

A Quantitative Operational Risk Management Model

ALEKSANDRA BRDAR TURK
 Doctoral candidate, Faculty of Economics
 University of Ljubljana
 Kardeljeva ploščad 17, Ljubljana
 SLOVENIA
 aleksandra.brdar@gmail.com

Abstract: A possible modified use of the New Basel Accord's LDA capital adequacy calculation method is proposed, including expert's estimates in addition to available historical data and using calculation methods from the Extreme Value Theory (EVT). In financial institutions with short histories the operational risk losses follow a fat-tailed distribution from the EVT, which is why an EVT-based model is most suitable for their analysis. In cases of small historical data samples the addition of experts' estimates and the use of simulated data provides for both a simple and a reliable model to be used for operational risk management by identifying key business areas and key risk factors in both smaller financial institution as well as larger financial institutions, sub-divided into smaller comprehensive sections.

Key-Words: Operational risk, Extreme value theory, Quantitative model

1 Introduction

In the last decade, risk management in the banking and insurance sectors has witnessed a rise in the importance of operational risk among other types of risks, rising from the position of »other risks« and placing itself alongside credit and market risks – the two risk categories deemed to be the most important in the industry, gaining most of the attention by risk managers in financial institutions and by regulators. The main reasons for such a change are a powerful growth of the financial markets, its increasing deregulation and globalization, the growing organizational complexity of these institutions, their corporate and capital partnerships, which increase their overall exposure to risk as well as the intense development of financial services, all of which are becoming more accessible to a wider circle of investors.

Operational risk (OR) can be defined as the risk remaining after eliminating market, credit, interest and exchange risks (Allen, Bali, 2007). The Basel Committee on Banking Supervision defines OR as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk (BIS, 2004, Van Greuning, Bratanovic, 2003). It is in the New Basel Accord (BIS, 2004) that OR is given a greater consideration and the methods for its identification, measurement and management are explored. It is also in this document that OR is

included in the calculation of minimum capital requirements for banks. According to Marshall (2001), the development of a comprehensive OR management system, which includes identification, evidence, analysis and use of OR management data, is the basis for the use of advanced measurement methods in a financial institution for the purpose of determining capital requirements as proposed by the Accord.

The problem with such data analysis models lies in extreme events, which are rare by nature and may have not yet occurred in the recent history of a financial institution, e.g. in the last 10 years. A logical, albeit erroneous, conclusion one would make based on such data is that extreme events do not happen and will not happen in the future, which may affect the underestimation of minimal capital requirements or capital reserves. Therefore, the development of an OR management model in such circumstances is an important issue for financial institutions with a short historical data background due to either their short history or only recent development of a data gathering system, which is the situation for most banks and other financial institutions in the Central and Eastern Europe, as well as for other institutions in developed Western countries which, due to changes in legislation, mode of operation, political or macroeconomic systems, consider any gathered data as an unreliable base for the development of such a model.

The European legislation is in the process of unifying the financial market regulation with the upcoming establishment of financial conglomerates and a significant increase in regulation centralization. Such developments could lead to a wider use of the Basel Committee directives concerning minimal capital requirements, as well as both the Second and Third Pillar of the New Basel Accord, throughout the financial system. Consequently, it would lead to a greater stability of the system while increasing the awareness of OR. Financial institutions with established OR management systems will incur significantly lower actual and opportunity costs in adopting the new regulatory requirements, which is another reason why immediate steps towards incorporating comprehensive risk management, including OR management, in non-banking financial institutions is imperative.

It is the author's opinion that the development of a quantitative OR measurement model in a non-banking financial institution would be considered an interesting development in the widening use of the Accord's proposed regulation, enabling non-banking institutions to adopt such models in practice as well as a possible tool in the partial operational risk management of a business segment in a more complex multinational financial conglomerate.

The paper is organized as follows: in the second section, the basis for minimal capital requirements in banks are reviewed. In the third section, we propose the OR measurement model in an environment of scarce data on OR events. In the following section, the model is implemented in a case study of Slovenian asset management companies where a theoretical OR losses model is developed and applied, followed by a summary of findings.

2 Capital adequacy measurement methods

2.1 The Loss Distribution Approach and its possible modifications

The New Basel Accord allows banks to choose one of the following proposed methods for the calculation of minimal capital requirements: the Basic Indicator Approach, the Standardized Approach or the Advanced Measurement Approach (AMA). Within the latter, there is the possibility of the bank developing its own specialized OR

measuring model, with the premises that the model be comprehensive, transparent and systematic. The AMA includes the Internal Measurement Approach, the Scorecard Approach and the Loss Distribution Approach (LDA). With the LDA, the bank creates a matrix of business processes and possible OR factors for each combination of risk factors and business processes, the probability of occurrence of an OR loss event as well as the severity of the loss is determined. This is the basis for the determination of the loss distribution function of the total loss incurred in a year (or other period). The bank then uses this loss distribution to calculate the Value at Risk (VaR) at a 99.9 % confidence level.

According to Rachev, Fabozzi, Chernobai (2007), the advantages of using the LDA are its high sensitivity, the possibility of integrating both internal and external data into the loss distribution model, as well as expert estimates, and high reliability of results with the usage of reliable data. Some disadvantages of the method include VaR's failure to meet the sub-additivity criteria in cases of fat-tailed distributions (Nešlehova et al., 2006), the interdependence and correlation between model input variables, the lack of a diversification effect with extreme event distributions (Embrechts, McNeil, Straumann, 2002; Ibragimov, 2005), the questionable reliability of high-quantile statistical indicators such as VaR with extreme events (McNeil et al., 2005), as well as the general problem of data gathering and data quality in an environment of scarce and extreme events, which are often well protected information (de Fontnouvelle et al., 2003). Many of these disadvantages can be averted by applying a few modifications to the LDA.

A key issue in constructing such a model is the identification of the correct loss distribution function for the gathered data. By correctly choosing the loss distribution function a bank can calculate the probability and total loss incurred by OR events and consequently maintain an adequate level of capital or provisions for OR losses. For the purpose of data analysis, the usage of classical statistical functions is quite adequate for data falling within the major part of the loss distribution, but may differ significantly in the tail of the data distribution, especially in the case of heavy-tailed data, where the use of the Extreme Value Theory (EVT) is much more suitable (Moscadelli, 2004). Due to the extreme nature, low frequency and high severity of operational loss events, which can cause

significant losses to a financial institution, it is imperative to achieve a good fit in the tail of the distribution function.

Especially in post-transitional Central and Eastern European countries, where the financial market has only been developing in the last few years or decades, the financial industry is too young to have a substantial history of operational loss events and a data background wide and reliable enough to supply an OR model in order to enable it to give reliable results. The usage of empirical OR measurement methods in such circumstances could lead to low quality and low reliability of results and estimates, which could, in turn, cause a bank to develop erroneous business strategies, over- or underestimate its necessary capital or loss provisions, causing the bank and its investors significant losses.

In such conditions, the LDA can be slightly modified and the data can be somewhat widened. Baud, Frachot in Roncalli (2002) point out the censorship problem in data gathering in banks, who in their databases only included OR events with losses of \$ 1 million or greater, as well as the correlation between the size of the financial institutions and the magnitude of its losses. They proposed combining external data into such databases, which was also proposed by Wei (2006). This is also a possibility included in BIS documents (2004). Chernobai, Rachev in Fabozzi (2007) propose the inclusion of near-miss losses in the model as a substitute for missing external data and as additional information on a financial institution's operations and business characteristics. Another possibility is the addition of expert estimates (Dell'Aquila, Embrechts, 2006; Ebnöther et al., 2001). These modifications somewhat expand the current LDA data requirement for a vast and reliable data source as a prerequisite for the use of the method. We agree on the inclusion of experts' estimates, as they can also provide an adaptation of existing historical data to unique shocks in the financial system due to changes in legislation and other external variables, which, in turn, cause different financial institutions to adapt to the new environment requirements in different time intervals. The expert estimates can significantly increase the reliability of empirical data and add an expectation dimension to the data in the sense of anticipated OR events, countering one important shortcoming of models relying solely on historical data (de Fontnouvelle et al., 2003).

A second possibility is a methodological modification of the LDA, which deals with the VaR's failure to meet the sub-additivity criteria in cases of heavy-tailed loss distributions. Instead of estimating the minimum capital requirement based on the sum of one-year VaRs at a 99.9 % confidence level, a simulation can be used to estimate the yearly sum of operational losses across processes and risk factors and this yearly sum can then be used to determine the most adequate distribution function and to estimate the one-year 99.9 % VaR and other capital charge indicators. This modification also reduces the risk of overestimating a financial institution's total VaR, which can occur in the original proposed methodology of summing individual business process-risk factor VaRs because the effect of diversification of risk among processes is not taken into consideration. Also, estimating VaR from yearly losses eliminates the danger of overestimating individual process-risk factor VaR, which can occur if the underlying data is distributed according to the normal distribution (Jorion, 2001; Embrechts et al. 2002). These modifications are discussed in more detail in the following sections of the paper.

2.2 Use of VaR

According to the New Basel Accord, the principal method of estimating of the capital charge for OR within the LDA is the Value at Risk measure. From a market risk measure, VaR has become a much more versatile measure of risk (Jorion, 2001; Manganelli, Engle, 2001) thanks to actuarial methods of estimating the loss distribution functions based on historical data (Larneback, 2006; Salzgeber in Maikranz, 2005). Its use is spreading to other fields of risk management as well, especially in the last decade with OR management gaining focus in financial institutions (Cruz, 2002; Ebnöther, Vanini, McNeil & Antolinez-Fehr, 2001; Crouhy, Galai & Mark, 2001; Dutta & Perry, 2007).

In a wider sense, one could state that VaR is a further development of classical derivatives valuation models such as the Black-Scholes model and refers to the volatility of a portfolio F_t within a timeframe t or on a target date:

$$VaR(\alpha, t) = E(F_t) - Q(F_t, \alpha), \quad (1)$$

where $Q(F_t, \alpha)$ is the quantile corresponding to the α confidence level.

Let us take into consideration the following case: if the aggregate OR losses are distributed

according to the afore mentioned actuarial model, X being the loss for a certain OR event and $N_{\Delta t}$ the number of risk loss events in a certain period of time Δt , e.g. one year, the cumulative losses occur according to the following stochastic process:

$$S_{\Delta t} = \sum_{k=1}^{N_{\Delta t}} X_k, \quad (2)$$

and the cumulative distribution function (CDF) can be written as:

$$F_{S_{\Delta t}}(s) = P(S_{\Delta t} \leq s) = \begin{cases} \sum_{n=1}^{\infty} P(N_{\Delta t} = n) F_X^{n*}(s); & s > 0 \\ P(N_{\Delta t} = 0); & s = 0 \end{cases} \quad (3)$$

where F_X is the distribution function of the stochastic variable X , while F_X^{n*} denotes the n -th convolution of F_X with itself:

$$F_X^{n*}(s) = P\left(\sum_{k=1}^n X_k \leq s\right). \quad (4)$$

Clearly, such a distribution function is non-linear by X and N and an analytical approach to determining its parameters is not viable. Instead, some of the alternative methods can be used (Klugman, Panjer, Willmot, 2004; Enrique, 2006; Padhye, 1999), such as the kernel method (Butler, Schachter, 1998) and the Monte Carlo simulation.

Within the actuarial models, the OR VaR can be calculated as follows (Chernobai, Rachev, Fabozzi, 2007):

$$1 - \alpha = F_{S_{\Delta t}}(VaR) = \sum_{n=1}^{\infty} P(N_{\Delta t} = n) F_X^{n*}(VaR), \quad (5)$$

or by using the inverse distribution function:

$$VaR = F_{S_{\Delta t}}^{-1}(1 - \alpha). \quad (6)$$

However, one must keep in mind some specifics when dealing with cumulative OR losses data within fat-tailed distribution, where the maximum observed value can significantly affect the cumulative loss S_n . For each $n > 2$, as x approaches infinity, the following is true (Embrechts, Klüppelberg, Mikosch, 1997):

$$P(S_n > x) \approx P(M_n > x), \quad (7)$$

where M_n is the maximum observed value in an n -sized sample of data. Also, for all sub-exponential distributions, the following approximation is true:

$$P(S_{\Delta t} > x) \approx E[N_{\Delta t}] \cdot \bar{F}_X(x). \quad (8)$$

By using these equations, the fat-tailed distribution VaR can be calculated as:

$$VaR \approx F_X^{-1}\left(1 - \frac{\alpha}{E[N_{\Delta t}]}\right). \quad (9)$$

Wilson (1998) lists some advantages of using VaR as a risk measure, including the possibility of comparing different types of risk and different subjects, i.e. financial institutions. It can be used directly as a measure for capital adequacy as stated in the Basel Accord. The use of VaR can also be applied to certain financial analysis measures, such as ROE or RAROC.

Some of the problems with using VaR (Yamai and Yoshiba, 2002) include the limitation of presenting only the 99.9-percentile loss and all higher losses, lying to the right of the 99.9-percentile threshold. It also fails to take into consideration the dependencies between risk factors and processes which can significantly affect the total size of potential OR losses. The use of VaR enables the financial institution to study operational loss data, but it cannot prevent high operational losses. Therefore, the use of VaR must be integrated within an efficient and comprehensive risk management system. Finally, the sub-additivity criteria, which VaR fails to meet, is very important from a methodological point of view, as discussed by Artzner et al. (1999), Chavez-Demoulin, Embrechts and Nešlehová (2006) and Embrechts, McNeil and Straumann (2002).

As an alternative to VaR, some authors are proposing the use of Expected Shortfall (ES) or Conditional Value at Risk (CVaR) (see Chernobai, Rachev, Fabozzi, 2007; Embrechts, Furrer, Kaufmann, 2008). It can be calculated as:

$$CVaR = E[S_{\Delta t} | S_{\Delta t} > VaR]. \quad (10)$$

CVaR calculates the potential loss in the case an event in the right tail of the distribution beyond VaR should occur. Unlike VaR, which may fail the sub-additivity property, CVaR is a sub-additive measure of risk suitable for use in fat-tailed and extreme event distributions.

2.3 The use of Extreme Value Theory (EVT)

Operational loss data are typically right-skewed, leptokurtic (i.e. concentrated around the value 0) and fat-tailed on the positive (right) side of the distribution (see Cruz, 2002; Moscadelli, 2004; De Fontnouvelle et al., 2006). The use of EVT methods is therefore recommended.

Among the two basic analytical methods, the block maxima model, which studies the most severe losses within a time interval-based block-

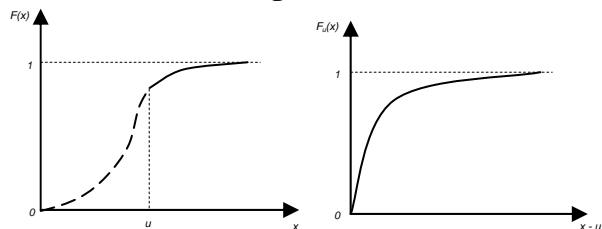
organized data, and the peak-over-threshold model (POT), which analyzes data above a high threshold, we have chosen the use of the POT method for the development of our quantitative OR measurement model.

Let u be a high threshold. F_u is the distribution function of data above this threshold, also known as the conditional excess distribution function (Chernobai, Rachev, Fabozzi, 2007):

$$F_u(x) = P(X - u \leq x | X > u) = \frac{F(u+x) - F(u)}{1 - F(u)} \tag{11}$$

as shown in Figure 1.

Fig.1: The conditional excess distribution function above the high threshold u .



Source: Chernobai, Rachev, Fabozzi, 2007, pg. 165.

Embrechts, Klüppelberg and Mikosch (1997) have shown that for high values of u the conditional excess distribution function F_u takes the shape of a two-parametric general Pareto distribution (GPD) with the following distribution function:

$$F(x) = \begin{cases} 1 - \left(1 + \xi \frac{x - \mu}{\beta}\right)^{-\frac{1}{\xi}}; \xi \neq 0 \\ 1 - e^{-\frac{x - \mu}{\beta}}; \xi = 0 \end{cases}, \tag{12}$$

where

$$\begin{aligned} x &\geq \mu; \xi \geq 0, \\ \mu \leq x \leq \mu - \frac{\beta}{\xi}; \xi < 0, \end{aligned} \tag{13}$$

where x is an individual OR loss above the threshold, $-\infty < \mu < +\infty$ is the location parameter, usually assumed to be 0, $\beta > 0$ is the size parameter and ξ is the distribution's shape parameter. The GPD becomes interesting for OR event data when $\xi > 0$, where it becomes fat-tailed. For $\xi < 1$ it is possible to calculate the mean value, whereas the variance and standard deviation can only be calculated for values of $\xi < 0.5$ (see also Lindskog, 2000).

Diebold, Schuermann and Stroughair (1998) and Embrechts (2000) list the following advantages of using EVT in modeling OR event data:

- the EVT describes the characteristics of the tail of a distribution function and is a direct tool for the analysis of high-severity-low-frequency extreme event data around and beyond a high threshold;
- the POT is very useful in the analysis of catastrophic losses lying beyond the high threshold for the use by other risk measures, such as VaR;
- computational and theoretic methods for determining the parameters of the entire extreme distribution function, as well as of the tail of the distribution function, are available;
- non-parametric measures for the determination of the shape of the distribution are available, such as the Hill estimator, which possesses interesting asymptotic qualities.

EVT weaknesses, as listed by the same authors, as well as others (Pritisker (1996), Smith (2000) and Kellezi and Gilli (2000)), are as following:

- the POT method includes a limited number of observations, which can lead to inaccuracy in parameter estimates;
- the choice of the high threshold is based on graphic methods, such as the mean excess plot; new, more accurate methods should be developed;
- in large data samples and complex models, the calculation times can be quite long;
- the EVT-based analysis of OR focuses mainly on high-severity events and tends to underestimate the importance of medium- and low-severity events.

Due to all the advantages of EVT and VaR listed above, we feel that by applying a few modifications to the OR VaR calculation method, which are described in the following sections of the paper, therefore lessening the shortcomings of the method, VaR is a suitable tool for the analysis of OR events and losses.

3 The OR measurement model

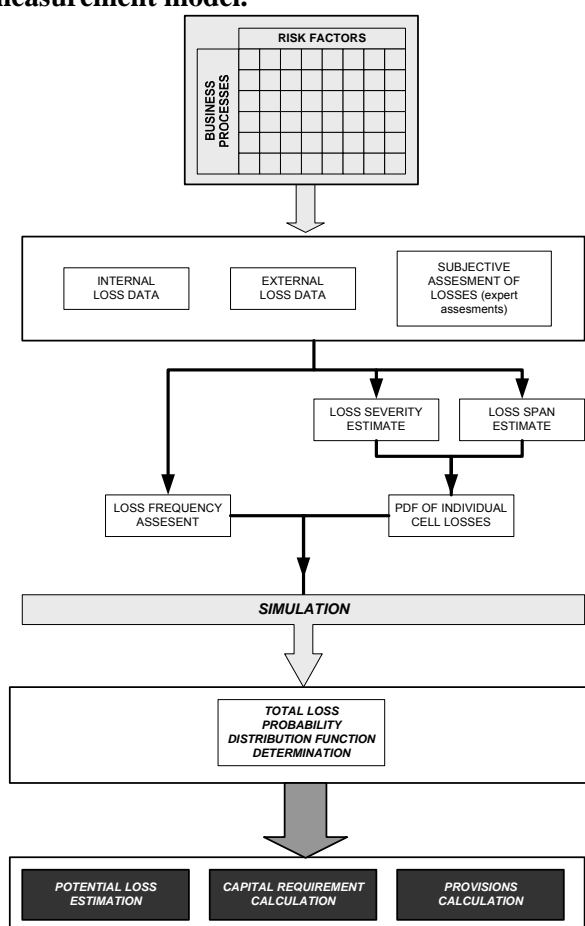
3.1 The basic building blocks of the model

The proposed model is based on the Loss Distribution Approach (LDA) of the New Basel Accord. It starts out as an analysis of business

processes, their classification, sorting and grouping into meaningful groups with common characteristics which can then be observed for OR events in practice in order to easily identify these events and study them further.

We propose the bottom-up approach to business-process mapping, keeping in mind that the complexity of a financial institution is its own worst enemy, potentially making a model too complicated and prone to modeling and interpretation errors. Therefore, we propose subdividing the financial institution into smaller sub-segments, possibly homogenous divisions or business lines (e.g. corporate banking, retail banking, treasury etc. for a bank).

Fig.2: The building elements of the OR measurement model.



The business processes of one such segment are then evaluated and the most likely risk factors for the specific business processes for the said segment are taken into consideration. The mapping of business processes and identification of risk factors should be generalized to such a level as to find some common ground between processes within

the sub-segment. The model of P processes and R risk factors is then a $P \times R$ matrix, where for each process-risk factor combination we estimate the frequency of the loss event ($F(\lambda)_P$) and the probability distribution function (PDF) of the loss severity ($F(a, s)_R$).

3.2 The practical approach

As mentioned before, a financial institution's complexity may present itself as an obstacle in creating a viable functioning OR risk measurement model. For the purpose of testing the proposed model, we could have chosen a business segment of a bank, e.g. investment banking or treasury, but in order to keep the model simple, we would have to ignore connections and dependencies between other business segments or back office operations. Instead, we chose a smaller financial institution, an asset management company. It basically encompasses all the characteristics of bigger financial institutions – the regulation, the legislation, the technological and human capital dependency, several business processes – as well as being a finite organization with clear boundaries and an individual interaction with its environment, being subject to several risk factors, common to the financial intermediaries' industry, but much more manageable in size.

Due to the relative young age of these intermediaries and their significantly smaller size compared to their banking counterparties, reducing the databases and availability of information on OR loss events, as well as the fact that they do not fall under Basel directives, we propose a few modifications to the original Basel II LDA method.

3.3 Modifications of the LDA method

The digression from the Basel "operating territory" into the field of asset management companies is the first modification of the method. By selecting a smaller individual organization, we are keeping the model simple while encompassing a full range of business processes and risk factors. The asset management industry, especially in fast growing economies of Central and Eastern Europe, possesses a relatively short history of stable economic, capital market and legislative environment ranging from ten to fifteen years in the most developed countries of the region to none at all other in countries, where the capital market is still in the first phases of development. This is obviously even truer for the individual asset management company. This lack of history,

meaning a lack of OR event data, causes a significant problem for the consistency and validity of the proposed model, since a large database of consistent quality is the basis of the method (Dell'Aquila, Embrechts, 2006; Ebnöther et al., 2001).

The second proposed modification of the LDA method is the widening of the database to include external data from the whole industry and the inclusion of subjective experts' assessments and estimates on OR losses.

In collecting experts' estimations, we adjust for the subjectivity of the estimations by instructing experts to base the estimations only on historical data available, we adjust the estimates for differences in asset management company's size while at the same time correcting or eliminating historical data, which are biased due to adjustment periods to legislation changes (e.g. tax or capital markets regulation which is known to change often in developing countries) within each company. By adding experts' estimates of OR losses and adjusting historical data, we not only increase the quality and size of the database, creating an adequate base on which to build our model, but we also include the expectation dimension or potential losses into the database, which significantly diminishes one of the problems of relying solely on historical data (de Fontnouvelle et al., 2003).

Within the second modification we also propose another similar deviation from the LDA model, namely the inclusion of potential or opportunity losses or costs of the financial institution, including the costs occurring during the process of detecting a potential (or actual) loss events and eliminating its potential consequences before they reach a significant severity by analyzing its sources or risk factors and limiting their influence by adding several additional internal control mechanisms.

The third proposed modification of the LDA model is methodological in its nature. It solves the problem of VaR potentially failing the sub-additivity property of coherent risk measures. Instead of determining a 99.9-confidence level VaR for each of the element of the business process/risk factor matrix and subsequent summation of these VaRs, we use a simulation to obtain the yearly OR losses for the entire financial institution which is then used for a yearly VaR calculation, that way eliminating both the sub-additivity problem and the potential overestimation of VaR which can occur in simple addition of several VaR measures which does not take into consideration the diversification

effect (Jorion, 2001; Embrechts, Kaufmann, Samorodnitsky, 2002). The proposed modification also eliminates the task of including correlation parameters into the final VaR estimate (Böcker, Klüppelberg, 2007; Chavez-Demoulin, Embrechts, Nešlehová, 2006). By simulating total yearly OR losses, we obtain a loss distribution function consistent with the sum of the individual business process/risk factor loss distribution functions, while simultaneously simplifying the model and the analysis and reducing calculation time and costs.

3.4 The simulation

Discrete events like OR loss events can be described by the function $\{N(t), t \geq 0\}$, its value being the number of events occurring in a given time interval $[0, t]$. Such a function is a Poisson process with a mean number of events λ , if the following conditions are met (Banks et al., 2001):

- the events occur separately;
- the function increases independently from the observation times, i.e. the difference between the number of events occurring within the interval $[0, t]$ and the number of events occurring in the interval $[0, t+s]$ is only dependent on the length of the time s and independent from t , meaning there are no peaks and no delays in the occurrences of events;
- the increases of the functions in different time intervals are random variables independent from each other.

There are several interesting characteristics of the Poisson process:

- the probability of exactly n events occurring in the time t is:

$$P[N(t) = n] = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$$

for $t \geq 0$ and $n = 0, 1, 2, \dots$ (14)

- the variance and mean are:

$$\mu[N(t)] = \lambda t = \sigma^2[N(t)]$$
 (15)

- the CDF of the times between two occurrences is the exponential function

$$f(t) = \lambda e^{-\lambda t}$$
 (16)

with a mean of $1/\lambda$,

which allow the Poisson process to be used for an elegant analysis of the frequencies of event occurrences, including the calculation of event occurrences in longer or shorter time periods and the estimation of event occurrences in two different business processes determined by two separate Poisson functions – the latter due to the sub-

additivity property of the Poisson function (Chernobai, Rachev, Fabozzi, 2007).

By using a Monte Carlo simulation where the event occurrences in each business process are determined by an independent stochastic Poisson process and with a yearly frequency of λ_i for each business process i , we can create a time series of yearly losses (calculated as the sum of individual processes' losses within the same year) with enough OR loss event data to include in the model. For the distribution function of the frequency of event occurrences, we use the available historic data and experts' estimates of loss severity (a_i) and the estimated span of the data determined by the standard deviation (s_i), obtaining a different distribution function for each business process-risk factor combination of the model matrix of varying shapes and sizes from normal distributions to asymmetric and skewed distributions like the log-normal distribution function.

4 Practical application of the model

4.1 Data gathering

Six Slovenian asset management companies, which at the end of the year 2007 managed 73 % of all assets under management in Slovenia were included in the OR loss event analysis. At the time, none of the companies in the sample had a computational model for determining OR event losses or incorporating OR loss data into current business strategies and decisions. All of them had, however, an early-stage internal control and OR management system of some kind.

By analyzing their business processes, we have identified four basic business segments: (1) *asset management*, (2) *back office operations*, (3) *sales of investment products* (mostly mutual funds) and (4) *compliance*. The risk factors identification was based on the Basel Accord and subsequently modified to the specifics of asset management. We identified the following seven risk factors occurring in each of the four business segments: (1) *internal fraud and theft*, (2) *external fraud and theft*, (3) *clients, services and practices*, (4) *IT error or failure*, (5) *employment and work environment safety*, (6) *execution and management of processes* and (7) *physical damage to assets*.

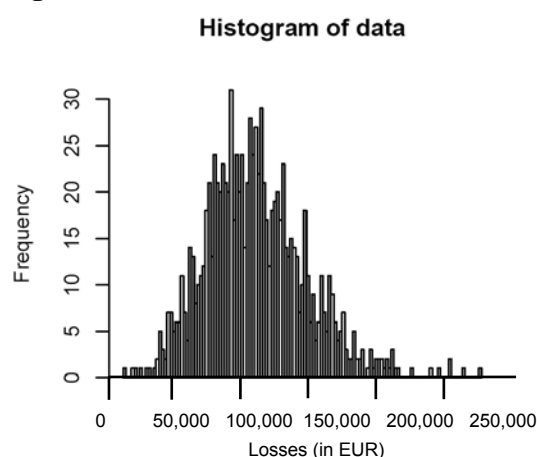
We gathered data on OR loss events in the years 2004 – 2007 from public sources (i.e. annual reports, public notices etc.). The short time span produced a poor database, which is why we

included one expert for each asset management company. The experts were financial or compliance managers with at least five years of experience working in the industry. The experts were asked to estimate actual and potential losses for each business process/risk factor combination of the model matrix, their span (standard deviation) and their frequency, taking into consideration their knowledge and experience and accounting also for opportunity losses. The data was then scaled by the size of the equity of each company, based on the 2007 year-end balance sheets, thus eliminating differences in size, number of employees, clients, investment funds and the size of their assets under management. The average loss severity, frequency and standard deviation for each matrix element was then calculated and input into the simulation. We ran the simulation to obtain a 1000 years' history and calculated the cumulative yearly losses for each of the 1000 years.

4.2 Basic data analysis

As shown in Fig. 3, the loss data is significantly asymmetric with a prolonged right tail, as also indicated by the skewness and kurtosis coefficients (0.63 and 0.93, respectively). The average yearly OR loss from 1000 years' history is estimated at 110,937.97 EUR. The individual yearly loss, however, spans from a minimum of 15,465.55 EUR to a maximum of 275,185.94 EUR with a median of 108,040.33 EUR.

Fig.3: Cumulative yearly OR event data histogram.



The frequency distribution analysis of the individual risk factors offers an interesting insight into the asset management companies' environment. The losses from *internal fraud and theft* differed significantly from the normal

distribution function, reaching the highest severities, followed by losses from *external fraud* and *physical damage to assets*. The most significantly asymmetric losses occurred due to the *clients, services and practices* risk factor.

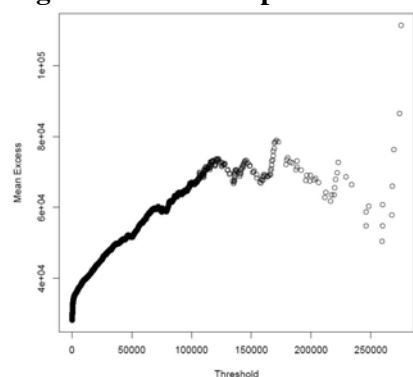
Similarly, the analysis of yearly losses from different business processes shows that losses from *compliance* are the most asymmetric and have a pronounced right tail, followed by *sales of investment products* and *asset management*. Losses from *back office operations* seem to be the least asymmetric, but, on the other hand, are potentially the largest.

4.3 Model results' analysis

4.3.1. Distribution function determination

We have confirmed the presence of a fat right tail in the cumulative yearly OR loss data by showing the presence of an increasing mean excess plot (Chernobai, Rachev, Fabozzi, 2007; Degen, Embrechts, Lambrigger, 2006), as shown in Fig. 4.

Fig. 4: Mean excess plot for simulated data.



Secondly, we fitted four theoretic distribution functions to the data: the generalized extreme value distribution – GEV, the Gumbell distribution, the Weibull distribution and the generalized Pareto distribution – GPD.

Based on the QQ-Plot (Cruz, 2002) of the empirical and theoretical data we were able to determine an adequate fit to the GEV within most of the distribution but detected a significant deviation from the central line in the largest 150 data or so. Subsequently, the Gumbell and Weibull distribution, as well as the GPD were fitted to the largest 150 data. The QQ-Plot shows the best fit with GPD.

4.3.2. GPD parameter estimation

For the GPD parameter estimation, the POT method, as described in section 2.3, was used.

Since the high threshold u can influence the characteristics of the parameter estimation (Einmahl, Li, Liu, 2006; Hongwei, Wei, 1999), we have dedicated significant attention to its determination.

Firstly, the Hill plot was used to determine the shape parameter ξ of the GPD (Chernobai, Rachev, Fabozzi, 2007; Cruz, 2002), which for each sample value k is calculated with the Hill function

$$\xi H = \frac{1}{k} \sum_{j=1}^k \ln X_j - \ln X_{(k)}. \quad (17)$$

The Hill plot stabilizes horizontally at the most likely value of the shape parameter. Unfortunately, the Hill plot for our sample data does not stabilize at any value.

The second graphic method of determining the high threshold u is a horizontally stable plot of the shape parameter ξ in relation to u . Based on this test, the high threshold was set at 150,000 EUR.

Fig.5: QQ-Plot for the largest 150 data and GPD.

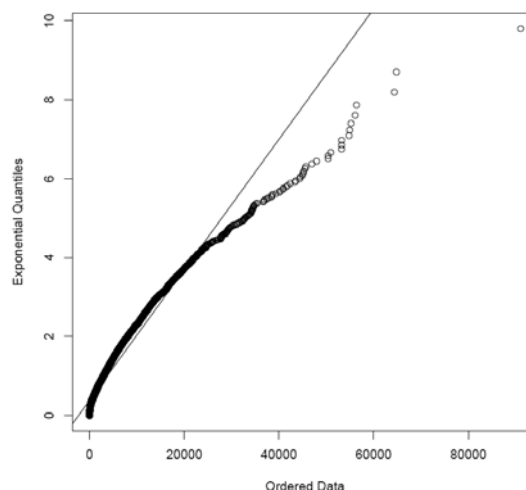
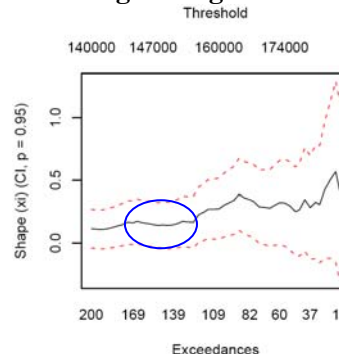


Fig.6: Determining the high threshold.



Among various mathematical GPD parameter calculation methods we have chosen the maximum likelihood estimate (MLE) method (Nylund, 2001),

which in a u -left-truncated sample converges to a single solution, as shown in Table 1.

Table 1: GPD parameters for sample data (right tail only).

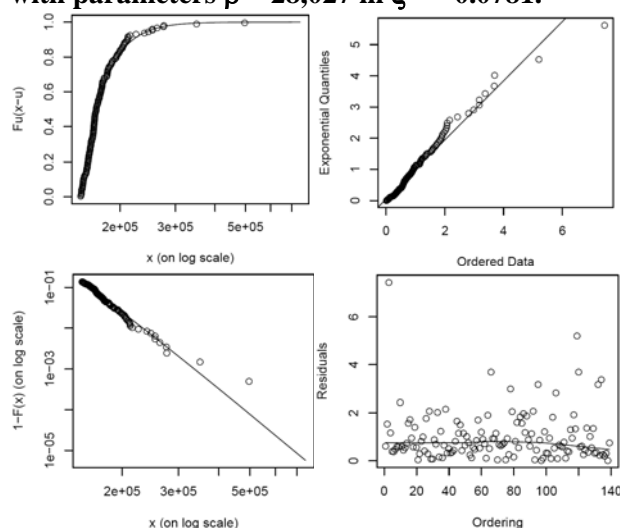
GPD parameter or sample data	value of parameter or data	stdev of parameter estimation
β	28,027	2,944.65
ξ	-0.0781	0.0755
u (threshold)	150,000	
n_u (number of events above threshold)	136	
$f < u$ (density of events below threshold)	0.861	

4.3.2. Goodness of fit analysis

The parameters obtained with the MLE method were tested for goodness of fit using graphic and mathematical methods.

The graphic methods show an adequate goodness of fit in all diagrams shown in Fig.7.

Fig.7: Graphic goodness-of-fit tests for GPD with parameters $\beta = 28,027$ in $\xi = - 0.0781$.



The first mathematical goodness-of-fit test used was the Pearson's χ^2 -test (D'Agostino, Stephens, 1986), a testing method most suitable for binominal and Poisson's distribution, where the data is divided into K classes and the test is calculated as

$$\chi^2 = \sum_{k=1}^K \frac{(n_k - En_k)^2}{En_k}, \quad (18)$$

with n_k being the actual frequency and En_k the expected or theoretic frequency of each class $k = \{1, 2, \dots, K\}$. An important shortcoming of the test is its dependency on the formation (number and size) of the K classes, which is especially problematic in continuous distributions. Also, the sample should be of significant size for the test to

adhere to the assumption of χ^2 being an asymptotic function.

The second group of goodness-of-fit tests used, the empirical distribution function (EDF) based tests, can be used for all distribution functions and determines the vertical differences between the theoretical and empirical distribution functions based on observed data (Anderson, Darling, 1952; D'Agostino, Stephens, 1986; Chernobai, Rachev, Fabozzi, 2007).

In our case, two supremum-type tests were used, namely the Kolmogorov-Smirnov test (KS) and the Anderson-Darling test (AD). The KS test focuses mostly on the center of the distribution function and gives values in the central range a higher significance, while the AD test focuses mainly on the tail of the distribution, therefore being more suited for fat-tailed distribution testing. Also, we performed a modified version of the AD test, the upper-tail AD test (AD_{up}), as proposed by Chernobai, Rachev in Fabozzi (2005), specifically designed for left-truncated distributions.

Table 2: Goodness of fit tests for GPD.

Test	value of test	df	confidence level (p)
χ^2	19,182	19,044	0.239
KS	0.864	-	0.875
AD	51.83	-	0.976
AD_{up}	372.16	-	0.764

The above results confirm our doubts about the adequacy of the Pearson's χ^2 -test as indicated by the high degrees of freedom in Table 2, which are the consequence of the data sample size and the sensitivity of the test statistic to the formation of the data classes. On the other hand, KS, AD and AD_{up} tests all confirm the fit of the GPD to the data with high confidence levels.

4.4 Capital requirements and OR losses provisions calculation

The capital requirement in financial institutions is supposed to shield the organization from unexpected OR losses and is under current legislation calculated as a 99.9 % VaR. Due to the shortcomings of the method, which were discussed in previous sections, the capital requirement could be underestimated, therefore exposing the financial institution to OR losses beyond its capital. Instead, we use the Conditional Value-at-Risk ($CVaR$), as defined in equation (10), as the main method for determining the adequate capital requirements or loss provisions.

The underestimation of capital requirements by using only the VaR method can also be tested on the sample data. We have compared capital requirements calculated with 99.9 % VaR directly from empirical data (VaR_{emp}) with 99.9 % VaR and CVaR from the fitted GPD (VaR_{GPD} and $CVaR_{GPD}$, respectively).

Based on these calculations, we can state that

$$VaR_{emp} < VaR_{GPD} < CVaR_{GPD}, \quad (19)$$

making CVaR calculated from GPD fitted data the most appropriate method for determining capital requirements or loss provisions.

Table 3: Capital requirements estimation using empirical and GPD fitted data.

Statistic	Value of statistic (€)
VaR_{emp}	211,951.71
VaR_{GPD}	216,152.57
$CVaR_{GPD}$	237,335.56

That way, a financial institution would ensure an adequate amount of capital or provisions for OR losses coverage, especially in times of increased market turmoil, structural changes, expansion of the organization or changes in the legislative environment.

5. Conclusion

The paper proposes a possible modified use of the New Basel Accord's LDA capital adequacy calculation method. Based on expert's estimates in addition to available historical data, an extreme value theory-based operational risk calculation model has been presented. We have shown that, in cases of small historical data samples with the addition of experts' estimates and the use of simulated data, such a model is both simple and reliable and can be used to measure, predict and manage operational risk by identifying key business areas and risk factors. Also, the paper shows that in financial institutions with short histories the OR losses follow a fat-tailed distribution from the extreme value theory, which is why an EVT-based model is most suitable for their analysis.

The model can be used in smaller as well as in larger financial institutions, which should be sub-divided into smaller comprehensive sections due to their organizational complexity. The model is specially suitable in cases of scarce historical data due to either the financial institution's short history or lack of organized OR loss event

documentation. Furthermore, the model can be incorporated into a larger-scale OR management system as the prediction and back-testing method for OR losses. The use of the model within existing OR management systems would also allow for greater accuracy and reliability in determining capital adequacy or loss provision criteria both for financial institutions and their regulators.

Unfortunately, the quality of such a system can only be determined by its use, by testing the quality of gathered data and of forecasts made on the basis of such data by comparing them to actual data arising from continuing operations. Obviously, it is imperative that in such a data gathering operation all levels of business be included by reaching a consensus of all levels of management in the company.

References:

- [1] Allen, L., Bali, T. G., Cyclicity in Catastrophic and Operational Risk Measurements, *Journal of Banking & Finance*, Vol. 31, No. 4, 2007, pp. 1191-1235.
- [2] Andersen, A., *Operational risk and financial institutions*, Risk Books, 1998.
- [3] Artzner, P., Delbaen, F., Eber, J., Heath, D., Coherent Measures Of Risk, *Mathematical Finance*, Vol. 9, No. 3, 1999, pp. 203 - 228.
- [4] Banks, J., Carson, J. S., Nelson, B. L., Nicol, D. M., *Discrete-Event System Simulation*, Third Edition, Prentice Hall, 2001.
- [5] Baud, N., Frachot, A., Roncalli, T., *How to Avoid Over-estimating Capital Charge for Operational Risk?* Working Paper, Crédit Lyonnais, Groupe de Recherche Opérationnelle, 2002.
- [6] Böcker, K., Klüppelberg, C., *Multivariate Models for Operational Risk*, Enterprise Risk Management Symposium, 2007.
- [7] Butler, J. S., Schachter, B., Estimating Value-At-Risk With A Precision Measure By Combining Kernel Estimation With Historical Simulation, *Review of Derivatives Research*, Vol 1, No. 4, 1998, pp. 371-390.
- [8] Chavez-Demoulin, V., Embrechts, P., Nešlehová, J., Quantitative Models for Operational Risk: Extremes, Dependence and Aggregation, *Journal of Banking and Finance*, Vol. 30, No. 10, 2006, pp. 2635 - 2658.
- [9] Chernobai, A. S., Rachev, S. T., Fabozzi, F. J., *Operational Risk. A Guide to Basel II Capital*

- Requirements, Models and Analysis*, Wiley & Sons, 2007.
- [10] Crouhy, M., Galai, D., Mark, R., *Risk Management*, McGraw-Hill, 2000.
- [11] Cruz, M. G., *Modeling, Measuring and Hedging Operational Risk*, Wiley & Sons, 2002.
- [12] D'Agostino, R. B., Stephens, M. A., *Goodness-Of-Fit Techniques*, Marcel-Dekker, 1986.
- [13] Dell'Aquila, R., Embrechts, P., Extremes and robustness: a contradiction? *Financial Markets and Portfolio Management*, Vol. 1, No. 20, 2006, 103 - 118.
- [14] De Fontnouvelle, P., DeJesus-Rueff, V., Jordan, J., Rosengren, E., *Using Loss Data to Quantify Operational Risk*, Working Paper. Federal Reserve Bank of Boston, 2003.
- [15] Degen, M., Embrechts, P., Lambrigger, D. D., *The Quantitative Modeling of Operational Risk: Between g-and-h and EVT*, Working Paper. ETH-Zentrum, 2006.
- [16] Dutta, K., Perry, J., *A Tale of Tails: An Empirical Analysis of Loss Distribution Models for Estimating Operational Risk Capital*, Working Paper, Federal Reserve Bank of Boston, J.P. Morgan Chase, 2007.
- [17] Ebnöther, S., Vanini, P., McNeil, A., Antolinez-Fehr, P., *Modelling Operational Risk*, Working Paper, Zurich Cantonal Bank, 2001.
- [18] Einmahl, J. H. J., Li, J., Liu, R. Y., *Extreme Value Theory Approach to Simultaneous Monitoring and Thresholding of Multiple Risk Indicators*, Discussion Paper No. 2006-104. CentER, University of Tilburg, 2006.
- [19] Enrique, N., Practical Calculation of Expected and Unexpected Losses in Operational Risk by Simulation Methods, *Banca & Finanzas: Documentos de Trabajo*, Vol. 1, No. 1, 2006, pp. 1 - 12.
- [20] Embrechts, P., Extreme Value Theory: Potential And Limitations As An Integrated Risk Management Tool, *Derivatives Use, Trading & Regulation*, Vol. 6, No. 4, 2000, pp. 449 - 456.
- [21] Embrechts, P., Furrer, H., Kaufmann, R., Different Kinds of Risk, in Andersen, T. G. (ed.), *Handbook of Financial Time Series*, Springer, 2008.
- [22] Embrechts, P., Kaufmann, R., Samorodnitsky, G., *Ruin theory revisited: stochastic models for operational risk. Risk Management for Central Bank Foreign Reserves*, European Central Bank, 2002.
- [23] Embrechts, P., Klüppelberg C., Mikosch, T., *Modeling Extremal Events for Insurance and Finance*. Springer-Verlag, 1997.
- [24] Embrechts, P., McNeil, A., Straumann, D., Correlation and dependence in risk management: Properties and pitfalls, in Dempster, M. A. H. (ed.), *Risk Management: Value at Risk and Beyond* (pp. 176 - 223), Cambridge University Press, 2002.
- [25] Hongwei, T., Wei, Z., *A New Method to Compute Value-at-Risk: Extreme Value Theory*, Working Paper, School of Management, Tianjin University, 1999.
- [26] Ibragimov, R., *Portfolio Diversification and Value At Risk Under Thick-Tailendness*, Working Paper No. 05-10, Yale International Centre of Finance, 2005.
- [27] *International Convergence of Capital Measurement and Capital Standards: A Revised Framework*, Bank For International Settlements (BIS), Basel Committee on Banking Supervision, 2004.
- [28] Jorion, P., *Value at Risk. The New Benchmark for Managing Financial Risk*, Second Edition, McGraw Hill, 2001.
- [29] Kellezi, E., Gilli, M., *Extreme Value Theory for Tail-Related Risk Measures*, University of Geneva, 2000.
- [30] Klugman, S. A., Panjer, H. H., Willmot, G. E., *Loss Models: From Data to Decision*, 2nd edition, Wiley & Sons, 2004.
- [31] Lindskog, F., *The Mathematics And Fundamental Ideas Of Extreme Value Theory*, Lecture Notes, ETH-Zentrum, 2000.
- [32] Manganelli, S., Engle, R. F., *Value At Risk Models In Finance*, Working Paper, European Central Bank, 2001.
- [33] Marshall, C. L., *Measuring and Managing Operational Risks in Financial Institutions: Tools, Techniques, and other Resources*, Wiley & Sons, 2001.
- [34] McNeil, A. J., Frey, R., Embrechts, P., *Quantitative Risk Management: Concepts, Techniques and Tools*, Princeton University Press, 2005.
- [35] Moscadelli, M., *The Modelling Of Operational Risk: Experience With The Analysis Of The Data Collected By The Basel Committee*, Economic Working Paper No. 517. Banca d'Italia, 2004.

- [36] Nešlehová, J., Embrechts, P., Chavez-Demoulin, V., Infinite-mean Models and the LDA for Operational Risk, *Journal of Operational Risk*, Vol. 1, No. 1, 2006, pp. 3 - 25.
- [37] Nylund, S., *Value-at-Risk Analysis for Heavy-Tailed Financial Returns*, Master Thesis, Helsinki University Of Tehnology, 2001.
- [38] Padhye, A., *Value at Risk*, Working Paper, ICICI Research Centre, 1999.
- [39] Smith, R. L., Measuring Risk With Extreme Value Teory, in Embrechts, P. (ed.), *Extremes and Integrated Risk Management* (pp. 19 - 36), Risk Books, 2000.
- [40] Van Greuning, H., Brajnovic Bratanovic, S., *Analyzing and Managing Banking Risk, A Framework for Assessing Corporate Governance and Financial Risk*, Second edition. World Bank, 2003.
- [41] Wei, R., *Quantification of Operational Losses Utilizing Firm-Specific Information*, PhD Disertaion. Wharton School, University of Pennsilvania, 2006.
- [42] Wilson, T., *Value At Risk*, in Alexander, C. (ed.), *Risk Management and Analysis, Measuring and Modelling Financial Risk*, Wiley & Sons, 1998.
- [43] Yamai, Y., Yoshiba, T. Comparative Analyses of Expected Shortfall and Value-At-Risk: Their Validity under Market Stress, *Monetary and Economic Studies*, Vol. 20, No. 3, 2002, pp. 181 – 237.