire road network, the choice probability of a used path, $\hat{p}_{ij}^m$, can be calculated by the following function:

$$\hat{p}_{ij}^m = u_{ij} \cdot p_{ij}^m \quad (6)$$

The sum of p_{ij}^m and the sum of $\hat{p}_{ij}^m$ are unitary for two layers, such as an OD pair and the total network, respectively.

By the law of total probability, when a sample space is the set of all used paths, the unblocked reliability of entire road network R, is expressed as follows:

$$R = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne i}}^{n} \sum_{m \in M} R_{ij}^m \cdot \hat{p}_{ij}^m \quad (7)$$

The same expression also can be deduced from the physical meaning of the entire road network unblocked reliability, which is the ratio of the unblocked trip times to the total trip times in the entire road network.

3 Upper-level Model of Bi-level Program

The upper-level model reflects the expectation of the planner that is maximum network surplus obtained by subtracting cost index associated with improving the road network from performance index. In this paper, the performance of the road network is expressed as an indicator of the unblocked reliability. The upper-level model must be able to respond to the change of the network property and cost of improvement. The upper-level model is established as follows:

$$\max F(y) = R - d \cdot \lambda \sum_{a \in A} \left(L_a \cdot y_a \right) \qquad (8)$$

where $y = (y_1, \cdots, y_a, \cdots, y_G)$ is a vector of the link capacity expansions, G is the total number of links in a road network, and $F(y)$ is the upper-level objective function, which is expected to be maximized when the link flows are provisionally fixed. R is the whole network unblocked reliability, which is a function of link flows.

The second term on the right-hand side of the upper-level model is called the penalty term, which represents the cost associated with improving the road network. Where d is a control factor of the road network expansion scale, λ is the unit transformation factor, L_a is the length of link a and y_a is the link capacity expansion of link a. The control factor, d, can magnify the impact of the cost of improving the road network, and $\sum_{a \in A} \left(L_a \cdot y_a \right)$ is the road network expansion scale. The conversion factor for the physical dimension, λ, which is a constant in a certain road network, is formulated as follows:

$$\lambda = \frac{1}{\sum_{a \in A} \left(L_a \cdot C_a \right)} \qquad (9)$$

where L_a and C_a are the length and capacity of each link in an existing road network, respectively. Thus, $\sum_{a \in A} \left(L_a \cdot C_a \right)$ indicates the existing road network scale, and $\lambda \sum_{a \in A} \left(L_a \cdot y_a \right)$ expresses the road network expansion ratio which implies cost of improving the road network. Then, the penalty term, $d \cdot \lambda \sum_{a \in A} \left(L_a \cdot y_a \right)$, becomes a dimensionless factor and ensures the network can be expanded to a reasonable degree.

4 Stochastic User Equilibrium Assignment as Lower-level Model of Bi-level Program

In this paper, the lower-level model is the SUE assignment. The SUE assignment is to find the link (or path) flows on a road network given a traffic demand matrix and a probabilistic route choice model.

Generally, the driver perception of travel time is assumed to be random. The stochastic network loading approach is called a logit-based loading model when the perceived link travel times follow a logit model. This model can be derived from the concepts of random perceived travel time and perceived travel time minimization by assuming that the random terms of each perceived travel time function are independently and identically distributed Gumbel variants. The aggregate share function gives the total number of individuals selecting a particular travel alternative. The proportion of traffic demand between the OD pair ij choosing path m is given by:

$$p_{ij}^{m} = \frac{\exp(-\theta T_{ij}^{m})}{\sum_{m=1}^{w} \exp(-\theta T_{ij}^{m})} \qquad (10)$$

where the dispersion parameter, θ, which scales the error of the perceived travel time, is a constant for a certain road network, the calculated travel time of path m, T_{ij}^{m}, is the sum of the travel time of each link on this path and w is the number of simple paths between an OD pair. The link travel time, $t_a(x_a)$, which represents the relationship between the flow and the travel time for link a, is calculated using the Bureau of Public Roads function [12], as shown below:

$$t_a(x_a) = t_a^e \left[1 + 0.15 \left(x_a / C_a \right)^4 \right] \qquad (11)$$

The path flows are expressed as a function of the OD flow and loading proportion as:

$$f_{ij}^{m} = q_{ij} \cdot p_{ij}^{m} \qquad (12)$$

The link flows are achieved by the indicator relationship of path-link:

$$x_a = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{m \in M} f_{ij}^{m} \delta_{ij}^{a,m} \qquad (13)$$

The stochastic network loading problem may be solved either in the space of the link flows or in the space of the path flows. An important disadvantage of path-based solutions is that they require explicit enumeration of the path choice set, which may be a huge number in large networks. However, this drawback has been relaxed tremendously in recent years with the rapid development of high-speed computers and large-capacity storage, and the proposed upper-level program needs path enumeration.

The depth-first search (DFS) [13] is adopted to find the total simple path in the network. DFS is a graph traversal algorithm, which extends the current path as far as possible before backtracking to the last choice node and trying the next alternative path. The graph traversal refers to the problem of visiting all the nodes in a graph. The DSF algorithm can be specialized to find a path when given a start node and end node. A stack is used to keep track of the path between the start node and the current node and avoid cycles. A stack is an abstract data type and data structure based on the principle of Last-In-First-Out. As an abstract data type, the stack is a container of nodes and has two basic operations, push and pop. Push adds a given node to the top of the stack leaving previous nodes below. Pop removes and returns the current top node of the stack.

The approach of path enumeration is illustrated by a road network as shown in Fig. 1, which consists of 7 nodes and 18 links. The network can be equivalently represented by an adjacency list as shown in Table 1. The DFS algorithm is moved along the adjacency list as the digital form of the network. The OD pair (2, 5) is studied to generate simple paths. Node 2 is the origin and node 5 is the destination. The generated simple paths are shown in Fig. 2.

This is known as the DFS since an adjacent node of a small code is searched first. Node 2 has two adjacent nodes, node 6 and node 7, and the current

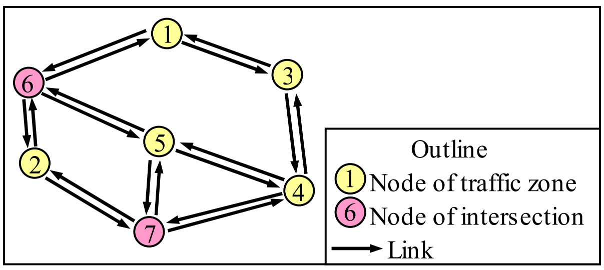

Fig. 1 Test road network

Table 1 Adjacency list

Node	Number of adjacent nodes	Adjacent nodes		
1	2	3	6	
2	2	6	7	
3	2	1	4	
4	3	3	5	7
5	3	4	6	7
6	3	1	2	5
7	3	2	4	5

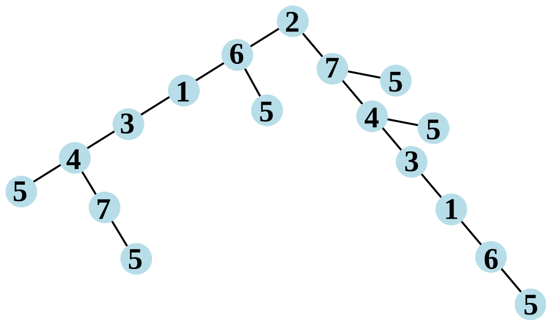

Fig. 2 Total simple paths of Origin-Destination pair (2-5)

stack is 2-6. Thus, the stack grows into 2-6-1-3. Node 3 has two adjacent nodes, node 1 and node 4. Node 1 has been in the stack, so node 4 is added to the stack and a loop is avoided. For node 4, the first adjacent node (first search time), node 3, has been in the stack, and the second adjacent node (second search time), node 5, is added to the stack. The first simple path, 2-6-1-3-4-5, is found since node 5 is the destination. When the search reaches the destination, it backtracks along the stack. Node 4 becomes the current top node and node 5 is not in the stack while the pop operand is executed. For the third search time of adjacent nodes of node 4, node 7 is added to the stack, and the stack is 2-6-1-3-4-7. For adjacent nodes of node 7, either node 2 or 4 has been in the stack, so node 5 is added to the stack. The second simple path, 2-6-1-3-4-7-5, is found. When all possible branches have been searched, six simple paths are found between the OD pair (2, 5). Thus, after all OD pairs have been searched, 116 simple paths are found in the road network.

Once these paths are generated and stored, the path choice proportions and the link flows can be computed according to Eqs. (10) to (13). That is, a single logit-based loading has been completed. It is necessary to formulate a minimization program since the solution by a single logit-based loading is not equilibrium. Consider the following unconstrained mathematical program [14]:

$$\begin{aligned}\min Z(x) = &-\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} q_{ij} E\left[\min_{m}\left\{T_{ij}^{m}\right\}\middle| T_{ij}(x,y)\right] \\ &+\sum_{a\in A} x_a t_a(x_a, y_a) \\ &-\sum_{a\in A}\int_0^{x_a} t_a(\omega, y_a) d\omega\end{aligned} \tag{14}$$

where $E\left[\min_{m}\left\{T_{ij}^{m}\right\}\middle| T_{ij}(x,y)\right]$ is the expected perceived travel time between each OD pair. The objective function of the lower-level program is given by $Z(x)$. The link flows $x=(x_1,\ \cdots,\ x_a,\ \cdots,\ x_G)$ is a vector. The total capacity of each link is composed of the link capacity expansion and the existing link capacity when travel time, t_a, is calculated by Eq. (11).

The iteration method is used to solve the program of Eq. (14). The minimization procedures can be written as:

$$x^{k+1} = x^k + \alpha^k (X^k - x^k) \tag{15}$$

In this equation, x^k is the value of the decision variable vector at the kth iteration, α^k is a scalar representing the move size, and X^k is a descending direction vector computed at x^k, where $x^k=(x_1^k,\ \cdots,\ x_a^k \cdots,\ x_G^k)$ is a vector variable of the link flows and $X^k=(X_1^k,\ \cdots,\ X_a^k \cdots,\ X_G^k)$ is a vector variable of the auxiliary link flows. Equation (15) means x^{k+1} is equal to the weighted average of x^k and X^k. It is difficult, however, to optimize the move size with the standard descent algorithm in the SUE objective function since the expected perceived travel time is difficult to calculate. Consequently, the MSA is applied to solve the SUE because it is based on a predetermined move size along the descending direction. In other words, the sequence of move sizes α^1, α^2, …, is determined a priori. For solving SUE, each move size is the reciprocal of the iteration time. The MSA algorithmic framework in conjunction with previously mentioned logit-based loading model is given as follows:

Step 0: Generate a set of simple paths using the DFS algorithm.

Step 1: Perform logit-based network loading with an empty road network to get link flow x^1. Set k = 1.

Step 2: Calculate the link travel times, $t_a(x^k)$, with respect to the current link flows.

Step 3: Perform logit-based network loading with the current set of link travel times, $t_a(x^k)$, to yield auxiliary link flows X^k, which is in a descending direction.

Step 4: Find the new link flows for all links:

$$x_a^{k+1} = x_a^k + \frac{1}{k}(X_a^k - x_a^k) \tag{16}$$

Step 5: If convergence is attained, stop. If not, set $k = k + 1$ and go to step 2. Convergence criterion is given:

$$\sqrt{\sum_{a\in A}(X_a^k - x_a^k)^2} < \varepsilon \tag{17}$$

5 Solving Bi-level Program

The H-J algorithm is a direct search method which includes an exploratory move and pattern move. The goal of solving the proposed bi-level program is to find a set of link capacity expansions to maximize the objective function of the upper-level program, which responds to the SUE assignment result. This method for given network topology is a common study since it can also solve the problem of how to add links. For example, a set of links which have infinitesimal capacity are added to an existing road network. Calculating this road network as given network topology problem, the planner can determine which links are worthy of improvement and the reasonable capacities of these improved links. In the planning scheme, a link need not be built if it has less than a certain threshold capacity.

There are three vectors used in the algorithm, the test link capacity expansion, $\hat{y}$, benchmark link capacity expansion, $\bar{y}$, and the success link capacity expansion, y^h. The four experiential parameters, s, ϕ, η and v, are provided for the case study, as shown in Table 2. The step-by-step procedure of this scheme is given below.

Table 2 Parameters of Hook-Jeeves algorithm

Means	Symbol	Value
Initial step length of exploratory move, pcu/h	s	1,000
Reduction factor of step length of exploratory move	ϕ	0.5
Convergence step length of exploratory move, pcu/h	η	100
Step length of pattern move	v	2

Step 0: Initialization.

Given an initial solution of y^0, solve the lower-level program with y^0. That is, the current capacity is the sum of the initial capacity expansion, y^0, and the capacity of the existing road network. Calculate the objective function of the upper-level to obtain $F(y^0)$. Set $\bar{y} = y^0$ and $F(\bar{y}) = F(y^0)$. Give the step length of the exploratory move, s, the reduction factor of the step length of the exploratory move, ϕ, the convergence step length of the exploratory move, η, and the step length of the pattern move, v. Set the direction factor, $\beta = 1$, counter, g = 1 (from the first link), for the exploratory move and counter, h =0, for the pattern move.

Step 1: Exploratory moves.

Step 1-1: If $g > G$, each link has been tested, so go to step 2. Let μ_g be a vector that contains a '1' in the gth position and '0' elsewhere.

Step 1-2: Set $\hat{y} = \bar{y} + \beta \cdot s \cdot \mu_g$. Solve the lower-level program with $\hat{y}$. Calculate the objective function of the upper-level to obtain $F(\hat{y})$.

Step 1-3: If $F(\hat{y}) > F(\bar{y})$ renew the benchmark link capacity expansions and objective function of the upper-level, $\bar{y} = \hat{y}$ and $F(\bar{y}) = F(\hat{y})$, respectively, set $g = g + 1$ and go to step 1-1. Otherwise, perform the next step.

Step 1-4: If β = 1, set β = -1 and go to step 1-2; otherwise $g = g + 1$, $\beta = 1$ and go to step 1-1.

Step 2: Pattern moves.

Step 2-1: If $F(\bar{y}) > F(y^h)$, renew the success solution and objective function value of the upper-level program, $y^{h+1} = \bar{y}$ and $F(y^{h+1}) = F(\bar{y})$, respectively. Perform the pattern move, $\bar{y} = y^h + v(y^{h+1} - y^h)$. Solve the lower-level program with $\bar{y}$. Calculate the objective function of the upper-level to obtain $F(\bar{y})$. Set $h = h+1$, g = 1 and go to step 1. Otherwise perform the next step.

Step 2-2: If the convergence criterion, $s < \eta$, is met, stop (the current solution, y^h, is the optimal link capacity expansion); otherwise, reduce the step length of the exploratory move, $s = \phi \cdot s$, return to the success solution and objective function value of the upper-level program, $\bar{y} = y^h$ and $F(\bar{y}) = F(y^h)$, respectively, set $g = 1$ and go to step 1.

6 Case Study of Local Road Network

6.1 Condition of Existing Road Network and Origin-Destination Matrix

The road network shown in Fig. 1 was analyzed to test the proposed optimization framework. Table 3 shows the link length of the existing road network. The 1st to 5th nodes express traffic zones regarded as the area of the origins and destinations of total trips. The 6th and the 7th nodes express the intersection of the roads. The capacity of each link is 1,600 pcu/h in the existing road network. The free flow speed is 60 km/h. Table 4 shows the OD matrix of the peak hour in the morning. An OD matrix displays the trips coming and arriving at respective traffic zones. Table 4 is the traffic demand for traffic assignment in this case study.

Table 3 Link length of existing road network

Link	Length (km)	Link	Length (km)
1-3	8.8	5-4	9.3
1-6	9.2	5-6	9.3
2-6	7.8	5-7	7.0
2-7	9.3	6-1	9.2
3-1	8.8	6-2	7.8
3-4	7.8	6-5	9.3
4-3	7.8	7-2	8.8
4-5	9.3	7-4	9.0
4-7	9.0	7-5	7.0

Table 4 Origin-Destination matrix

Unit: pcu/h

O \ D	1	2	3	4	5	Sum
1	-	210	900	710	250	2,070
2	210	-	210	650	1100	2,170
3	900	210	-	260	150	1,520
4	710	650	260	-	290	1,910
5	250	1100	150	290	-	1,790
Sum	2,070	2,170	1,520	1,910	1,790	9,460

6.2 Route Choice and Loading Proportion

There is a dispersion parameter, θ, which scales the perceived travel time in Eq. (10) when the loading proportion of traffic demand is calculated. If θ rises, the perception error will decrease. Drivers will tend to select the minimum measured travel-time path when θ is very large. If θ drops, the perception error will increase. In the limit of θ approaching zero, all paths are used.

The total travel time of the entire road network with respect to the different values of θ is considered to determine an approximation value of the parameter θ. The total travel time of the entire road network in one hour equals the sum-product of each link travel time and traffic volume on it. The curve of the total travel time with respect to different values of θ is shown in Fig. 3. In order to display clearly, the curve is cut off at a travel time of 2,600 hours. When $\theta = 0, 0.1, 0.2$, the total travel times are 8,167, 3,287 and 2,707 hours, respectively. The curve in Fig. 3 reaches the lowest point, that is, 2,511 hours, at about $\theta = 0.95$. In this case study, the approximation value of the parameter θ is 0.95.

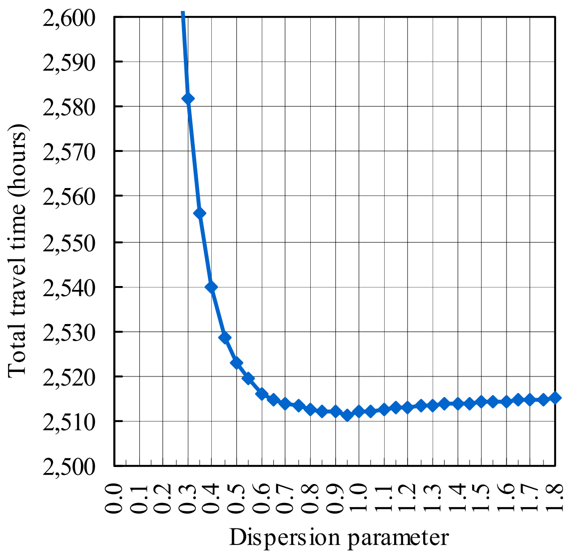

Fig. 3 Total travel times with respect to the different values of dispersion parameter

When the DFS algorithm is completed, 116 simple paths are found. The traffic assignment is performed by the MAS algorithm in conjunction with the logit loading model and only 28 paths are used. Due to space limitations, only the OD pair (2, 5) is used as an example to show the paths used and the loading proportions, as shown in table 5. This case study confirms that the lower-level program can determine the paths used in accord with the SUE criterion. Of course, the link flows have also been calculated. The unblocked reliability in the upper-level program can then be calculated by the proposed model.

Table 5 Paths used and loading proportion of OD pair (2, 5)

Order	Loading proportion in existing road network	Loading proportion in improved road network	Passing nodes		
1	45.2%	37.6%	2	6	5
2	54.8%	62.4%	2	7	5

6.3 Optimization Result

The control factor of road network expansion scale, d, significantly influences the objective function of the upper-level program. The proposed bi-level program can comply with various limits of environmental and financial resources by setting the control factor to different degrees. When d increases, the results of the bi-level program show the decreasing objective function value of the upper-level program, lower unblocked reliability and network expansion scale. The contribution of more network expansions to road network performance improvement decreases with decreasing d. In this paper, the special constraints of the environmental and financial resource are not considered, and the controllable factor $d = 1$.

The H-J algorithm is applied, and the result is a set of link capacity expansions as shown in table 6. Only links that will be improved are shown, and the other link capacities are not changed. The network expansion ratio, $\lambda \sum_{a \in A} (L_a \cdot y_a)$, reaches 10.6% in the existing road network scale by application of the H-J algorithm to solve the proposed bi-level program when the objective function of the upper-level program reaches maximum. In table 6, a set of symmetrical link capacity expansions can be obtained when there is symmetry between the OD matrix and performance of links in existing network. The symmetrical feature means the opposing links have the same capacity and opposing OD pairs have same traffic demand.

Table 6 Link capacity expansion

Link	Expansion capacity (pcu/h)
1-3	1,000
2-7	375
3-1	1,000
3-4	125
4-3	125
7-2	375

When the planning scheme of Table 6 is completed, the unblocked reliability of each level, such as, the link, path, OD pair and the entire network in a road network, is improved. The grade of links according to unblocked reliability are shown in Fig. 4. The red, brown and green arrows

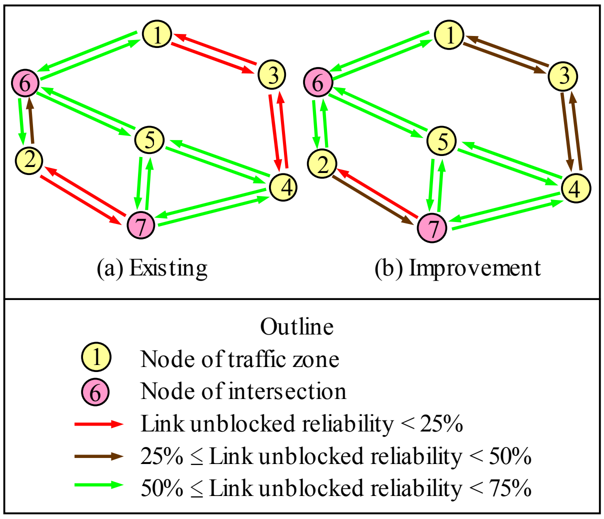

Fig. 4 Grading links by unblocked reliability

express the grade of reliability from low to high, corresponding to R_a< 25%, 25% ≤R_a < 50%, 50% ≤R_a < 75%, respectively. There is no link in which the unblocked reliability is greater than 75%. Since the capacities of the links are improved, the color of five links changes from red to brown and the unblocked reliability of link 2-7 is also increased from 9.2% to 23.7%. Link 2-6 is promoted to green due to the decreasing flow on this link, although its capacity does not change. Twenty-eight used-path unblocked reliabilities of the existing road network and the improved road network are shown in Fig. 5. The abscissa is the used path that is denoted by a series of nodes. The ordinate is the unblocked reliability of the path. The lower the path reliability is, the more the path can be promoted. A driver focuses his attention on the performance of an OD pair. The desire line of the unblocked reliability is shown in Fig. 6, which symbolizes the feature of a

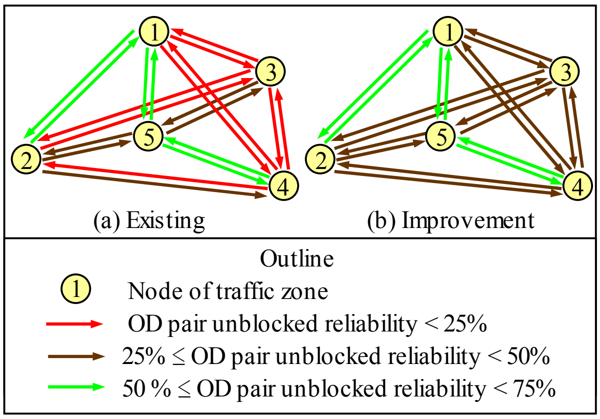

Fig. 6 Desire a line of OD pair unblocked reliability

trip between an OD pair, not necessarily the actual route followed. The OD pair unblocked reliability is graded by the same colors as Fig. 4. Total red desire lines are promoted to brown desire lines, that is, most unreliable OD pairs disappear. There is a distinct improvement in the unblocked reliability of the entire road network from 25.8% of the existing road network to 40.1% of the improved road network.

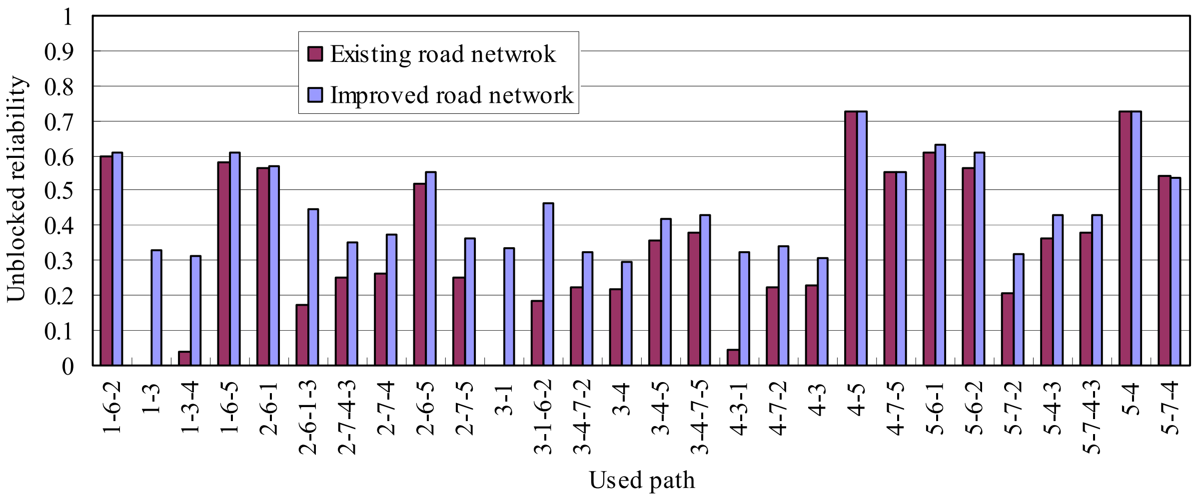

Fig. 5 Unblocked reliability of path

7 Conclusion

To optimize road networks, a new model of a bi-level program is established in which the upper-level program maximizes the objective function by subtracting the network expansion from the entire road network unblocked reliability, and the SUE assignment is regarded as the lower-level program. A planning scheme represented as a set of link capacities is obtained by solving this bi-level program. The proposed unblocked reliability model takes full advantage of the route choice behavior of drivers since the loading proportion of the paths, which is calculated by the SUE assignment, enters both the OD pair unblocked reliability and the entire road network unblocked reliability. The improvement of unblocked reliability comes from the link capacity expansion and the fact that the OD matrix is newly and reasonably assigned. The lower unblocked reliability units of the road network, respectively corresponding to links, paths and OD pairs, are improved. The proposed optimization framework is capable of improving the entire road network to achieve its highest possible reliability level with a minimal scale of road network expansion. The established bi-level program is an effective and advantageous tool for optimizing a road network.

References

[1] Iida, Y. and Wakabayashi, H., An Approximation Method of Terminal Reliability of a Road Network Using Partial Minimal Path and Cut Set, *Proceedings of the Fifth World Conference on Transport Research*, Vol. 4, 1989, pp. 367-380.

[2] Iida, Y., Basic Concepts and Future Directions of Road Network Reliability Analysis, *Journal of Advanced Transportation*, Vol. 33, No. 2, 1999, pp. 125-134.

[3] Chen, A., Yang, H., Lo, H. K., and Tang, W., A Capacity Related Reliability for Transportation Networks, *Journal of Advanced Transportation*, Vol. 33, No. 2, 1999, pp. 183-200.

[4] Chen, Y., Liang, Y., and Du, H., The Application of Reliability in the Road Network Performance Evaluation (in Chinese), *China Civil Engineering Journal*, Vol. 36, No. 1, 2003, pp. 36-40.

[5] Hou, S., Maruyama, N., and Kato, S., A Study on Unblocked Reliability Assessing Road Network Operation Performance for Planning, *International Symposium on EcoTopia Science 2007*, No. 1193, 2007, pp. 1376-1379.

[6] Daganzo, C.F., Sheffi, Y., On Stochastic Models of Traffic Assignment, *Transportation Science*, Vol. 11, 1977, pp. 351-372.

[7] Dial, R. B., A Probabilistic Multi-path Traffic Assignment Algorithm which Obviates Path Enumeration, *Transportation Research*, Vol. 5, 1971, pp. 83-111.

[8] Powell, W. B. and Sheffi, Y., The Convergence of Equilibrium Algorithms with Predetermined Step Sizes, *Transportation Science*, Vol. 16, 1982, pp. 45-55.

[9] Abdulaal, M., and LeBlanc, L., Continuous Equilibrium Network Design Models, *Transportation Research Part B*, Vol. 13, 1979, pp. 19-32.

[10] Zhu, S., Wang, W., Deng, W., Tang, Y., and Wang, B., Research on Traffic Network Reliability and Access Road Algorithm (in Chinese), *China Journal of Highway and Transport*, Vol. 13, No. 1, 2000, pp. 91-94.

[11] Hou, S., Maruyama, N., and Kato, S., New Methodology to Calculate Unblocked Reliability to Assess Road Network Operation Performance, *Journal of Mechanical Systems for Transportation and Logistics*, Vol. 1, No. 3, 2008, pp. 240-251.

[12] Bureau of Public Roads, Traffic Assignment Manual, U. S. Department of Commerce, Urban Planning Division, 1964.

[13] Cormen, T. H., Leiserson, C. E., Rivest, R. L. and Stein, C., Introduction to Algorithms, Second Edition, MIT Press, 2001.

[14] Sheffi, Y., Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods, Prentice-Hall, 1984.