Subtractive Fuzzy C-means Clustering Approach with Applications to Fuzzy Predictive Control

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Abstract: - To identify T-S models, this paper presents a so-called "subtractive fuzzy C-means clustering" approach, in which the results of subtractive clustering are applied to initialize clustering centers and the number of rules in order to perform adaptive clustering. This method not only regulates the division of fuzzy inference system input and output space and determines the relative member function parameters, but also overcomes the impacts of initial values on clustering performance. Additionally, the orthogonal least square algorithm is employed to identify the parameters of consequents and linearize the systems over every sample time, ultimately resulting in the entire T-S fuzzy models. With this approach available, a fuzzy model predictive control system is established, along with corresponding control algorithms derived, as well as control system simulations carried out which explicitly demonstrate the effectiveness of the proposed method.

Keywords: - T-S mode identification; subtractive clustering; fuzzy C-means clustering; Generalized Predictive Control

1 Introduction

Conventional control strategies usually demand well structured process models and explicit model parameters. However, practical processes considered are becoming more complicated due to lack of formal knowledge, strong nonlinearity and involvement of diverse sources of uncertainty. In the sense of a process which is unable to be described by precise mathematical terms, fuzzy models become potentially very useful.

Tskagi-Sugeno (T-S) fuzzy model [1] has been one of the most popular fuzzy types circulated in the literature ever since. Consisting of a number of local input-output linear regression models in each subspace, a T-S model can be built by means of fuzzy rule based descriptions of input-output measurements of the continuous plants. Existing researches show that fuzzy C-means clustering (FCM) is an effective way to establish fuzzy inference rules [2-5]. However, due to multiple iterations employed and a large number of eigen vectors computed, the algorithm suffers heavy computational burdens, becoming very time-consuming. Additionally, it is strongly sensitive to the initialization treatment, which usually requires a priori knowledge of the cluster numbers to form the initial cluster centers. To our disappointment, inappropriate initial values readily lead to an undesired local minimum or suboptimal solution.

Previously, some alternative approaches have been made to avoid these limitations. Taking advantage of global searching, Lucio Ippolite [6] proposed a fuzzy clustering approach combined genetic algorithm (GA) with FCM, which can effectively avoid local optimum and reduce computational effort. Li Wang, et al. [7] involved Particle Swarm Optimization (PSO) algorithms into FCM, where the gradient descent iterations are replaced by PSO operations, enabling FCM a strong global searching ability as well as avoiding local convergence. Aiming at resolving the problem that FCM algorithms strongly depend on initialization, Emani et al. [8] developed an agglomerative hierarchical clustering scheme as an introductory procedure to find properly identified hard-clusters as the initial locations of the cluster prototype in the fuzzy c-means algorithm.

It is conceivable that the aforementioned approaches can help avoid local suboptimal solutions at the expense of increasing computations. However, they become more dependent on the initial assignments of cluster number and clustering centers due to the attempts of accelerating the convergence speeds. Subtractive clustering (SC) is recognized as a fast clustering algorithm [9]. It can be utilized to estimate the number of clusters and clustering centers, designed for high dimension problem with a moderate number of data points. Inspired by this idea, Liu et al [10] suggested an approach to merge SC and FCM, in which, however, assignments of initial associated membership functions were scarcely mentioned, let alone its applications.

Motivated by effective identification of T-S models, an improved subtractive fuzzy C-means clustering (SFCM) technique is introduced in this paper. The key idea behind this approach is that the results of subtractive clustering are designated as the initial values of FCM parameters, which leads to a high clustering speed as well as avoids local suboptimal solutions. Moreover, the proper number of clusters is automatically generated according to the impacts of system values on the clustering centers, greatly reducing the computational complexity. Inspired by these observations, the proposed method can surely achieve higher accuracy than traditional FCM to deal with T-S model identification. In what follows, the SFCM will be explicitly introduced along with numerical examples. To demonstrate the power of the contributions, the approach is applied to fuzzy predictive control systems where fuzzy predictive control algorithms are derived and identified T-S models are employed, resulting in satisfactory control performances..

The rest of the paper is organized as follows. Section 2 briefly describes the T-S model identification based on FCM. In section 3, SFCM algorithm is introduced and simulation investigations are carried out. This is followed in section 4 by applications to model predictive control system, in which the fuzzy model predictive control algorithm is accordingly derived and the simulation work is performed. Finally, the concluding remarks are given in section 5.

2 FCM based T-S model Identification

Consider a nonlinear MISO system with p inputs: $u \in U \subset R^p$, and 1 output, $y \in Y \subset R$. The corresponding T-S fuzzy models is expressed as n rules, in which, the ith fuzzy rule for kth time instant data is described as follows [11] :

$$\begin{aligned} R^i: \quad & \text{if} \quad x_1 \text{ is } A_1^i, x_2 \text{ is } A_2^i, \ldots, x_m \text{ is } A_m^i \\ & \text{then} \quad y^i(k) = p_0^i + p_1^i x_1 + p_2^i x_2 + \ldots + p_m^i x_m \\ & \qquad = p_0^i + \sum_{j=1}^{m} p_j^i x_j \qquad i = 1, 2, \ldots, n \end{aligned} \tag{1}$$

where A_j^i is the fuzzy set of the jth input variable of the antecedent of the ith fuzzy rule; $x(k) = [x_1, x_2, ..., x_m]$ is the vector of the input variables; y^i is the output variable of ith rule; p_j^i are the consequent parameters.

The final output of T-S model can be expressed by a weighted mean defuzzification at kth time as follows:

$$y = \sum_{i=1}^{n} \mu^i y^i \Big/ \sum_{i=1}^{n} \mu^i \tag{2}$$

where n corresponds to the number of fuzzy rules, y^i is the output variable of ith rule; μ^i represents the firing strength of the ith rule, which is defined as:

$$\mu^i(x) = \prod_{j=1}^{m} A_j^i(x_j) \tag{3}$$

where $\prod$ is the fuzzy operator, usually performing minimizing or product operation; $A_j^i(x_j)$ is the grade of membership function.

Note $$\beta_i \triangleq \mu^i \Big/ \sum_{i=1}^{n} \mu^i \tag{4}$$

The current estimated output may be expressed generally as follows:

$$\begin{aligned} y &= \sum_{i=1}^{n} \mu^i y^i \Big/ \sum_{i=1}^{n} \mu^i \\ &= \sum_{i=1}^{n} \beta_i y^i = \sum_{i=1}^{n} \beta_i (p_0^i + p_1^i x_1 + \cdots + p_m^i x_m) \qquad i = 1, 2, \ldots, n \end{aligned} \tag{5}$$

Regarding T-S models, both the clusters (fuzzy regions) and the linear sub-models' parameters valid are requested. By premise structure identification, we mean to determine the specific input variables and partition the input space properly. The clusters can be identified using clustering algorithms such as fuzzy C-means.

The objective function of the FCM is defined by:

$$J_b(U, Z) = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^m \left\| x_k - z_i \right\|^2 \tag{6}$$

Where x_k signifies the point in data space, $k = 1, 2, \ldots, N$; N signifies the number of data points; z_i stands for the final cluster center, $i = 1, 2, \ldots, c$; c corresponds to the number of fuzzy rules; $\mu_{ik} \in [0,1]$ is the fuzzy membership degree of the kth data pair pertaining to the ith fuzzy subset. It is assumed that μ_{ik} is constrained with following equation:

$$\sum_{i=1}^{c} \mu_{ik} = 1, \; k = 1, 2, \ldots, N \tag{7}$$

The C-means algorithm for clustering in n dimensions produces C-means vectors that present c classes of data. The problem of finding the fuzzy clusters in the data set is now solved as a constrained optimization problem using FCM algorithm, considering the minimization of the function in Eq.(6) over the domain data set and taking into account the constrains in Eq.(7). The results of FCM imply the clustering centers together with the corresponding membership degrees. The main steps for identifying the T-S fuzzy model based on FCM are given as follows:

Step 1 Given c, m, and the initial clustering centers for all $k = 1, 2, \ldots, N$ and $i = 1, 2, \ldots, c$. Set an initial fuzzy c-partition matrix $U = [\mu_{ik}]$ to indicate the membership value for the ith cluster representatives.

Step 2 Calculate the following equation:

$$z_i = \frac{\sum_{k=1}^{N} z_k (\mu_{ik})^m}{\sum_{k=1}^{N} (\mu_{ik})^m} \qquad \text{i} = 1,2,\ldots,\text{c} \tag{8}$$

Step 3 Update U to adjust

$$\mu_{ik} = \Big[\sum_{j=1}^{c} \Big(\frac{x_k - z_i}{x_k - z_j}\Big)^{\frac{2}{m-1}}\Big]^{-1} \qquad \text{i=1,2,...,c} \quad \text{k=1,2,...N} \tag{9}$$

Step 4 Check for termination. If

$$\left\| U_k - U_{k-1} \right\| < \varepsilon \qquad (10)$$

stop; otherwise, let $k = k+1$ and return to step 2.

Step 5 Identify the consequent parameters using orthogonal least-squares (OLS) method [12]. Rewrite Eq. (5) in a vector form:

$$y = \phi\theta \qquad (11)$$

where

$\phi = [\beta_1, ..., \beta_n, \beta_1 x_1, ..., \beta_1 x_1, ..., \beta_n x_m, ...\beta_n x_m]$,

$\theta = [p_0^1, ..., p_m^1, p_0^2, ..., p_m^2, p_0^n, ..., p_m^n]^T$ signifies the consequent parameters.

In regard to the least squares solutions,

$$\theta = (\phi^T \phi)^{-1} \phi^T y , \qquad (12)$$

we convert $[\phi^T \phi]$ into an orthogonal matrix $[W^T W]$. By implementing iteration and conversion algorithms, the $(m+1)*n$ coupled equations become mutually independent, thereby calculating the consequent parameters θ.

3 SFCM based T-S Model Identification

3.1 Subtractive Clustering

Subtractive clustering (SC) is an effective method that searches for the number of clusters and cluster centers, which starts off with generating a number of clusters in the dimensional input space. The aim of the clustering approach is to group data by using a similarity measurement which assumes each data point is a potential clustering center and calculates a measure of the likelihood that each data point would define the clustering center based on the density of surrounding data points. Each point of the input vector $[x_1, x_2, ..., x_N]$ is considered as a potential clustering center. The density measurement at a data point x_i is calculated by:

$$D_i = \sum_{j=1}^{N} \exp\left(-\frac{\left\| x_i - x_j \right\|^2}{(r_a/2)^2}\right) \qquad (13)$$

where N is the total number of data points; $[x_1, x_2, ..., x_N]$ are data points; $r_a \in [0, \infty)$ is the neighborhood range of the cluster implying the radius of hypercube cluster in data space. Thus, the potential associated with each cluster depends on its distance to all points, leading to clusters with high potential where neighborhood is dense. The density value of *i*th data point will be larger one if it has many neighboring data points and the distance between the data points and its location is small. The first clustering center is defined as x_{c1} which has the largest density value D_{c1}. For the second and other cluster centers, the effect of the first cluster centering is updated in determination of the new density values, as follow:

$$D_i = D_i - D_{c1} \sum_{j=1}^{N} \exp\left(-\frac{\left\| x_i - x_{c1} \right\|^2}{(r_b/2)^2}\right) \qquad (14)$$

where $r_b \in [0, \infty)$ has measurable reduction in density measurement. Typically, $r_b = 1.5 r_a$. According to Eq.(13), the data points which are near the first cluster center x_{c1} will have reduced the density measurement strongly, and the probability for those points to be chosen as the next cluster is lower. This procedure selecting centers and reducing their potential is carried out iteratively until the stop criterion is satisfied. Additionally two threshold levels are defined. If the one is greater than a higher threshold, then the *i*th data is selected for a clustering center. If the one is below the lower threshold, the point is rejected.

3.2 SFCM Algorithms

Step 1 According to the specific system, the number of data points, the radius of neighborhood and the error should be given.

Step 2 Calculate the density of every data point, and the highest density of the point is chosen as x_{c1}

Step 3 According to Eq.(14), the density of all data points are updated. The data point x_{c2} which corresponding to the larger density value is chosen as the second cluster center. The selection is carried out iteratively, until the stopping criteria achieved. The results of the clustering are clustering number and cluster centers, all of which are adaptive formulated according to the effect of the cluster centers in each dimension.

Step 4 The results of aforementioned including the clustering number and cluster centers are chosen as the FCM initial values. The initial fuzzy partition matrix $U(0)$ is also set contemporaneously as follows:

$$\mu_{ik}(0)=\frac{1}{\sum_{j=1}^{c}\left(\frac{D_{ik}}{D_{jk}}\right)^{\frac{2}{m-1}}} \qquad i=1,2,...,c \quad k=1,2,...N \tag{15}$$

where D_{jk}, which is calculated firstly, signifies the distances between k th data point and j th initial cluster center.

Step 5 Calculate the center values according to Eq.(8);

Step 6 Update the fuzzy partition matrix U(k) according to Eq.(9);

Step 7 If Eq.(10) is satisfied, then stop; otherwise, $k=k+1$, return to step (5);

Step 8 The consequent parameters are identified using orthogonal least-square method (OLS), eventually resulting in the T-S models.

3.3 Experimental Discussion

We applied both FCM and SFCM to deal with 100 data points which are generated randomly within [0, 1] by in two-dimensional space. The radius of SFCM was specified as 0.5; the weighting exponent m=2; a termination criterion min_impro=0.00001. The clustering number of FCM was initiated to 4, which means 4 rules are available. SFCM automatically generates appropriate clustering numbers according to the impacts of each dimension of data on cluster centers, rather than demands the number of clusters ahead. On the contrary, inappropriate initial clustering number of FCM can lead to undesired results. Fig.1 and Fig.2 reveal that the data points are more intensive and obvious in the boundaries of SFCM.

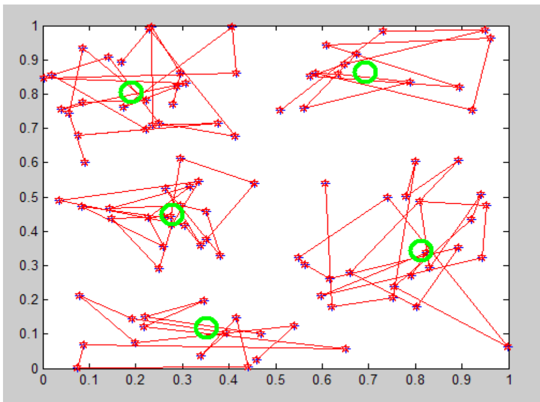

Fig.1 Clustering results of SFCM

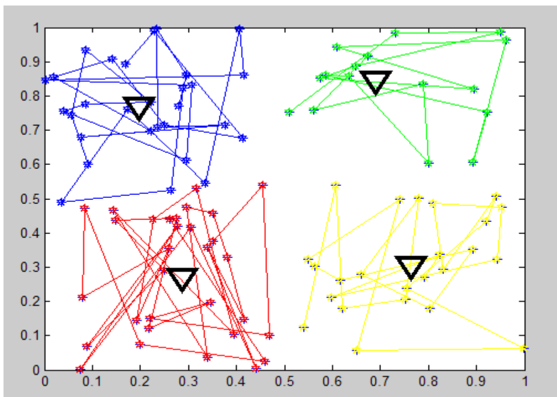

Fig.2 Clustering results of FCM

The objective function evolutions associated with the two methods are shown in Fig.3, which indicates that SFCM method not only performs less iteration, but also achieves smaller value of objective function, implying that SFCM conducts faster convergence and higher accuracy.

Assign the first and the second dimension data to the outputs and inputs of the system, respectively, and then the T-S models can be built. Fig.4 shows the 5 membership functions associated with SFCM. The consequent parameters of the T-S model are obtained as follows:

$$\theta = \begin{pmatrix} -0.041 & 0.855 & -0.566 & 0.348 \\ 0.929 & -0.004 & -0.215 & 0.309 \\ -0.067 & -0.125 & -0.078 & -0.095 \\ 0.193 & 0.085 & 0.077 & 0.726 \\ 0.386 & 0.48 & -0.036 & -0.188 \end{pmatrix},$$

which implies the following T-S modes:

R^1: $if\ y(k-1)\ is\ A_1\ and\ y(k-2)\ B_1\ and\ u(k-1)\ is\ C_1$

$then\ y(k) = -0.041 + 0.855y(k-1) - 0.566y(k-2) + 0.348u(k-1)$

R^2: $if\ y(k-1)\ is\ A_2\ and\ y(k-2)\ B_2\ and\ u(k-1)\ is\ C_2$

$then\ y(k) = 0.929 - 0.004y(k-1) - 0.215y(k-2) + 0.309u(k-1)$

R^3: $if\ y(k-1)\ is\ A_3\ and\ y(k-2)\ B_3\ and\ u(k-1)\ is\ C_3$

$then\ y(k) = -0.067 - 0.125y(k-1) - 0.078y(k-2) - 0.095u(k-1)$

R^4: $if\ y(k-1)\ is\ A_4\ and\ y(k-2)\ B_4\ and\ u(k-1)\ is\ C_4$

$then\ y(k) = 0.193 + 0.085y(k-1) + 0.077y(k-2) + 0.726u(k-1)$

R^5: $if\ y(k-1)\ is\ A_5\ and\ y(k-2)\ B_5\ and\ u(k-1)\ is\ C_5$

$then\ y(k) = 0.386 + 0.48y(k-1) - 0.036y(k-2) - 0.188u(k-1)$

Here, Root Mean Square Error (RMSE) is employed to evaluate the accuracy of the model identification. It reveals that RMSE of SFCM is 0.0752 and RMSE of FCM goes to 0.1033., which demonstrates the benefits of SFCM obviously.

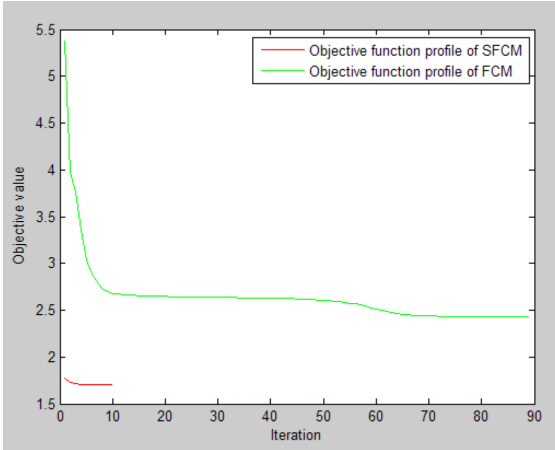

Fig.3 Objective function profiles of the two methods

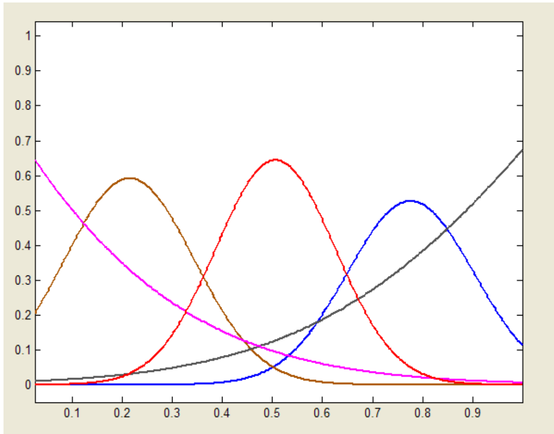

Fig.4 Membership functions of SFCM

4 Applications to Fuzzy Predictive Control

4.1 Control Algorithms

Linear model based predictive control (MPC) has increasing popularity in applications. However, most industrial processes are nonlinear, time-variant and uncertain. It is an effective way to solve these problems by combining fuzzy models with the MPC strategies [13,14]. It is acknowledged that the T-S fuzzy models can approximate nonlinear systems with arbitrary precision. Taking advantage of local dynamic linearization of nonlinear systems process, the generalized predictive control (GPC) algorithms can be implemented.

The design of fuzzy predictive control is characterized by fuzzy modeling and predictive control, where the off-line identified T-S models serve as the predictive models of GPC. Fig.5 shows the basic scheme of the control system.

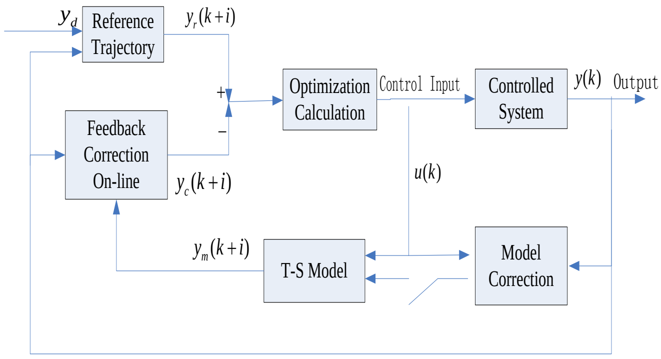

Fig.5 Schematic diagram of fuzzy GPC systems

T-S fuzzy predictive model is described by:

$$y(k+1)=\sum_{l=1}^{n_a}\bar{a}_l(k)y(k-l+1)+\sum_{l=1}^{n_b}\bar{b}_l(k)u(k-l+1)+e \quad (16)$$

Where $\bar{a}_l(k)=\sum_{i=1}^{n}\beta_i(k)a_{i,l}$ and $\bar{b}_l(k)=\sum_{i=1}^{n}\beta_i(k)b_{i,l}$,

a_{il} is first n_a columns about the ith row in the whole consequent parameters matrix θ_i; whereas, b_{il} is the other columns about the same row; e means the error. Because of the consequent parameters depend on the time-varying coefficient β_i, the T-S model a state-space linear time-varying (LTV) model. The sub-model consequent parameters θ_i according to the ith rule are determined, so the T-S model is divided into some linear time-invariant (LTI) models.

The multi-steps predictive outputs of global T-S model are defined by:

$$y_m(k+j)=Px(k)+Su(k)+\mathrm{Re} \quad (17)$$

where

$$y_m(k+j)=[y_m(k+1),y_m(k+2),...,y_m(k+N_y)]^T$$

with N_y indicating predictive steps;

$$x(k)=[y(k),...,y(k-n_a+1),u(k-1),...,u(k-n_b+1)]^T;$$

$$u(k)=[u(k),...,u(k+N_u-1)]^T;$$

$$R=[r_1,...,r_j]^T;$$

$$P=\begin{pmatrix} p_{11} & \cdots & p_{1(n_a+n_b-1)} \\ \vdots & & \vdots \\ p_{N_y1} & \cdots & p_{N_y(n_a+n_b-1)} \end{pmatrix}_{j*n_a+n_b+1};$$

$$S=\begin{pmatrix} s_{11} & 0 & 0 & \cdots & 0 \\ s_{21} & s_{22} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ s_{N_u1} & s_{N_u2} & s_{N_u3} & \cdots & s_{N_uN_u} \\ \vdots & \vdots & \vdots & & \vdots \\ s_{N_y1} & s_{N_y2} & s_{N_y3} & \cdots & s_{N_yN_u} \end{pmatrix}_{N_y*N_u};$$

$y_m(k+j)$ denotes the model predictions based on the linearized model over the prediction horizon and $u(k)$ is a vector of future controller outputs.

T-S models are used to predict the future behavior of the process output signal over a certain finite horizon and to evaluate control actions to minimize a certain cost function. The considered predictive control law is in general obtained by minimizing of the following criterion:

$$J_P=E[\sum_{j=N_1}^{p}q_j[y_c(k+j)-y_r(k+j)]^2+\sum_{j=1}^{N_u}\lambda_j[\Delta u(k+j-1)]^2] \quad (18)$$

where $y_c(k+j)=y_m(k+j)+e(k+h)$, $y_c(k+j)$, $y_m(k+j)$ and $e(k+h)$ stand for the on-line correction of j-step-ahead prediction of the process output signal, process output signal and the model predictive error, respectively; $q_j>0$, $\lambda_j>0$ determine the relative importance of the different terms in the cost function. N_1, P, N_u are the minimum, maximum prediction and control horizon. Note that in addition to the standard terms, this cost function also includes the variation in the predicted output and the deviation of the control input from an input reference.

$y_r(k+j)$ is the reference output trajectory,

Note

$$\begin{cases} y_r(k+j)=\alpha_r^i y_r(k+j-1)+(1-\alpha_r^i)\omega \quad (j=1,2,...) \\ y_r(k)=y(k) \end{cases} \tag{19}$$

where

$y_r(k+j)=[y_{r,1}(k+h), y_{r,2}(k+h),. \quad y_{r,n}(k+.h)]^T$;

$y_r(k+j-1)=[y_{r,1}(k), y_{r,2}(k),..., y_{r,3}(k)]^T$;

It ensures that the reference output tracks a constant reference signal ω. $y_{r,i}(k+h)$ denotes the ith value of output reference trajectory at the k+h times. $y_{r,i}(k)$ means the output measurement value. α_r determines the velocity of convergence in the reference trajectory and $\alpha_r = e^{-\frac{T_0}{\tau}}$, 0< α <1. T_0 and τ stands for sample time and respond time of the output trajectory respectively.

Differentiate Eq. (18) and let $\frac{\partial J_P}{\partial u(k)}=0$, leading to the following optimal solutions:

$$\Delta u(k)=(S^T QS+\lambda)^{-1} S^T Q[Y_r(k+1)-Px(k)-\mathrm{Re}] \tag{20}$$

4.2 Experimental Discussions

The following example is taken from Narendra and Parthasarathy [15] in which the plant to be identified is given by the second-order highly nonlinear difference equation

$$y(k)=\frac{y(k-1)y(k-2)[y(k-1)-2.5]}{1+y^2(k-1)+y^2(k-2)}+u(k-1) \tag{21}$$

(1) Case 1:

To estimate the T-S models, a sinusoidal input signal, $u(k-1)=5\sin(\pi*(k-1)/100)$, is designated in the presence of white noise disturbances over (0,1). Triangle membership functions together with 4 clustering centers are initially selected for FCM, while SFCM does not demand prior knowledge of the membership functions and rule number. The parameters associated with GPC were specified as that the predictive step was 8; the control step was 3; the weight of the output error was 1; the weight of the control increment was 0.1; gentle factor was 0.3 and the reference trajectory was square.

Fig.6 shows the comparison between the outputs of T-S models identified by SCFM and FCM, along with the system; Fig.7 and Fig.8 indicate the model predictive outputs; the corresponding controller's outputs were given in Fig.9 and Fig.10; Fig.11 and 12 present the updated membership functions of the predictive models. Obviously, better performances are achieved by SFCM.

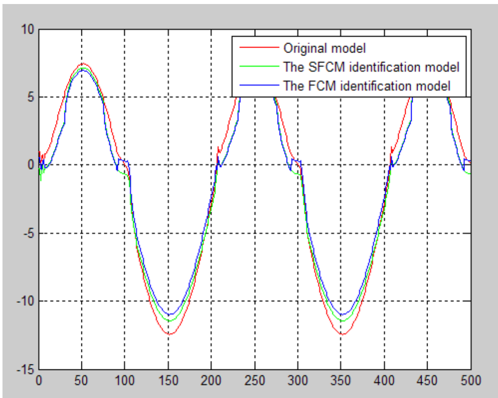

Fig.6 Modeling performances of the two identification methods

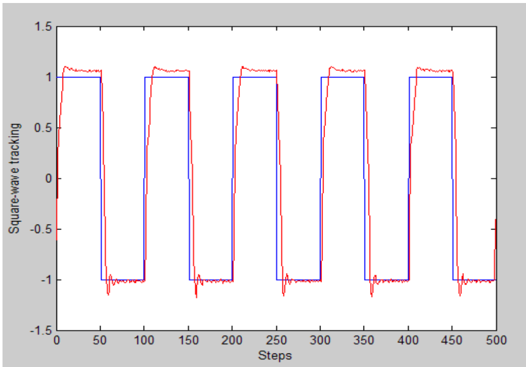

Fig.7 Output profile of the FCM based control systems

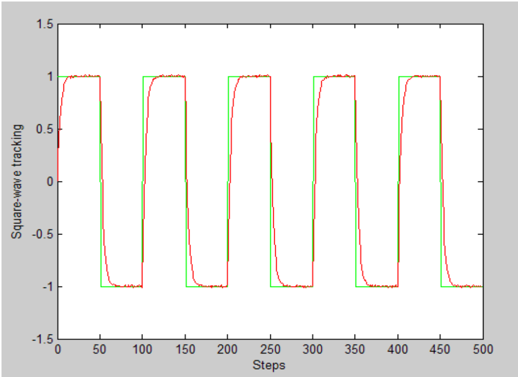

Fig.8 Output profile of the SFCM based control systems

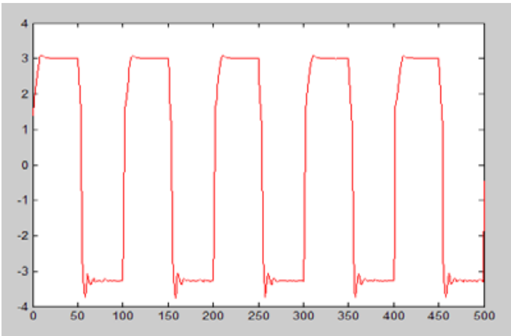

Fig.9 Controller's output of the FCM based control systems

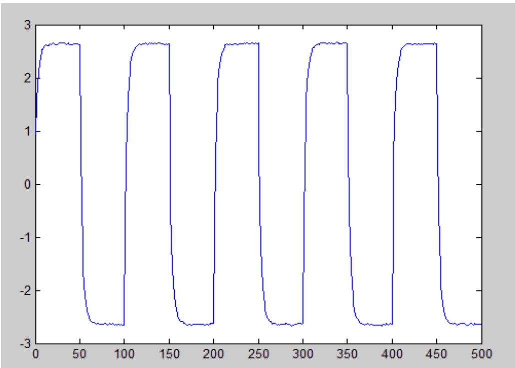

Fig.10 Controller's output of the SFCM based control systems

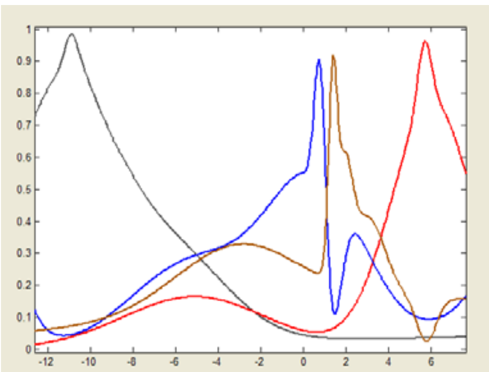

Fig.11 Membership functions of the FCM based predictive models

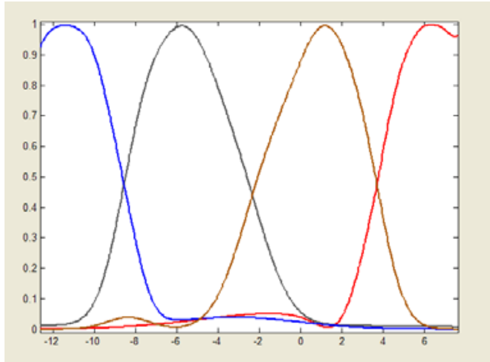

Fig.12 Membership functions of the SFCM based predictive models

(2) Case 2

A random signal $u(k)$ uniformly distributed over [-1, 1] together with 500 data points were selected as the input to identify the T-S models. In regard to FCM, the initial the clustering center vector was selected as [-4 -3 -2 -1 0 1], together with 6 rules. Eventually, the identified cluster center vector was updated to [-3.52,-2.643,-1.848,-0.871,-0.029,0.862] together with the corresponding consequent parameters

$$\theta = \begin{pmatrix} 10.303 & 0.254 & 0.088 & -0.144 \\ 0.01 & 0.058 & 1.672 & 0.584 \\ 0.708 & 0.777 & 0.032 & -0.963 \\ 1.635 & 0.369 & 0.307 & 0.154 \\ 0.172 & 0.118 & -1.251 & 1.011 \\ 0.986 & 0.924 & 1.036 & 0.986 \end{pmatrix}$$

In comparison, the input was partitioned into 3 parts by the SFCM and the cluster centers were located at [-0.016 -0.724 0.669]. The output consists of 3 groups whose cluster centers were [-0.938 -2.431 0.448]. As the rule number of the fuzzy models was 3*3=9, the consequent parameters were given by

$$\theta = \begin{pmatrix} -1.531 & 0.407 & 0.339 & -0.608 \\ 0.721 & 0.533 & -0.142 & 0.182 \\ -0.053 & -2.295 & 1.277 & 0.992 \\ 0.558 & -1.056 & -0.65 & 0.727 \\ 0.759 & 1.398 & 2.064 & 0.596 \\ 0.261 & 0.091 & -0.967 & -0.902 \\ 0.268 & 0.177 & -0.086 & 1.305 \\ 0.914 & 1.009 & 1.25 & 1.109 \\ 0.767 & 0.529 & 0.922 & 0.899 \end{pmatrix}$$

Fig.13 and Fig.14 imply that SFCM adaptively generated the membership functions based on input-output data. With a sinusoidal reference trajectory available, the outputs of corresponding predictive control systems were shown in Fig.15 and 16. In this regard, a conclusion similar to that of the former case is obtained.

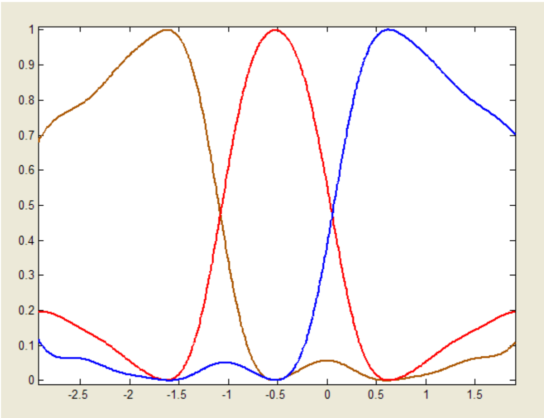

Fig.13 Membership functions of the predictive model outputs

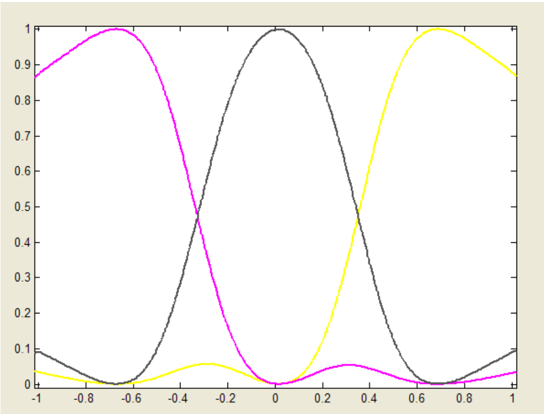

Fig.14 Membership functions of the predictive model inputs

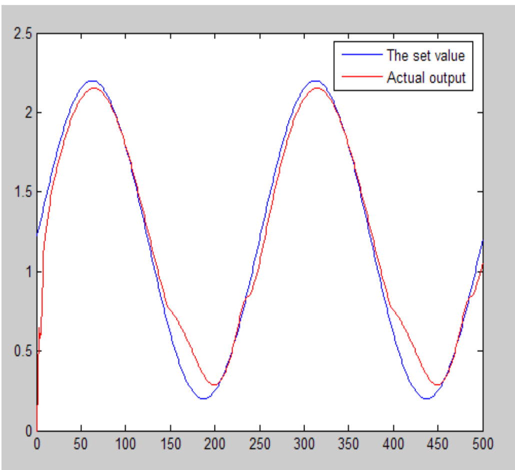

Fig.15 Output profile of the FCM based control systems

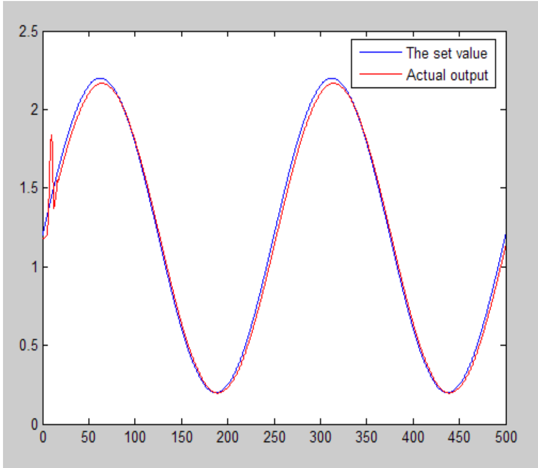

Fig.16 Output profile of the SFCM based control systems

5 Conclusions

A so-called "subtractive fuzzy C-means clustering" method was proposed, where the results of subtractive clustering are applied to initialize clustering centers and the number of rules in order to perform adaptive clustering. Particularly, a detailed coverage of how to set the initial membership functions is provided to meet the needs of T-S model identification. This method can avoid the problems of blindly and randomly assigning the membership functions of the fuzzy inference systems. In order to reduce the calculation, Orthogonal Least Squares method is employed to identify the consequent parameters of T-S models. Additionally, well-established T-S models are applied to fuzzy model predictive control systems, thereby developing compatible fuzzy predictive control algorithms. As shown with experimental results, the approaches are able to achieve more satisfied control performances. Currently, close attention is being paid to more practical problems such as fuzzy predictive control with constraints.

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