Decison Support System for Module Type Products : From the Perspective of Problems of Utility and Applicability

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Abstract: - Companies have to develop core competitive technologies in a very short time. Therefore, product development and research and development must proceed with the concept of management. This study proposes a method and decision support system for selection of R&D projects utilized to the modular type of PD. First, it should be clarified the relationship among R&D investment, its project, modular technology and the product. Second, it is helped the decision making about how many modules of technology introduced the product considered to the cost restriction. Finally, it will be made the support system for its decision making from the outputs of figures. In this way, it becomes possible to effectively develop products while analyzing the conditions of the industry as well as the market and competing companies. It is also able to allow the proposal of future critical management strategy or tactical strategy.

Key-Words: - Product development, R&D strategy, R&D investment, Technology management, Decision support system, Integer programming

1 Introduction

In recent years, customer requirements for various products and services have started to demand shorter wait times. For that reason, companies have to develop core competitive technologies in a very short time. Companies also need to advance technology from the perspective of overall strategy [1]. Therefore, product development (hereafter called PD) or research and development (hereafter called R&D), in which finishing times and successful outcomes are unknown, must proceed coupled with the concept of management.

There are many research fields of PD and R&D, referred to as "Management of Technology" or "Technology Management." One of the fields is "the study of R&D investment." One study found that the way to optimize R&D investment is through integer programming. Another proposed a way to allocate resources following the evaluation of R&D projects [2-5]. While these methods are useful for deciding the fiscal year's investment, they have a few issues:

1) This previous research focused primarily on the amount of R&D investment itself, with less attention paid to the relationship between R&D activity and PD.

2) The final goal of R&D activity is to develop a core technology, and introduce it into new products. We therefore need to start discussions with product strategy.

Subsequently, when we look at PD and R&D activities in Japan, the current style of products in the manufacturing industry is mainly the module type [6-7]. For example, such products as digital cameras, cellular phones and printers, are constituted from various parts or technologies, combined to create one product. Each component unit manufacturer has a competitive edge and sells to customers or to other companies. This method gives an advantage by decentralizing the risk in R&D investment in the case of introducing this module type of product. In other words, if similar unit technology exists in other companies, the risk can be lowered through purchase or alliance with that company. Consequently, it becomes economical over time.

There are a few useful applications for researches on module products. For instance, after the multiple parts or technologies have been evaluated by AHP or scaling method, the most suitable combination is chosen by integer programming with cost restrictions [8-10]. These studies are effective for extracting core parts or technologies, and proposing the most valuable assemblage of parts. They have, however, the following problems:

- 1) They evaluated only parts or technologies and only at a point in time. More recently, it has been recognized that it is necessary to discuss the process, from the R&D project to the final technology and the product. As a result, the studies should be argued from a more realistic situation of time and condition.
- 2) They look at many modules for one product, and they also have cost restrictions. The same as in (1) above, in a real situation, they do not require that all of the modules or technology be introduced to the product. It will advance the discussion to decide which technologies are selected or eliminated due to cost restrictions.
- 3) There is no consideration of the relationship between R&D investment and the PD.

On the other hand, there is the "Modeling approach," which models the process of decision-making in PD, which allows for easy discussion from tools or diagrams. Tacit knowledge in the decision making process of PD is presented concretely, clarified objectively, and the relationship among the various parts of the procedure is specified. Finally, the structure of the process becomes clear, and the figures are useful in expressing the situation. Kusaka is adopted the modeling approach at the R&D research filed [11-14]. They have suffered from the following problem;

- 1) Because the time factor is not considered, it is difficult to apply to mid or long-term decision making.
- 2) It is not clear what kinds of business strategy or information offering is possible using this tool.

Considering these backgrounds and previous research, this study proposes a method and decision support system for selection of R&D projects utilized to the modular type of PD. That is, at the scene of R&D situation through the plural generations for modular type of PD, first, it should be clarified the relationship among R&D investment, its project, modular technology and the product. Second, considered to the cost restriction, it is helped the decision making about how many modules of technology introduced the product and which projects selected or rejection. For that, it is set the integer programming. Finally, it will be made the support system for its decision making, and extracted a lot of useful information from the figure. Through this study, it must be useful for business manager and R&D manger to discuss the mid term PD strategies.

2 Prerequisites of This Study and its Flow

2.1 Prerequisites

In order to proceed with this study, the following prerequisites are to be set.

- 1) Target markets where the products will be sold, consumer needs, as well as information regarding competitor companies, have already been understood.
- 2) The target industry is the manufacturing industry, and all products are to be of the module type. As a concrete example, a digital camera is a single product that consists of a collection of several units of technology. This unit-based technology shall be called a "module." There are several kinds of modules such as resolution adjustment software technology, hardware technology with telescopic lens functions, or long-term battery technology (Fig.1).
- The option to improve, or otherwise develop anew, module technology shall be called the "R&D Project." This consists of information compiling the "Technology Name," "R&D Investment," "Technical Strength (Performance)," and "Estimated Completion Period."
- 4) In order to quantitatively express the relationship between R&D investment and its performance, a functional relationship between

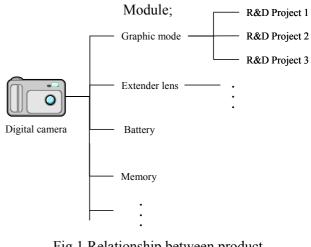
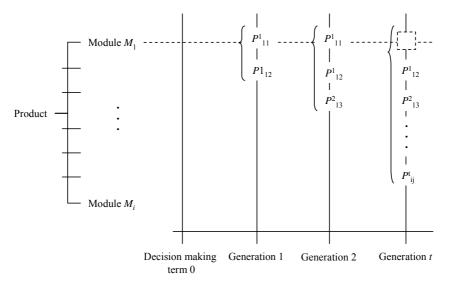
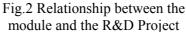


Fig.1 Relationship between product and its module





the two shall be set. In which case, information regarding "what level of performance can be achieved relative to the investment" must already be understood.

- 5) The PD has been continued even after the product has been launched so long as the suspension decision has not been made. Thus, the performance level is improved over time.
- 6) Each function for the PD can be developed independently. Thus, the total cost of the PD is the sum of costs for all selected functions. The total performance of the PD is the sum of realized technology evaluation values of selected project for all selected functions.

2.2 Inputting information on the R&D Project

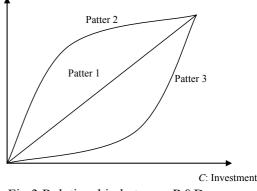
There are several kinds of modules, from those that should be minimally furnished, to those that will need to be rapidly improved. It is also insufficient to consider only one module, but the mutual interactions with other modules while accounting for the limitations on resources, as well as time factors such as a "three-year-later release," must also be considered. And so the relationship between the module and the R&D Project must be explained including the time axis (Fig.2).

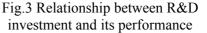
The product is composed of the Modular M_i . In each period (called a "generation"), there is an R&D Project P_{ij}^t . The decision-making period is 0, and the R&D Project is completed when P appears, and then

that output can be introduced to the product. Only one project may be selected for each module. For example in the case of M_1 , in Generation 1, two research projects (P_{11}^1, P_{12}^1) can be completed. And in Generation 2, P_{13}^2 can be completed. At the point of Generation 2, P_{11}^1 and P_{12}^1 , which had been completed in Generation 1, as well as P_{13}^2 , completed in Generation 2, may become candidates for product adoption. The selection is dependent on factors such as the expense-performance relationships of other projects, or relationships with other modules. And as time passes, sometimes it is the case that the technology will have grown old, or becomes standardized. In which case it will be terminated as with P_{11}^1 in Generation *t*.

By explaining the module and project by considering the time axis in these ways, various circumstances can be set.

P: Module's performance





This study also discusses the relationship between R&D investments and its PD. In order to do so, the relationship between R&D investment and the performance of the research project is set as a functional relationship (Fig.3). Specifically, if the development sequence of the "Resolution module" is known to be Pattern 1, a pixel resolution of 1,000,000 can be achieved with an R&D investment of \$100,000. Similarly, 2,000,000-pixel resolution can be achieved with \$200,000, and the results of the R&D investment can continue to be modeled in this way. In addition to the direct ratio, depending on the targeted R&D the pattern 2) or increasing returns (Pattern 3), so the selection will take place with a support system.

2.3 Formularization of the problem of R&D Project Selection

The above information should make it possible to discus how many modules can be included in a product and which R&D project can be selected, based on the performance (referring to technical strength) and investment limitations. To this end, integer programming that uses the R&D investment as a constraint is set. The results would show the optimum combination for R&D project.

Max
$$\sum_{i} \sum_{j} p'_{ij} x'_{i}$$
 for $t=1,\cdots,T$ (1)

s.t.

$$p_{ij}^{\prime} = \begin{cases} 0 & \text{if project is not finished in terms } t \\ \text{Numeric value} & \text{if project is already finished in terms } t \\ \sum_{i} c_{ij}^{\prime} x_{ij} \leq C^{\prime} & (3) \\ \sum_{j} x_{ij}^{\prime} = 1 & \text{if modular } t \text{ is selected} & (4) \end{cases}$$

Note;

 $t (t = 1 \sim T) : \text{time, generation}$ $i (i = 1 \sim I) : \text{a number of module}$ $j (j = 1 \sim J) : \text{a number of project}$ $x_{ij} = \{0, 1\}$ $p^{t}_{ij} (p^{t}_{ij} \ge 0) : \text{project } j\text{'s performance of } i \text{ module's in}$ t term $c^{t}_{ij} (c^{t}_{ij} \ge 0) : \text{ project } j\text{'s R\&D expenditure of } i$

 $\begin{array}{c} c \\ ij \\ c \\ i$

 $C^{t}(C^{t} > 0)$: the amounts of R&D budget in t term

Formula (1) calculates the optimum combinations for performance, by each generation. In other words, the solution is that in which the combination of the performances for the projects completed in each generation is greatest. However, as a constraint, so that C^{t} is not surpassed like formula (3). The calculations are performed in order, beginning with i = 1. Accordingly, as a tacit rule, the smaller the number *i* in M_{i} , the more valuable the module is. Finally, only one project is selected out of one module like formula (4).

3 Decision Support System

It was constructed the support system for the calculation of the suitable combination and the analysis the information from the outputs. Support system is composed from input part, processing part and output parts. This time, we made the system by the VBA, because it is easy and wide to use. Fig.4 is top and input screen of the system. The system consists of the input and graph chart screens. First, we have to input the data on the functions and the technology alternatives. Right of the Fig. 4 appears after clicking the button "Input DATA." The process is as follows:

Step 1: Input the project name, the R&D cost, the performance and the finishing time, and the weight of function F1.

Step 2: Click the button "Input" and input another alternative. Repeat Steps 1 through 2 until the input operations for all alternatives in F1 are completed.

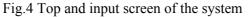
Step 3: If there is a new function, F2, click the button "Other Function" and repeat Steps 1 through 2 again.

Step 4: When all alternatives for the last function Fn, are finished click the "Finish" button.

This system is intended to provide useful information for decision-makers through graphs and charts. Thus, it is able to make two patterns of time series graphs and radar charts for each function.

At the output parts, they have one graph and one radar chart, which are obtained the useful information for decision making. Fig.5 shows a Cost-Performance curve (CPC) at each generation. The system calculates the optimal PD solution and makes a graph of the CPC. For output the combination alternative, the system calculates according to the amounts of R&D budget.

Menu	
Input DATA Selection Help	Input DATA
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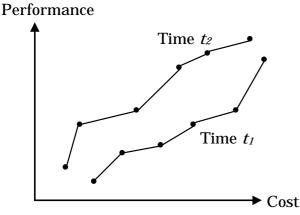


Fig.5 Cost-Performance curve

From that, it shows how much time and costs are needed to attain a desired performance.

Fig.6 is an example of a "radar chart." It represents the upper limit of attainable performance for each generation.

4 Numerical Analysis and Discussion

The three scenarios shown in Table 1, each of which has five module and four alternatives for each function, show how the system can provide useful graphical information. Scenario 1 is a case where, through spending the time and cost, PD is steadily introduced over the long term and thus long-range competitive advantage is achievable. As the PD steadily evolves under the planned roadmap only small revisions are needed in the short term. Scenario 2 is a case where main technological developments occur in the early generations of PD. The PD then becomes incremental in the last stage and thus, at this stage, strong

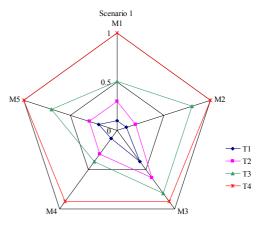


Fig.6 Radar chart

competitions occur, making it difficult to maintain competitive advantage. A radical revision of the roadmap, through adding a new function and/or new technology development, is needed to regain competitive advantage. Scenario 3 is a case where difficulty in developing the necessary technology means that remarkable results are not achieved in the short or mid-term. In the later stage, competitive advantage is great, but in order to support such difficult and expensive PD, other parallel PD plans, which support the funding of the roadmap, could be considered. Table 2 (see Appendix) is the result of integer programming. Through the support system, where the total investment constraint was increased from 0, the optimum combination achievable can be output. For example in Scenario 1, with an investment of 15, the P_{11}^1 project, same connotation as a product using only Modular M_1 , can be made. In this case, the performance is 0.1. Likewise, if the investment constraint was 50 and the investment could not go over 50, the combinations $p_{11}^1 p_{21}^1 p_{32}^1 p_{41}^1$ will be selected.

M _i	p ^t _{ij}	Scenario 1		Scen	ario 2	Scenario 3		
		Investment	Performance	Investment	Performance	Investment	Performance	
<i>M</i> ₁	p_{11}^{1}	15	0.10	15	0.17	15	0.06	
	p_{12}^2	30	0.30	30	0.77	30	0.07	
	p_{13}^{3}	40	0.50	40	0.84	40	0.11	
	p_{14}^{4}	60	1.00	60	0.87	60	0.92	
<i>M</i> ₂	p_{21}^{1}	12	0.10	12	0.30	12	0.08	
	p^{2}_{22}	20	0.20	20	0.81	20	0.09	
	p_{23}^{3}	40	0.80	40	0.84	40	0.12	
	p^{4}_{24}	80	1.00	80	0.95	80	1.00	
<i>M</i> ₃	p_{31}^{1}	5	0.40	5	0.39	5	0.11	
	p_{32}^{2}	20	0.60	20	0.95	20	0.14	
	p_{33}^{3}	45	0.80	45	0.97	45	0.24	
	p^{4}_{34}	90	0.90	90	1.00	90	0.84	
M_4	p_{41}^{1}	6	0.10	6	0.17	6	0.09	
	p_{42}^2	40	0.30	40	0.87	40	0.12	
	p_{43}^{3}	45	0.40	45	0.92	45	0.33	
	p^{4}_{44}	56	0.90	56	0.95	56	0.92	
M 5	p_{51}^{1}	30	0.20	30	0.39	30	0.11	
	p^{2}_{52}	40	0.30	40	0.90	40	0.14	
	p_{53}^{3}	60	0.70	60	0.95	60	0.16	
	p^{4}_{54}	90	1.00	90	1.00	90	1.00	

Table 1 Numerical examples for scenario analyses

However, an investment of 46 will suffice. The modules selected in this case go from M_1 to M_4 . If the constraint is 100, $p_{13}^3 p_{23}^3 p_{32}^2$ will be selected. In this case, the projects completed in Generations 2 and 3 are simultaneously selected.

Fig.7 shows CPC for scenarios 1~3. Scenario 1 shows that performance increases uniformly in terms of both investment and time. It is easier to discuss form the figure than the table. For example, curves T3 and T4 intersect at 100 investments. If the aim is PD with a long-range competitive advantage, investments of more than 100 should be invested in order to be able to continue research until generation T4, and thus be able to launch a higher performance product.

Scenario 2 is a case where PD achieves high performance at an early stage. That is, high performance is achieved at the T2 generation but performance would not improve much after that time. In such a case, it is meaningless to continue PD without any changes before the T3 and T4 generations, and immediate further development should be considered to achieve higher performances.

Scenario 3 is a case where there are considerable technological challenges and PD takes an extremely long time to achieve the desirable product performance. Bearing in mind that most radically innovative technological products take a long time to develop, innovative producers need to be able to continue with non-profitable patience until generation T4.

Radar charts, as shown in Fig.8. For Scenario 1, function M_3 achieves an ideal performance in an early stage. It also shows that functions M_2 and M_5 are increasing steadily in performance over time, that is, these alternative are growing. The chart shows that the functions M_2 and M_3 are important for the product.

For Scenario 2, because alternatives achieve high performance in the early stages, the growth in quality for each function is limited in later stages.

For Scenario 3, where a great deal of time is needed to achieve high performance, the radar chart shows the functions that become obstacles for product evolution. In this case, M_3 and M_4 are approaching high performance in relatively early stages. On the other hand, the growth of alternatives for M_1 , M_2 and M_5 is slow. As a result, high costs are incurred to raise the performance of M_1 , M_2 and M_5 .

From here, it gives the example of information from the Fig.7's Scenario 1. The information obtained from this system is considered. Specifically, it allows us to understand how much increase or decrease in investment or time is necessary in order to obtain the desired performance. In the curved line of Fig.7, the manager may wish for a performance of over 3. With an investment of 205, this is achieved at T3, but with an investment of only 161, it is achieved at T4. However, if the competition for this product is heavy

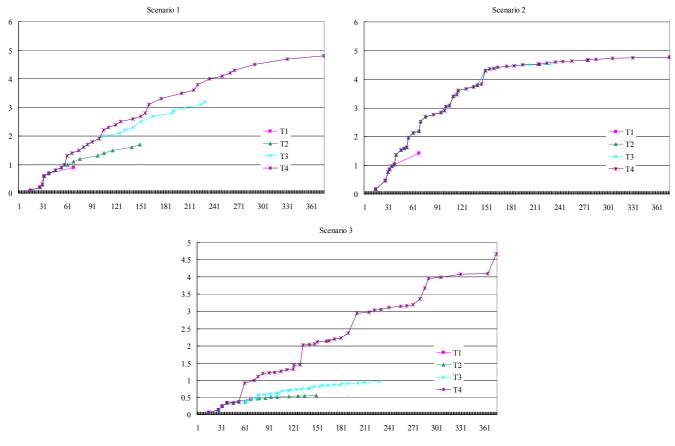


Fig.7 Outputs of CPC graphs in each scenarios

and there is a need to quickly improve, the following may be discussed.

- 1) If the investment is increased to 205, the time can be shortened by one generation. That is to say, it can be discussed how much one must budget in order to sell a satisfactory product.
- 2) If the investment cannot be increased, it can be achieved in t3 by lowering the desired performance to 2.8 or 2.7. That is to say, the relationship between performance and time can be discussed.

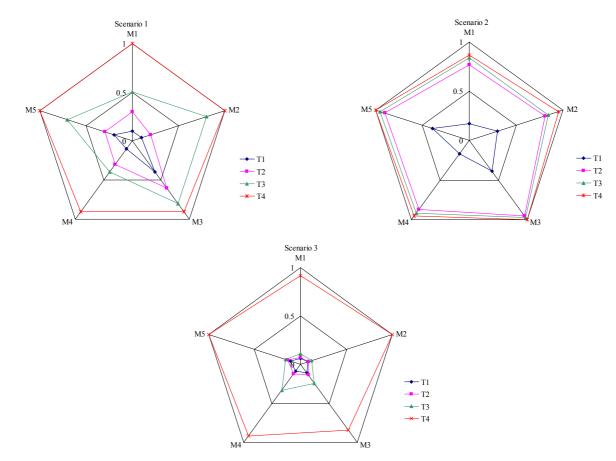
By using the support system in this way, it becomes possible to effectively develop products while analyzing the conditions of the industry as well as the market and competing companies along with investment constraints and temporal fluctuations.

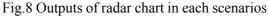
4 Conclusion

From the need for a management theory taking into consideration the time factors in the PD as well, we set the formulation of the product part selection problem with the constraining condition being R&D. Also, I built a decision making support system for deliberating the results of such.

Specifically, with the subject being the module products and assuming a need for improving such modules, I clarified the relationship with the R&D projects for such. Also, from the information of the investment amount, technical capacity, performance in the project, and the term of completion, I formulated the integer programming problem for selecting the optimum project, proposing a proposal for an optimum project combination. Also, I was able to perform various numerical simulations from the results and tables obtained through the system. Further, I was also able to deliberate the increase/decrease in the investment amount and performance.

From all of the above, this research is able to clarify the relationship between the R&D investment and PD, as well as build a support system allowing the proposal of future critical management strategy or tactical strategy. However, there are future issues that exist. They are 1) considering the market and consumer needs in measuring performance, 2) considering the uncertainty of research and





development projects, and 3) building a web based support system, etc.

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Yoshiki Nakamura

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Appendix

Table 2 Result of integer programming

Scenario 1			Scenario 2			Scenario 3		
Alternative plan Sum of Sum of		Alternative plan Sum of		Sum of	Alternative plan	Sum of Sum of		
×		performance	*		performance	1		performance
p_{11}^{1}	15	0.1	p_{11}^{1}	15	0.17	p_{11}^{1}	15	0.06
$p_{11}^{1}p_{21}^{1}$	27	0.2	$p_{11}^{1}p_{21}^{1}$	27	0.47	$p_{11}^{1}p_{21}^{1}$	27	0.14
p_{12}^2	30	0.3	p_{12}^2	30	0.77	$p_{11}^{1}p_{21}^{1}p_{31}^{1}$	32	0.25
$p_{11}^{1}p_{21}^{1}p_{31}^{1}$	32	0.6	$p_{11}^{1}p_{21}^{1}p_{31}^{1}$	32	0.86	$p_{11}^{1}p_{21}^{1}p_{31}^{1}p_{41}^{1}$	38	0.34
$p^{1}_{11}p^{1}_{21}p^{1}_{31}p^{1}_{41}$	38	0.7	$p_{11}^{1}p_{22}^{2}$	35	0.98	$p_{11}^{1}p_{22}^{1}p_{31}^{1}p_{41}^{1}$	46	0.35
$p_{11}^{1}p_{22}^{1}p_{31}^{1}p_{41}^{1}$	46	0.8	$p_{11}^{1}p_{21}^{1}p_{31}^{1}p_{41}^{1}$	38	1.03	$p_{11}^{1}p_{21}^{1}p_{32}^{1}p_{41}^{1}$	53	0.37
$p_{11}^{1}p_{21}^{1}p_{32}^{1}p_{41}^{1}$	53	0.9	$p_{11}^{1}p_{22}^{2}p_{31}^{1}$	40	1.37	p_{14}^{4}	60	0.92
$p_{13}^{3}p_{21}^{1}p_{31}^{1}$	57	1	$p_{11}^{1}p_{22}^{1}p_{31}^{1}p_{41}^{1}$	46	1.54	$p^{4}_{14}p^{1}_{21}$	72	1
$p_{11}^{1}p_{23}^{3}p_{31}^{1}$	60	1.3	$p_{12}^2 p_{22}^2$	50	1.58	$p^{4}_{14}p^{1}_{21}p^{1}_{31}$	77	1.11
$p_{11}^{1}p_{23}^{1}p_{31}^{1}p_{41}^{1}$	66	1.4	$p^{1}_{12}p^{1}_{23}p^{1}_{31}p^{1}_{41}$	53	1.63	$p^{4}_{14}p^{1}_{21}p^{1}_{31}p^{1}_{41}$	83	1.2
$p_{11}^{1}p_{23}^{3}p_{32}^{2}$	75	1.5	$p_{12}^2 p_{22}^2 p_{31}^1$	55	1.97	$p^{4}_{14}p^{2}_{22}p^{1}_{31}p^{1}_{41}$	91	1.21
$p_{11}^{1}p_{23}^{3}p_{32}^{2}p_{41}^{1}$	81	1.6	$p_{12}^2 p_{22}^2 p_{31}^1 p_{41}^1$	61	2.14	$p^{4}_{14}p^{1}_{21}p^{2}_{32}p^{1}_{41}$	98	1.23
$p_{13}^{3}p_{23}^{3}p_{31}^{1}$	85	1.7	$p_{12}^2 p_{21}^1 p_{32}^2 p_{41}^1$	68	2.19	$p_{11}^{1}p_{24}^{4}p_{31}^{1}p_{41}^{1}$	106	1.26
$p_{13}^{3}p_{23}^{3}p_{31}^{1}p_{41}^{1}$	91	1.8	$p_{12}^2 p_{22}^2 p_{32}^2$	70	2.53	$p^{4}_{14}p^{1}_{21}p^{1}_{31}p^{1}_{41}p^{1}_{51}$	113	1.31
$p_{13}^{3}p_{23}^{3}p_{32}^{2}$	100	1.9	$p_{12}^2 p_{22}^2 p_{32}^2 p_{41}^1$	76	2.7	$p_{12}^2 p_{22}^2 p_{31}^1 p_{41}^1 p_{52}^2$	121	1.32
$p^{4}_{14}p^{3}_{23}p^{1}_{31}$	105	2.2	$p_{13}^{3}p_{22}^{2}p_{32}^{2}p_{41}^{1}$	86	2.77	$p^{4}_{14}p^{1}_{21}p^{1}_{31}p^{3}_{43}$	122	1.44
$p^{4}_{14}p^{3}_{23}p^{1}_{31}p^{1}_{41}$	111	2.3	$p_{12}^2 p_{22}^2 p_{31}^1 p_{42}^2$	95	2.84	$p_{14}^4 p_{22}^2 p_{31}^1 p_{43}^3$	130	1.45
$p_{14}^4 p_{23}^3 p_{32}^2$	120	2.4	$p_{12}^2 p_{22}^2 p_{31}^1 p_{43}^3$	100	2.89	$p^{4}_{14}p^{1}_{21}p^{1}_{31}p^{4}_{44}$	133	2.03
$p_{14}^4 p_{23}^3 p_{32}^2 p_{41}^1$	126	2.5	$p_{12}^2 p_{22}^2 p_{31}^1 p_{41}^1 p_{52}^2$	101	3.04	$p_{14}^4 p_{22}^2 p_{31}^1 p_{44}^4$	141	2.04
$p_{13}^{3}p_{23}^{3}p_{31}^{1}p_{44}^{4}$	141	2.6	$p_{12}^2 p_{22}^2 p_{32}^2 p_{41}^1 p_{51}^1$	106	3.09	$p^{4}_{14}p^{1}_{21}p^{2}_{32}p^{4}_{44}$	148	2.06
$p^{4}_{14}p^{3}_{23}p^{3}_{33}p^{1}_{41}$	151	2.7	$p_{12}^2 p_{22}^2 p_{32}^2 p_{42}^2$	110	3.4	$p^{4}_{14}p^{4}_{24}p^{1}_{31}p^{1}_{41}$	151	2.12
$p_{13}^{3}p_{23}^{3}p_{32}^{2}p_{44}^{4}$	156	2.8	$p_{12}^2 p_{22}^2 p_{32}^2 p_{43}^3$	115	3.45	$p^{4}_{14}p^{1}_{21}p^{1}_{31}p^{4}_{44}p^{1}_{51}$	163	2.14
$p^{4}{}_{14}p^{3}{}_{23}p^{1}{}_{31}p^{4}{}_{44}$	161	3.1	$p_{12}^2 p_{22}^2 p_{32}^2 p_{41}^1 p_{52}^2$	116	3.6	$p^{4}_{14}p^{4}_{24}p^{2}_{32}p^{1}_{41}$	166	2.15
$p^{4}_{14}p^{3}_{23}p^{2}_{32}p^{4}_{44}$	176	3.3	$\frac{p_{13}^{3}p_{22}^{2}p_{32}^{2}p_{32}^{2}p_{41}^{1}p_{52}^{2}}{p_{141}^{3}p_{52}^{2}}$	126	3.67	$p^{4}_{14}p^{1}_{21}p^{1}_{31}p^{1}_{41}p^{4}_{54}$	173	2.2
$p^{4}_{14}p^{3}_{23}p^{3}_{33}p^{4}_{44}$	201	3.5	$p^{2}_{12}p^{2}_{22}p^{1}_{31}p^{2}_{42}p^{2}_{52}$	135	3.74	$p^{4}_{14}p^{4}_{24}p^{1}_{31}p^{1}_{41}p^{1}_{51}$	181	2.23
$p^{4}_{14}p^{3}_{23}p^{2}_{32}p^{4}_{44}p^{2}_{52}$	216	3.6	$p^{2}_{12}p^{2}_{22}p^{2}_{31}p^{3}_{43}p^{2}_{52}$	140	3.79	$\frac{p^{4}}{p^{4}} \frac{4}{4} \frac{p^{4}}{24} \frac{p^{3}}{31} \frac{p^{4}}{43} \frac{p^{4}}{43}$	190	2.36
$\frac{p^{4}}{p^{4}} \frac{14p^{2}}{23} \frac{23p^{3}}{14p^{3}} \frac{14p^{4}}{31p^{4}} \frac{32}{44p^{3}} \frac{14p^{3}}{53}$	221	3.8	$\frac{p^{2}}{12}p^{2}\frac{22p^{2}}{22}p^{2}\frac{3p^{2}}{32}p^{3}\frac{43p^{2}}{43}p^{5}$	145	3.84	$p^{4}_{14}p^{4}_{24}p^{1}_{31}p^{4}_{44}$	201	2.95
$p^{4}_{14}p^{3}_{23}p^{2}_{32}p^{4}_{44}p^{3}_{53}$	236	4	$p^{2}_{12}p^{2}_{22}p^{2}_{32}p^{2}_{32}p^{2}_{42}p^{2}_{52}$	150	4.3	$p^{4}_{14}p^{4}_{24}p^{2}_{32}p^{4}_{44}$	216	2.98
$p^{4}_{14}p^{3}_{23}p^{1}_{31}p^{4}_{44}p^{4}_{54}$	251	4.1	$p^{2}_{12}p^{2}_{22}p^{2}_{32}p^{3}_{43}p^{2}_{52}$	155	4.35	$p^{4}_{14}p^{1}_{21}p^{1}_{31}p^{4}_{34}p^{4}_{54}$	223	3.03
$p^{4}_{14}p^{2}_{23}p^{3}_{33}p^{4}_{44}p^{3}_{53}$	261	4.2	$p^{-1}_{13}p^{-2}_{22}p^{-3}_{32}p^{-4}_{43}p^{-5}_{52}$	160	4.37	$p^{4} 14p^{2} 11p^{3} 11p^{4} 44p^{54}$ $p^{4} 14p^{4} 24p^{3} 11p^{4} 44p^{51}$	231	3.06
$p^{4}_{14}p^{2}_{23}p^{2}_{32}p^{4}_{44}p^{4}_{54}$	266	4.3	$p^{-13p^{-22p^{-32p^{-42p^{-52}}}}$ $p^{-13p^{-22p^{-2}}}_{-13p^{-22p^{-2}}} p^{-3}_{-32p^{-42p^{-52}}} p^{-3}_{-43p^{-52}}$	165	4.42	$p \frac{14p}{14p} \frac{24p}{24p} \frac{31p}{31p} \frac{44p}{41p} \frac{51}{54}$	241	3.12
$p^{4}_{14}p^{3}_{23}p^{3}_{33}p^{4}_{44}p^{4}_{54}$	291	4.5	$\frac{p^{-1}}{13}p^{-2}2p^{-3}2p^{-4}4p^{-5}2$	176	4.45	$p^{4}_{14}p^{4}_{24}p^{2}_{32}p^{1}_{41}p^{4}_{54}$	256	3.15
$p^{4}_{14}p^{4}_{24}p^{3}_{33}p^{4}_{44}p^{4}_{54}$	331	4.7	$\frac{p^{-1}}{p^{-3}} \frac{1}{22} p^{-2} \frac{2}{22} p^{-3} \frac{2}{32} p^{-44} p^{-52} \frac{1}{53} p^{-53} \frac{1}{53} \frac{1}{53} p^{-53} \frac{1}{53} \frac{1}{53} \frac{1}{53} \frac{1}{53} 1$	185	4.47	$p^{4}_{14}p^{1}_{21}p^{3}_{33}p^{4}_{44}p^{4}_{54}$	263	3.16
$p^{4}_{14}p^{4}_{24}p^{4}_{34}p^{4}_{44}p^{4}_{44}p^{4}_{54}$	376	4.8	$p^{-1}_{13}p^{-2}_{22}p^{-3}_{32}p^{-4}_{44}p^{-5}_{53}$ $p^{-3}_{13}p^{-2}_{22}p^{-2}_{32}p^{-4}_{44}p^{-5}_{53}$	196	4.5	$p^{4} \frac{4}{14p} \frac{3}{24p} \frac{3}{33p} \frac{44p}{44p} \frac{54}{51}$	271	3.19
P 14P 24P 34P 44P 54	510	1.0	$p^{-13p} 22p^{-32p} 44p^{-53}$ $p^{-3} p^{-2} 2p^{-2} 2p^{-3} 43p^{-4} 54$	215	4.52	$\frac{p}{p} \frac{14p}{14p} \frac{24p}{24p} \frac{33p}{31p} \frac{44p}{43p} \frac{51}{54}$	280	3.36
			$p^{1}_{13}p^{2}_{22}p^{2}_{32}p^{4}_{44}p^{3}_{53}$	215	4.53	$p^{4}_{14}p^{4}_{24}p^{4}_{34}p^{4}_{44}p^{4}_{44}$	286	3.68
			$\frac{p^{-1}3p^{-2}3p^{-3}2p^{-3}2p^{-4}4p^{-5}3}{p^{-3}_{13}p^{-4}_{-2}q^{-2}_{-3}2p^{-3}_{-4}3p^{-2}_{-5}2}$	225	4.56	$\frac{p^{4}}{p^{4}} \frac{14p^{2}}{24p^{2}} \frac{24p^{3}}{31p^{4}} \frac{34p^{4}}{44p^{4}} \frac{4}{54}$	291	3.95
			3 4 2 4 2	236	4.59	4 4 2 4 4	306	3.98
			$\frac{p_{13}p_{24}p_{32}p_{44}p_{52}}{p_{13}^3p_{24}^4p_{32}^2p_{32}^3p_{43}^3p_{53}^3}$	230	4.59	$\frac{p_{14}p_{24}p_{32}p_{44}p_{54}}{p_{14}^4p_{24}^4p_{33}^3p_{44}^4p_{54}^4}$	331	4.08
	、 、		$\frac{p_{13}p_{24}p_{32}p_{43}p_{53}}{p_{13}^3p_{24}^4p_{32}^2p_{44}^4p_{53}^3}$	243	4.64	$\frac{p_{14}p_{24}p_{33}p_{44}p_{54}}{p_{14}^4p_{24}^4p_{34}^4p_{43}^3p_{43}^4}$	365	4.08
	\mathbf{i}		$\frac{p_{13}p_{24}p_{32}p_{44}p_{53}}{p_{13}^3p_{24}^4p_{32}^2p_{32}^3p_{43}^4p_{54}^4}$	230	4.66	$\frac{p_{14}p_{24}p_{34}p_{43}p_{43}p_{54}}{p_{14}^4p_{24}^4p_{34}^4p_{44}^4p_{54}^4}$	376	4.69
			$\frac{p_{13}p_{24}p_{32}p_{43}p_{54}}{p_{14}^4p_{24}^4p_{32}^2p_{44}^4p_{53}^3}$	273	4.60	P 14P 24P 34P 44P 54	570	00
	\backslash		$\frac{p_{14}p_{24}p_{32}p_{44}p_{53}}{p_{13}^3p_{24}^4p_{32}^2p_{44}^4p_{54}^4}$	276	4.67			
		\backslash	$p_{13}p_{24}p_{32}p_{44}p_{54}$					
		\mathbf{i}	$\frac{p_{14}^4 p_{24}^4 p_{32}^2 p_{44}^4 p_{54}^4}{p_{4}^4 p_{4}^4 p_{3}^4 p_{4}^4 p_{54}^4}$	306 331	4.72	ł		
			$p^{4}_{14}p^{4}_{24}p^{3}_{33}p^{4}_{44}p^{4}_{54}$		4.74	ł		
			$p^{4}_{14}p^{4}_{24}p^{4}_{34}p^{4}_{44}p^{4}_{54}$	376	4.77			