Optimizing Containership Size and Speed: Model Formulation and Implementation

HSIEN-LUN WONG[1], SHANG-HSING HSIEH[2], CHI-CHEN WANG[3]
[1,3] Department of International Business, Minghsin University of Science and Technology
[2] Department of Transportation Technology and Management, National Chiao Tung University
[1]No1 Hsin-Hsing Road, Hsinchu, 30401, TAIWAN, R.O.C. E-mail: alan@mail.must.edu.tw
[1]http://acade.must.edu.tw/03_teacher_detail.aspx?UnitID=30204&Mainid=397&ID=1054

Abstract: - One of the most important decisions for shipowners is to determine how big a containership to order. The optimal containership problem represents a trade-off between the cost and revenue resulting from size and speed. In reality, there is a tendency toward increasing containership size and speed, which is resulted from some factors related to profit. However, most of the past studies devoted their attention to the problem from the cost perspective. These models omitted the effect of ship speed on profit, and might result in inadequate solutions to the problem. Based on the cost-volume-profit analysis, a nonlinear programming model is formulated to approach the problem. The objective function is a strictly concave function with a globally unique optimal. An example of the Trans-Pacific Route is employed to test the model's formulation. The results provide shipowners with a beneficial reference for planning deployment and routing of big containerships.

Key-Words: - Optimal containership; nonlinear programming; ship speed; concave function

1 Introduction

One of the most important decisions for shipowners is to determine how big a containership to order for liner services. This problem represents an optimal trade-off between the cost and revenue, resulting from the different combination of ship size and speed. However, most of the past studies devoted their attention to the problem from the cost perspective. These models omitted the effect of ship speed on cost and revenue, and probably result in inadequate solutions to the problem. In this paper, we propose an adequate model for addressing the characteristics of the traditional problem in the industry, with achieving the purpose of profit maximization for shipowners.

Since Jansson and Shneerson proposed a mathematical model for solving the containership size problem, some advances have been made on this issue in the maritime transportation field [1]. For example, Jansson and Shneerson contend that the main check on the optimal ship size problem depends on port costs [2]. They conclude that the deployment of containerships implies that economies of ship size are enjoyed at sea and diseconomies of ship size suffered in port. Cullinane and Khanna develop a disaggregated function to determine the optimal containership size [3, 4]. The results reveal that economies of containership operation are crucially dependent on port productivity, and suggest that the deployment of larger containership depends on are special nodes of serving as intermediate switching and consolidation points for connecting many origins and destinations voyage distance. Tally proposes a cost model to investigate the effect of the changes in ports calls, sailing distance, and time in port on the same problem [5]. Later, Pope and Tally formulate a periodic-review inventory model for testing the work of [2], and conclude that one may not make general statements about optimal containership size [6]. Lim formulates a revenue model to examine the ship size problem [7]. Some of models focus on the factors of size growth and forecast a limit to the advantages of larger ship [8, 9]. These studies workout minimal unit cost; however the lowest cost does not guarantee that a ship can maximize profit to shipowners.

In addition to ship size involved in analyzing the problem, one key factor to take into account is ship speed [10]. The ship speed can easily be neglected in analyzing the problem. The rationale for the concern is that: (1) for two same size containerships, one that has more power of speed costs more to purchase; (2) for a given route a containership that makes faster speed can execute more roundtrips within a horizon planning time, thus increasing amount of the cargo carried and revenue; (3) for a given route a containership making faster speed needs more fuel consumption within a horizon planning time, thus increasing bunker cost. As ship speed will highly affect both revenue and cost, it supposes not to be a constant in the problem.

In this paper we propose a model optimizing ship size with speed being considered, based on the cost-volume-profit method for approaching the problem. The model is a strictly concave function with a globally optimal and unique solution. The model employs two-fold computing stages: "one variable" and "two variables", while maximizing the transportation profit to solve the problem. Finally, this paper utilizes an empirical example that tests the model formulation, utilizing six major container ports on the Trans-Pacific Route.

The next section presents the model formulation and solution process. Section three provides the economies analysis for costs related to ship size and speed. Section four provides a real case along with implication of the problems. Finally, we present our conclusions and directions for future research.

2 Model Formulations

To specify the problem for model formulation, some postulates are used in this process as set forth below:

- The fixed cost of a containership depends on its ship size and ship speed.
- Tariff between ports are given and constant.
- Loading factor for a containership between ports is available and given.
- No consideration for empty container leasing cost and shipper inventory cost.

2.1 Objective Function

Max

$$\psi = \sum_{i=1}^{n} R_i^y - \sum_{i=1}^{n} C_i^y \quad (1)$$

$$\sum_{i=1}^{n} R_i^y = \sum_{i=1}^{n} Q_i^y S_i (F_{ab} L_a + F_{ba} L_b) \quad (2)$$

$$\sum_{i=1}^{n} C_i^y = \sum_{i=1}^{n} C_{ci}^y + \sum_{i=1}^{n} C_{pi}^y + \sum_{i=1}^{n} Q_i^y S_i (L_a + L_b)(O_i^a + O_i^b) + \sum_{i=1}^{n} Q_i^y C_{wi}^k E_i^k + \sum_{i=1}^{n} Q_i^y E_i^s C_{fi} \quad (3)$$

$$\sum_{i=1}^{n} Q_i^y = \sum_{i=1}^{n} 360 \div [(2D_{ab} \div 24V_i) + E_i^y] \quad (4)$$

$$\sum_{i=1}^{n} E_i^y = \sum_{i=1}^{n} [2S_i (L_a + L_b) \div 24N_i^y] \quad (5)$$

Subject to

$$10 \le V_i \le 40 \quad (6)$$

$$1000 \le S_i \le 16000 \quad (7)$$

Where the decision variables of the problem are defined as V_i, representing the average speed of a containership i in knot on a voyage leg, and S_i, size for containership i in TEU (twenty-foot equivalent unit). Input data for the mode are below:

R_i^y = Revenue for containership i within one-year planning horizon in USD,

C_i^y = Total transportation cost for containership i within one-year planning horizon in USD,

Q_i^y = Frequency of service for containership i within one-year planning horizon,

F_{ab} = Freight rate between ports a and b in USD per TEU,

F_{ba} = Freight rate between ports b and a in USD per TEU,

L_k = Loading factor for containership i at port k, $k \in$ (a, b) in percentage,

D_{ab} = Sailing distance between ports a and b in nautical mile,

C_{fi} = Bunker cost for containership i in USD per ton,

N_i^k = Number of cargo loading for containership i at port k per hour,

O_i^k = Loading fee for containership i at port k in USD per TEU /hour,

C_{ci} = Purchasing prices for containership i in USD,

C_{ci}^y = Annual capital cost for containership i in USD,

C_{pi}^y = Annual operation cost for containership i in USD,

C_{wi}^k = Daily wharfing fees for ship i at port k in USD per day,

E_i^k = Number of berthing days for containership i berthing at port k in day,

E_i^s = Number of sailing days for containership i per voyage leg (a, b) in day.

The objective function (1) represents the profit obtained from revenue minus cost. The model (2) is the revenue function and (3) the cost function. The revenue is decided by freight rate, number of cargo carried and frequency of service, which is related to ship size and ship speed. The cost item will be classified into capital cost, operation cost, bunker cost, and wharfing cost, which are related to ship size and ship speed. The model (4) is the frequency of service depending on ship speed and (5) the berthing time for a ship at port. The time is determined by number of cargo carried and cargo loading per hour in port, which is related to ship size. Constraints (6) and (7) ensure that the observed range of the size and speed of a containership is restricted.

2.2 Solution Method
The procedures of model formulation to the problem are as described below: Step 1. Define the decision variables, parameters and cost items related to the problem. Step 2. Formulate a profit maximization model based on a cost-volume-profit approach. Step 3. Formulate regression model of costs related to the decision variables. Step 4. Simplify the profit maximization model with two decision variables. Step 5. Interpret the ship size and speed economies and the objective value.

The solution processes to the problem are outline below: Step 1. Examine if the optima of the functions exists, by given the first order condition, $\nabla f = \phi$, then locating the value at $\mathbf{X}^* = (V_i, S_i)$. Step 2. Compute the second order condition, by Hessian Matrix donated H. Step 3. If $\left|H_1(\mathbf{X}^*)\right|$ <0 and $\left|H_2(\mathbf{X}^*)\right|$ >0, then $d^2f(\mathbf{X}^*) < 0$, the value at $\mathbf{X}^* = (V_i, S_i)$ is the global optimum. Step 4. Compute the objective value of the model at different locations of $\mathbf{X}^* = (V_i, S_i)$.

3 Regression Formulations
This section will use regression analysis to simplify the nonlinear programming model and interpret the economies of ship size and speed [1, 5]. The equation of $\ln C_{ci}^{y} = \alpha + \beta \ln S_i + \gamma \ln V_i$ is equivalent to the power model as below:

$C_{ci}^{y} = AS_i^{\beta} V_i^{\gamma}$ ……………………………….……(8)

Where β is the elasticity of capital cost (C_{ci}^{y}) related to ship size (S_i), meaning that a 1% increase in the ship size results in a β % increase in its capital cost, regardless of ship speed (V_i). When the value of β is less than 1, there are economies of capital cost related to ship size; γ is the elasticity of capital cost related to ship speed.

3.1 Capital Cost
Capital costs shares about 59 % of total cost for a containership [11]. The annual capital costs depend on ship useful life (n) and the cost to purchase (P) [5]. The capital cost is through capital recovery factor (CRF) to obtain. This equation can be expressed as CRF = [r (1+r) n]/ [(1+r) n-1], where r is interest rate. For example, if the ship cost to purchase is P, then the annual capital cost is obtained by multiplying CRF by P [12]. The cost data used for computation consists of 107 different containerships from *Containerization International* [13]. In the model, it is assumed that average interest rate (r) is to be 10 % and ship useful life (n) 20 years, similar to that as assumed by Talley [5]. After regression estimating, the capital cost equation is obtained as below:

$C_{ci}^{y} = e^{-3.170} S_i^{0.598} V_i^{0.586}$ (R^2 = 0.930, F = 721.159, P< 0.001)……………………………………….(9)

The regression (9) certainly supports that there is economies of capital cost related to ship size and ship speed, respectively as the statistics of regression are significant.

3.2 Operation Cost
The operating cost of containerships is the sum of wages, subsistence, stores, supplies, maintenance, repairs and insurance. Facing a similar data obtained problem as Cullinane and Khanna and Lim [3,7], we use the estimation of Buxton, who asserts that the operation cost are approximately 10~15 % of the total cost for containerships [14], close to the cost construct from *Lloyd's Shipping Economist* [15]. In doing so, we employ the data of annual capital cost obtained for further computing of operation cost [15]. The effective operation cost equation is obtained as below:

$C_{pi}^{y} = e^{8.461} S_i^{0.603} V_i^{0.506}$ (R^2 = 0.902, F = 499.526, P< 0.001)…………………….……………….….(10)

The regression (10) also supports that there is economies of operation cost related to ship size and ship speed, respectively as the regressions are statistically significant.

3.3 Bunker Cost
The data used for bunker cost test is based on bunker oil consumption of 655 different containerships from *Containerization International* [13], omitting lubricating oil because it shares only 3 % of the total consumption [3]. After regression estimating, the bunker cost equation is obtained as below:

$C_{fi} = e^{-0.462} S_i^{0.493} V_i^{2.001}$ (R^2 = 0.947, F = 5821.341, P < 0.001)……….…………………………….....(11)

It is noticeable that the equation is not consistent in coefficient of ship size and ship speed in the regression estimation. As the regression (11) is statistically significant, it supports that there is economies of bunker cost with related to ship size; but as the value of β is 2.001, higher than 1, there is diseconomies of bunker cost to ship speed.

Table1 Elasticity value of costs with related to ship size and ship speed

Elasticity	Capital cost	Operation cost	Bunker cost	Wharfing fee
β	0.598	0.603	0.493	0.582
γ	0.586	0.506	2.001	--

3.4 Wharf Cost

Wharf fee is the major factor to be related to ship size among port charges, and the rest of the charges may be only negligible because of no significance [3]. In the model, it is assumed that there is not significant difference in wharf fee offered from ports.

The cost data used for test is from the Kaoshiung Harbor Bureau, Taiwan. As wharf fee is related to ship size only, the effective equation is obtained as below: $C_{wi}^{k} = 23.9517 S_i^{0.582}$ ($R^2 = 0.943$, $F = 83.34$, $P < 0.001$)……………………..……...………….(12)

As the regression figure is statistically significant, it supports that there is economies of wharf cost related to ship size. Finally, these cost regressions (9~12) resulted in high value of R^2 (> 0.9), i.e., a better fit to the data, both explanatory variables of ship size and speed to costs are significant at the 0.01 level or better.

The elasticity value with related to ship size and ship speed is listed in Table 1. Holding ship speed constant, there are cost economies of ship size since all β values to ship size are less than 1. However, it is unable to know if the cost economies of ship speed exist. Table 1 shows that there is a different elasticity value of γ in cost regressions related to ship speed. That is, the γ value of capital cost and operation cost with related to speed are less than 1; however, the γ value of bunker cost is found to be 2.001, more than 1, which means that a 10% increase in ship speed will result in 5.86 % increase in capital cost, 5.06 % in operation cost, and 20.01 % in bunker cost

From the statistics in Table 1, it is easily seen that larger ship size may achieve lower unit transportation cost because of economies of costs. As for faster speed, we need to compare the changes in increase in bunker cost with the decrease in capital cost and operation cost. Therefore, it is the key point for observing ship speed to affect the trade-off problem. As previously discussed, high bunker price should be one of important factors to drive the result. This interesting finding has not been mentioned in past literature.

4 Numerical Examples

4.1 Kaoshiung-Los Angeles

We now test the model employing an experimental data set from the Trans-Pacific Route. The data with six ports[1]includes freight rate, loading factor, bunker price, loading fee, labor hours and traffic flow. The paper uses Mathematica 4.0 to implement the solution algorithms. The data set related for test is from various sources[2]. To easily interpret the results, this paper takes a voyage leg from Kaoshiung to Los Angeles for model test. It is assumed i to be 1 (i.e., one containership) and loading factor 0.7 for two ports.

[1]*Ports:* (1) Kaoshiung, (2) Hong Kong, (3) Shanghai, (4) Pusan, (5) Tokyo, (6) Los Angeles.

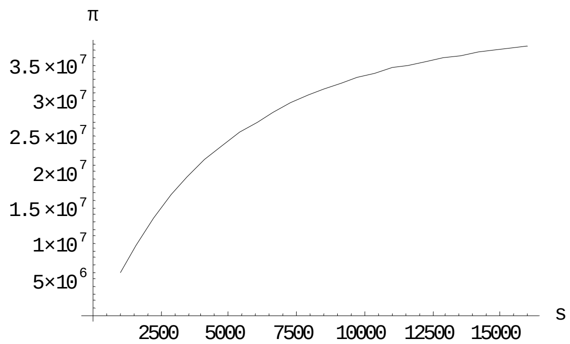

Fig.1 The relationship between the optimal value (π) and ship size (s) holding ship speed constant

First, holding ship speed constant, we change the value of ship size to observe the objective value. The relationship between the optimal value (π) and ship size (s) is shown in Fig.1. The pattern shows that the objective value rises with the increases of ship size. This increase seems intuitively to be that there is no limit with ship size given. The outcome appears to be supported by economies of ship size. As previous discussion, all of β values of cost regressions are less than 1, meaning that large ship can enjoy the lower unit transportation cost.

Secondly, holding ship size constant, we adjust ship speed to check the objective value of the model. Fig.2 shows the relationship between the objective value (π) and ship speed (v). It can be seen that the objective value rises with the increases of ship speed between 10~20 knots, leveling at speed 24~26 knots, and then sharply declining when speed over 26 knots. This curve reveals that the highest point is obtained when ship speed is about 24 knots. The point means that margin revenue equals margin cost. If ship speed is faster than 24 knots, it means margin revenue is less than margin cost, thus resulting in the objective value declines. The same rationale can explain the situation when ship speed is lower than 24 knots. From the Fig.2, it is interesting to note that fast speed over 24 knots would not be easy to produce the profit.

Next, we observe the objective value with two variables of ship size and speed taking into account.

[2] *Sources*: (1) Institute of Transportation, Ministry of Transportation & Communciations, Taiwan, R.O.C, (2) Containerization International Yearbook, 2005, (3) Official U.S. Waterborne Transportation Statistics, Maritime Administration, Department of Transportation, 2005.

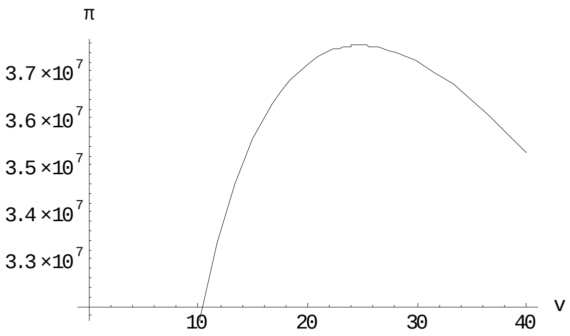

Fig. 2 The relationship between the optimal value (π) and ship speed (v) holding ship size constant

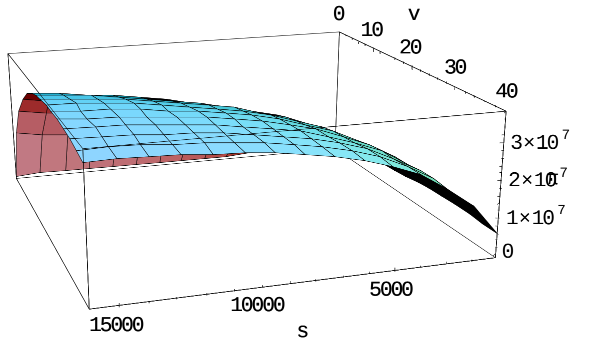

Fig. 3 The three dimensions plot of a model with ship size (s) and ship speed (v)

Fig. 3 is the three-dimension plot of the model with ship size and ship speed. As it can be seen, the optimal point rises up as ship size increases; however, the point moves up to a stationary point and declines as ship speed increases. With size and speed taking into account, the highest point is obtained at the location of X* ={s =16,000 TEU, v = 24 knot}, meaning that it is a global optimal. As in previous discussion, the reason resulted from the different elasticity value of ship size and speed. Observing the Fig.1 and Fig.2, we understand that an increase in margin cost after ship speed over 24 knots is higher than an decrease in margin cost as ship size increases, resulting in the objective value decreases. High bunker price should be reasonable one to explain this outcome. From the result of the example, it reveals that if shipowners will deploy ships for service from profit prospective, an optimal strategy for suggestion would be the ship size of 16,000 TEU with speed at 24 knots on the Kaoshiung to Los Angeles route.

4.2 Sensitivity Analysis

The postulate in the model is that market demand is given and constant, meaning that shipping liners are only price followers. The issue regarding the market demand is not included in this study. Next, we will examine if the optimal value is influenced by the parameters, such as the changes in loading factor, sailing distance, labor work time, and bunker price.

Next, taking a voyage leg from Kaoshiung to Tokyo as an example, we alter the loading factor for a ship to examine the objective value. The result is shown Table 2. The result reveals that as the loading factor decreases, the profit also decreases, but ship size increases and ship speed decreases. To achieve the model, if the loading factor increases, the model tends towards utilizing smaller ships to enhance slot capacity and increasing speed to raise ship roundtrip.

Table 3 shows the result of sailing distances. It reveals that as sailing distance increases, the profit, ship size and ship speed increase. A review of Table 2 shows that there are cost economies of ship size to sailing distance. However, Table 3 also shows that an increase in profit and ship speed on the short voyage from Kaoshiung to Shanghai is higher than those of Kaoshiung to Pusan and Tokyo. The reason is because Shanghai offers lower loading fee than that of other ports. A similar result was also found in our previous study [16].

Table 4 shows the relationship between the objective value, ship size and ship speed related to labor work time. It should be noted that as labor work time increases, the profit increases; but ship speed increases and ship size remains unchanged. This result is similar to real-world cases.

The result of sensitivity analysis for bunker price is shown in Table 5. It can be seen that as bunker price increases, the profit decreases; but ship size increases and ship speed decreases. The rationale for this outcome is because of diseconomies of bunker price to ship speed as previously discussed.

From the result of sensitivity analysis, it should be noted: (1) Liner operators may consider choosing optimal hubs or operation alliances for satisfying the needs of bigger containerships. (2) Liner operators may plan a network system for their fleet, such as hub-and-spoke marine networks, in which bigger containerships run in hub links; whereas, smaller ones operate on spoke links[17].(3) Management of ports may provide an effective policy (e.g., loading fee, operation time) to attract ship port-calling.

5 Conclusions

In this paper, we propose a nonlinear programming model to capture the characteristics of the optimal containership problem, with achieving the profit maximization for shipowners. The model is a strictly concave function with a global unique optimal. An example of the Trans-Pacific Route is employed to test the model's formulation and stability. This study technically has achieved the objectives of problem solving.

The results indicate that bigger containerships can enjoy benefits of size economies; however, faster speed does not guarantee that it will increase profit to shipowners. Other results exhibit that an optimal ship size and speed is sensitive to loading factor, sailing distance, labor work time, and bunker price. The model introduced can be expanded to examine the optimal number of ships and frequency of service for a fleet. An advantage of the study would provide shipowners with a reference for the deployment and routing of big containerships.

Table 2 Sensitivity analysis of the changes in loading factor

Loading factor	Ship size (TEU)	Increase (%) in ship size	Ship speed (knot)	Decrease (%) in ship size	Profit (USD)
0.9	6,998	--	20.61	--	15,442,000
0.8	7,416	5.63%	19.71	-6.57%	14,493,900
0.7	7,709	5.89%	18.34	-6.64%	13,469,500
0.6	8,016	5.95%	17.60	-6.67%	12,296,100

Table3 Sensitivity analysis of the changes in sailing distance

Item/Voyage	Kaoshiung-Hong Kong	Kaoshiung-Shanghai	Kaoshiung-Pusan	Kaoshiung - Tokyo	Kaoshiung - Los Angeles
Ship speed (knot)	18.01	20.31	19.80	20.61	24.00
Ship size (TEU)	3,651	3,894	5,284	6,998	16,000
Profit (USD)	12,657,600	15,823,700	14,341,300	15,442,000	37,534,700

Table 4 Sensitivity analysis of the changes in labor work time

Labor work time Per day	Ship size (TEU)	Ship speed (knot)	Profit (USD)
18 hours	16,000	19	23,825,300
20 hours	16,000	22	28,297,400
22 hours	16,000	23	32,877,900
24 hours	16,000	24	37,534,700

Table 5 Sensitivity analysis of the changes in bunker price

Price/Item	Ship size (TEU)	Increase in Ship size	Ship speed (knot)	Decrease in Ship speed	Profit (USD)
$120/Ton	5,927	5.63%	24.45	-6.57%	15,651,500
$140/Ton	6,299	5.89%	22.82	-6.64%	15,568,300
$160/Ton	6,698	5.95%	21.30	-6.67%	15,484,100
$180/Ton	7,140	6.18%	19.81	-6.70%	15,397,800

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