Hysteresis model parameters identification for the SAS rotational MR damper

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Abstract: Comfort, reliability, functionality performance which provide a longer life cycle requires thourogh understanding and analysis of the vibrations, this is a general rule for most of the static and dynamic when studying the functionality performance of any application. Vibrations is an extremely important issue to consider when designing various systems.The hysteresis in the dampers is very important issue when characterizing the damper used to suppress the vibrations, it is a very complex phenomena but very important to understand and consider during the design phase.
The hysteresis equations of Bouc-Wen, Lugre, and Dahl have been modeled and simulated in Matlab/Simulink. Afterward, the different parameters in the models was manipulated and their effects on the outcome was analyzed. The hysteresis models of Bouc-Wen, Dahl and LuGre have been analyzed and compared analytically to really show the difference in the models. At last the Bouc-Wen model was implemented together with the SAS(Semi Active Suspension) system. The model parameters were tuned manually to try to fit the response of the system.
In this paper a predefined methodology has been applied for determining the hysteresis loop parameters using the data collected for vibration analysis under predefined test specifications. The following data has been used later to regenerate the vibration signal, so on get as closer to the real signal.In the coming work, advanced method will be used to determine the exact parameters for the hysteresis loop as well as using the inverse hysteresis to improve the of the vibration suspension in the Semi Active Suspension system.The behavior of MR dampers can be presented with different mathematical models. The Bouc-Wen model was found to be model to both illustrate the MR damper and recreate the behavior of the SAS system.

Key–Words: Bouc-Wen model, Dahl model, Hysteresis, Lugre model, Magnetorheological damper,Parameters identification, Semi-Active Suspension System

1 Vibration analysis

The hysteresis identification can be a complex process, understanding the static and the dynamic vibration in the system enable facilitating the process of the hysteresis identification, In this work different vibration responses of the MR damper. When using fluid dampers the exerted force, or torque, will respond differently to vibrations with respect to the system's natural frequency and the exerted vibration on the damper. We will discuss the static response compared to the dynamic response using the models discussed in 3.

1.1 Static vibration

Static vibration on the MR damper is interpreted as when the damper is working without the mechanical dynamics of the system. The damper's exerted force, or torque, is then a function fully described by the current and velocity. To illustrate the effects of the vibration, the Matlab Simulink models are used to simulate the maximum torque of the damper with respect to frequency when the displacement is forced in a sine motion. The result using the Bouc Wen model is shown in Figure 1

When using a dynamic model, here Bouc Wen, someone can observe high torque even at low frequencies.

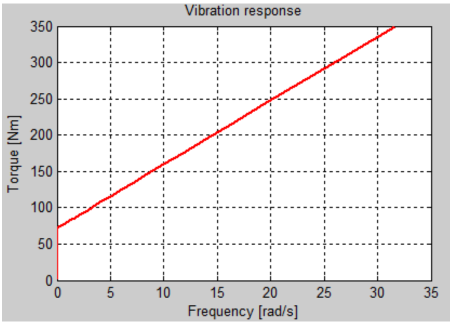


Figure 1: Vibration response with respect to frequency using the Bouc Wen model. Displacement is forced in a sine motion creating a frequency dependent torque.

1.2 Dynamic vibration

Considering the dynamics in a system, it is important to have knowledge of its vibrational response. The system will have a natural frequency and applying a harmonic excitation near the system's natural frequency can create a highly unstable system. We use the semi-active damper to control and mute these critical vibrations. To analyze, the mechanical system is built up using

$$T = I\ddot{\theta} = -\left(k_s\theta + T_{mr}\dot{\theta} + T_{in}\right) \tag{1}$$

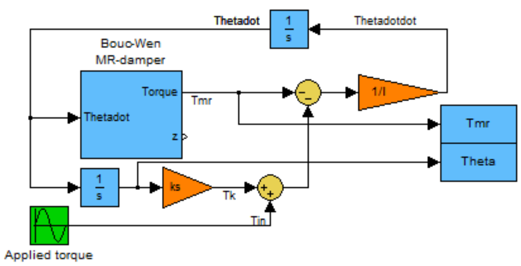


Figure 2: The physical system with the MR damper

The model is shown in Figure 2 with fictitious values. The system has a natural frequency of

$$\omega_0 = \sqrt{ks/m} \tag{2}$$

To make a resonance plot, it is required to give the input torque in Figure 2 a frequency, measure the maximum displacement and redo the process with a new frequency. The result is shown in Figure 3.

Note how the peak is shifting towards higher frequency ratios as the current is increased. For frequency ratios higher than 2.2, the best result may come with lower or no current applied.

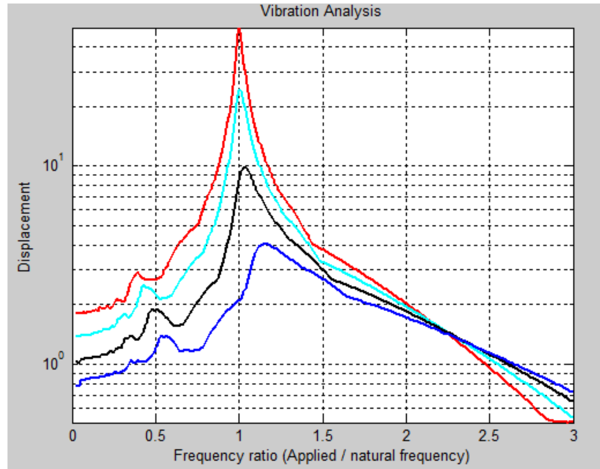


Figure 3: Vibration analysis of the Dynamic system using the Bouc Wen model with current $[0, \frac{1}{3}, \frac{2}{3}, 1]A$

In Figure 4 the actual result retrieved from the SAS system is shown. The frequency increases linearly from low (wheel slowly turning) to high. At around 30 seconds, the system is near the its natural frequency ($\sim 1.6Hz$) resulting in maximum displacement change. Passing this critical frequency will again calm the system.

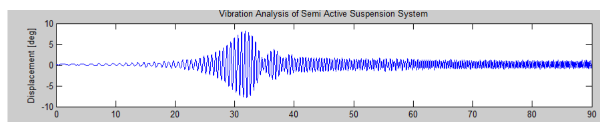


Figure 4: Vibration Response of SAS system with no current applied

2 Types of friction

The MR damper can be modeled with different types of mathematical models. Each model describes different aspects of friction and/or dynamic properties of the MR damper. In this section, some friction aspects together with the models will be later simulated, compared and analyzed.

2.1 Hysteresis

Hysteresis is a dynamic friction phenomena which represents the history dependence of physical systems.[1] We get a model for the systems nonlinear behavior at low velocities. Figure 5 show an example of hysteresis. The force versus velocity curve is not coincide for increasing and decreasing velocities.

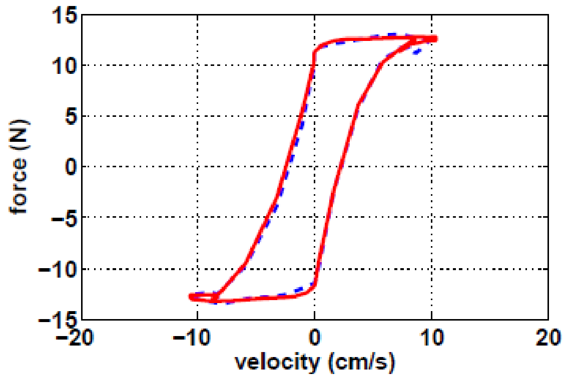


Figure 5: Example of hysteric behavior[2]

2.2 Coloumb friction

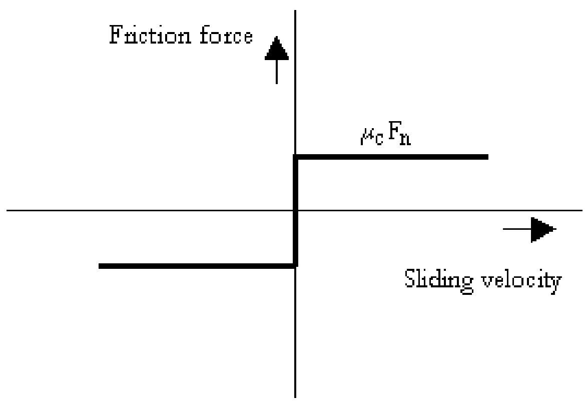


Figure 6: Columb friction model[3]

The Columb friction model represents the static relationship between friction forces and velocities. This model cannot reproduce friction characteristics that depend on time.[4]

2.3 Viscous friction

Viscous fricton is static and lineary dependent to the sliding velocity.

2.4 Stiction

If someone combine static, Coulomb and the viscous friction model, someone will get a model of stiction. A stiction model includes the threshold force that is needed to start a movement between two bodies, the constant Coulomb friction and the velocity dependent viscous friction.

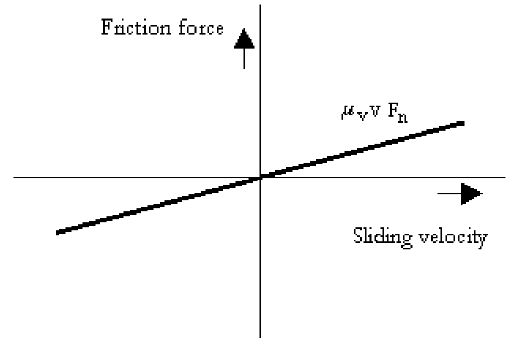


Figure 7: Viscous friction[3]

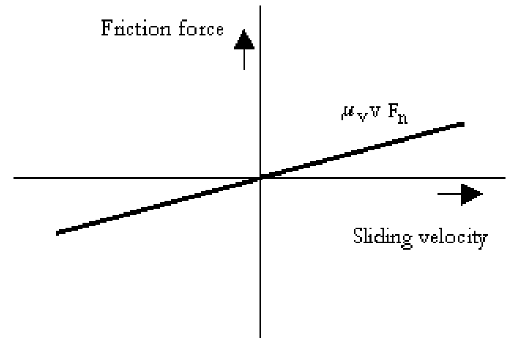
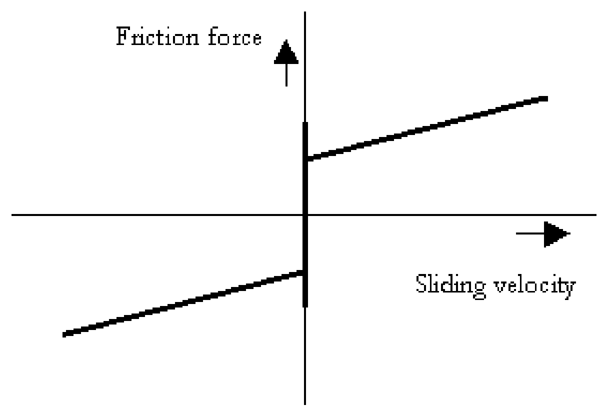


Figure 8: Viscous friction[3]

2.5 Viscous friction

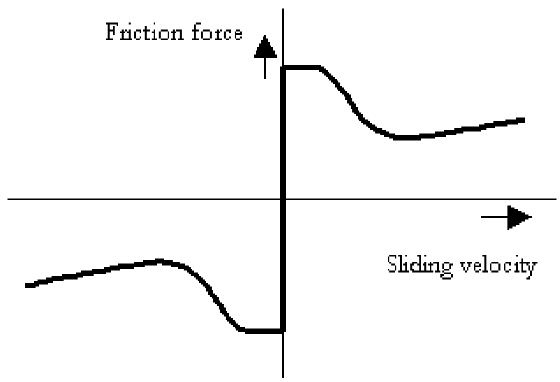
Viscous friction is static and linearly dependent to the sliding velocity.

2.6 Stiction



If someone combine static, Coulomb and the viscous friction model, someone will get a model of stiction. A stiction model includes the threshold force that is needed to start a movement between two bodies, the constant Coulomb friction and the velocity dependent viscous friction.

2.7 Stribeck effect



The static, Coulomb, viscous plus Stribeck friction model.

Figure 9: Stribeck effect[3]

The Stribeck effect appears when the friction is decreasing and later is increasing with increasing velocities starting at zero velocity. This effect is seen in for example bearings. The oil thickness is building up in the beginning, causing the drop in the friction force.

2.8 Stick-slip motion

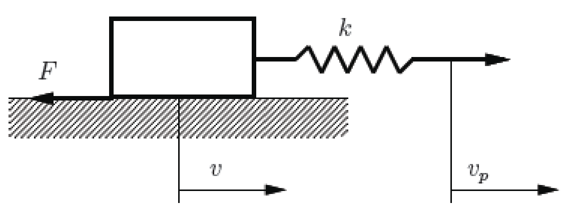


Figure 10: Model of stick slip motion[5]

If a mass m is pulled with a spring at a constant speed v_p along a surface; the mass will have a periodic motion where the mass varies between sticks and slips.

2.9 Zero-slip motion

$$m \cdot a = F_d - F \tag{3}$$

The applied force F_d is smaller than the stiction force. This occurs when a masses is dragged along a surface with a low constant velocity while the friction keeps the mass from slipping.

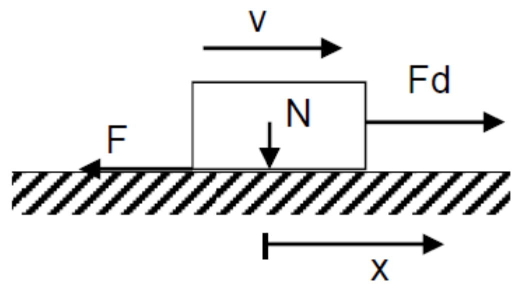


Figure 11: Zero-slip motion[5]

3 Models

3.1 The Bingham model

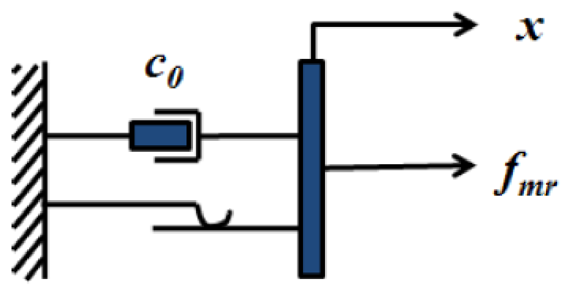


Figure 12: Bingham mechanical model[2]

$$F_{mr} = F_c \cdot sign(\dot{x}) + c_0\dot{x} + F_0 \tag{4}$$

$\dot{x}$: Piston velocity
F_c : Frictional force
c_0 : Damping constant
F_0 : Offset value (Constant force)

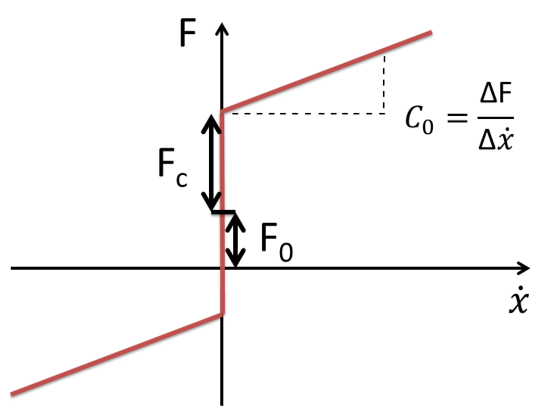


Figure 13: Response of the Bingham model

From equation 4 it can assumed that the shape of the Bingham force profile will be equal to the coloumb plus viscous friction as seen in Figure 13. The friction force F_c equals the starting point of the graph on the y-axis. The damping constant c_0 equals the linear relationship $\frac{\Delta F}{\Delta \dot{x}}$ which appears when the velocity $\dot{x}$ is not zero.

The Bingham model is clearly linear and since the MR damper is highly nonlinear, this model will not be an area of focus and detailed study.

4 The MR damper hysteresis models

4.1 Dahl model

Using the Dahl model of the MR damper [6]

$$\begin{aligned} F_{mr} &= k\dot{x} + (k_{wa} + k_{wb}v)w, \\ \dot{w} &= \rho(\dot{x} - |\dot{x}|w) \end{aligned} \tag{5}$$

, we obtain the expression:

$$\begin{aligned} T_{mr} &= k\dot{\theta} + (k_{wa} + k_{wb}v)w, \\ \dot{w} &= \rho(\dot{\theta} - |\dot{\theta}|w) \end{aligned} \tag{6}$$

, with new parameter values. T_{mr} is the exerted torque, θ is the angle, v is the control voltage, w is a dynamic hysteresis coefficient, and k,k_{wa}, k_{wb}, and ρ are parameters that control the shape of the hysteresis loop.

Dahl's first paper states: "The origin of friction is in quasi static bonds that are continuously formed and subsequently broken" [7]. This can be seen in hysteresis loops for torque vs velocity; when the velocity changes sign, the torque does not change instantly. This does not happen until a certain change in displacement allows these "bonds to be broken".

4.2 Dahl model: Parameters effect

These plots are used for further reference when analysing the parameters. Notice that the "knees" of the hysteresis are at points $(0, \pm 80)$ in the velocity graph. This value is decided by v, and k_w, which is now represented by $T(\dot{\theta}) = T_0 \approx k_{wa} + v \cdot k_{wb}$. Now, when increasing the value of k, with the result showing in figure 14. The knees are still in the area of $(0, \pm 80)$. This establish that the hysteresis does not change much, but the linear part of the graph has a greater slope. This results in higher torque with respect to velocity.

From equation 6 it can be shown that increasing either voltage, v, or k_w will have the same impact on the shape. An increase in either one will result in an increase of the torque in the hysteresis loop. This is shown in figure 15.

Inreasing ρ will change the width of the hysteresis loop, giving a fast change in torque. This is illustrated in figure 16 where ρ is increased from 2 to 15.

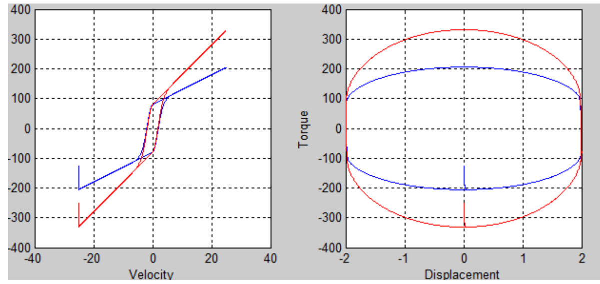


Figure 14: Plot of the Dahl model with k=5(blue) and 15(red), k_{wa}=80 = k_{wb}, v=0,ρ=15

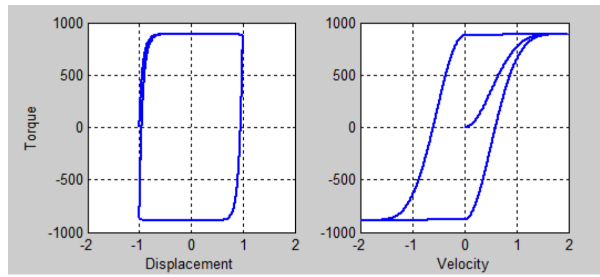


Figure 15: Plot of the Dahl model with k=5, k_{wa}=80 = k_{wb}, v=10,ρ=15

4.3 Lugre model

In *Modeling of MR damper with hysteresis for adaptive vibration control*[8] an MR damper model based on the earlier mentioned LuGre model is described. This model expresses the dynamic friction characteristics and the hysteresis effect. The equation looks like this:

$$T = \sigma_a z + \sigma_0 z v + \sigma_1 \dot{z} + \sigma_2 \dot{x} + \sigma_b \dot{x} v \tag{7}$$

$$\dot{z} = \dot{x} - \sigma_0 a_0 |\dot{x}| z \tag{8}$$

σ_0: stiffness of z(t) influenced bu v(t), $(N/(m \cdot V))$
σ_1: damping coefficient of z(t),$(N \cdot s/m)$
σ_2: viscous damping coefficient,$(N \cdot s/m)$
σ_a: stiffness of z(t), (N/m)
σ_b: v(t) dependent viscous damping,$(N \cdot s/(m \cdot V))$
a_0: constant value, (V/N)

In simulink, the model looks like this:

A sine wave and its derivative is used as an input to the model:

There are outputs connected to the different terms of the equation. These values are plotted with respect to the speed. The model parameters used are from table I in [8] Using this model, we can find out what the different parts of the model adds to the result. This helps us to understand how the model works and how

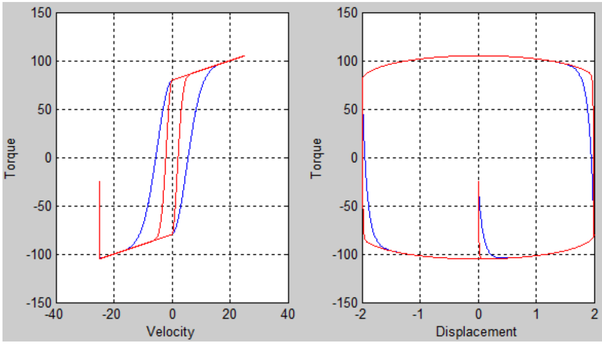


Figure 16: Plot of the Dahl model with k=5, k_{wa}=80 = k_{wb}, v=0,ρ=2(blue) and 15(red)

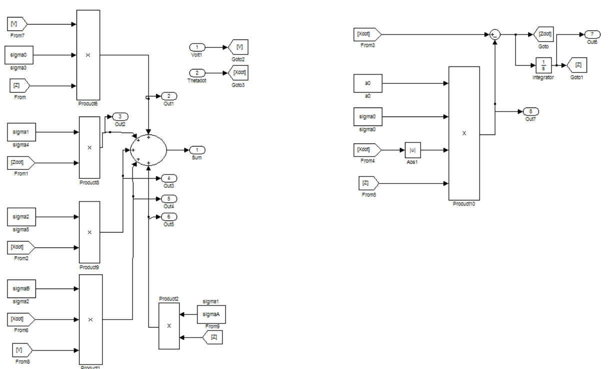


Figure 17: MR-damper subsystem

we can adjust the model. The plots is viewed starting with output 1 from the left

Out1 and Out5 and captures the nonlinear effect. Out1 is the active part of the damping, and Out5 is the passive part.
Out2 gives the stribeck effect (without the viscous friction), Out3 gives the passive linear damping and Out4 gives the active linear damping.
Out6 is the state variable Z and Out7 is the state variable Z multiplied with σ_0, a_0 and $|\dot{x}|$

Sigma0 affects both the gain of the state variable Z and the active nonlinear damping
Sigma1 affects the gain of the "stribeck"-part
Sigma2 affects the gain of the passive linear damping
SigmaA affects the gain of the passive nonlinear damping
SigmaB affects the gain of the active linear damping
a0 affects the gain of the state variable Z
The voltage affects the gain of all the active parts

4.4 Bouc-Wen model

The Bouc-Wen model is used to describe a hysteric effect. By applying the hysteric effect of Bouc-Wen, we can establish a good model of the MR damper. In

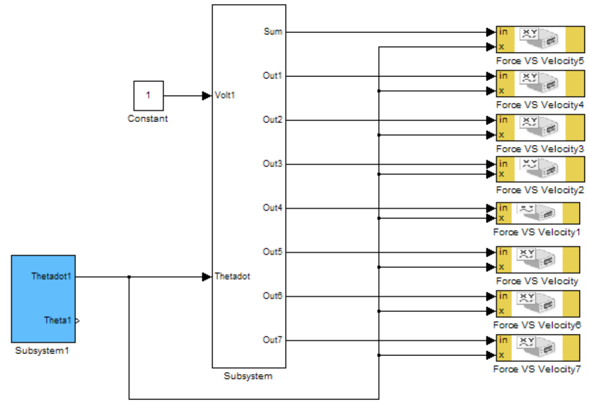


Figure 18: Lugre system

this section we model the Bouc-Wen and test the effect of changing the implemented parameters.

To model the Bouc-Wen we used Matlab Simulink. To make sure that the model was correct, the model was set up in a system identically to the one in file "Characterization of a commercial magnetorheological brake/damper in oscillatory motion" [9]. By doing this we could use the same parameters and thereby confirm that the model was correct. The system used was a simple system with Bouc-Wen and a linear damper. In translational dampers the model also contains a spring. In the rotational MR damper it can be neglected.

Formulas used to model the mr-damper[9]:

$$T = \alpha(i)z + c(i)\dot{\theta} \tag{9}$$

$$\dot{z} = -\gamma|\dot{\theta}|z|z|^n - \beta\dot{\theta}|z|^n + \delta\dot{\theta} \tag{10}$$

i: The current applied to the mr-damper
z: The hysteretic parameter from Bouc-Wen
The constants α and c are linear to the current [10]:

$$c(i) = c_1 + c_2 \cdot i \tag{11}$$

$$\alpha(i) = \alpha_1 + \alpha_2 \cdot i \tag{12}$$

The α_1 and c_1 are constants for the passive damping. α_2 and c_2 are parameters for the active damping.

To show the hysteresis and Bouc-Wen, the model was implemented in simulink.

The simulink model in fig22 corresponds to the equations 9,10,15 and 16. In fig23 compressed into the subsystem "Bouc-Wen MR damper".

4.5 Bouc-Wen model: The current effect

By changing the current from 0 to 1, a significant change in the two hysteresis appears. In this test a sinusoidal velocity profile i used.

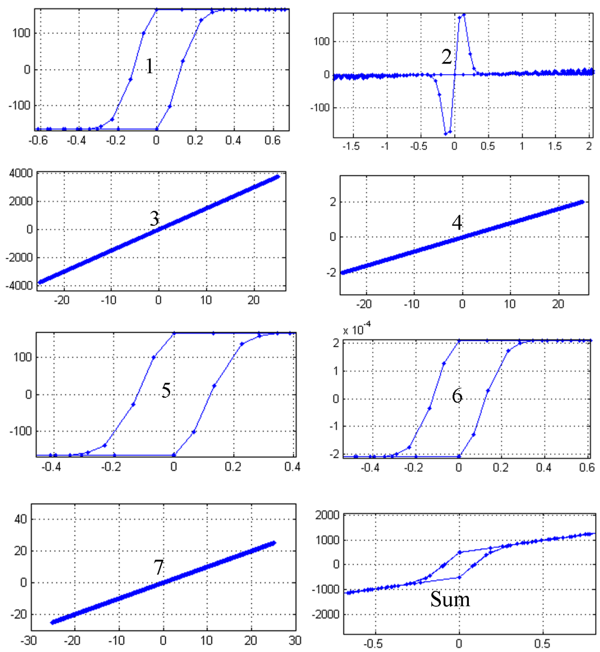


Figure 19: LuGre equation speed

In Figure 24 we see that the increased current increases the maximum torque. It also increases the slope of the linear damping outside the hysteresis. When looking at the torque vs displacement, we see that the shape of the curve is kept the same. Compared with the linear damper we see a noticeable difference(26).

From eq.16 we should have a linear dependency of the parameter α_1 and the applied torque from the hysteresis effect.

5 Testing hysteresis-models on the SAS system

In this section, the zooming and investigating if the Dahl and the bouc-wen can fit the MR damper in the suspension system in the lab. We have not been focusing on the Lugre model because this is just an extension to the Dahl model and much harder to tune because of the many parameters. The SAS system is simplified and modeled in matlab/simulink.

6 SAS model

Since a static vibration analysis with no motion of the wheel, in this situation, it is assumed the wheel to be stiff and with no damping.

The spring represents the physical spring on the model. The damper represents the constant damping

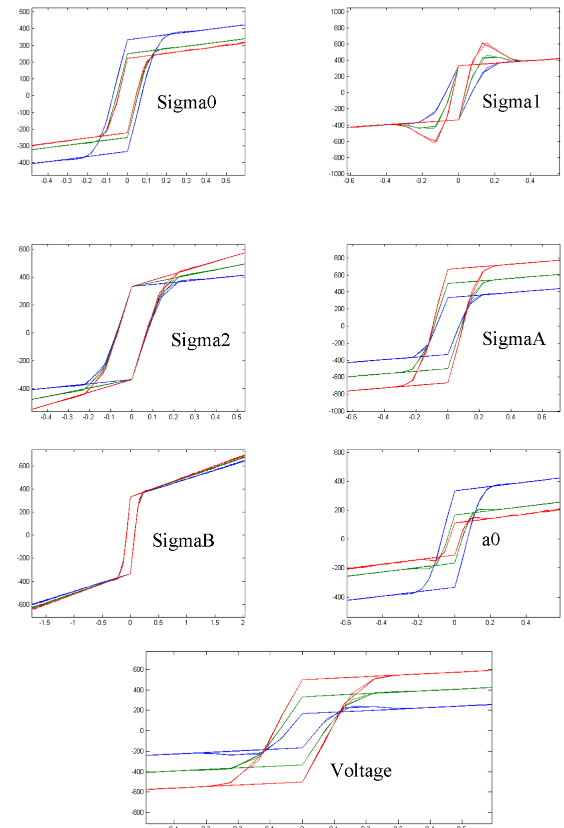


Figure 20: Parameter change speed

in the MR damper plus other external things as friction and so on. The last part is representing the force from the nonlinear damping in the MR damper. θ_1 and $\theta(\theta = \theta_2 - \theta_1)$ is measured trough a angle sensor on the model and plotted on the computer.

7 Equations for system with Bouc-Wen and Dahl

Model-equations with Bouc-Wen:

$$T = J\ddot{\theta} \quad = \quad -k\theta - c(i)\dot{\theta} - \alpha(i)z \tag{13}$$

J is redundant:

$$\ddot{\theta} \quad = \quad -k_1\theta - c(i)\dot{\theta} - \alpha(i)z \tag{14}$$

$\alpha(i)$ and c(i) are defined in the following:

$$c(i) \quad = \quad c_1 + c_2 \cdot i \tag{15}$$
$$\alpha(i) \quad = \quad \alpha_1 + \alpha_2 \cdot i \tag{16}$$

The Bouc-Wen non linearity is a function of z.
Z can be calculated from the differential equation:

$$\dot{z} \quad = \quad -\gamma|\dot{\theta}|z|z|^n - \beta\dot{\theta}|z|^n + \delta\dot{\theta} \tag{17}$$

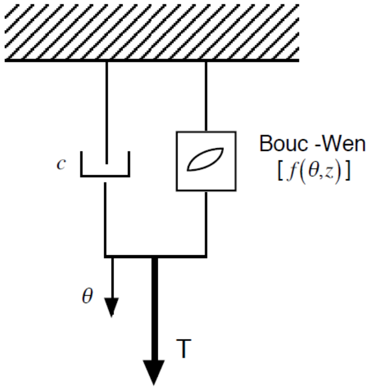


Figure 21: Bouc-Wen model of MR-damper[9]

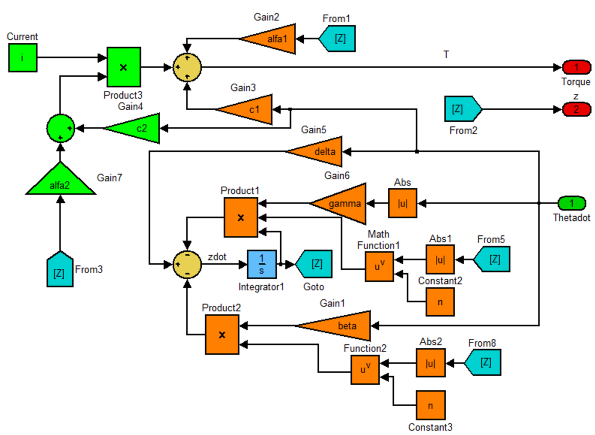


Figure 22: MR-damper model in simulink

This equation was modeled in matlab/simulink and then merged together with the rest of the system.

On the left side, the bouc-wen model(green system) for calculating the parameter z. The yellow system is the contribution from the active damping, dependent on the current. The white blocks illustrates the physical system with a linear spring and damper. The purple block multiplied by z is the passive nonlinear damping.

Model-equations with Dahl:

$$T = J\ddot{\theta} = -k\theta - k_x \cdot \dot{\theta} + (k_{wa} + k_{wb} \cdot v) \cdot w \quad (18)$$

J is redundant:

$$\ddot{\theta} = -k\theta - k_x \cdot \dot{\theta} + (k_{wa} + k_{wb} \cdot v) \cdot w \quad (19)$$

$$\dot{w} = \rho \cdot (\dot{\theta} - |\dot{\theta}|w) \quad (20)$$

The orange blocks represents the dahl-model for MR damper and the white blocks represents the SAS-model.

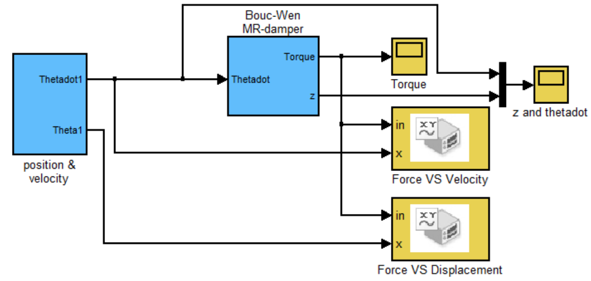


Figure 23: System design in simulink

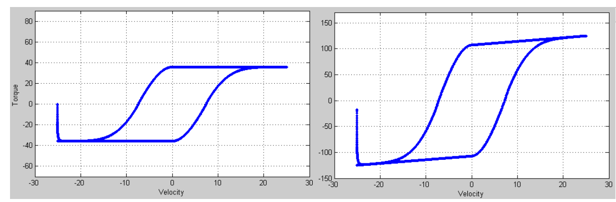


Figure 24: Torque vs Velocity. Hysteresis for 0 and 1 ampere.

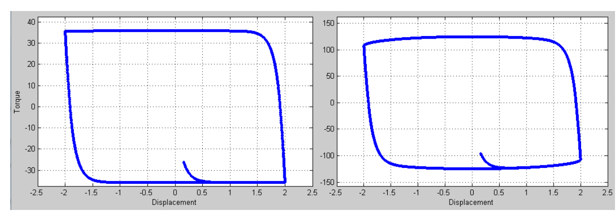


Figure 25: Torque vs displacement. Hysteresis for 0 and 1 ampere.

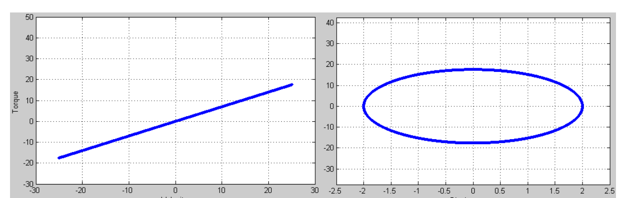


Figure 26: Linear damper

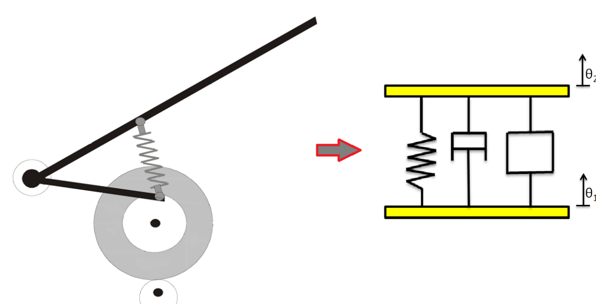


Figure 27: Model of system

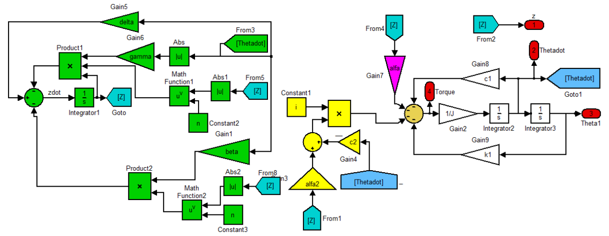


Figure 28: Total system with Bouc-Wen

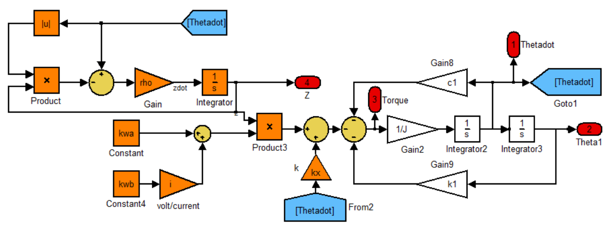


Figure 29: Dahl and SAS system in matlab/simulink

8 Experimental testing

The experimental tests done using the SAS test setup as shown in figure(30) when pressing the ”arm” down to an initial $\theta(0) = \theta_0 = -5deg$ and then drop. The arm would then oscillate around equilibrium.

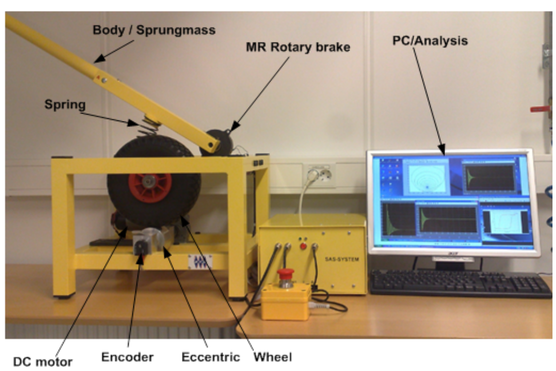


Figure 30: The mechatronics setup for the SAS

The results with 0A and 1A on the MR damper gave us these curves:

First, trying to fit the dahl and bouc-wen into the passive system with no current.

Bouc-Wen(passive)	Dahl(passive)
$n = 0.099$	$\rho = 10$
$\gamma = 1.0$	$kwa = 0.6$
$\beta = 873$	$k1 = 109$
$\delta = 700$	$c1 = 0.7$
$c1 = 0.7$	$kx = 0.15$
$\alpha_1 = 1$	
$k1 = 109$	

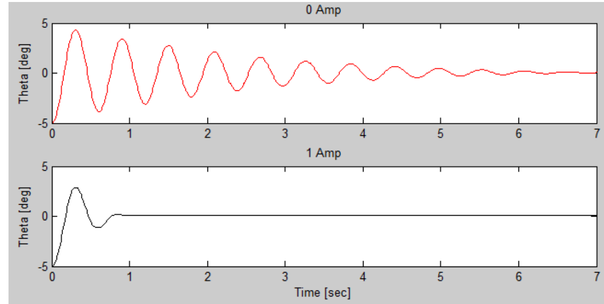


Figure 31: Results with 1 and 0 amp.

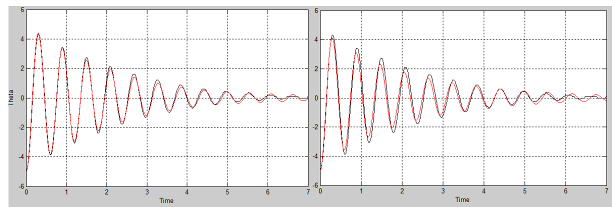


Figure 32: Left: Bouc-Wen. Right: Dahl

After trying to fit the model by changing the parameters for the hysteresis models, it is shown that the Bouc-Wen actually is a better fit than Dahl. Fitting the Dahl model was more difficult. The problem was that the nonlinear damping leads to a varying frequency, which the Dahl has a bigger problem with than Bouc-Wen. The Bouc-Wen is more adjustable because of the nonlinear terms with the exponential n-parameter.

Next, the same test with 1 amp into the MR damper is performed.

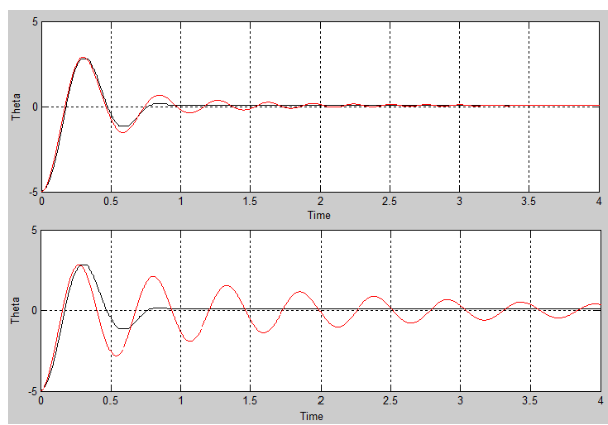


Figure 33: Top: Bouc-Wen. Bottom: Dahl

Bouc-Wen(active)	Dahl(active)
$c_2 = 1.5$	$k_{wb} = 2.8$
$\alpha_2 = 1$	

When looking at the Figure(33), it is shown that the Bouc-Wen model has a much better fit than the Dahl.

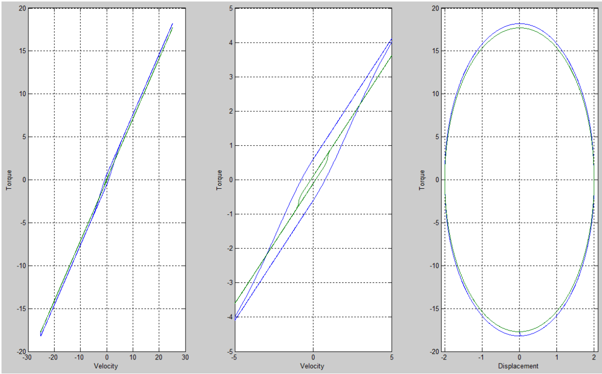


Figure 34: comparing hysteresis with 0A (Bouc-Wen:[green],Dahl:[blue])

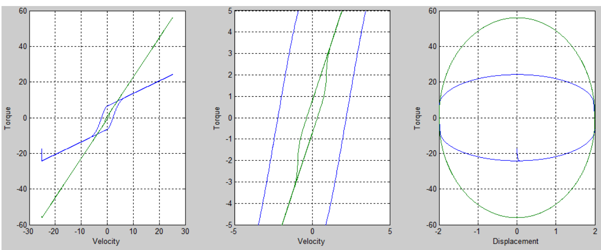


Figure 35: comparing hysteresis with 1A (Bouc-Wen:[green],Dahl:[blue])

Seen from Figures(34) and (35) the hysteresis of Dahl is a bit larger than the Bouc-Wen hysteresis(Figure(34)). When the current is set to one, the Changes are increasing dramatically. The Bouc When which is the most correct model for this MR damper has increased to about two times of the Dahl model. This is mainly because the linear damping in the used Dahl model, does not have a linear damping coefficient proportional to the current such as our Bouc-Wen.

9 Results

These experiments show us that Bouc-Wen is more adaptable to our system. Even though the Dahl hysteresis is quite similar to the Bouc-Wen, it could not fit the system as good. With no current on the damper the differences were quite small, and could probably been even smaller if someone had used a specific scientific method to estimate the parameters. When the current was set to one, the Dahl model could not manage to increase the linear slope of the hysteresis. This problem could easily be solved by adding a term for the linear damping dependent on current. Like, $k_i \cdot i$

A source of error in addition to inaccurate parameters could be that the neglect of the tire damping makes our simplified model too inaccurate.

10 Conclusion

The determined parameters using different models has be utilized to determine the dynamic hysteresis loop, the process was done using different models which simulate the hysteresis implemented in a SAS system.

- The methodology of identifying the hysteresis parameters has been implemented using two different models : Dahl and Bouc-Wen
- The Semi Active Suspension system has proven to be efficient for studying the rotational MR damper behavior, thus including the vibration and hysteresis analysis.
- The identified parameters which has been harvested from the passive and active vibration data has been used to determine the hysteresis dynamics in different models
- The Dahl, Lugre and Bouc-Wen have all an adaptable hysteresis, but the Bouc-Wen is found to be the best model for illustrating the MR damper. The Dahl is also pretty good in the passive part but it needs to be modified to fit the active properties. This may be done by adding a voltage dependent damping coefficient similar to the Bouc-Wen and Lugre models.

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