Morphing Motion in Modular Robots

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Abstract: -Two kinds of basic modules are presented to set up the modular robots. One is the cuboid module, the other is the cubic module. Four connecting types of cuboid modules and two connecting types of cubic modules are analyzed based on their geometric feature. For deriving the transformation matrix of each connecting type, the concepts of basic group are proposed. Then, the geometric relationships of each connecting type are derived with the group theory. The basic motion of modules is presented. With the rotary motion around the neighboring module, the morphing motion from a quadruped to two-leg modular robot is shown and the kinematics of modular robots is generated with product of matrix exponentials. Examples of kinematics on a quadruped and a two-leg modular robot are given to demonstrate the applicability of the proposed methods.

Key-Words: - Modules, Robot, Kinematics, Morphing, Motion

1 Introduction

Modular robots consist of a set of homogeneous basic modules that can connect to, disconnect from and relocate relative to adjacent modules. It can autonomously and dynamically adapt its geometric structures and functions to different given tasks and unknown or unpredictable environment.

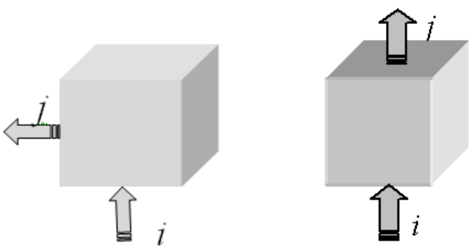
In early years, two dimensional modular robot systems were studied. Fukuda developed a cellular robotic system [1], which was the first modular robot proposed. Fractum [2] was a homogeneous system. Its mechanism was simple so it can only achieve motion in the plane.

Recently, research on three dimensional models has made remarkable progress and many kinds of robots have been proposed. There are two classes of three dimensional systems, a class based on lattice systems and a class of linear or string systems. In the former class, the shape of a robot is determined by the lattice systems, such as a cubic structure [3], M-TRAN [4]. The latter class is a linear or string system, such as Polybot [5], Conro [6]. Their researches were mostly about the structure design, motion planning and control. The analysis of automatically generating kinematics on modular robots is minimum. Kelmar [7] proposed an algorithm for automatically generating kinematics with the standard D-H method. In their technique, only rotary joints were covered and some constraints were imposed improperly. Benhabib [8] improved the above technique. However, their work needed to be further extended to consider a more general modular structure. Chen [9,10] used the cube as the module and 1-DOF joint modules were studied on the kinematics. Fei [11,12] used prism as the module and analyzed the kinematics.

In this article, the basic modules are classified and the motion on the morphing progress among modular robots is analyzed, then their kinematics is generated. First, the cuboid modules and cubic modules are classified into six connecting types. Three coordinate systems are set up on each module. Second, the concepts of basic group are proposed. Thus, the description of each module can be simplified. When the configuration of each modular robot is known, the connecting type of each module is chosen. Third, the basic motion of modules is presented. With the rotary motion around the neighboring module, the morphing motion from a quadruped robot to a two-leg modular robot is shown. Their kinematic models can be generated automatically by using the product of matrix exponentials.

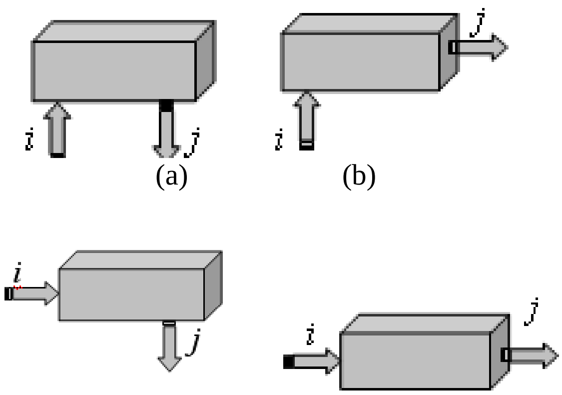
2 Basic Module Description

Modular robots consist of a set of homogeneous units. The cuboid module and the cubic module are general units in modular robots. According to the flowing direction of power and information, there are two connecting types for cubic modules (Figure 1). If the module is assumed to be square prism whose section is square, the module is classified into four connecting types (Figure 2). With the group theory, the transformation matrix $T_{Li\text{-}j}$ (port j relative to port i) of the cuboid module and $T_{Ci\text{-}j}$ of the cubic module can be generated.



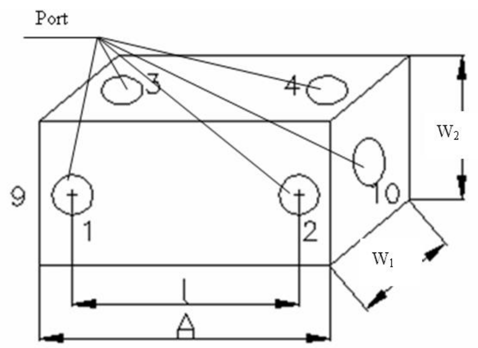
(a)   (b)

Figure 1. Cubic module.

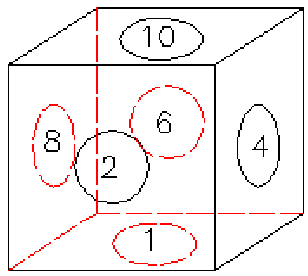


(a) (b) (c) (d)

Figure 2. Cuboid module.



(a) (l-port separation, A- length, $w_1(=w_2)$- width)



(b) (width=w)

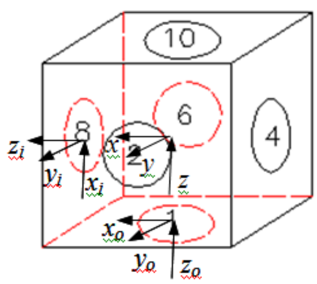
Figure 3. Structures of the cuboid module and the cubic module.

2.1 Module Structure

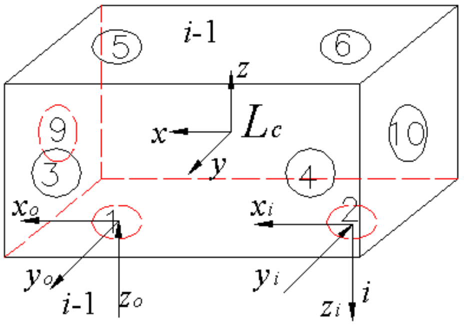
With the bottom-up manner, the module ports are numbered. The input ports are given odd numbers and the output ports even numbers (Figure 3).

2.2 Mathematical Description

We take the connecting type of Figure 1(a) and Figure 2(a) as an example. Three coordinate frames (the center coordinate system (x y z), the input port (connecting port) coordinate system (x_i y_i z_i) and the output port (connecting port) coordinate system (x_o y_o z_o)) can be set up (Figure 4).The i-th z axis is defined to be along the direction of the axis in the i-th connecting port, which points to the i-th module.



（a）



(b)

Figure 4. One of the module'connecting types.

Definition 1. The module is based on a quadrivalent circular, whose twist angle is $\frac{2\pi}{4}$. The basic cycle group of the transformation matrix is $\left(\sigma(\frac{2\pi}{4})^0 \quad \sigma(\frac{2\pi}{4})^1 \quad \sigma(\frac{2\pi}{4})^2 \quad \sigma(\frac{2\pi}{4})^3\right)$.

$$\sigma(\frac{2\pi}{4})^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

According to $\sigma(\frac{2\pi}{4})^0$, it rotates $\frac{2\pi}{4}$ about the x_i axis and translates $\begin{pmatrix} 0 & -\frac{w_{i-1}}{2} & -\frac{w_{i-1}}{2} & \end{pmatrix}^T$ to $\sigma(\frac{2\pi}{4})^1$.

Thus,

$$\sigma(\frac{2\pi}{4})^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{w_{i-1}}{2} \\ 0 & 0 & 1 & -\frac{w_{i-1}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\frac{w_{i-1}}{2} \\ 0 & 1 & 0 & -\frac{w_{i-1}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\sigma(\frac{2\pi}{4})^2 = \sigma(\frac{2\pi}{4})^1 \cdot \sigma(\frac{2\pi}{4})^1 \tag{3}$$

$$\sigma(\frac{2\pi}{4})^3 = \sigma(\frac{2\pi}{4})^1 \cdot \sigma(\frac{2\pi}{4})^1 \cdot \sigma(\frac{2\pi}{4})^1 \tag{4}$$

Definition 2. Basic Group (G): In G, each set is a single cell base which forms other similar models with a relative basic cycle group.

Definition 3. Twist equivalence: After the configuration $\boldsymbol{C}_1$ is transformed by $\boldsymbol{T}$ (the transformation matrix), the configuration $\boldsymbol{C}_2$ can be obtained. $\boldsymbol{C}_1\boldsymbol{T}= \boldsymbol{C}_2$, thus $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are in twist equivalence.

For example, for the cuboid module, there are four kinds of connecting types, G={{$T_{L2\text{-}1}$}, {$T_{L10\text{-}1}$},{$T_{L2\text{-}9}$},{$T_{L10\text{-}9}$}}. See Figure 2, $T_{Lj\text{-}i}$ stands for the transformation matrix of the port j relative to the port i. {$T_{L2\text{-}1}$} is a single cell base. Other transformation matrixes $T_{L4\text{-}1}$, $T_{L6\text{-}1}$, $T_{L8\text{-}1}$can be formed with the basic cycle group, such as $\boldsymbol{T}_{L4\text{-}1} = \boldsymbol{T}_{L2\text{-}1} \cdot \sigma(\frac{2\pi}{4})^1$, $\boldsymbol{T}_{L6\text{-}1}=\boldsymbol{T}_{L2\text{-}1} \cdot \sigma(\frac{2\pi}{4})^2$, $\boldsymbol{T}_{L8\text{-}1}=\boldsymbol{T}_{L2\text{-}1} \cdot \sigma(\frac{2\pi}{4})^3$, …. $\boldsymbol{T}_{L2\text{-}1}$, $\boldsymbol{T}_{L4\text{-}1}$, $\boldsymbol{T}_{L6\text{-}1}$ and $\boldsymbol{T}_{L8\text{-}1}$ are in twist equivalence. According to the above definition and relation, the transformation matrix of other connecting types can also be obtained. So we only need to study the basic expressions about modular robots. Other connecting types' expressions can be obtained with the group theory.

For example,

$$\boldsymbol{T}_{L2-1} = \begin{bmatrix} 1 & 0 & 0 & -l_{i-1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

With the basic cycle group, the following equation can be obtained.

$$\boldsymbol{T}_{L4\text{-}1}=\boldsymbol{T}_{L2\text{-}1} \cdot \sigma(\frac{2\pi}{4})^1 = \begin{bmatrix} 1 & 0 & 0 & -l_{i-1} \\ 0 & 0 & 1 & \frac{w_{i-1}}{2} \\ 0 & -1 & 0 & \frac{w_{i-1}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$\boldsymbol{T}_{L6\text{-}1}=\boldsymbol{T}_{L2\text{-}1} \cdot \sigma(\frac{2\pi}{4})^2 = \begin{bmatrix} 1 & 0 & 0 & -l_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & w_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$\boldsymbol{T}_{L\,8\text{-}1}=\boldsymbol{T}_{L2\text{-}1}\cdot\sigma(\frac{2\pi}{4})^3=\begin{bmatrix}1 & 0 & 0 & -l_{i-1}\\ 0 & 0 & -1 & -\frac{w_{i-1}}{2}\\ 0 & 1 & 0 & \frac{w_{i-1}}{2}\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (8)$$

……

3 Kinematics

3.1 POE Method

According to the above analysis, when the serial configuration of the modular robot is determined, the modules are also determined and the transformation matrix T_L are also obtained. For the linear modular robot with several modules, our approach is the use of POE (product of matrix exponentials) to represent modular robots' forward kinematics

$$\boldsymbol{T}(\theta)=\boldsymbol{T}(0)\cdot e^{\hat{\xi}_1^1\cdot\theta_1}\cdot\boldsymbol{T}_{L1}\cdot e^{\hat{\xi}_2^2\cdot\theta_2}\cdot\boldsymbol{T}_{L2}\cdots \quad (9)$$

For a tree type system with several branches, the forward kinematics is $\boldsymbol{T}(\theta)=\left[\boldsymbol{T}^1(\theta)\quad\boldsymbol{T}^2(\theta)......\right]^T$.

$$\boldsymbol{T}^k(\theta)=\boldsymbol{T}^k(0)\prod_{i=1}^{n}e^{\hat{\xi}_i^i\cdot\theta_i}\cdot\boldsymbol{T}_{Li}{}^k \quad (10)$$

Thus, the kinematic model can be generated automatically.

$T(0)$ is the transformation matrix of the 1-th module's input port relative to the given coordinates. T_{Li} is the transformation matrix of one connecting type. θ_i is the rotary angle between two neighboring modules， $e^{\hat{\xi}_i^i\theta_i}$ is the rotary transformation matrix of two neighboring modules.

$$e^{\hat{\xi}_i^i\theta_i}=\begin{bmatrix}c\theta_i & -s\theta_i & 0 & 0\\ s\theta_i & c\theta_i & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (11)$$

According to the structure of the modular robot, for the cuboid modules, the connecting types of Figure 2 (b) and Figure 2 (d) are chosen to be the end-modules.

$$T_{end-b}=\begin{bmatrix}0 & 0 & -1 & -\frac{A_{i-1}+l_{i-1}}{2}\\ 1 & 0 & 0 & 0\\ 0 & -1 & 0 & \frac{w_{i-1}}{2}\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (12)$$

$$T_{end-d}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & A_{i-1}\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (13)$$

$$T_{Lc-1}^{cuboid}=\begin{bmatrix}1 & 0 & 0 & -\frac{l_{i-1}}{2}\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & \frac{w_{i-1}}{2}\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (14)$$

For the cubic modules, each connecting type can be chosen as the end-modules.

$$T_{Lc-1}^{cubic}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & \frac{w_{i-1}}{2}\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (15)$$

Where T_{Lc-1}^{cuboid} or T_{Lc-1}^{cubic} is the transformation matrix of port 1 relative to the module's center for the cuboid module or the cubic module.

3.2 Kinematics of a Quadruped

A quadruped is composed of eight identical cuboid modules and one center module (Figure 5 (a)). The coordinate system （*oxyz*）is set up at the center module (Figure 5). The graph representation of the quadruped is shown in Figure 5 (b). The line stands for the module and the small circle stands for the rotary joint.

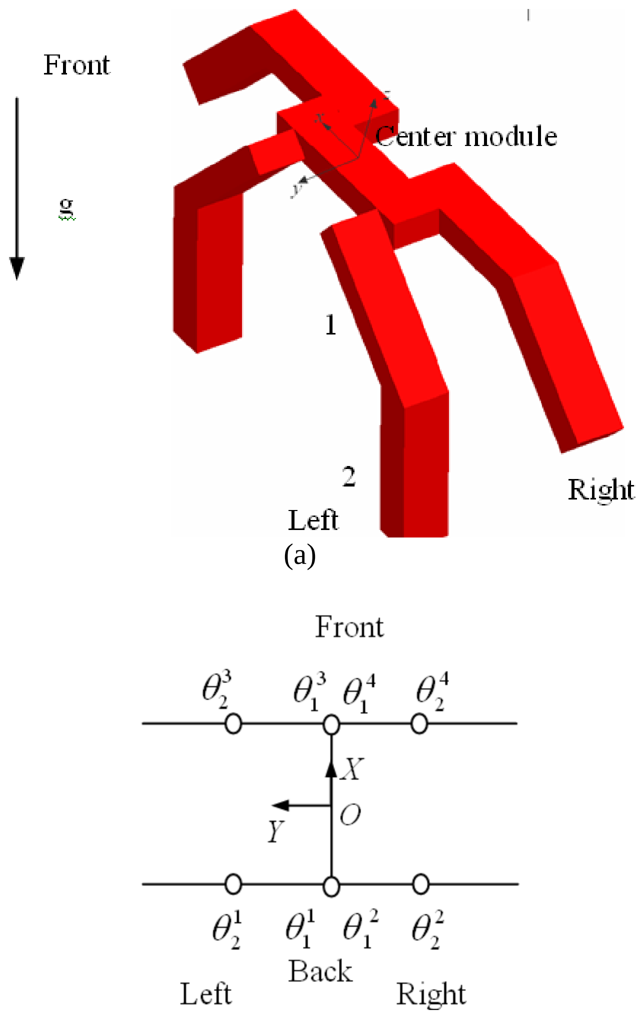


Figure 5. A quadruped composed of eight identical cuboid modules and one center module.

The forward kinematics of its back-left leg is generated automatically with the above method.

$$\boldsymbol{T}^1(\theta_1^1,\theta_2^1)=\boldsymbol{T}_{c-4}^1 e^{\hat{\xi}_1^1\theta_1^1}\boldsymbol{T}_1^1 e^{\hat{\xi}_2^2\theta_2^1}\boldsymbol{T}_2^1 \qquad (16)$$

With the basic cycle group, the following equations can be obtained.

$$\boldsymbol{T}_{c-4}^1=\boldsymbol{T}_{c-2}^1\cdot\sigma\left(\frac{2\pi}{4}\right)^1=\begin{bmatrix}1 & 0 & 0 & -\frac{l_0'}{2}\\ 0 & 0 & 1 & \frac{w_0'}{2}\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad (17)$$

$$e^{\hat{\xi}_1^1\theta_1^1}=\begin{bmatrix}c\theta_1^1 & -s\theta_1^1 & 0 & 0\\ s\theta_1^1 & c\theta_1^1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{aligned}T_1^1&=T_{4-7}=T_{4-1}\cdot\sigma\left(\frac{2\pi}{4}\right)\\ &=T_{2-1}\cdot\sigma\left(\frac{2\pi}{4}\right)\cdot\sigma\left(\frac{2\pi}{4}\right)\\ &=\begin{bmatrix}1 & 0 & 0 & -l_1^1\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & w_1^1\\ 0 & 0 & 0 & 1\end{bmatrix}\end{aligned} \qquad (18)$$

$$e^{\hat{\xi}_2^2\theta_2^1}=\begin{bmatrix}c\theta_2^1 & -s\theta_2^1 & 0 & 0\\ s\theta_2^1 & c\theta_2^1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\boldsymbol{T}_2^1=\begin{bmatrix}0 & 0 & -1 & -\frac{A_2^1+l_2^1}{2}\\ 1 & 0 & 0 & 0\\ 0 & -1 & 0 & \frac{w_2^1}{2}\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad (19)$$

Thus, the kinematics of the quadruped's back-left leg can be obtained.

$$T^1(\theta_1^1,\theta_2^1)=\begin{bmatrix}-s\theta_{12}^1 & 0 & -c\theta_{12}^1 & x\\ 0 & -1 & 0 & y\\ -c\theta_{12}^1 & 0 & s\theta_{12}^1 & z\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad (20)$$

$$x=-\frac{A_2^1+l_2^1}{2}c\theta_{12}^1 - l_1^1 c\theta_1^1-\frac{l_0}{2}$$

$$y=\frac{w_2^1}{2}+w_1^1+\frac{w_0}{2}$$

$$z=\frac{A_2^1+l_2^1}{2}s\theta_{12}^1+l_1^1 s\theta_1^1$$

The dimensions of each module are given as follows: $A_2^1 = A_1^1 = 15cm$, $l_0 = 20cm$ (the center module), $l_2^1 = l_1^1 = 10cm$, $w_2^1 = w_1^1 = 5cm$, $\theta_1^1 = t$ rad, $\theta_2^1 = t$ rad. The space trajectory of its back-left leg is shown in Figure 6.

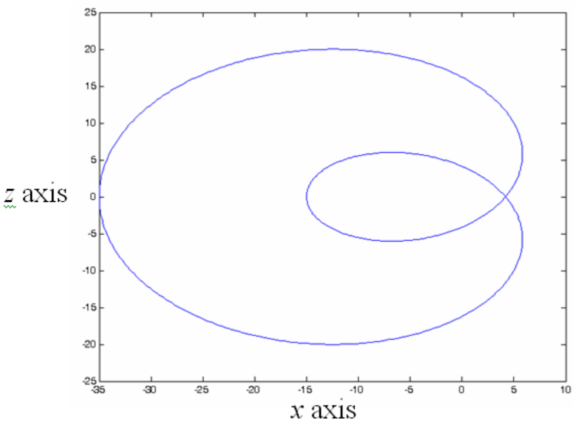
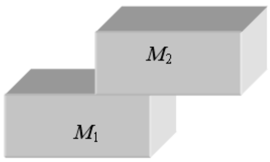


Figure 6. The space trajectory of a quadruped's back-left leg in the *oxz* plane.

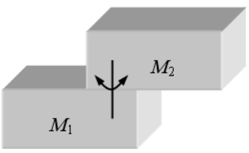
4 Morphing Motion

4.1 Basic Motion

During the morphing process of the modular robot, the basic constraints are shown: 1) the connectivity of the whole system should be maintained; 2) the fixed base must always link to at least one module; 3) each module must always remain linked to the side of at least one other module; 4) the moving module do not collide with other modules, environment and obstacle.



(a)



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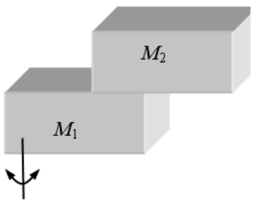

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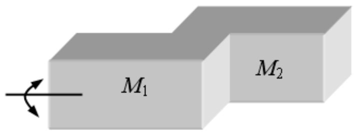
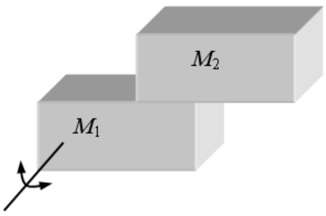


(d)



(e)

Figure 7. Basic motions.

According to the analysis, the possible motion of two modules (Figure7. a) are shown as follows:

1) The module M_2 connects to/disconnects from the module M_1.
2) The module M_2 rotates $\pm\frac{\pi}{2}n°$ about the axes of two neighboring modules M_1 and M_2 (Figure 7. b).
3) Two modules M_1 、 M_2 have same motion direction. There are two kinds: 2.1) horizontal plane motion, two connecting modules M_1 and M_2 rotate together about the axes of M_1 and another module, Figure 7.c; 2.2) vertical plane motion, two connecting modules rotate in the same vertical plane about the axis of M_1, Figure 7. d, e.

With the rotations of the modules (Figure7 d, e), the movement (translation) of the robot system can be obtained. For example, a system consists of two connecting modules M_1 and M_2. They rotate about the axis of M_1 (Figure 8 a). The system moves forward. Then, they rotate about the axis of M_2 (Figure 8 b). At last, the system moves forward 4*A*-4*w* (Figure 8 c). Where *A* is the length of a module, *w* is the width of a module (in Figure 3.).

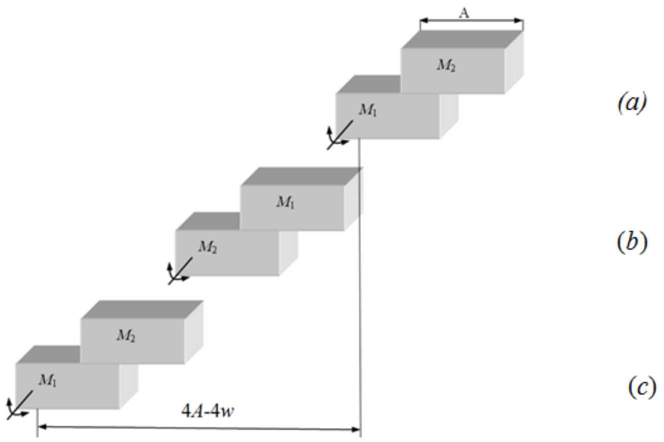
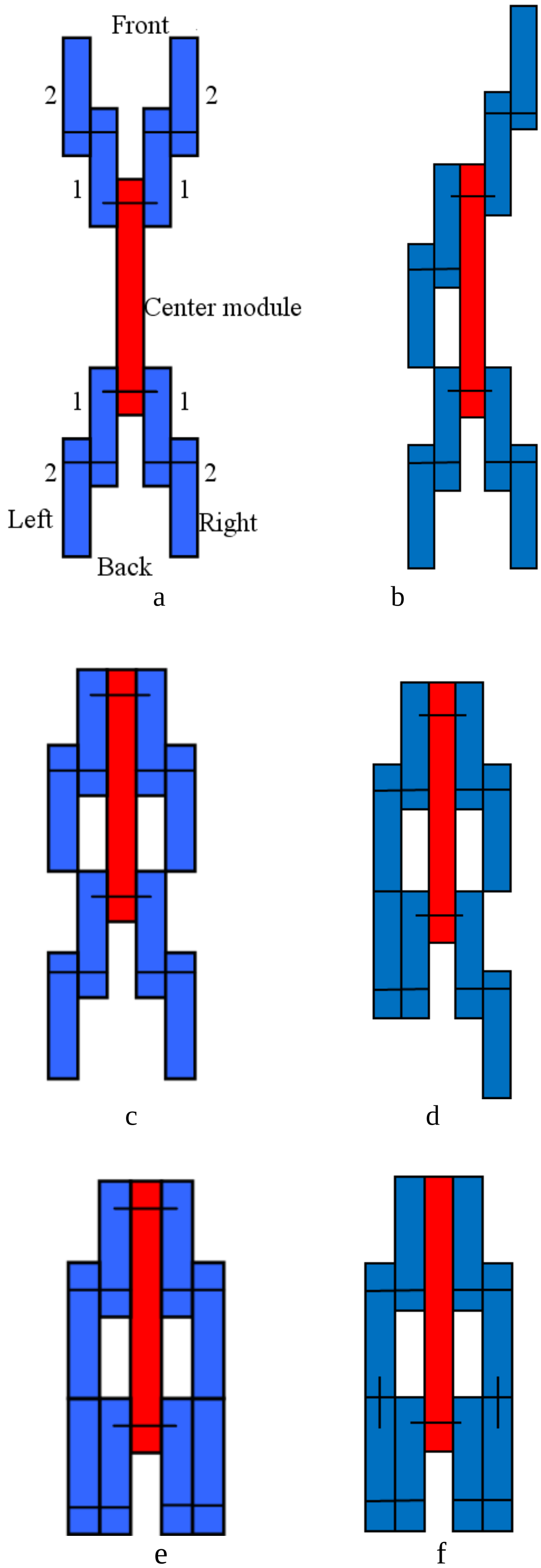
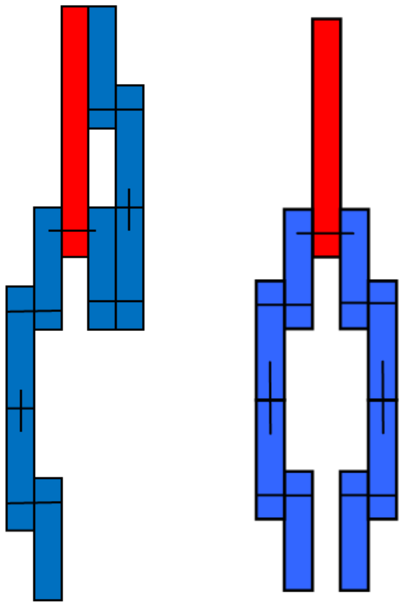


Figure 8. Movement (translation) of the modules

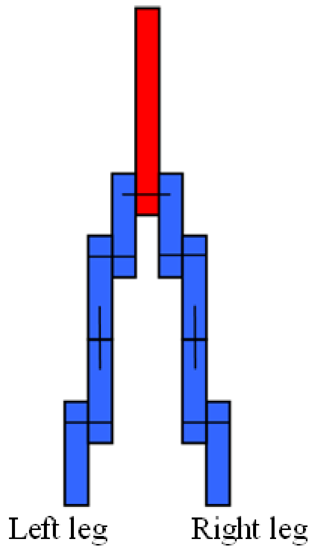
4.2 Morphing Analysis of a Two-leg Modular Robot

With the rotary motion around the neighboring module, the above quadruped (Figure 5.a) can morph from a quadruped (Figure 9.a) to a two-leg modular robot (Figure 9. h) by connecting to or disconnecting from the adjacent modules. The morphing motion is shown in Figure 9 (the top view of the two-leg modular robot). The front module 1 connects to the front module 2, and they rotate around the center module (Figure 9 b，c). The back module 2 rotates around the back module 1 (Figure 9 d，e). The front module 2 connects to the back module 2, the front module 1 disconnect from the center module (Figure 9 f). The front module 1, module 2 and the back module 2 rotate around the back module 1(Figure 9 g，h). The front module 1 and the front module 2 rotate around the back module 2 (Figure 9 i). Then the configuration of a two-leg modular robot is determined (Figure 9 i). The graph representation of the top view of the two-leg modular robot is shown in Figure 9 j. The forward kinematics of the left leg can be generated with the POE method.

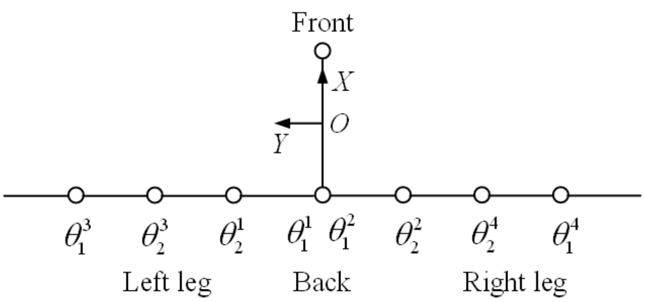




g h



i



j

Figure 9. Morphing motion from a quadruped to a two-leg modular robot.

$$T^1(\theta_1,\theta_2,\theta_3,\theta_4)=T_{c-4}^1 e^{\hat{\xi}_1^1\theta_1^1}T_1^1 e^{\hat{\xi}_2^2\theta_2^1}T_2^1 e^{\hat{\xi}_3^3\theta_2^3}T_3^1 e^{\hat{\xi}_4^4\theta_1^3}T_4^1 \tag{21}$$

$$\boldsymbol{T}_{c-4}^1=\boldsymbol{T}_{c-2}^1\cdot\sigma\left(\frac{2\pi}{4}\right)^1=\begin{bmatrix}1&0&0&-\frac{l_0'}{2}\\0&0&1&\frac{w_0'}{2}\\0&-1&0&0\\0&0&0&1\end{bmatrix}$$

$$T_1^1=T_{4-7}=T_{4-1}\cdot\sigma\left(\frac{2\pi}{4}\right)$$
$$=\begin{bmatrix}1&0&0&-l_1^1\\0&1&0&0\\0&0&1&w_1^1\\0&0&0&1\end{bmatrix}$$

$$\boldsymbol{T}_2^1=\begin{bmatrix}0&0&-1&-\frac{A_2^1+l_2^1}{2}\\1&0&0&0\\0&-1&0&\frac{w_2^1}{2}\\0&0&0&1\end{bmatrix}$$
$$\boldsymbol{T}_3^1=\begin{bmatrix}0&1&0&0\\0&0&-1&-\frac{w_3^1}{2}\\-1&0&0&\frac{A_3^1+l_3^1}{2}\\0&0&0&1\end{bmatrix} \tag{22}$$

$$T_4^1=\begin{bmatrix}0&0&-1&-\frac{A_4^1+l_4^1}{2}\\1&0&0&0\\0&-1&0&\frac{w_4^1}{2}\\0&0&0&1\end{bmatrix} \tag{23}$$

The two-leg modular robot consists of one center module and eight identical cuboid modules. The kinematics of the left leg can be obtained, too.

$$T^1\left(\theta_1^1,\theta_2^1,\theta_1^3,\theta_2^3\right)=\begin{bmatrix}\Theta_1&s\theta_{12}^1 s\theta_2^3&\Theta_2&x\\-s\theta_2^3 c\theta_1^3&-c\theta_2^3&s\theta_2^3 s\theta_1^3&y\\\Theta_3&c\theta_{12}^1 s\theta_2^3&\Theta_4&z\\0&0&0&1\end{bmatrix} \tag{24}$$

$$\Theta_1 = -s\theta_{12}^1 c\theta_2^3 c\theta_1^3 - c\theta_{12}^1 s\theta_1^3$$
$$\Theta_2 = s\theta_{12}^1 c\theta_2^3 s\theta_1^3 - c\theta_{12}^1 c\theta_1^3$$
$$\Theta_3 = -c\theta_{12}^1 c\theta_2^3 c\theta_1^3 + s\theta_{12}^1 s\theta_1^3$$
$$\Theta_4 = c\theta_{12}^1 c\theta_2^3 s\theta_1^3 + s\theta_{12}^1 c\theta_1^3$$

$$x = \frac{A_4^1 + l_4^1}{2}(\sin\theta_{12}^1 \cos\theta_2^3 \sin\theta_1^3 - \cos\theta_{12}^1 \cos\theta_1^3)$$
$$-\frac{w_3^1 + w_4^1}{2}\sin\theta_{12}^1 \sin\theta_2^3$$
$$-\frac{A_3^1 + l_3^1 + A_2^1 + l_2^1}{2}\cos\theta_{12}^1 - l_1^1 \cos\theta_1^1 - \frac{l_0}{2}$$

$$y = \frac{A_4^1 + l_4^1}{2}\sin\theta_2^3 \sin\theta_1^3 + \frac{w_3^1 + w_4^1}{2}\cos\theta_2^3$$
$$+\frac{w_2^1}{2} + w_1^1 + \frac{w_0}{2}$$

$$z = \frac{A_4^1 + l_4^1}{2}(\cos\theta_{12}^1 \cos\theta_2^3 \sin\theta_1^3 + \sin\theta_{12}^1 \cos\theta_1^3)$$
$$-\frac{w_3^1 + w_4^1}{2}\cos\theta_{12}^1 \sin\theta_2^3$$
$$+\frac{A_3^1 + l_3^1 + A_2^1 + l_2^1}{2}\sin\theta_{12}^1 + l_1^1 \sin\theta_1^1$$

With the same dimensions, the space trajectory of its left leg is shown in Figure 10.

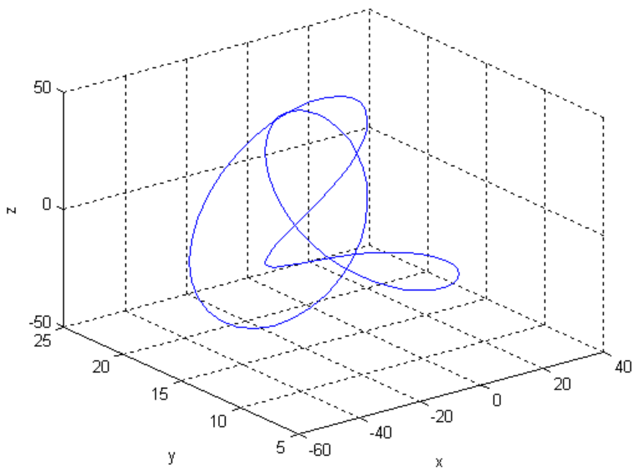


Figure 10. The space trajectory of a two-leg modular robot's left leg.

5 Conclusion

In the paper, due to having a more general configuration, a cuboid module and a cubic module of modular robots are described. According to the flow direction of power or information, six kinds of connecting types are identified, four types are the cuboid module and two types are the cubic module. Basic group is proposed to derive the geometric relationships and the transformation matrix T_L of each type of modules iteratively and simply. For the purpose of automatic generation of the forward kinematics, an approach has been adopted by a series of elemental matrix multiplication with product of matrix exponentials. Then, the basic motion of modules is presented. With the rotary motion around the neighboring module, the morphing motion from a quadruped to a two-leg modular robot is presented. Examples of the kinematics for a quadruped and a two-leg modular robot, which consist of eight same cuboid modules and one center module, are given to demonstrate the effectiveness of the proposed methods.

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