Decision Processes in Public Organizations

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Abstract: Application of binary Analytic Hierarchy Process in a decision process related to public sector is in the main focus of this work. We argue that the standard nine point scale based rating system used by the classical Analytic Hierarchy Process is not the most optimal one with respect to decision processes in the majority of public organizations. This mainly because it requires a lot of resources that such organizations do not have. Therefore a simpler rating system like binary or ternary point scale can be used instead. At the same time simpler rating systems are not very effective when a large number of criteria is involved since pairwise comparisons with respect to different criteria might end up with the same evaluation due to lack of alternatives.

Key–Words: Cooperation, multi-criteria decision-making, AHP

1 Introduction

Inconsistencies in decision making are considered to be internal and substantive according to behavioral decision theories, [8]. The first category involves violation of one or several axioms of rational decision theory like procedure invariance, descriptive invariance, cancellation, and transitivity, where the latter appears when ones decision is suboptimal according to some external substantive criteria [8].

Consistency in multy criteria decision-making means that if a basic amount of raw data is available than all other data can be logically deduced from it. When relations among all criteria involved in a decision-making are graphically represented by a directed complete graph, a cycle of length 3 in the complete graph results in a consistency ratio smaller than the accepted limit, while introducing a cycle of length 4 makes the consistency ratio goes above the accepted limit.

The Analytic Hierarchy Process (AHP) facilitates development of a hierarchical structure of a complex evaluation problem. Thus subjective judgment errors can be avoided and an increase of the likelihood for obtaining reliable results can be achieved.

’Situations that call for an ordinal scale are generally those involving subjective, non measurable qualities, as in sensory testing, personnel rating, or the study of preference and choice behavior, for example. Ordinal, ”pick-the-winner” type data also occur naturally in sporting contexts or when attempting to elicit preferences from subjects who are incapable of quantifying their judgments (animals in food testing, say) or for whom the task of comparing alternatives on a ratio scale would be too tedious or time-consuming’, [4].

The AHP has been advocated as an approach that not only can deal with both tangibles and intangibles but also helps organizing all aspects involved in a hierarchic structure where the benefit or cost aspects act as criteria and the projects as alternatives, [27]. Modelling risk and uncertainty with the AHP is discussed in [10]. Other intersting applications of AHP can be found in [2], [9], and [13].

Application of binary AHP in a decision process related to public organizations is in the main focus of this work. The decision process involves three levels, three alternatives and six criteria. We argue that the standard nine point scale based rating system used by the classical Analytic Hierarchy Process is not the most optimal one with respect to decision processes in the majority of public organizations. This mainly because it requires a lot of resources that such organizations do not have. Therefore a simpler rating system like binary or ternary can be used instead.

The rest of the paper is organized as follows. Related work and supporting theory may be found in Section 2. The decision process is presented in Section 3. The paper ends with a conclusion in Section 4.

2 Related Work

Activities in AHP are evaluated by a given number of decision makers, [14]. Judgements on the activities' relative importance are to be provided under the condition that they are quantified to an extent permitting their quantitative interpretation among the activities. The quantified judgements are further employed to derive a set of weights $w_i, \ \ i = 1, 2, ..., n$ associated with the activities.

A matrix $A = (a_{ij}), \ \ i, j = 1, 2, ..., n$ represents the quantified judgements on pairs of activities $\mathcal{C}_i, \mathcal{C}_j$, where $\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_n$ is the set of activities. The elements a_{ij} satisfy the conditions

- $a_{ij} \neq 0, a_{ij} = a_{ji}^{-1}, a_{ii} = 1 \ \ i, j = 1, 2, ..., n,$
- $a_{ij} = a_{j1} = 1$ implies $\mathcal{C}_i$ being of equal relative importance to $\mathcal{C}_j$.

The matrix $A = (a_{ij}), \ \ i, j = 1, 2, ..., n$ can than be written in the form

$$A = \begin{pmatrix} 1 & a_{12} & ... & a_{1n} \\ \frac{1}{a_{12}} & 1 & ... & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & ... & 1 \end{pmatrix}$$

The relations among weights w_i and judgements a_{ij} in the ideal case of exact measurements can be presented as

$$a_{ij} = \frac{w_i}{w_j}, \ \ i, j = 1, 2, ..., n.$$

The following unique solution is used in the general case

$$w_i = \frac{1}{\lambda_{max}} \sum_{j=1}^{n} a_{ij} w_j, \ \ i, j = 1, 2, ..., n$$

where λ_{max} is the maximum eigenvalue [18] of the reciprocal matrix [23] A' of the consistent matrix A, $Aw = nw$ and $w = (w_1, w_2, ..., w_n)$.

Decision makers' judgements are consistent if

$$a_{ij} a_{jk} = a_{ik}, \ \ \ i, j, k = 1, 2, ..., n.$$

In this content consistency means that if a basic amount of row data is available than all other data can be logically deduced from it. Application of eigen vectors leads to a very useful consistency measure called consistency index CI, [14]. The CI of a comparison matrix is defined as

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

Table 1: Random indexces for matrices of size n

n	1	2	3	4	5	6	7
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32

where n is the order of the comparison matrix and λ_{max} is its maximum eigen value. CI measures the transitivity of a preference that is a part of the pairwise comparisons.

The following notations are used in an a_{ij} matrix comparison:

- 'item i is more important than item j' is denoted by θ, $\theta > 1$,
- 'item j is less important than item i' is denoted by $\frac{1}{\theta}$, respectively.

A random index RI, [14] is the mean CI value of random generated matrices of size n, see Table 1. A consistency ratio CR is defined as

$$CR = \frac{CI}{RI}$$

and is a measure of comparison between a given matrix and a random generated matrix in terms of consistency indexes, [14]. The upper bound for an acceptable CR is 0.1 and a ratio exceeding 0.1 implies that the judgments could be too inconsistent to be reliable. A revision of judgements is required if larger values are obtained. In practice the limit 0.1 appears often to be too restrictive.

Right and left eigenvector inconsistency is discussed in [22].

An alternative way to measure consistency is proposed in [12]. A comparison matrix A can be represented by a complete graph if binary rating scale is applied. Inconsistencies in A are then represented by the number of directed cycles in the graph. Misjudgments are further on assumed to generate cycles in the graph. What is needed is to calculate the number of cycles of length 3 since a cycle of f. ex. length 4 implies cycles of length 3, [12].

The number of cycles of length 3 in a given complete directed graph can be calculated as in Theorem 1 and Theorem 2, [12].

Theorem 1 *Given a comparison matrix A in a binary AHP, the trace $tr(V)$ of the third power of the vertex matrix V, $tr(V)$ corresponding to A is three times the number $\mathcal{N}$ of cyclic graphs of length 3, i.e.*

$$\frac{tr(V^3)}{3} = \mathcal{N}$$

Let $S = v^2 \star V^t$ be a matrix where operation $'\star'$ is defined as an element-wise multiplication and V^t is the transposed of matrix V.

Theorem 2 *[12] The number of cycles that includes arc (i, j) in the corresponding graph is the element $s_{ij} \in S$.*

If $a_{ij} > 1$ than $i \rightarrow j$ notation is applied in the complete graph. In a vertex matrix V an element $v_{ij} = 1$ if $a_{ij} > 1$, [24].

Suppose $s_{ij} = k$. The total number of cycles can be reduced by k by reversing the direction of the arc (i, j).

Decision makers find AHP to be a very useful tool. However, an increase of the number of alternatives and criteria results in a larger amount of pairwise comparisons. The latter is time consuming and thus increases the loads of the involved decision makers. Binary and ternary AHP have been proposed for solving problems that do not require a larger scale of values representing the intensities of judgments, [5] and [20].

Addition or deletion of alternatives can lead to possible rank reversal [17], [19], and [21]. Change of local priorities can cause rank reversal before and after an alternative is added or deleted, [25]. In order to avoid rank reversal the authors suggest an approach where the local priorities should be kept unchanged.

The distributive mode normalizes alternative scores under each criterion so that they sum to one. This creates a dependency on how well all other alternatives perform and hence the potential for rank reversal. In contrast, the ideal mode preserves rank by dividing the score of each alternative only by the score of the best alternative under each criterion, [10].

3 Decision Process

Three departments belonging to one faculty should elect a person to serve as a research leader of the faculty. Each department suggests its candidate for the leader position and a member for the evaluation committee. The job of the ones elected evaluation committee is to select criteria first and then evaluate the candidates.

The decision process involves three levels, three alternatives and six criteria, Fig. 1. The notations are as follows:

Goal - 'Select the most appropriate candidate'

Criteria -

- C1 - scientific degrees,
- C2 - obtained research results within last five years,

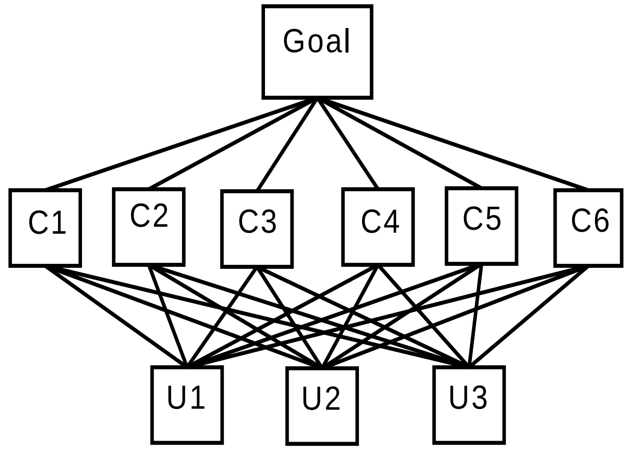

Figure 1: Three levels decision process

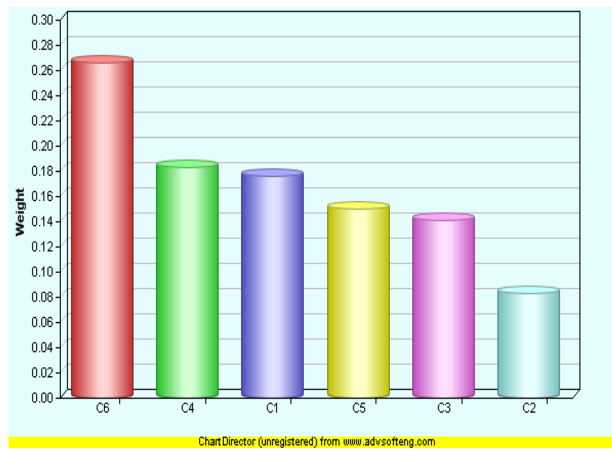

Figure 2: Criteria distribution

- C3 - experience in leading institutionally based research projects,
- C4 - experience in leading international research projects,
- C5 - communication skills,
- C6 - proficiency in English.

Alternatives - U1, U2 and U3.

Data included in Table 2 and Tables 3- 8 is obtained from the evaluation committee's final decisions.

General criteria distribution is shown in Fig. 2.

A distribution for alternatives U1, U2, and U3 is shown in Fig. 3.

Summary of scores for the three alternatives with respect to the six criteria is shown in Fig. 4

Global priority distributive mode is calculated in Table 10. According to that, alternative U2 is the best choice.

Table 2: Pairwise comparison of the criteria C1, C2, C3, C4, C5, and C6 where $CR = 0.08 < 0.1$

	C1	C2	C3	C4	C5	C6	Local priority
C1	1	2	2	$\frac{1}{2}$	2	$\frac{1}{2}$	0.176
C2	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0.083
C3	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	2	$\frac{1}{2}$	0.141
C4	2	2	2	1	$\frac{1}{2}$	$\frac{1}{2}$	0.183
C5	$\frac{1}{2}$	$\frac{1}{2}$	2	2	1	$\frac{1}{2}$	0.150
C6	$\frac{1}{2}$	2	$\frac{1}{2}$	$\frac{1}{2}$	2	1	0.266

Table 3: Pairwise comparison of alternatives U1, U2 and U3 with respect criterion C1

C1	U1	U2	U3
U1	1	$\frac{1}{2}$	$\frac{1}{2}$
U2	2	1	$\frac{1}{2}$
U3	2	2	1

Table 4: Pairwise comparison of alternatives U1, U2 and U3 with respect criterion C2

C2	U1	U2	U3
U1	1	2	2
U2	$\frac{1}{2}$	1	$\frac{1}{2}$
U3	$\frac{1}{2}$	2	1

Table 5: Pairwise comparison of alternatives U1, U2 and U3 with respect criterion C3

C3	U1	U2	U3
U1	1	2	2
U2	$\frac{1}{2}$	1	2
U3	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 6: Pairwise comparison of alternatives U1, U2 and U3 with respect criterion C4

C4	U1	U2	U3
U1	1	$\frac{1}{2}$	2
U2	2	1	2
U3	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 7: Pairwise comparison of alternatives U1, U2 and U3 with respect criterion C5

C5	U1	U2	U3
U1	1	$\frac{1}{2}$	$\frac{1}{2}$
U2	2	1	2
U3	2	$\frac{1}{2}$	1

Table 8: Pairwise comparison of alternatives U1, U2 and U3 with respect criterion C6

C6	U1	U2	U3
U1	1	2	$\frac{1}{2}$
U2	$\frac{1}{2}$	1	$\frac{1}{2}$
U3	2	2	1

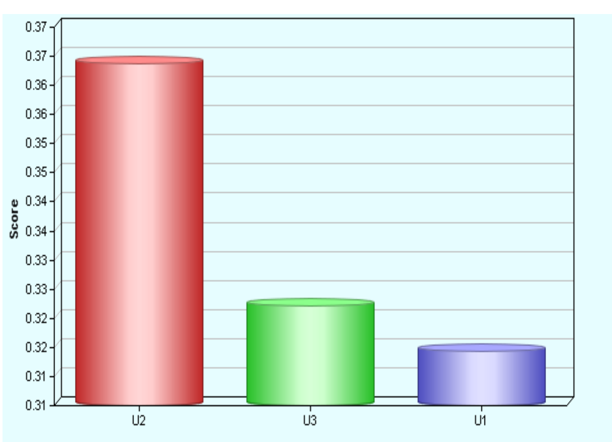

Figure 3: Distribution for alternatives U1, U2, and U3

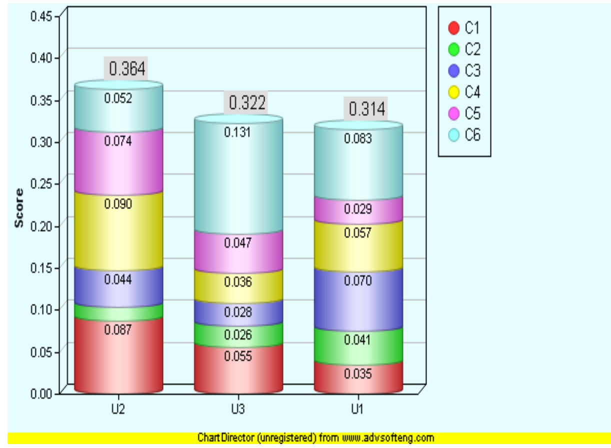

Figure 4: Criteria distribution

Table 9: Local priority summary

	C1 0.176	C2 0.083	C3 0.141	C4 0.183	C5 0.150	C6 0.266
U1	0.199	0.494	0.496	0.311	0.193	0.688
U2	0.494	0.205	0.053	0.638	0.404	0.347
U3	0.312	0.313	0.198	0.196	0.313	0.196

Table 10: Global priority of alternatives

Alternatives	Global priority distributive mode
U1	0.314
U2	0.364
U3	0.322

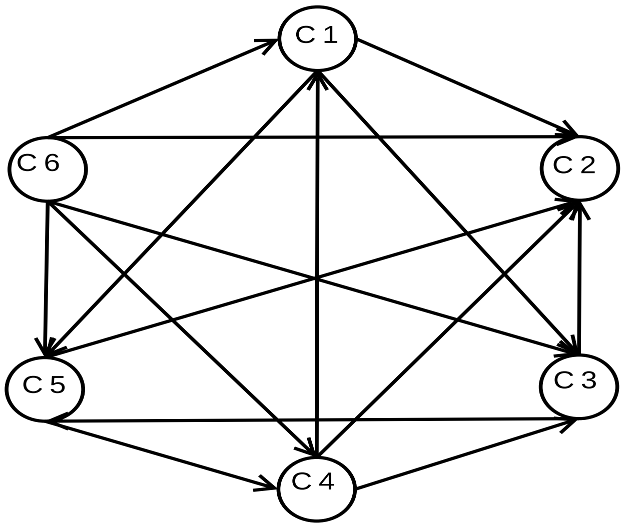

Figure 5: A complete graph with a cycle of length 3

The distributive mode is recommended if rank reversal due to the addition or deletion of alternatives is acceptable, [19].

3.1 Graph Representation

A complete graph is a graph in which each vertex is connected to each of the others (with one edge between each pair of vertices). A directed graph (digraph) is a set of vertices and ordered pairs of edges. The first element in an ordered pair is the initial vertex of the edge and the second element is the terminal vertex, [26]. A path where the start vertex and the end vertex coincide is a cycle, [3].

The relations among the six criteria in Table 2 can be graphically represented by a directed complete graph (see Fig. 5).

Observe that a cycle of length 3 in the complete graph in Fig. 5 results in a consistency ratio smaller than 0.1, while introducing a cycle of length 4 makes the consistency ratio go above the accepted 0.1 limit, Fig. 6.

Graph representations of pairwise comparison of alternatives U1, U2 and U3 with respect to criteria C1, C2, C3, c4, C5, and C6 are shown in Fig. 7, Fig. 8, Fig. 9, Fig. 10, Fig. 11, and Fig. 12. These figures illustrate some of the limitations of a binary scale, i.e. if a larger number of criteria would lead to repetition of some evaluations and or inconsistency.

3.2 Sensitivity Analysis

The sensitivity analysis with respect to criterion C1 (see Fig. 13) illustrates that alternative U2 dominates over both alternative U1 and alternative U3 practically

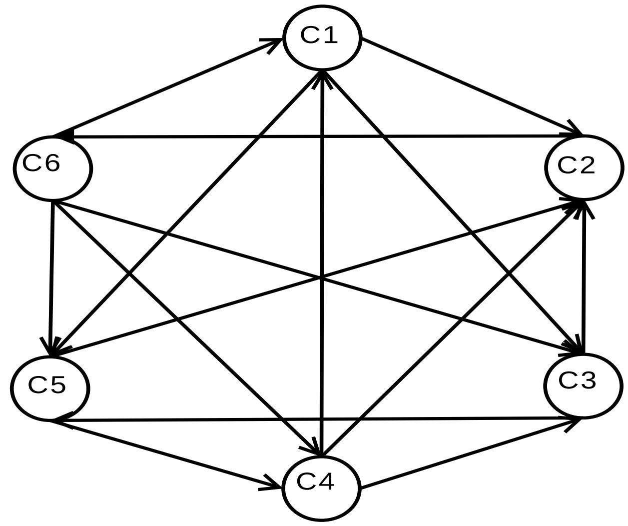

Figure 6: A complete graph with a cycle of length 4

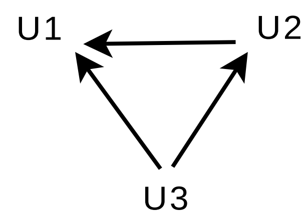

Figure 7: A graph representation of pairwise comparison of alternatives U1, U2 and U3 with respect to criterion C1

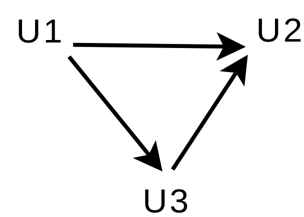

Figure 8: A graph representation of pairwise comparison of alternatives U1, U2 and U3 with respect to criterion C2

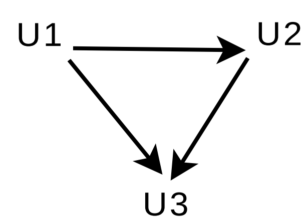

Figure 9: A graph representation of pairwise comparison of alternatives U1, U2 and U3 with respect to criterion C3

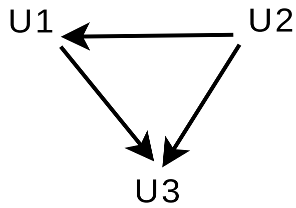

Figure 10: A graph representation of pairwise comparison of alternatives U1, U2 and U3 with respect to criterion C4

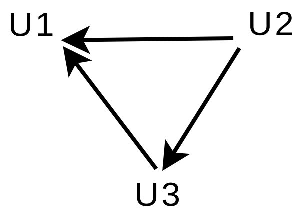

Figure 11: A graph representation of pairwise comparison of alternatives U1, U2 and U3 with respect to criterion C5

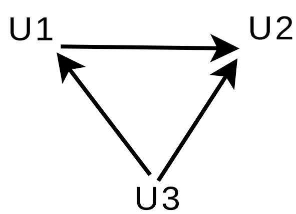

Figure 12: A graph representation of pairwise comparison of alternatives U1, U2 and U3 with respect to criterion C6

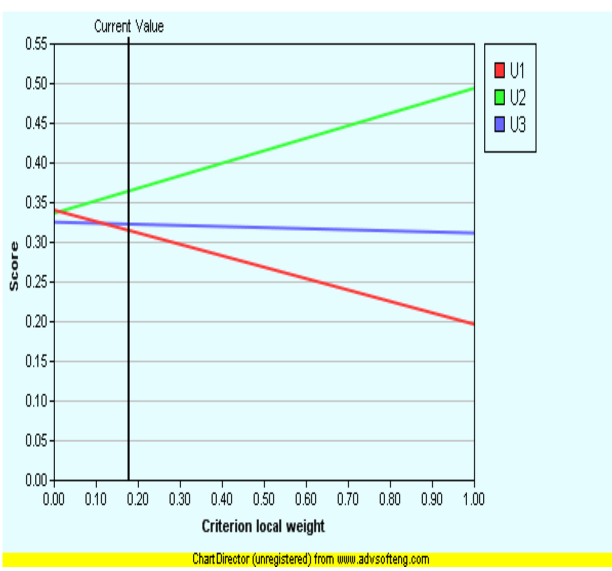

Figure 13: Sensitivity presentation of alternatives U1, U2 and U3 with respect to criterion C1.

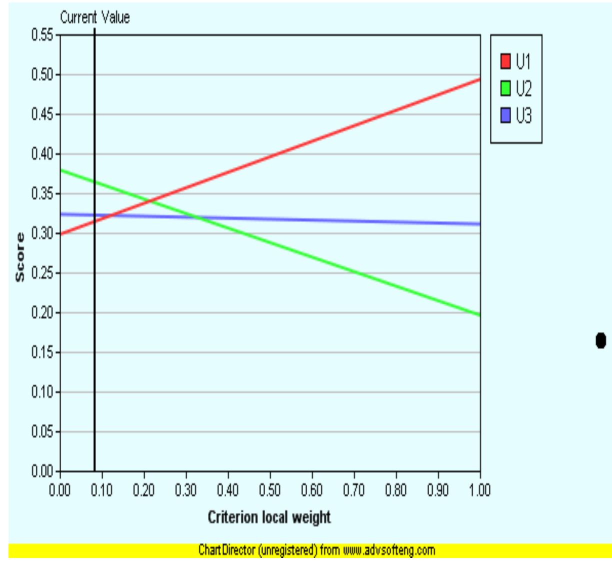

Figure 14: Sensitivity presentation of alternatives U1, U2 and U3 with respect to criterion C2.

trough the entire interval, while alternative U1 dominates over alternative U3 only at the beginning of the discussed interval.

The sensitivity analysis with respect to criterion C2 (see Fig. 14) illustrates that alternative U2 dominates over both alternative U1 and alternative U3 only at the beginning of the discussed interval, while alternative U1 dominates completely afterwards.

The sensitivity analysis with respect to criterion C3 (see Fig. 15) shows similar development but after weight 0.32 the dominance is U1, U2 and U3 in decreasing order.

The sensitivity analysis with respect to criterion C4 (see Fig. 16) illustrates that alternative U2 dominates over both alternative U1 and alternative U3 practically through the entire interval, the behavior of alternative U1 is more or less constant and the influence of alternative U2 decreases over the interval.

The sensitivity analysis with respect to criterion C5 (see Fig. 17) illustrates that alternative U2 dominates over both alternative U1 and alternative U3 practically through the entire interval, the behavior of alternative U3 is more or less constant and the influence of alternative U1 decreases over the interval.

The sensitivity analysis with respect to criterion C6 (see Fig. 18) is similar to the one of criterion C5.

3.3 Head-to-Head Analysis

The Head-to-Head Analysis of alternatives U1 and U2 (see Fig. 19) indicates that

- alternative U1 dominates over alternative U2 considerably with respect to criteria C2 and C3,

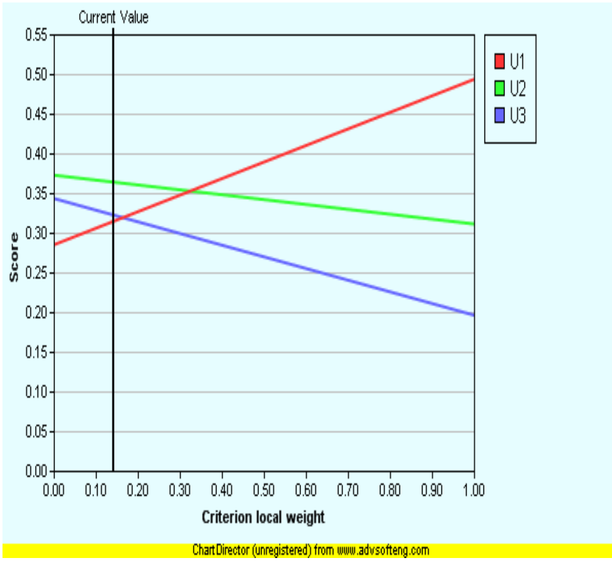

Figure 15: Sensitivity presentation of alternatives U1, U2 and U3 with respect to criterion C3.

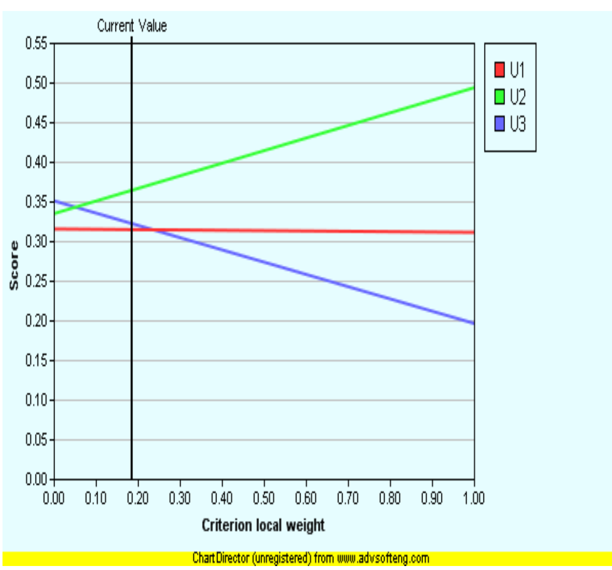

Figure 16: Sensitivity presentation of alternatives U1, U2 and U3 with respect to criterion C4.

Figure 17: Sensitivity presentation of alternatives U1, U2 and U3 with respect to criterion C5.

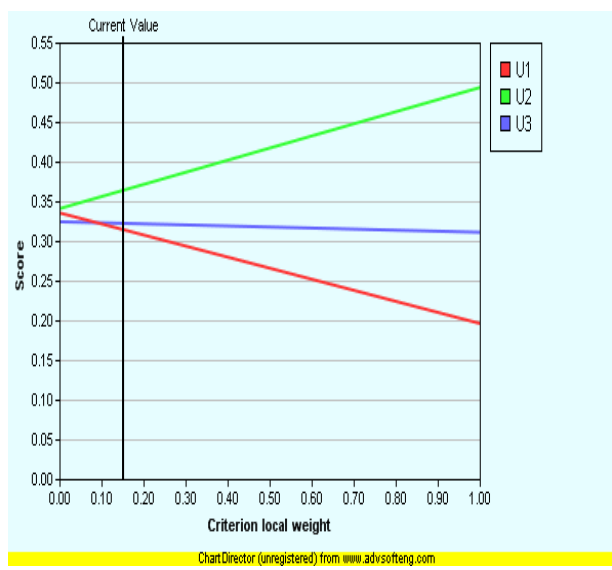

Figure 18: Sensitivity presentation of alternatives U1, U2 and U3 with respect to criterion C6.

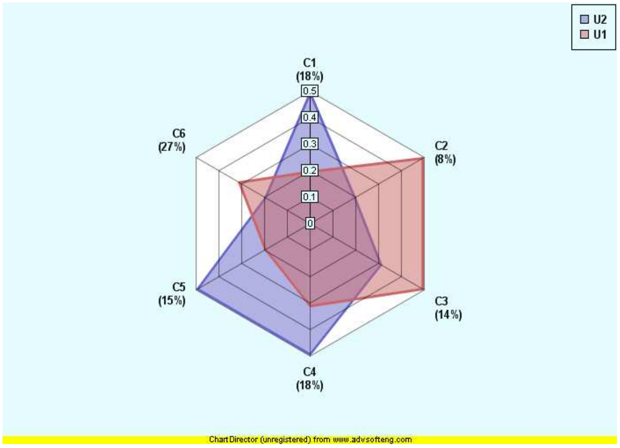

Figure 19: Head-to-Head Analysis of alternatives U1 and U2 with respect to criteria C1, C2, C3, C4, C5, and C6.

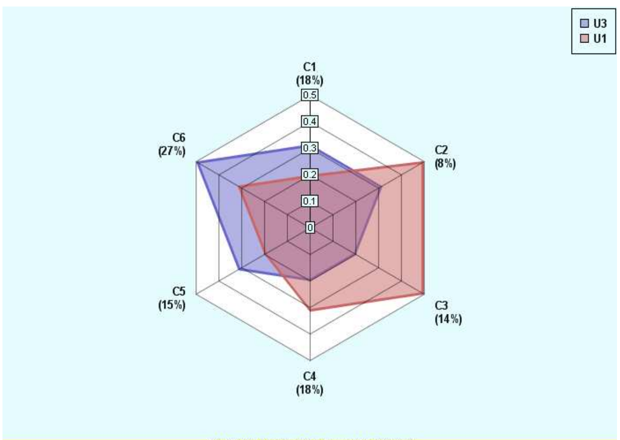

Figure 20: Head-to-Head Analysis of alternatives U1 and U3 with respect to criteria C1, C2, C3, C4, C5, and C6.

and slightly with respect to criterion C6, while

- alternative U2 dominates over alternative U1 considerably with respect to criteria C1, C4 and C5.

The Head-to-Head Analysis of alternatives U1 and U3 (see Fig. 20) indicates that

- alternative U1 dominates over alternative U3 considerably with respect to criterion C3, and slightly with respect to criteria C2 and C4, while
- alternative U3 dominates over alternative U1 considerably with respect to criteria C6, and slightly with respect to criterion C1.

The Head-to-Head Analysis of alternatives U2 and U3 (see Fig. 21) indicates that

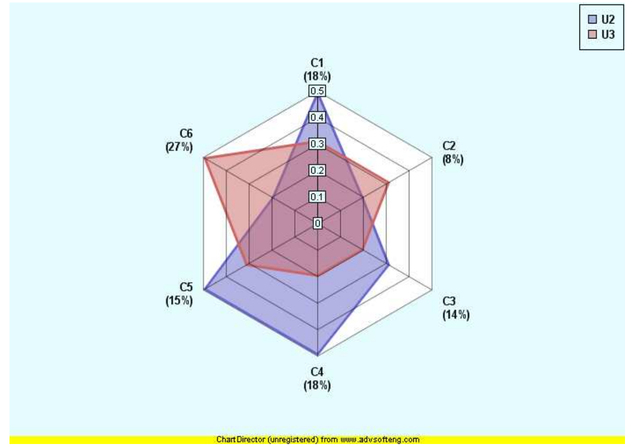

Figure 21: Head-to-Head Analysis of alternatives U2 and U3 with respect to criteria C1, C2, C3, C4, C5, and C6.

- alternative U2 dominates over alternative U3 considerably with respect to criteria C4 and C5, and slightly with respect to criteria C1 and C3, while
- alternative U3 dominates over alternative U2 considerably with respect to criterion C6, and slightly with respect to criterion C2.

The AHPproject free Web-based decision support tool [7] is used to create figures related to Sensitivity analysis and Head-to-Head analysis.

3.4 Discussion

The classical AHP employs a 1 to 9 scale. Binary AHP restricts the grading in the evaluation process but shortens considerably the time used by the decision committee members for pairwise comparison. In our opinion the most distinguished features of binary AHP are to find the exact cause (location) of inconsistencies via graph representation and to considerably reduce the level of inconsistencies by reversing directions of minimum number of arcs.

4 Conclusion and Future Research

Binary AHP is sufficient for evaluating reasonably small cases in public organizations. However the binary AHP is not very effective in case of a large number of criteria since pairwise comparisons with respect to different criteria might end up with the same evaluation due to lack of alternatives. As a consequence some criteria will be excluded from the decision process.

The AHP tolerates inconsistency with consistency ratio smaller than 0.1. An interesting topic for future research is to find the maximum number of cycles of given length that would preserve the accepted by the AHP theory value of the corresponding consistency ratio.

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