Modeling and Simulation of a Novel Active Vibration Control System for Flexible Structures

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Abstract: - This paper investigates the feasibility of using active force control (AFC) method for the problem of active vibration control (AVC) of a flexible thin plate structure through simulation. The plate system was first modeled and simulated using Finite Difference (FD) method. Then, the validity of the obtained computational model was investigated through comparative studies between the plate natural frequencies predicted by the model and the exact values of resonance modes found in literature. The AFC algorithm was then embedded within the FD simulation platform to study the performance of the control scheme. An appropriate value of the mass gain in the AFC algorithm was estimated to be 14 kg through crude approximation technique which was later shown to be close to the real mass of the plate system. Different types of disturbances were applied to excite the plate system at different excitation points so that the robustness of the proposed control algorithm can be evaluated in terms of its performance in reducing the vibration at observation point. Results of the study clearly demonstrated the effectiveness of the proposed control strategy to attenuate the unwanted vibrations of the flexible thin plate system.

Key-Words: - Active force control; finite difference; flexible plate; vibrations

1 Introduction

The interest to employ light weight, stronger and more flexible structures in various fields such as aerospace and marine industries has been considerably increased during the last two decades. However, light weight flexible structures can be more easily influenced by unwanted vibrations, which may eventually lead to problems including fatigue, instability and performance reduction. These problems may eventually cause damage to the highly stressed structures. Vibration problems may also cause acoustic disturbance due to the coupling of structural vibration and acoustic fields. Hence, controlling the unwanted vibrations of flexible structures is of practical importance. Amongst the vast spectrum of flexible structures, thin plates are known as the most commonly used flexible elements in mechanical structures and machines. In designing a structure, plates are usually specified only to withstand applied static loads. However, this is inadequate for more flexible plates. Dynamic forces and random cyclic loads also threaten the stability of such systems. There exist a large number of discrete frequencies at which a flexible plate will undergo large amplitude vibration by sustained time varying forces of matching frequencies. Thus, the possibility of large displacement and stresses due to this recent type of excitation must be taken into account.

In the past few decades, different control strategies have been devised by researchers to attenuate the structural vibrations. Among these control strategies, traditional passive control is used widely in industries and commercial products. The passive control method consists of mounting passive material on the structure in order to change its dynamic characteristics such as stiffness and damping coefficient. This method is efficient at high frequencies but expensive and bulky at low frequencies [1]. Passive vibration control usually leads to an increase in the overall weight of structure, which makes it less transportable especially for space applications.

On the other hand, Active Vibration Control (AVC) techniques use external active devices to generate a second set of vibratory signal of equal amplitude but opposite phase to cancel the targeted unwanted vibrations at an observation point. Lueg [2] is among the first who used AVC in order to cancel the vibrations. Since then, a large number of researchers have concentrated on developing methodologies for the design and implementation of AVC systems. Various control methodologies have been developed in literature for AVC problem of the flexible structures.

Jenifene [3] proposed a simple position control system for the purpose of AVC of a single-link flexible manipulator. A delayed position feedback signal was used to actively control the vibrations of the flexible structure. The stability of the controller was investigated. This method was found to be acceptable in lightly damped dynamic systems. Xie *et al.* [4] applied H_∞ robust vibration control to a thin plate covered with a controllable constrained damping layer. The numerical simulations and experimental results indicated that the control scheme suppressed effectively low-frequency modal vibrations of a thin plate covered with a controllable constrained damping layer. Kar and his co-workers [5] presented a method for controlling the bending and torsional vibration modes of a flexible plate structure through the use of H_∞-based robust control. A three degree-of-freedom (DOF) reduced order lumped mass model of a plate structure was derived by considering first three vibration modes and neglecting the all other high-frequency modes. They showed the efficiency of the proposed feedback controller through simulation and experimental studies. Xianmin *et al.* [6] employed reduced modal controller, the classical and the robust H_∞ controller for the high-speed flexible linkage mechanism systems. Their simulation results showed that the vibration of the flexible system was significantly suppressed with permitted actuator input voltages by each of the three controllers. Madkour and his co-workers [7] investigated the performance of different intelligent AVC strategies for suppressing the unwanted vibration in a flexible beam structure. Finite Difference (FD) method was used to model and simulate the flexible beam system. They then employed different learning algorithms including genetic algorithm (GA), artificial neural network (ANN) and adaptive neuro-fuzzy inference system to develop the mechanisms of an AVC system. Comparative studies were carried out and the effectiveness of each technique was investigated. Tokhi and Hossain [8] proposed an active control mechanism for vibration suppression of a flexible beam within an adaptive control framework. A control mechanism was designed within a feedforward control structure on the basis of optimum cancellation at an observation point. The design relations were formulated such that it allows on-line design and implementation and thus results in a self-tuning control algorithm. They employed a FD method to simulate the flexible system. The simulation results verified the performance of the proposed algorithm. Later, Mat Darus and Tokhi [9] applied a similar adaptive control strategy to a square flexible thin plate structure. They modeled the flexible plate system using a FD approach. Then, a feedforward self-tuning adaptive controller was applied to compensate the unwanted vibration of the plate system. The efficiency of the control technique was demonstrated through a simulation studies using the FD simulation platform.

The research about Active Force Control (AFC) was initiated by Johnson [10] and later Davison [11] based on the principle of invariance and the classic Newton's second law of motion. It was shown that it is possible to design a feedback controller that ensures the system set-point remains unchanged in the presence of the disturbances. Hewit and Burdess [12] presented a more complete package of the AFC system such that the nature of disturbances was oblivious to the dynamic system. Thus, an efficient disturbance rejection technique was established to facilitate the robust motion control of dynamical systems in the presence of disturbances, parametric uncertainties and changes that are commonly prevalent in the real-world environment. The first attempts to employ the AFC strategy for the purpose of vibration control was reported by Hewit *et al.* [13]. They applied the AFC incorporated with PD control to the problem of accurate motion control of a flexible manipulator. They used AFC for deformation and vibratory disturbance attenuation and PD control for trajectory tracking. Hewit and his co-workers extended the powerful method of AFC which had proved so effective in controlling the rigid arms in the face of disturbances to flexible arms. Mailah *et al.* [14, 15, 16] investigated the usefulness of the AFC method by introducing intelligent mechanisms to approximate the mass or inertia matrix of the dynamic system to trigger the compensation effect of the controller. They applied the AFC technique to different dynamical systems such as robot arms and mobile manipulators for the purpose of effective motion control. It was recognized that the AFC was robust and effective both in theory as well as in practice. Priyandoko *et al.* [17] utilized the AFC methodology for a vehicle active suspension system. Their theoretical and experimental results revealed the effectiveness of AFC in reduction the amplitude of the unwanted vibrations affecting the suspension system.

Based on the previously outlined literature, there is no published report in which the AFC technique is employed for the purpose of AVC of a flexible plate structure. Thus, this study has been devoted to the AVC of a rectangular flexible thin plate with all clamped edges through the use of AFC methodology. First, a rectangular flexible plate system is modeled using the FD approach. Then, the validity of the obtained model is demonstrated. The AFC scheme is then applied to the flexible plate under vibratory disturbances to cancel the unwanted vibration of an observation point. Finally, the performance of the proposed control strategy to compensate different disturbing signals will be evaluated within the FD simulation platform.

2 Computational model of the flexible plate system

It is important initially to recognize the flexible nature of the plate and construct a mathematical model for the system. In order to control the vibration of a plate efficiently, it is required to obtain an accurate model of the plate structure. An accurate model will result in satisfactory and good control. Such a model can be constructed using a partial differential equations (PDE) formulation of the dynamics of the flexible plate.

The vibration problem of a flexible thin plate can be formulated as a PDE together with the corresponding boundary conditions. The plate is assumed to undergo a small lateral deflection. Using Kirchhoff's plate theory, this yields the following equation [18]:

$$\frac{\partial^4 w(x,y,t)}{\partial x^4} + 2\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y,t)}{\partial y^4} + \\ + \frac{\rho h}{D}\frac{\partial^2 w(x,y,t)}{\partial t^2} = \frac{q(x,y,t)}{D} \tag{1}$$

where w is the lateral deflection in z direction, ρ is the density of plate with dimension mass per unit volume, h is the thickness, q is the transverse external force with dimensions of force per unit area, $D=(Eh^3)/(12(1-\upsilon^2))$ is the flexural rigidity, E is the modulus of elasticity and υ is the Poisson ratio. For a clamped edge (at $x=a$), the boundary condition is as follows:

$$w\Big|_{x=a} = \frac{\partial w}{\partial x} = 0 \tag{2}$$

For simulation purpose, it is natural to assume that initially the plate has no deflection. In other words, the forces and moments of the plate due to its weight are neglected. Thus, for every point located on the plate:

$$w\Big|_{t=0} = 0 \tag{3}$$

A commonly used approach for solving the PDE, representing the dynamics of the plate, is to utilize a representation of the PDE, obtained through a simplification process, by a finite set of ordinary differential equations. Various approaches have previously been developed for modelling of flexible plate systems. Finite element method (FEM) has been previously utilized to simulate the flexible plates by many researchers [19]. An advantage of the FEM is that it allows irregularities in the structure, to be accounted for. However, the computational complexity and consequent software coding involved in the FEM constitute major disadvantages of these technique, especially in real-time applications. In applications involving uniform structures, such as the plate system considered here, the finite difference (FD) method is found to be more appropriate, and the relatively reduced amount of computation involved in the FD method makes the technique more suitable in real-time applications.

Thus, to simulate the plate system, FD method was employed to numerically solve Eq. (1). In FD method, the entire solution domain was divided into a grid of cells. Then, the derivatives in the governing partial deferential equations were written in terms of difference equations. FD was applied to each interior point so that the displacement of each node is related to the values at the other nodes in the grid connected to it. Considering the boundary conditions of the problem, a unique solution can be obtained for the overall system. In the case of a flexible plate, a three dimensional coordinate system was considered.

The x-axis is represented with the reference index i, the y-axis with reference index j and the time axis with index k, where $x=i\Delta x$, $y=j\Delta y$ and $t=k\Delta t$. For each nodal point in the interior of the grid (x_i, y_j, t_k) where i = 0,1, …n; j = 0, 1,…m and k = 0, 1…p, a Taylor series expansion was used to generate the central FD formulae for the partial derivative terms of the deflection, $w(x, y, t) = w_{i,j,k}$ of the plate at point $x = i\,\Delta x$, $y = j\,\Delta y$ and $t = k\,\Delta t$.

Thus, using the central difference approximations, Eq. (1) can be written as the following difference equation [20, 21]:

$$w_{i,j,k+1} = -\frac{D\Delta t^2}{\rho h}(Pw_{i,j,k} + Q(w_{i+1,j,k} + w_{i-1,j,k}) + \\ + (w_{i,j+1,k} + w_{i,j-1,k}) + S(w_{i+1,j+1,k} + w_{i-1,j+1,k} + \\ + w_{i-1,j-1,k} + w_{i+1,j-1,k}) + T(w_{i+2,j,k} + w_{i-2,j,k}) + \\ + U(w_{i,j+2,k} + w_{i,j-2,k})) + 2w_{i,j,k} - w_{i,j,k-1} + \frac{\Delta t^2 q_{i,j}}{\rho h} \quad (4)$$

where

$$P = \frac{6}{\Delta x^4} + \frac{8}{\Delta x^2 \Delta y^2} + \frac{6}{\Delta y^4},\ Q = -\frac{4}{\Delta x^4} - \frac{4}{\Delta x^2 \Delta y^2},$$
$$R = -\frac{4}{\Delta y^4} - \frac{4}{\Delta x^2 \Delta y^2},\ S = \frac{2}{\Delta x^2 \Delta y^2},\ T = \frac{1}{\Delta x^4},$$
$$U = \frac{1}{\Delta y^4}.$$

The central difference equation for boundary and initial conditions defined by Eqs. (2) and (3) for a clamped edge (at $x=a$) can be written as:

$$w\big|_{x=a} = \frac{\partial w}{\partial x}\Big|_{x=a} = \frac{1}{2\Delta x}\left(w_{i+1,j,k} - w_{i-1,j,k}\right) = 0 \quad (5)$$

According to Eq. (3), the displacement of entire nodes on the plate at $t=0$ is assumed to be zero:

$$w_{i,j,k}\big|_{t=0} = 0 \quad (6)$$

The stability of the algorithm can be examined by ensuring that the iterative scheme described in Eq. (4) converges to a solution. According to the stability rules for convergence of a finite difference equation, Eq. (7) can be derived to satisfy the necessary condition for the convergence of Eq. (8) as follows [9, 20, 21]:

$$0 \le c \le \frac{1}{4} \quad (7)$$

where

$$c = \left(\frac{D\Delta t^2}{\rho \Delta x^2 \Delta y^2}\right)\left(2 + \frac{\Delta y^2}{\Delta x^2} + \frac{\Delta x^2}{\Delta y^2}\right)$$

To study the dynamic behaviour of the rectangular thin plate system, a computer program was written within the MATLAB environment to numerically solve Eq. (1) based on the FD solution presented in Eq. (4). Fig. 1 shows the flowchart of the FD simulation program. The mentioned computer program has been arranged so as to reduce the processing time for real time applications.

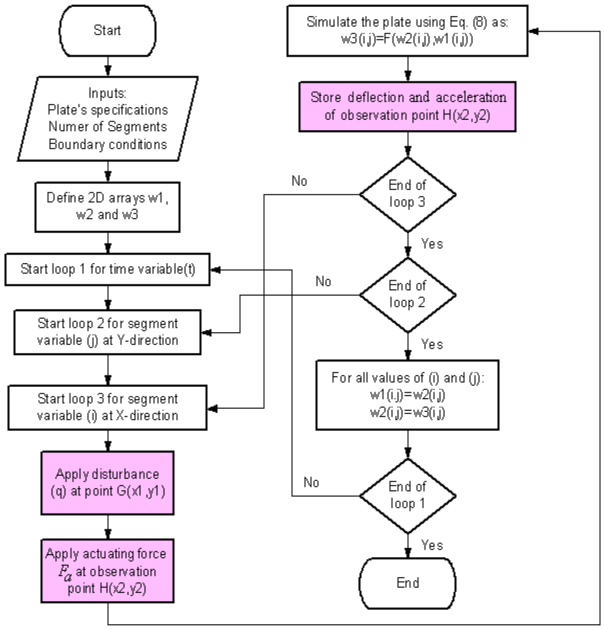


Fig. 1. Schematic flowchart of the FD program

The proposed FD program allows the user to place a disturbing force (q) and the actuating force (if any) at any arbitrary point on the plate. The dynamic response of the plate system at any observation point can also be saved in an array for later analysis.

For implementation of the simulation algorithm, a 3.2 mm thick aluminium plate with the parameters given in Table 1 and with all clamped edges was considered in this study (Fig. 2). Using the FD method, the plate was divided into 90 and 60 sections in the x and the y directions, respectively (to produce square difference element), with spacing between successive nodal points being 0.016 m. The value of c, in Eq. (7) was chosen as 0.11, which is less than half of its maximum allowable value. This value of c results in the required sampling time of $\Delta t = 9.26\times10^{-6}$ s, which satisfies the stability requirement and is sufficient to cover a broad range of dynamics of the flexible plate.

TABLE I
Plate specifications

Parameter	Value
Length (a)	1.5(m)
Width (b)	1.0(m)
Thickness (h)	0.0032(m)
Density (ρ)	2700(Kgm^{-3})
Modulus of elasticity (E)	7.11×10^{10}(Nm^{-2})
Poisson ratio (υ)	0.3

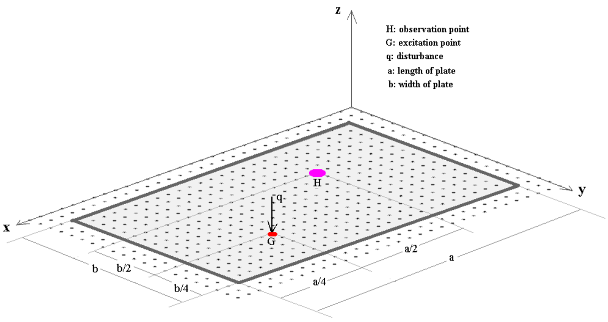


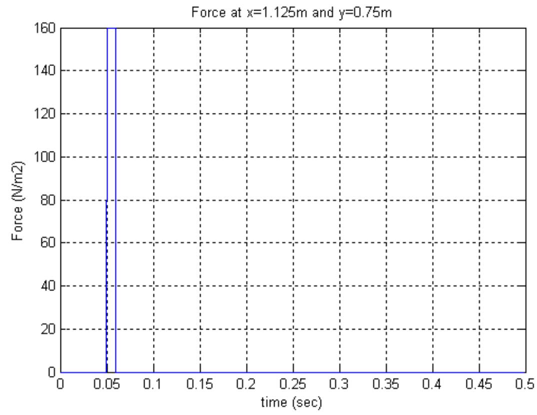
Fig. 2. Schematic diagram of the rectangular thin plate

A finite duration step input force (q) with amplitude of 160 Nm^{-2} (Fig. 3a) was applied to point G (Fig. 2) from t = 0.05 s to t =0.06 s and the dynamic response of the plate system was investigated during 0.5 s. Fig. 3b represents the plate response at observation point H.

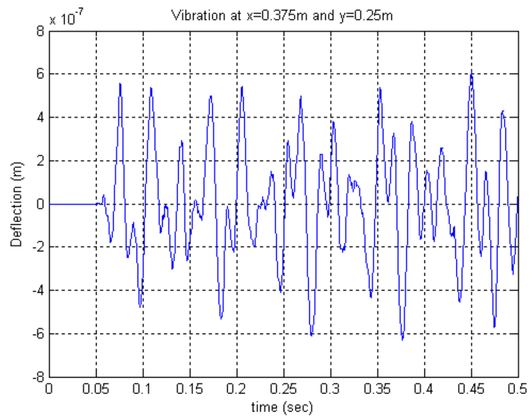
To investigate the validity of the proposed FD model, the spectral density of the deflection of the plate (based on system eigen-values) at point H was calculated by Fast Fourier Transform (FFT) analysis using MATLAB. Fig. 4 shows the corresponding frequency-domain characteristic of the plate's vibration subjected to a finite duration step input using the FD model.

The first five resonance modes of the plate system were studied. As can be observed, the first five resonance modes of vibration have occurred at 21.39 Hz, 32.92 Hz, 52.67 Hz, 62.55 Hz and 81.48 Hz respectively. To present the validity of the model, the predicted natural frequencies were compared to the exact values of resonance modes found in literature [22].

According to Table 2, it can be seen that the proposed model has effectively predicted the first five resonance modes of the flexible plate system. Furthermore, the first resonance mode predicted by the FD model is exactly the same as the real value of the first mode. Thus, the validity of the proposed model can be confirmed.



(a)



(b)

Fig. 3. Simulation results of the FD model (a) Input force applied to point G (b) Dynamic response of the plate at point H

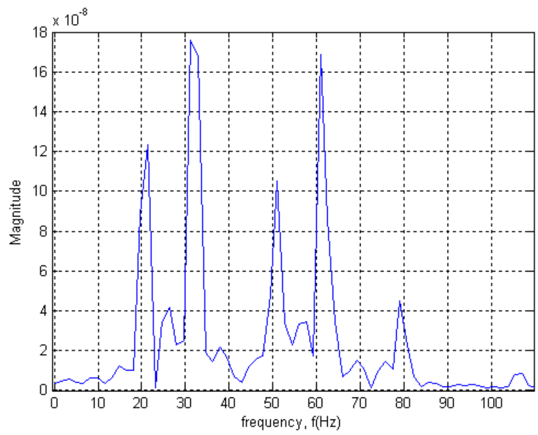


Fig. 4. Frequency-domain response of the plate at point H

TABLE II
Comparative modes of vibrations between the FD model and experimental data

Mode	Exact value (Blevins, 1979) (Hz)	Predicted value by FD model (Hz)	Error (%)
1	21.39	21.39	0.00
2	32.93	32.92	0.03
3	52.70	52.67	0.05
4	63.25	62.55	1.10
5	79.80	81.48	2.10

3 Active force control method

AFC is a well known strategy for rejecting the disturbances affecting the dynamical systems. The effectiveness of AFC as an excellent disturbance rejection scheme has been addressed in [12, 23]. In AFC, it has been shown that the system subjecting to a number of disturbances remains stable and robust [12] via the compensating action of the control strategy. The detailed mathematical analysis of the AFC scheme can also be found in [23]. Fig. 5 shows a schematic block diagram of the AFC scheme considered in many references [12, 14, 17].

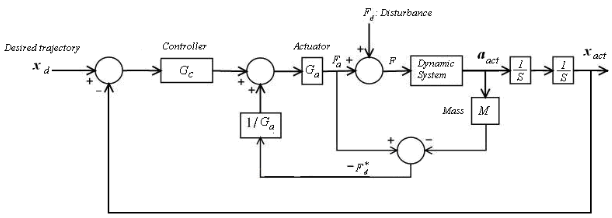


Fig. 5. Schematic loop of conventional AFC scheme

As can be seen in Fig. 5, two distinct feedback loops can be distinguished which form a two degree-of-freedom control system. The outer loop is a simple PID controller through which the reference trajectory tracking will be assured. Whereas, to compensate the effect of disturbing forces affecting the dynamical system, the inner loop namely AFC is added to reject the disturbances. The main computational burden in the AFC loop is about the computation of the estimated mass of the dynamical systems.

4 Active vibration control scheme

4.1 Theory of the control scheme

The control problems are usually categorized into regulation and servo problems. In servo problems, the fundamental desired behavior is to make the system output follow a reference trajectory closely (e.g. controlling a robot). On the other hand, in regulation problems, the fundamental desired behavior is to keep the output of the system at a constant level regardless of the disturbances acting on the system. The AFC technique has been previously employed in the servo control problems [12, 14, 15]. However, for AVC, the desired behavior is to eliminate the unwanted vibration. In other words, the AVC attempts to reduce the amplitude of unwanted vibrations to zero. Thus, AVC would be a regulation problem. Since in AVC, the main objective is to remove the effect of vibratory disturbances and there is no reference trajectory, the inner loop shown in Fig. 5 was found to be sufficient to meet the demand of AVC. Fig. 6 represents the schematic of the AFC scheme proposed in this investigation for active vibration control applications.

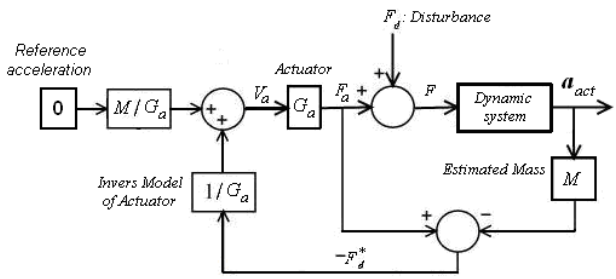


Fig. 6. Schematic loop of the proposed AVC scheme

In this figure, V_a, G_a, F_d, F_a, F, a_{act}, M and F^*_d are Input voltage to the actuator, transfer function of the actuator, disturbing force, actuating force, input force to dynamic system, actual acceleration, mass of dynamic system and estimated disturbance respectively. Using the control procedure of Fig. 6 the unknown disturbance F_d at instant t can be estimated using Newton's second law of motion and based on the knowledge about the actual mass value (M_{act}) of the dynamical system as:

$$F_d + F_a = M_{act} a_{act} \quad (8)$$

Thus, the unknown disturbance F_d can be calculated as:

$$F_d = M_{act} a_{act} - F_a \quad (9)$$

From Eq. (9), it is obvious that if one can find the estimated mass (M) of the dynamical system appropriately and the acceleration and force signals are measured accurately, it is possible to calculate the real value of the unknown disturbance F_d irrespective of its characteristics. The acceleration signal can be measured with a good accuracy using an accelerometer. The force signal can also be measured directly through a force sensor or can be found indirectly by multiplying the controlled voltage by the actuator transfer function. However, in the complex systems such as the flexible structures considered in this thesis, finding the effective mass of the flexible system would be a rigorous task which may lead one into employing an estimated mass value M. Consequently, the estimated disturbance (F^*_d) can be found by substituting the estimated mass value (M) into Eq. (9) as:

$$F_d^* = Ma_{act} - F_a \quad (10)$$

According to Fig. 6 the input force to the dynamical system (F) at instant t can be calculated as follows:

$$F = F_a + F_d \approx F_a + F_d^* \quad (11)$$

Thus, if one wants to have the force cancellation ($F = 0$), the following criterion must be fulfilled:

$$F_a = -F_d \approx -F_d^* = F_a - Ma_{act} \quad (12)$$

And it is the case which has been considered in the AFC loop for AVC represented in Fig. 6. For a very small value of sampling time dt, the actuator voltage $V_a(t)$ can be approximately calculated as (see Fig. 6):

$$V_a(t) = (F_a(t-dt) - Ma_{act}(t-dt))(1/G_a) \quad (13)$$

The actuator force $F_a(t\text{-}dt)$ may be estimated indirectly using the applied voltage to the actuator and the actuator transfer function as:

$$F_a(t-dt) = G_a V_a(t-dt) \quad (14)$$

Substituting Eq. (14) into Eq. (13), the actuator voltage V_a at time instant t can be computed:

$$\begin{aligned} V_a(t) &= (G_a V_a(t-dt) - Ma_{act}(t-dt))(1/G_a) = \\ &= V_a(t-dt) - \frac{M}{G_a} a_{act}(t-dt) \end{aligned} \quad (15)$$

Therefore, the actuator force F_a at time instant t can be expressed as:

$$\begin{aligned} F_a(t) &= V_a(t)G_a = G_a V_a(t-dt) - Ma_{act}(t-dt) = \\ &= F_a(t-dt) - Ma_{act}(t-dt) \end{aligned} \quad (16)$$

Throughout the simulation study which is presented in the next subsection, Eq. (16) is employed to calculate the actuator force.

4.2 Simulation of the control scheme

The validity of the proposed FD model was shown previously in section 2. Thus, to simulate the control algorithm, the computational model presented earlier in section 2 was employed. Based on the FD flowchart depicted in Fig. 1, it is possible to easily apply a disturbing force q at any arbitrary point (excitation point) on the plate. Furthermore, the actuating force F_a generated by an appropriate actuator (linear actuator) can be applied to observation point and the lateral deflection as well as the acceleration of observation point can be picked up and stored.

For the simulation purpose, a linear actuator and a linear sensor (accelerometer) with unity transfer functions were assumed. Using the FD simulation flowchart (Fig. 1), the computer program of section 2 was developed so as to embed the control algorithm depicted in Fig. 6 within the proposed FD flowchart. First, a disturbing force q was applied to point G_1 (through which the un-known disturbing force F_d was created at observation point). Then, the necessary actuating force F_a was computed using Eq. (16) and substituted into the FD flowchart (see Fig. 1). Perhaps, approximating an appropriate mass value (the crude approximation technique is used here) for the control algorithm is the most important design consideration. Therefore, different values of M were tested within the simulation algorithm to find the best performance of the control system. A sine disturbing force q (Fig. 8a) with the frequency equivalent to the first natural frequency of the plate (21 Hz) was used to excite the plate at point G_1 (see Fig. 7) and the performance of the controller in attenuating the first resonance mode of the plate for different values of *M* was investigated.

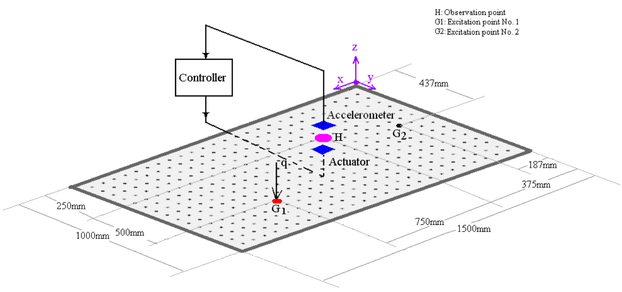
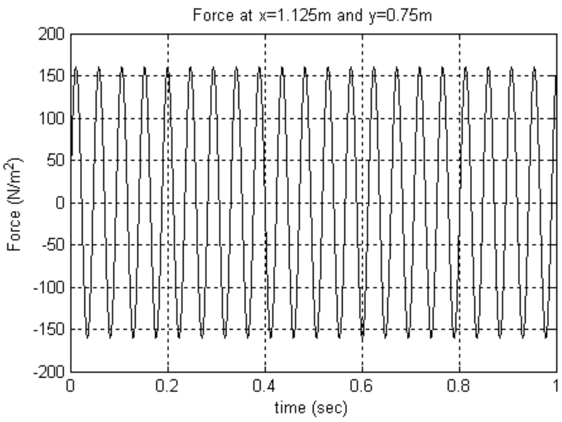
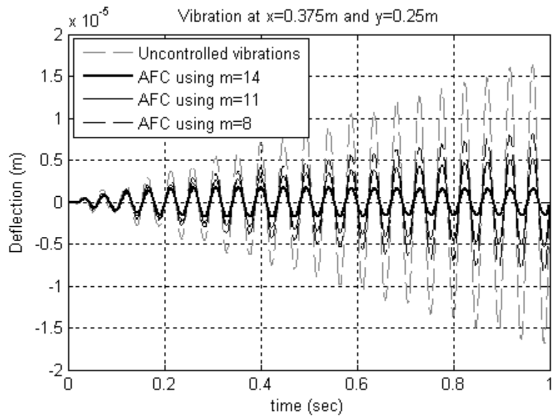


Fig. 7. Active vibration control system



(a)



(b)

Fig. 8. Performance of the AFC in attenuating the first resonance mode of the plate for different values of *M* (a) sinusoidal excitation (b) Plate response at observation point

As can be observed in Fig. 8b the best performance of the control system was found using the optimized value of *M=14* kg which might initially lead one into believing that a larger value of *M* can suppress the vibration much better. However, it can be shown that increasing the value of *M* beyond the optimized value (*M=14*) results in the instability of the control scheme. Fig. 9 reveals the system instability for *M=15*.

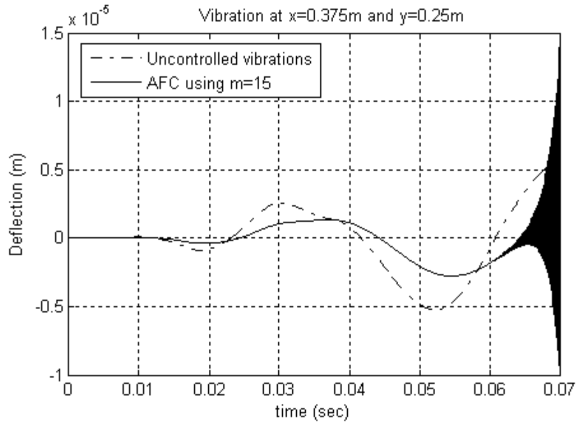
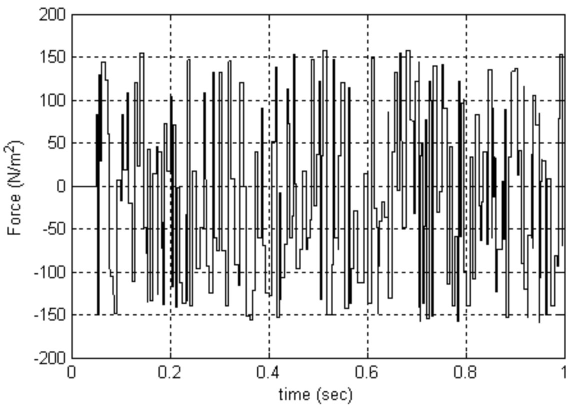


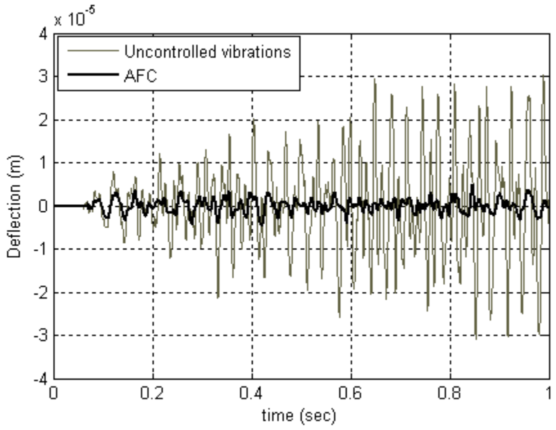
Fig. 9. Performance of the control scheme for *M=15*

It can be clearly observed from Fig. 8b that for an optimum value of *M* the control strategy has effectively suppressed the increasing trend of the amplitude of unwanted vibration due to resonance phenomenon. In other words, the first resonance mode of vibration of the flexible plate system has been effectively controlled by approximating a suitable mass value for the dynamic system.

Hence, for simulation study, the optimized value of the gain *M* was obtained to be 14. To present more evidences about the effectiveness of the control system different types of disturbances (*q*) were applied to excite the plate system at point G_1 (Fig. 7) and the performance of the controller in rejecting the unwanted vibrations was studied. Figs. 10b and 10c demonstrate the effectiveness of the proposed AVC scheme to reduce the amplitude of unwanted vibration under a random type disturbance (Fig. 10a). This type of excitation was chosen to ensure that all the non-linearities presented in the simulated plate system were captured. The performance of the control system was also examined using a chirp disturbance (Fig. 11a) with starting frequency of 1 Hz and final frequency of 50 Hz during a period of 1 s. The dynamic response of the plate to this recent type of excitation has been represented in Figs. 11b and 11c. It can be seen that for both types of disturbances the AVC scheme has considerably decreased the level of unwanted vibration.



(a)



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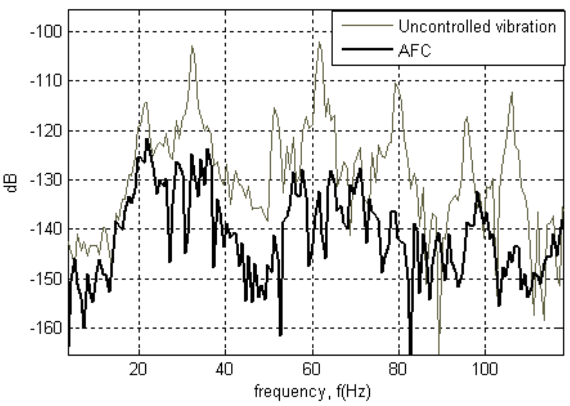
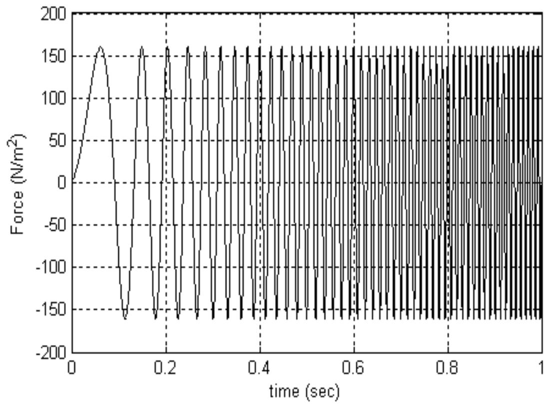
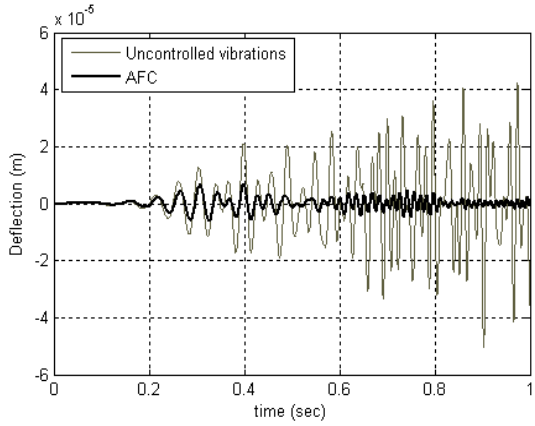


(c)

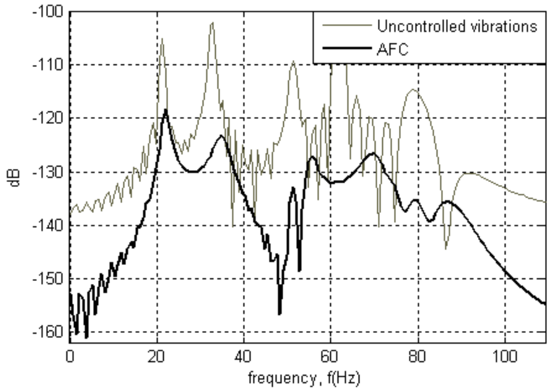
Fig. 10. Performance of the AVC system to reduce the unwanted vibrations at observation point H (375mm, 250mm) under vibratory excitations at point G_1(1125 mm,750mm). (a) Uniformly distributed random disturbance with amplitude of 160 Nm^{-2}, (b) Plate response to random disturbance, (c) Spectral density for random disturbance



(a)



(b)



(c)

Fig. 11. Performance of the AVC system to reduce the unwanted vibrations at observation point H (375mm, 250mm) under vibratory excitations at point G_1(1125 mm,750mm). (a) chirp disturbance with amplitude of 160 Nm^{-2}, (b) Plate response to random disturbance, (c) Spectral density for random disturbance

4.3 Stability evaluation of the proposed control scheme over a longer period of time

Since in this study a computational technique was employed for modeling the control system and there was no transfer function for the dynamical system, it was impossible to use the conventional analysis for stability evaluation of the control system such as pole placement technique. Thus, the stability problem of the controller was just investigated through the simulation studies. It was previously found that the control algorithm was stable over a period of 1 s for different types of disturbances at point G_1. To evaluate the stability of the control algorithm over a longer period of time, a uniformly distributed random disturbance was exerted to another point namely G_2 (see Fig.7) and the performance of the controller in reducing the vibration was investigated during a period of 10 s. Fig. 12 shows the stability of the control algorithm over 10 s for the given random disturbance. Based on the obtained results demonstrated in Fig. 12b over 10 s, more convincing evidences about the stability of the control system were determined.

4.4 Discussion

The simulation study presented in the previous sections showed that the AFC procedure can be employed for active vibration control of a flexible plate structure. According to the AFC theory and Newton's second law of motion, finding an appropriate mass value for the dynamical system is a key issue that assesses the efficiency of the control system. In this investigation a trial and error method, namely, crude approximation technique was used to find the approximate mass of the system. The best performance of the control system was achieved using the estimated mass value of 14 kg. However, a question about the exact mass value of the plate system may arise. To address this question, Eq. (17) can be used to find the exact mass of the plate system M_{exact} based on the knowledge about the plate density (ρ) as well as the corresponding dimensions (a, b and h) of the plate as:

$$M_{exact} = \rho \times a \times b \times h \quad (17)$$

Using the plate specifications depicted in Table 1, the exact mass M_{exact} of the plate system was calculated to be 13 Kg (M_{exact}=2700(Kgm^{-3})×1.5(m)×1(m)×0.0032(m)) which is close to the estimated mass value in simulation. This finding can confirm the correctness of the proposed control algorithm based on the principles of AFC and Newton's second law of motion.

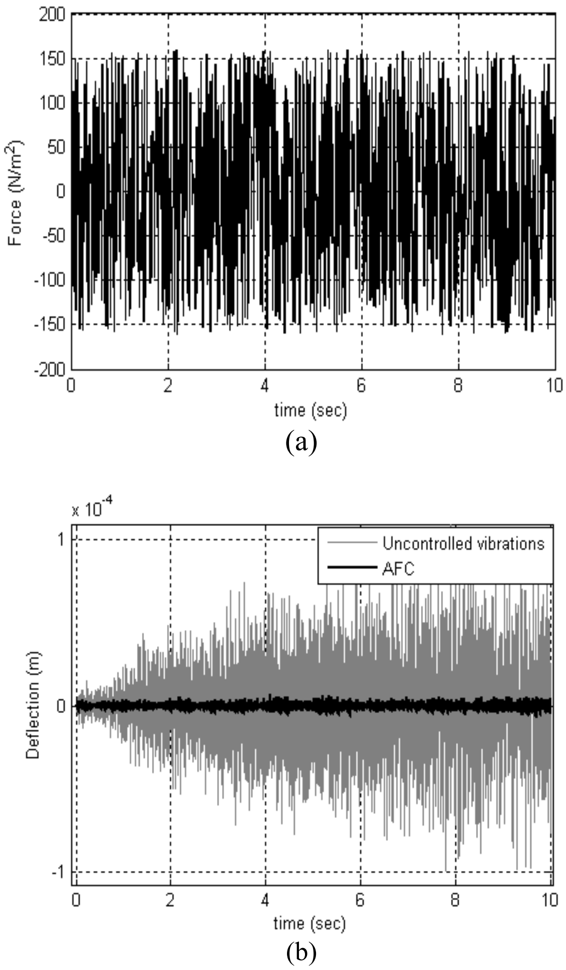


Fig. 12. Stability of the control algorithm over 10 seconds (observation point: H (375mm, 250mm), excitation point: G_2 (187mm, 437mm)). (a) Random input, (b) Dynamic response of the plate

5 Conclusion

The AFC methodology was developed and implemented for the problem of AVC of a flexible thin plate structure through a simulation study using the FD approach. A FD simulation platform was then developed to evaluate the control algorithm. The FD method was shown to be a very effective technique to model the flexible structures such as thin plates with uniform geometry. It was found that using 90 and 60 segments along the length and width of the plate respectively, the FD model accurately predicted the first five resonance modes of the plate system and thus, the validity of the proposed model was confirmed.

The AFC algorithm was carried out within the FD simulation environment. The best value of the mass gain, *M* was found to be 14 through crude approximation technique which is roughly equivalent to the real mass of the plate system. The simulation results revealed the effectiveness of the AFC technique in attenuating the unwanted vibration of the flexible plate structure. The stability of the control algorithm was also investigated during a period of 10 s in the presence of a random disturbance. It was also demonstrated that the proposed active force controller effectively controlled the first resonance mode of vibration of the flexible plate system.

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