ASM and Genetic Algorithm for Risk Abatement Actions Selection

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Abstract: The risk abatement actions selection is a complex problem, so we consider the genetic algorithm the most adequate to solve it, especially in the case of big projects. This paper contains an optimization of project risk management efforts, based on an evolutionary algorithm. It uses the mathematical model of the project risk management made by Ben-David, Rabinowitz and Raz in [3]. An abstract state machine (ASM), inspired by [8], is build to design the economic optimization. A numerical example illustrates the efficiency of the algorithm.

Key–Words: Genetic Algorithm, ASM, Project Risk Management

1 Introduction

Project risk is the possibility that something may go wrong, or at least not turn out as planned. It is unrealistic to assume that everything will work as expected [6]. Risks are different for each project, and risks change as a project progresses. Project-specific risks could include, for example, the following: lack of staff buy-in, loss of key employees, questionable vendor availability and skills, insufficient time, inadequate project budgets, funding cuts, cost overruns, etc [1].

Risk Management is a process for organized assessment and control of risks. It is one of the seven project processes identified by international standard ISO/IEC 15288:2002 Systems engineering - System life cycle processes [16]. The other six processes are: Project Planning, Project Assessment, Project Control, Decision Making, Configuration Management and Informational Management.

Project risk management [9] is the systematic process of identifying, analyzing, evaluating, and responding to risk by applying risk management principles and processes at the project level. It seeks to maximize the probability and consequences of positive events and to minimize the probability and consequences of adverse events. The objective of project risk management is to apply a systematic process to reduce cost effectively the effects of uncertainties that compromise project or business objectives.

Effective risk management requires an investment of resources. Risk management for a particular project cannot be efficiently addressed without an organizational context to provide policies and guidance. Good management requires an organization to have documented and approved policies for risk management. Risks are significant uncertainties about outcomes, the uncertainty is in two dimensions: the likelihood of the risk event occurring, and the extent of the consequences if it does.

Risk events give rise to problems, some of which may be absorbed or accommodated, but others have impacts that affect project objectives. Treatments are actions which reduce the probability of risk apparition and/or the impact on objectives, see the Figure 1 from [16]:

Figure 1: Sources of risks and impacts on objectives

The risks associated with a project can be: inherent, which result from the nature of the project objectives and scope; acquired, which result from the selected organization, approach, technology, methods, tools, techniques, skills and experience, and contex-
ual, which result from events, circumstances or interrelationships outside or across the project [1]. Key elements of the project risk management plan include the following [15]:

1. Risk identification (identifying and documenting all the risks that can affect the project)
2. Qualitative risk analysis (determine the consequences of identified risks on project objectives)
3. Quantitative risk analysis (assign numeric probabilities to each risk and their impact on project objectives)
4. Risk exposure planning (decide what actions are needed to reduce threats)
5. Risk monitoring and control (respond to risks as they occur and ensure proper risk management procedures are being followed).

The last works on project risk analysis have utilized the work breakdown structure (WBS) as the basis for risk identification. Following this approach, Ben-David and Raz, in [4], have developed a model that integrates the project’s scope into the risk management process, and allows focusing on causes and effects of risks as they are distributed among the project activities. Later, Ben-David, Rabinowitz and Raz [3] extended that work in three directions: they provided a complete mathematical formulation of the model and of the actions selection problem; they presented the optimal and heuristic algorithms for solving the risk abatement selection problem; and they reported the results obtained in the experiment.

We take the mathematical model from [3] and we make an experiment using an evolutionary algorithm.

2 Mathematical Model

The objective of the model is to find the economically optimal combination of risk abatement actions. The principal factors are: the project work elements, risk events which threaten the works elements and a set of possible risk abatement actions which can be selected.

The project work elements are all the components of the WBS. The set of work elements is denoted by \( W = \{ w_1, \ldots, w_W \} \).

The set of risk events is \( R = \{ r_1, \ldots, r_R \} \). When a risk event materializes it affects some work elements of the project. An event has three attributes: source, probability of occurrence, and impact.

There are two types of risk sources: internal and external. The internal sources are the work elements of the project. The set of sources is \( S = \{ s_1, \ldots, s_S \} \).

The first \( W \) are the work elements of the project and the remaining \( S \setminus W \) are external.

Each risk event has a single source, but a risk source can generate multiple risk events.

The probability of occurrence of a risk event depends on its source. The probability matrix \( P \) has the elements \( p_{r,s} \), where \( p_{r,s} \) is the probability that source \( s \) will cause a risk event \( r \).

The occurrence of a risk event may impact one or more work elements of the project. The impact matrix \( M \) has the elements \( m_{r,w} \), where \( m_{r,w} \) is a monetary loss to work element \( w \) caused by risk event \( r \).

Risk abatement actions modify the probability and/or the impact of risk events. The set of risk abatement actions is denoted by \( A = \{ a_1, \ldots, a_A \} \). \( X_a \) is the selection decision variable of action \( a \), \( X_a = 1 \) if \( a \) is selected, and \( X_a = 0 \) otherwise. The abatement action cost is denoted by \( c_a \). The effect factor of action \( a \) on the probability of risk \( r \) originated from work element \( w \) is \( v_{r,w} = (v_{r,w,1}, \ldots, v_{r,w,A}) \). The effect of action \( a \) on the impact of risk \( r \) originated from risk source \( s \) is \( u_{r,s} = (u_{r,s,1}, \ldots, u_{r,s,A}) \). If an effect attribute is 0 it has no effect.

\[
X_{(A \times A)} = \text{the diagonal matrix with } X_{a,a} = 1 \text{ if the action } a \text{ is chosen and 0 otherwise. } \forall \text{ } Xv_{r,w} \text{ is the resulted probability vector with the chosen actions, and } Xu_{r,s} \text{ the impact effect vector with the chosen actions.}
\]

The modified probability of risk event \( r \) from source \( w \) is given by \( f(p_{r,w}, Xv_{r,w}) \).

The modified impact of the risk event \( r \) from source \( s \) is given by \( h(m_{r,s}, Xu_{r,s}) \). \( f \) and \( h \) are functions of arity \( A + 1 \). A risk abatement action cannot affect the probability of a risk event that originates from external sources, but it can affects its impact.

2.1 Objective function

The total expected costs (TEC), which are risk related, consist of two components: abatement actions costs (AAC), and expected risks loss (ERL):

\[
AAC(X) = \sum_{a=1}^{A} c_a X_{a,a} = cXe
\]

Where \( e \) is a row vector of \( c_a \) and \( e \) is a column vector of appropriate size of 1’s. If no abatement action is chosen:

\[
ERL = \sum_{r=1}^{R} \left( \sum_{s=1}^{S} p_{r,s} \right) \left( \sum_{w=1}^{W} m_{r,w} \right) = (Pe)(Me)'
\]

Where ‘ denote the matrix transpose operator.
If some actions have been selected, the expected risks loss becomes:

\[ ERL(X) = \sum_{r=1}^{R} (\sum_{s=1}^{W} f(p_{r,w}, Xv_{r,w}) + \sum_{s=W+1}^{S} p_{r,s}) \]

\[ \cdot (\sum_{w=1}^{W} h(m_{r,s}, Xu_{r,s})) = (f(P, Xv)e)(h(M, XU)e)' \]

Where \( f \) and \( h \) are matrix versions of the functions \( f \) and \( h \). The first component of the product is split into \( W \) modifiable probabilities and the remaining \((S \setminus W)\), that are not. In the matrix representation these two component are combined with \( v_{r,s} = 0 \) for \( s = W + 1, \ldots, S \).

The general risk management problem can be expressed as:

\[ \text{Minimize } TEC(X) = AAC(X) + ERL(X) \]

The model allows two types of pairwise constraints: exclusion and implication. The first is defined by \( q_{i,j} = 1 \) if actions \( i \) and \( j \) exclude each other, and \( b_{i,j} = 1 \) if selection of action \( i \) implies selection of action \( j \), and 0 otherwise. The conditions can be expressed as follows:

\[ X_{i,i} + X_{j,j} \leq 1, \quad \forall q_{i,j} = 1, i,j \in A \]
\[ X_{i,i} \leq X_{j,j}, \quad \forall b_{i,j} = 1, i,j \in A \]
\[ X_{i,i} \in \{0,1\}, \quad \forall i \in A \]

Optionally, a budget constraint can be included on the abatement actions spending: \( AAC(X) \leq B \).

An important special case is studied in [2] and we use the same operators to compute the objective function. To modify the function \( f \) of the probabilities is used the multiplication. To modify the function \( h \) of impacts is used the minimization. The minimum and multiplication operators have the advantage of being simple to comprehend and implement. In formal terms:

\[ f(p_{r,w}, Xv_{r,w}) = \text{Product}(p_{r,w}, Xv_{r,w}) \]
\[ h(m_{r,s}, Xu_{r,s}) = \text{Minimum}(m_{r,s}, Xu_{r,s}) \]

Meaning that \( f \) is the product of its \((A + 1)\) non-zero arguments and \( h \) is the minimum of its \((A + 1)\) non-zero arguments.

3 Why genetic algorithm?

The problem is to select the risk abatement actions so that the total expected costs is minimum possible. Usually, the possible combinations of selected actions are a lot and it is impossible to calculate for each one the objective function and to choose the appropriate combination, especially when the project has a big number of elements.

Due to the broad variety of requirements a lot of different optimization techniques have been developed in recent years. Basically there are two main groups of optimization techniques: gradient based optimization techniques, and stochastic or guided search techniques without gradient utilization. Gradient based optimization techniques have been developed for all kinds of applications. They are able to handle linear, nonlinear and discrete optimization. The property all these algorithms have in common is that they use, in addition to the process model, gradient information to find the optimal solution. The search for the optimum is very efficient, the computation time is low and the inclusion of inequality constraints is quite simple. The drawback of the gradient based approaches is that most of the commercial simulation tools, which are mainly used for process model development, do not provide gradient information. Therefore existing models can not be used for process optimization.

Another class of optimization techniques are the stochastic or guided search techniques without gradient utilization [12]. These techniques use either random or some kind of guided variations of the optimization variables to detect the optimum [5]. As an optimization criterion only the objective function value is considered. The main advantage of these approaches is that they don’t need any gradient information. Therefore an existing process model can be used as a black box simulator to compute the objective function value. Only an interface is needed where the optimization variables are passed on from the optimization level to the simulation level where the corresponding objective function is computed and passed on back to the optimization level. Basically these approaches are able to avoid local minima and find the global optimum of the optimization problem [12]. On the other hand these approaches usually need a large number of function evaluations (simulation runs) to find the optimal solution and therefore they need a lot of computation performance and computation time. To chose the right approach, the optimization problem has to be analyzed in order to identify the requirements on the solution approach.

One approach that belongs to this class of optimization is the evolutionary approach, see [7] and [13], which has been chosen for this task. This algorithm is the appropriate technique for the economic optimization of project risk management because it has the following characteristics: very large scale, very complex, large scale of integration, strongly non-
linear. The basic concept of this approach can be characterized as an adaptive search, a permanent process of finding and evaluating information [11].

The evolutionary strategy is an iterative procedure which uses the mechanisms of biological evolution to solve an optimization problem [10]. Thereby the propagation of different individuals is used in the optimization process. The vector of the selected actions $A$ represents the characteristics of an individual. An iterative procedure of evaluation, selection, recombination and mutation is used to identify the optimal characteristics of the individual and therefore the optimal solution of the optimization problem.

**Evaluation**: To determine the quality of the individual, the objective function value is evaluated based on the corresponding set of characteristics. This means that for a given set of risk abatement actions a simulation is carried out and the objective function value is calculated and assigned to that individual.

**Selection**: Based on the quality (objective function value) the $\lambda$ best individuals are selected from a set $\mu$ of individuals to build the next generation.

**Recombination**: To create a new set of individuals, the characteristics of the chosen individuals (the so-called parents) from the previous generation are recombined. Because we used the binary representation of variables, we made the recombination of the chromosomes.

**Mutation**: To increase the diversity of the individuals, random mutation is applied to the characteristics (decision variables) to create $\lambda$ descendants. The mutation is the main force in evolutionary strategies.

The evolutionary strategy has the theoretical ability of finding the global optimum, but the success of the approach strongly depends on some parameters. These parameters have to be predetermined and influence the result of the optimization as well as the computation time needed. The most important parameters to be chosen are:

- **Number of parents selected** ($\mu$). The algorithm gets more robust the more parents are selected, but also needs more generations to find the solution, because strong improvements are diluted by recombination.

- **Number of descendants created** ($\lambda$). The number of descendants influences the ability of the algorithm to find the optimal solution. The more descendants are created the more robust the algorithm gets, but also the number of function evaluations increases.

- **Standard deviation** ($\sigma$). The standard deviation of the normal distribution influences the search region. For large standard deviations the algorithm has the ability to search a larger region. This enables algorithm to escape local optima. Near the optimal solution only small steps lead to a further improvement.

- **Stopping criterion** ($\varepsilon$). As no gradient information is available as a convergence criterion, a stopping criterion has to be formulated. Some possibilities are: maximum number of iterations or the best individual stays the same for a given number of iterations or the difference between best and worst individual of one generation is below a certain stopping criterion.

A genetic algorithm is the appropriate method to find an optimal solution to select the risk abatement actions.

### 4 ASM for Economic Optimization of Project Risk Management

Modeling the entities of the project risk management as in section 2, we obtained abstract relationships between them. Any project can be analyzed in this way and the optimization schema is generally applicable. The abstract state machine (ASM) method describe algorithms without compromising the abstract level.

The ASM method [2] offers a uniform conceptual framework to model the project’s algorithmic content as distinct from the description of technological, managerial, financial or other non-functional system features. The requirements can be captured by constructing ground model ASM, in which non-functional features can be formulated as constraints or assumptions. The ground model ASM represents succinct process-oriented models of the to-be-implemented piece of "real world", transparent for both the customer and the software designer. Ground model comes with a sufficiently precise yet abstract, unambiguous meaning to carry out an implementation-independent, application-oriented requirements analysis prior to coding.

With a couple of examples, Gurevich, Veanes and Wallace prove in [8] that abstract states machines can be useful in evolutionary algorithms. The idea is to promote ASM as a better alternative to pseudocode specification of algorithms. It is common to specify algorithms in the pseudocode form, and the case of evolutionary algorithms is no exception. Typical drawbacks of pseudocode include unintended ambiguities, inconsistent abstraction levels, unnecessary details, missing essential information, insufficient clarity, and insufficient precision [14]. ASM
helps to keep a consistent abstraction level and it imposes certain precision without forcing unnecessary details or determinism.

The project entities: the work elements, the risks, the risk sources, their probabilities and impacts, once established, are the static data of the optimization problem and there are necessary only to evaluate the objective function.

The risk abatement actions are the variables which will be selected by the evolutionary algorithm. An individual represents a sequence of selection decisions. A Population $P$ is a set of individuals. $\mu$ and $\lambda$ denote parent and offspring population size.

In ASM terms, the functions $evaluate$, $initialize$, $mutate$, $recombine$, $select$, and $terminate$ are external functions. They are evaluated by the environment and they depend on the characteristic parameters $\theta_s$, $\theta_m$, $\theta_r$, $\theta_i$. The algorithm compute the best population $P^*$.

The main ASM contains three submachines that are defined below:

**ACTIONS EVOLUTION**

**InitializeComputation**

while not terminate($P$, $evaluate(P)$, $\theta_i$) 
CreateNextGeneration 
UpdateBestPopulation

The first submachine returns a set of individuals of size $\mu$:

**InitializeComputation**

$P := initialize(\mu)$
$P^* := P$

The second submachine uses the four external functions with the intuitive meaning:

**CreateNextGeneration**

let $P' = recombine(P, \theta_r)$
let $P'' = mutate(P', \theta_m)$
let $F = evaluate(P'')$
$P := select(P'', F, \mu, \theta_s)$

The third submachine provides the best population found at the moment:

**UpdateBestPopulation**

if $evaluate(P) < evaluate(P^*)$ then $P^* := P$

When the algorithm terminates, the variable $P^*$ gives the best population and the objective function was minimized.

5 Numerical Experiment Using a Genetic Algorithm

In order to evaluate the performance of the evolutionary algorithm in the project risk management optimization, we performed a numerical experiment. The project’s entities were defined and the relations established: 13 work elements, 12 risk events with work element source and 3 risks with external sources. Every risk has one source, but it can impact more work elements. The number of risk abatement actions was 16, 4 of them to reduce probabilities of the risks, 7 with influence in impact reduction, and the others with both effects. The problem was to select the appropriate actions so that the total cost be minimum.

The risk abatement actions were binary codified. The algorithm generates a population consisting of different combinations of selected actions. It is evaluated, and, after selection, it supports recombination and mutation. The number of generations (termination criterion) was 100. The algorithm give at the end the set of risk abatement actions which ensure the minimal costs found during the run.

5.1 Principal factors

The project entities are defined as follows:

The set of **work elements** is denoted by:

$W = \{w_1, \ldots, w_{13}\}$

The set of **risk events** is:

$R = \{r_1, \ldots, r_{15}\}$

There are two types of risk sources: internal and external. The internal sources are the work elements of the project. The set of **risk sources** is:

$S = \{w_1, \ldots, w_{13}, s_1, s_2, s_3\}$

Each risk event has a single source, but a risk source can generate multiple risk events. The occurrence of a risk event may **impact** one or more work elements of the project. Table 1 contains the risk sources, the risk events and the affected work elements.

5.2 Probability of occurrence

The **probability of occurrence** of a risk event depends on its source. The probability matrix $P$ has the elements $p_{r,s}$, where $p_{r,s}$ is the probability that source $s$ will cause a risk event $r$, see Figure 2.
Table 1: Risk sources, risk events, affected work elements

<table>
<thead>
<tr>
<th>Risk Source</th>
<th>Risk Event</th>
<th>Affected work elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$r_1$</td>
<td>$w_9$</td>
</tr>
<tr>
<td>$w_1$</td>
<td>$r_2$</td>
<td>$w_{10}$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$r_3$</td>
<td>$w_6$</td>
</tr>
<tr>
<td>$w_3$</td>
<td>$r_4$</td>
<td>$w_1$, $w_2$</td>
</tr>
<tr>
<td>$w_4$</td>
<td>$r_5$</td>
<td>$w_3$</td>
</tr>
<tr>
<td>$w_4$</td>
<td>$r_6$</td>
<td>$w_4$, $w_5$</td>
</tr>
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<td>$w_4$</td>
<td>$r_7$</td>
<td>$w_{10}$</td>
</tr>
<tr>
<td>$w_5$</td>
<td>$r_8$</td>
<td>$w_2$, $w_3$, $w_{11}$</td>
</tr>
<tr>
<td>$w_6$</td>
<td>$r_9$</td>
<td>$w_{12}$</td>
</tr>
<tr>
<td>$w_7$</td>
<td>$r_{10}$</td>
<td>$w_6$, $w_7$</td>
</tr>
<tr>
<td>$w_7$</td>
<td>$r_{11}$</td>
<td>$w_8$</td>
</tr>
<tr>
<td>$w_8$</td>
<td>$r_{12}$</td>
<td>$w_9$, $w_{13}$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$r_{13}$</td>
<td>$w_3$, $w_6$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$r_{14}$</td>
<td>$w_7$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>$r_{15}$</td>
<td>$w_7$</td>
</tr>
</tbody>
</table>

$P =$

$$
\begin{pmatrix}
0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 \\
\end{pmatrix}
$$

Figure 2: The probability matrix $P$
5.3 Risk event impact

The occurrence of a risk event may impact one or more work elements of the project. The impact matrix $M$ has the elements $m_{r,w}$, where $m_{r,w}$ is a monetary loss to work element $w$ caused by risk event $r$, see Figure 3.

5.4 Risk abatement actions

Risk abatement actions modify the probability and/or the impact of risk events. The set of risk abatement actions is denoted by:

$$A = \{a_1, \ldots, a_{16}\}$$

Table 2 specifies the effect of reducing the probability of occurrence and $f$ or the impact of risk events for each risk abatement action.

The abatement action cost is denoted by $c_a$ and is specified in Table 3. $X_a$ is the selection decision variable of action $a$, $X_a = 1$ if $a$ is selected, and $X_a = 0$ otherwise. The risk abatement actions costs (AAC) is:

$$AAC(X) = \sum_{a=1}^{16} c_a X_{a,a}$$

The effect factor of action $a$ on the probability of risk $r$ originated from work element $w$ is $v_{r,w} = (v_{r,w,1}, \ldots, v_{r,w,16})$, see Table 4. The actions without effect on the risk probability were omitted. The value 1 means that the action has no effect on the risk, because the function $f$ is the product.

The effect of action $a$ on the impact of risk $r$ originated from risk source $s$ is $u_{r,s} = (u_{r,s,1}, \ldots, u_{r,s,16})$, as depicted in Table 5. If an effect attribute is 0, it has no effect. The table contains only the actions $a_5 - a_{16}$, which reduce the risks impact.

5.5 Objective function

The total expected costs (TEC), which are risk related, is the following:

$$ERL(X) = \sum_{r=1}^{15} \left( \sum_{w=1}^{13} f(p_{r,w}, Xv_{r,w}) + \sum_{s=1}^{3} p_{r,s} \right) \cdot \left( \sum_{w=1}^{13} h(m_{r,s}, Xu_{r,s}) \right)$$

To modify the function $f$ of the probabilities is used the multiplication, and to modify the function $h$ of impacts is used the minimization:

$$f(p_{r,w}, Xv_{r,w}) = \text{Product}(p_{r,w}, Xv_{r,w})$$

$$h(m_{r,s}, Xu_{r,s}) = \text{Minimum}(m_{r,s}, Xu_{r,s})$$

The general risk management problem can be expressed as:

$$\text{Minimize} \quad TEC(X) = AAC(X) + ERL(X)$$

5.6 Result

The values of the objective function (total expected costs) obtained during the optimization run are represented in Figure 4. As can be seen on the graphic (where the first 15 generations are not represented), after about 20 generations, a good value was found, and the next searches decreased it lightly.

The solution can be seen in Table 6, where $X(a) = 1$ if $a$ is selected, and $X(a) = 0$ otherwise.

In this case, the values of the objective function are:

$$AAC = 330, \quad ERL = 109.54, \quad TEC = 439.54$$

6 Conclusion

We consider the mathematical model presented by Ben-David, Rabinowitz and Raz in [3] very helpful for the economic optimization of project risk management.

We improved its efficiency using a genetic algorithm. This approach is useful in optimization problems because it allows the adaptive search of optimal solutions in a large space.

The abstract state machine (ASM) is the first step in the computerization process, and very important, because it is dynamic, succinct, precise and unambiguous.

The genetic algorithm for economic optimization is a good opportunity in the case of big projects. It can be applied with good results not only at the beginning of the project, but any moment when a decision problem appears.

References:

\[
M = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 6000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3500 & 5000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 6000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4000 & 3000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2500 & 3000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 4800 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4500 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 4000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4000 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

Figure 3: The impact matrix $M$

<table>
<thead>
<tr>
<th>Action</th>
<th>Reduces probability of occurrence of risks</th>
<th>Reduces impact of risks</th>
</tr>
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Table 2: Effects of risk abatement actions

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<th>$a_{10}$</th>
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Table 3: Risk abatement actions costs
### Table 4: The effect factor on the probability of risks

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### Table 5: The effect of actions on the impact of risks

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### Table 6: Solution obtained in an optimization run

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Figure 4: An optimization run