Measuring Process Capability for Bivariate Non-Normal Process Using the Bivariate Burr Distribution

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Abstract: - As is well known, process capability analysis for more than one quality variables is a complicated and sometimes contentious area with several quality measures vying for recognition. When these variables exhibit non-normal characteristics, the situation becomes even more complex. The aim of this paper is to measure Process Capability Indices (PCIs) for bivariate non-normal process using the bivariate Burr distribution. The univariate Burr distribution has been shown to improve the accuracy of estimates of PCIs for univariate non-normal distributions (see for example, [7] and [16]). Here, we will estimate the PCIs of bivariate non-normal distributions using the bivariate Burr distribution. The process of obtaining these PCIs will be accomplished in a series of steps involving estimating the unknown parameters of the process using maximum likelihood estimation coupled with simulated annealing. Finally, the Proportion of Non-Conformance (PNC) obtained using this method will be compared with those obtained from variables distributed under the bivariate Beta, Weibull, Gamma and Weibull-Gamma distributions.

Key-Words: - Process Capability Index (PCI), bivariate Burr distribution, simulated annealing algorithm, non-normal distribution, multivariate processes.

1 Introduction

In the field of statistical quality control, it is generally assumed that the distributions of quality characteristics are normal. But, in most practical cases this assumption is not valid and the distribution of the quality characteristics may follow non-normal distributions such as Gamma, Beta, and Weibull distributions.

Many industries are using a quantitative measure called Process Capability Indices (PCIs) for the purpose of process assessment and improvement. The objective of these statistical measures is to estimate process variability relative to process specifications. Additionally, process capability provides a common standard of product quality for suppliers and customers. The standard Process Capability Index is based on certain assumptions which are as follow:

Data are collected from an in-control process.

Collected process data are independent and identically distributed.

Collected process data are normally distributed.

For non-normal stable processes, capability ratio Cp and process capability ratio for off center process Cpk , defined by Kane (1986) (equation (1) and (2)), are not appropriate.

$$C_p = \frac{USL - LSL}{6\sigma} \quad (1)$$

$$C_{pk} = \min(C_{pu}, C_{pl}) \quad (2)$$

where

$$C_{pu} = \frac{\mu - LSL}{3\sigma} \quad (3)$$

and

$$C_{pl} = \frac{USL - \mu}{3\sigma} \quad (4)$$

USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ is the process standard deviation. If μ and σ are not known, one can replace them by $\overline{X}$ and S respectively, where $\overline{X}$ is the sample mean and S is the sample standard deviation.

In the past decade, several modifications of classical PCIs have been proposed to resolve the issue of non-normality of quality characteristics data. Castagliola (1996) presented a new approach to compute process capability. This approach is based on using probability distribution to compute the proportion of non-conforming items and then use these to estimate the capability index. This approach is straightforward, logical and easy to deploy by engineers and managers, for normal as well as for non-normal data. Castagliola et al. (2005) have also extended univariate method in Castagliola (1996) to bivariate distribution but again limited it to bivariate normal data, and compared the results against existing methods for multivariate normal processes.

In this paper, we will use method presented by Castagliola et al. (2005) to evaluate PCI for bivariate non-normal quality characteristics data. Preliminary to this, we also use the bivariate Burr distribution with three parameters (Durling (1975)) to fit our bivariate non-normal data.

This paper is organized in the following manner. A capability analysis for univariate non-normal data and multivariate normal data is discussed in Section 2. A review of the bivariate Burr distribution is discussed in Section 3. Section 4 explains our proposed method to estimate the Burr parameters using simulated annealing algorithm (SA). Simulation studies for different bivariate non normal distributions are presented in Section 5 and, finally, we conclude the paper with suggestions for future works.

2 PCI for Non-Normal Data and Multivariate Normal Data

Many researchers have proposed several methods to handle the issue of non-normality in the quality characteristics data. Most of these efforts have been devoted to estimate PCI for multivariate normal data. In case of multivariate non-normal quality data, this field is still wide open for researchers due to the complex nature of the problem. In the proceeding section; we will review research literature related to the subject mater.

2.1 PCI for Univariate Non-Normal Processes

One simple method to handle non normal data is to transform the data into normal form using mathematical functions and then use traditional normal methods to estimate PCI. For transformation purpose, Johnson (1949) built a system of distributions based on moment method, called the Johnson transformation system. Box & Cox (1964) presented a useful family of power transformations which transform non-normal data into normal ones. Somerville & Montgomery (1996) also used a square-root transformation to transform a skewed distribution into a normal one. Niaki & Abbasi (2007) presented the transformation called "root transformation" to transform skewed discrete multivariate data to multivariate normal data. Another conceptually simple way to treat the non-normal data is to use non-normal percentiles to modify classical PCIs. Clements (1989) proposed the method of non-normal percentiles to calculate process capability C_p and process capability for off center process C_{pk} indices for a distribution of any shape, using the Pearson family of curves. Clements's method is widely used in industry. Pearn & Kotz (1994) applied Clements's method to construct the second generation index C_{pm} and the third generation C_{pmk} for non normal data. Pearn et al. (1999) presented a generalization of Clements' method with asymmetric tolerances. These quantile-based indices, C_p and C_{pk} for non normal data are defined as follow:

$$C_p = \frac{USL - LSL}{X_{99.865} - X_{0.135}} \tag{5}$$

$$C_{pk} = \min(C_{pu}, C_{pl}) \tag{6}$$

where

$$C_{pu} = \frac{USL - X_{50}}{X_{99.865} - X_{50}} \tag{7}$$

and

$$C_{pl} = \frac{X_{50} - LSL}{X_{50} - X_{0.135}} \tag{8}$$

In the above equations, X_p is $p*100^{th}$ percentile value of the data. Although Clements's method is commonly used in industry, a research study by Wu et al. (1998) indicated that the Clements's method can not accurately measure the PCI values, especially when the underlying data distribution is skewed. Tang & Than (1999) also did a comprehensive review of the process capability indices for non normal processes.

Liu & Chen (2006) proposed a modified Clements method to evaluate PCI for non-normal data. They suggested that accuracy of the estimated PCI for non normal data can be improved by using Burr distribution instead of the Pearson curves percentiles. The parameters of a Burr probability density distribution function can be set as to fit the normal, Gamma, Beta, Weibull, log-normal, extreme value type I distribution. Wang et al. (1996), Zimmer et al. (1998), Kan and Yazici

(2006) and Mousa & Jaheen (2002) have presented a comprehensive review of Burr distribution and its application to many non-normal situations.

Liu & Chen (2006) has used 3rd and 4th sample moments of the data to get the standardized moments and used Burr tables to fit Burr distribution to process data.

Using simulation study, Liu & Chen (2006) showed that Burr distribution is superior to Clements's method in estimation C_{pu} but both methods over estimate the C_{pu} in cases of highly skewed distributions (skewness ≥1.5).

Bai & Choi (1997) have developed a "weighted variance" (WV) approach to measure PCIs for skewed distributions. Pal (2005) evaluated PCI using process capability of non normal to generalized Lambda distribution. Parchami et. al (2005) used a fuzzy approach to estimate PCI.

Castagliola (1996) defined the relationship between process capability and proportion of non-conforming items and presented a new approach to evaluate PCI for non normal data. Castagliola used a method based on the generalized Burr distribution to assess the capability of the process data. Through the sample empirical distribution function, he used a polynomial function to approximate a Burr distribution and from this obtained the Process Capability Indices.

For normal data, it is easily shown that

$$C_p = \frac{\Phi^{-1}(0.5 + 0.5\int_{lsl}^{usl} f(x)dx)}{3} \quad (9)$$

$$C_{pk} = \min(C_{pu}, C_{pl}) \quad (10)$$

where

$$C_{pl} = \frac{\Phi^{-1}(0.5 + \int_{lsl}^{T} f(x)dx)}{3} \quad (11)$$

and

$$C_{pu} = \frac{\Phi^{-1}(0.5 + \int_{T}^{usl} f(x)dx)}{3} \quad (12)$$

where *f(x)* represents the probability density function of the process and T its mean. For non-normal distribution, the above equations can still be used to obtain PCIs but T now represents the median.

2.2 PCI for Multivariate Normal Process Data

Multivariate process capability indices, in general, can be obtained from (a) the ratio of a tolerance region to a process region, (b) the probability of nonconforming product, and (c) other approaches using loss functions. Hubele et al. (1991,) using multivariate normal distribution, defined the PCI as the ratio of the rectangular tolerance region to modified process region which is the smallest rectangle around the ellipse with $\alpha = 0.0027$. The number of quality characteristics in the process is taken into account by taking the υ^{th} root of the ratio where υ present the number of quality characteristics.

$$C_{PM} = \left[\frac{\text{vol. of the engineering tolerance region}}{\text{vol. of the engineering process region}}\right]^{\frac{1}{\upsilon}} \quad (13)$$

Here the modified tolerance region is the largest ellipsoid centered at the target which falls completely within the original tolerance region.

Another method for estimating PCI for multivariate normal was proposed by Chen (1994). In that paper, a tolerance zone is defined by $V = \{X \in R^V : h(X - \mu_0) \le r_0\}$, where r_0 is a positive number, μ_0 is a target value and $h(x)$ is a positive function. The process is capable if $P(X \in V) \ge 1 - \alpha$.

Let $r = \min\{c : P(h(X - \mu_0) \le c) \ge 1 - \alpha\}$. If the cumulative distribution function of $h(X - \mu_0)$ is increasing in a neighborhood of r, then r is simply the unique root of equation $P(h(X - \mu_0) \le c) = 1 - \alpha$. The process is deemed capable if $r \le r_0$. Here r_0 is the half-width of the tolerance interval centered at the target value, μ_0. Here, r, is the half width of an interval centered on the target value such that the probability of a process realization falling within this interval is $1 - \alpha$.

Wang et al. (2000) compared the above three multivariate process capability indices and presented some graphical examples to illustrate them. Chen et al. (2006) extend Boyles' work (1994) for normal distribution and Liao et al.'s work (2002) for non normal distribution. They have also extended Huang et al.'s (2002) work for multivariate data but they have not considered the correlation between the variables. They computed process capability for multivariate data (without correlation) and for each individual variable. Castagliola and Castellanos (2005) extended the univariate method developed in Castagliola (1996) to multivariate normal distribution by replacing the univariate probability density function $f(x)$ with the multivariate normal probability density function. They used equation (9) but replaced f(x) with a multivariate normal pdf $f(x_1, x_2, ..., x_p)$, i.e.

$$C_p = \frac{\Phi^{-1}(0.5 + 0.5\int_{lsl_1}^{usl_1}\int_{lsl_2}^{usl_2} \cdots \int_{lsl_p}^{usl_p} f(x_1, x_2, ..., x_p)dx_1 dx_2 ... dx_p)}{3} \quad (14)$$

Keeping in view the above literature survey, there is an opportunity for researchers to explore a suitable PCI evaluation method that can address the complex situation of multivariate non-normal data. In this paper, we replace probability density function in equation (14) with a Burr distribution. The efficacy of the proposed method will be assessed by using the Proportion of Non-Conformance (PNC) criterion.

3 Review of the Bivariate Burr Distributions

Durling (1975) introduced the bivariate Burr distribution as follows:

$$f(x_1,x_2)=\frac{\Gamma(p+2)}{\Gamma(p)}b_1b_2x_1^{b_1-1}x_2^{b_2-1}$$
$$*(1+x_1^{b_1}+x_2^{b_2})^{-(p+2)}, \qquad (15)$$
$$x_1,x_2\geq 0,\ b_1,b_2,p\geq 0$$

The cumulative distribution function has the form:

$$F(x_1,x_2)=1-(1+x_1^{b_1})^{-p}-$$
$$(1+x_2^{b_2})^{-p}+(1+x_1^{b_1}+x_2^{b_2})^{-p}, \qquad (16)$$
$$x_1,x_2\geq 0,\ b_1,b_2,p\geq 0$$

In the bivariate Burr distribution there are three parameters, b_1,b_2 and p, to be estimated. These parameters can be estimated by maximizing the log likelihood function based on a sample of size n given by:

$$L(b_1,b_2,p:x_1,...,x_n)=n(\ln b_1)+n(\ln b_2)+$$
$$n(\ln p)+n(\ln(1+p))+(b_1-1)$$
$$\sum_{j=1}^{n}\ln x_{1j}+(b_2-1)\sum_{j=1}^{n}\ln x_{2j}- \qquad (17)$$
$$(2+p)\sum_{j=1}^{n}\ln(1+x_{1j}^{b_1}+x_{2j}^{b_2})$$

$(x_{1j},x_{2j}),\ j=1,2,...,n$ is an observed bivariate sample. The first order condition for maximizing L with respect to b_1,b_2 and p lead to the following differential equations:

$$\frac{\partial L}{\partial b_1}=\frac{n}{b_1}+\sum_{j=1}^{n}\ln x_{1j}-$$
$$(2+p)\sum_{j=1}^{n}\frac{\ln x_{1j}x_{1j}^{b_1}}{1+x_{1j}^{b_1}+x_{2j}^{b_2}}=0 \qquad (18)$$

$$\frac{\partial L}{\partial b_2}=\frac{n}{b_2}+\sum_{j=1}^{n}\ln x_{2j}-$$
$$(2+p)\sum_{j=1}^{n}\frac{\ln x_{2j}x_{2j}^{b_2}}{1+x_{1j}^{b_1}+x_{2j}^{b_2}}=0 \qquad (19)$$

$$\frac{\partial L}{\partial p}=\frac{n}{p}+\frac{n}{1+p}-$$
$$\sum_{j=1}^{n}\ln(1+x_{1j}^{b_1}+x_{2j}^{b_2})=0 \qquad (20)$$

In this paper, instead of solving equations (18), (19) and (20) for our maximum likelihood estimators of b_1,b_2 and p, we will use, directly from equation (17), a systematic random search algorithm called "Simulated Annealing" (see Abbasi et al. (2006)) to obtain the estimated parameters.

4. Computing CPI for Bivariate Non-Normal Data Using Burr Distribution

To use equation (14), one needs to calculate the probability of quality characteristics falling between specification limits. In order to calculate this probability we need to know the distribution of the data. In this paper we use bivariate Burr distribution to calculate the probability of non-conforming products in a bivariate non-normal process. Maximum likelihood estimation (MLE) method is used to estimate its unknown parameters b_1,b_2 and p .Since the maximum likelihood function (MLF) of bivariate Burr is complex and may have some local optima, and numerical method to solve differential equations may also give local optima, we will maximize likelihood function by using Simulated Annealing algorithm (SA). Abbasi et al. (2006) used simulated annealing algorithm to estimate three parameters of Weibull distribution through MLE method and observed that it was fast and the results were very accurate. Having obtained the Burr distribution, we will then use equation (14), and replace $f(x_1,x_2,...,x_p)$ in the numerator with the bivariate Burr distribution (equation (15)) to compute process capability (C_p). Table 1 outlines the procedure of the proposed procedure.

5. Simulation studies

The purpose of this section is to show the capacity of the proposed method for estimating the C_p value of non-normal bivariate processes. Simulation studies have been conducted for bivariate non-normal processes. For this simulation study, bivariate non normal distributions such as Gamma, Beta and Weibull and Weibull- Gamma are used.

Table 2 depicts the simulation methodology for this research study. (We used NORTA method to generate simulation bivariate data. Refer to Cario & Nelson (1997) and Niaki & Abbasi (2007) for a discussion of the procedures used to generate these data.)

Table (1) – the procedure for C_p computation

Step 1	Select a sample from the process.
Step 2	Write down the maximum likelihood function (MLF) for sample based on bivariate Burr distribution.
Step 3	Maximize MLF by using Simulated Annealing and obtain estimates of b_1, b_2 and p.
Step 4	From Eq (16) compute the difference between cumulative density function at the upper specification limits (usl_1, usl_2) and the lower specification limits (lsl_1, lsl_2), i.e. $B = F(usl_1, usl_2) - F(lsl_1, lsl_2)$.
Step 5	$C_p = \frac{\Phi^{-1}(0.5 + 0.5B)}{3}$.
Step 6	Compute the corresponding PNC= $\Phi(-3cp)$ and compare it with the PNC obtained from the exact distribution for example Gamma.

Table (3) presents the parameters of the bivariate non-normal distributions used in the simulation study. The C_p value computed using the exact bivariate distributions, for example Gamma, is presented under heading Exact Distribution. We have generated m=30 samples of size n=100 and fitted a bivariate Burr distribution to each sample. The parameters b_1, b_2 and p for the fitted Burr distribution are estimated using SA algorithm. The C_p's of these 30 samples are then calculated using equation (14). The mean and standard deviation for 30 computed C_p's are presented in the last two columns of Table (3). The results in Table (3) show that the mean C_p values for different bivariate non-normal distributions are very close to the exact C_p value. Therefore one can conclude that the proposed method enables one to estimate C_p value of the bivariate non-normal data reasonably accurately. To further assess the efficacy of the proposed method, we have also calculated the Proportion of Non-Conformance (PNC) data using PNC$=\Phi$(-3Cp) in Table (4). This table also indicates that the proportions of non-conformance based on the proposed method are close to the proportion of non-conformance obtained using the exact distributions.

Table (2) – the procedure for simulation methodology

Step 1	Generate 100 vectors from bivariate non-normal using one of above distributions. (Compute expected proportion of non-conformance (p*) by using 1,000,000 data from the corresponding distribution e.g., Gamma and calculate the proportion of data falling out side the given USL.)
Step 2-1	Fit MLF of Bivariate Burr distribution to data.
Step 2-2	Estimate parameters of the fitted bivariate Burr distribution using SA.
Step 3	Use Castagliola method to compute Cp for Bivariate Burr distribution Eq (14)
Step 4	Compute proportion of non-conforming for Cp $\text{PNC} = \Phi(-3C_{pu})$ say p**
Step 5	Compare p*and p** to evaluate the accuracy of the proposed method

Table (3) – Simulation study for bivariate non-normal distribution

Distribution	Parameters[1]	Σ
Gamma	α=[3,4] β=[2.5,3]	$\begin{pmatrix} 8.71 & 13.10 \\ 13.10 & 80.49 \end{pmatrix}$
Gamma	α =[3,6] β=[8,10]	$\begin{pmatrix} 191.46 & -95.12 \\ -95.12 & 596 \end{pmatrix}$
Beta	α =[2,4] β=[5,4]	$\begin{pmatrix} 0.0256 & 0.0104 \\ 0.0104 & 50.0276 \end{pmatrix}$
Weibull	α =[3,4] β =[1,2]	$\begin{pmatrix} 9.0534 & 3.6666 \\ 3.6666 & 3.4469 \end{pmatrix}$
Gamma, Weibull	α =[5,2] β=[3,5]	$\begin{pmatrix} 45.18 & -4.20 \\ -4.20 & 1.11 \end{pmatrix}$

Follow of Table (3)

Specification Limits[2]		C_p		
		Exact Distribution	Burr (n=100) (m=30)	
LSL	USL		Mean	Std
[0,0]	[20,45]	0.7761	0.7124	0.1117
[0,0]	[100,160]	1.0405	1.0109	0.2371
[0.0025 ,0.005]	[0.9,0.92]	0.8645	0.8100	0.0403
[0,0]	[15,12]	0.8943	0.9089	0.3830
[0,0]	[45,6.8]	0.8541	0.8260	0.3381

[1] Note that each value in the pair represents the corresponding marginal distribution.
[2] Specification limits are selected to represent almost natural specification limits

Table (4) – proportion of non-conformance

Distribution	Burr p**	Expected PNC In the process p*
Gamma	0.0325	0.0199
Gamma	0.0025	0.0018
Beta	0.0151	0.0095
Weibull	0.0064	0.0073
Gamma, Weibull	0.0132	0.0104

6. Conclusion

In this paper, the method proposed in Castagliola et al. is used to estimate the process capability index for bivariate non-normal quality characteristic data. We used the bivariate Burr distribution to fit the probability density function of the data. The process combines both MLE and simulated annealing algorithm to estimate the parameters of the bivariate Burr distribution. We have presented the results using simulated data from bivariate distribution such as Gamma, Beta and Weibull. Thirty samples of size 100 from each distribution are generated and used to assess the accuracy of the proposed approach.

The simulation study results for different non-normal bivariate distributions revealed that the proposed method perform well. Using the expected non-conformance proportion criterion, the results also indicate that proportion of non-conformance obtained using the proposed method is close to that obtained under the exact distributions.

The extension of the proposed method to more than two non-normal multivariate quality data is straightforward and is recommended as a future research area. Using other metaheuristic algorithms such as Genetic Algorithm (GA) and Tabu Search (TS) to estimate bivariate Burr parameters could be another future research.

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