A Generalised Minimum Variance Controller for Time-Varying MIMO Linear Systems with Multiple Delays

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Abstract: - In this paper we study optimal control of time-varying multi-input multi-output stochastic systems and develop a generalised minimum variance controller. The system to be controlled is described using a multi-input multi-output time-varying autoregressive moving average model that has multiple delays between the output and input. The controller minimises the sum of output tracking error variances and squared current control variables.

Key-Words: - MIMO systems, minimum variance control; multiple delays, time-varying systems.

1 Introduction

The multi-input multi-output (MIMO) generalised minimum variance controller (GMVC) minimises a generalised minimum variance cost functional for stochastic optimal control. It is based on a MIMO controlled autoregressive moving average (CARMA) model and extends the MIMO minimum variance controller (MVC) by adding quadratic control variables to the minimum variance cost functional for a generalised minimum variance cost functional. Although making the output tracking suboptimal the inclusion of the control variables in the cost functional improves the stability of the closed-loop control system by penalising large control actions. Consequently, large fluctuations in control variables are reduced and the controller is no longer restricted to systems with a stable inverse.

The MIMO GMVC for linear time-invariant (LTI) systems was developed by Koivo [1]. It extends the MIMO minimum variance controller of Borisson [2] from minimum phase systems to non-minimum phase systems. This LTI GMVC extends also the LTI GMVC of Clarke and Gawthrop [3], [4] from single-input and single-output (SISO) case to MIMO LTI systems. Both the SISO and MIMO LTI GMVCs are very useful and have seen many applications in stochastic adaptive control.

The basic difficulty in extending the SISO LTI GMVC to MIMO LTI GMVC is the noncommutativity of MIMO transfers functions. A pseudocommutation technique is used for the development of the MIMO LTI GMVC for overcoming noncommutativity. This technique was developed by Wolovich [5] and was introduced to the design of stochastic optimal controllers by Borisson [2]. The pseudocommutation has seen many applications in analysis and design of MIMO LTI systems.

There are many industrial systems that have time-varying dynamics [6], [7]. In this paper we will develop a generalised minimum variance controller for MIMO linear time-varying (LTV) systems. Noncommutativity is also an obstacle for extending a GMVC from SISO LTI plants for SISO LTV systems. A pseudocommutation technique was developed for SISO LTV transfer operators in the previous work [8] on minimum variance prediction of SISO LTV systems. Based on this technique the SISO LTI GMVC was extended for SISO LTV plants under a generalised cost functional that uses time-varying filters and weighting functions [9]. However, a pseudocommutation technique is unavailable for MIMO LTV transfer operators for extending the SISO LTV GMVC for MIMO LTV plants.

An LTV GMVC was developed without using the pseudocommutation for an exponentially stable SISO LTV CARMA model [10]. However, when the LTV filters are involved in the cost functional the pseudocommutation cannot be avoided [11], [12]. An MIMO LTV GMVC [13] was recently developed for MIMO LTV systems as an extension of the LTV MIMO MVC [14], [15]. However, this LTV GMVC can only be applied to the MIMO CARMA model with a uniform single delay and cannot be used for the MIMO plants that have different delays between the inputs and outputs.

It is very common that MIMO processes have multiple delays and cannot be controlled using a MIMO GMVC that has a uniform single delay. In this paper, we extend the LTV GMVC for MIMO LTV plants from the uniform delay case for multiple delay systems. We consider the case where the delay can be described using a diagonal matrix such that each output can have a different delay to its inputs.

The reminder of this paper is organised as the following. Section 2 describes the MIMO LTV CARMA model, the LTV operators, the multiple time delays and the generalised minimum variance cost functional. Section 3 develops the MIMO LTV GMVC, analyses its closed-loop stability and compares it with other LTV GMVCs. Section 4 presents design and simulation examples followed by a conclusion in Section 5.

2 Problem Formulation

The MIMO LTV plants to be controlled is described by the LTV CARMA model

$$A(k,q^{-1})Y'(k)=B(k,q^{-1})U(k)+C(k,q^{-1})W'(k)\,, \quad (1)$$

where $W'(k)$ is a $p\times 1$ vector of zero mean independent Gaussian processes that represent the stochastic disturbances. The variance of $W'(k)$ is time varying and uniformly bounded away from infinite. $U(k)$ and $Y'(k)$ are the plant input and output. They are all $p\times 1$ vectors. In the CARMA model, q is the one-step-advance operator and

$$\begin{aligned}A(k,q^{-1})&=I+A_1(k)q^{-1}+\cdots+A_n(k)q^{-n}\\B(k,q^{-1})&=B_0(k)+B_1(k)q^{-1}+\cdots+B_m(k)q^{-m}\\C(k,q^{-1})&=I+C_1(k)q^{-1}+\cdots+C_h(k)q^{-h}\end{aligned} \quad (2)$$

are LTV moving average operators (MAO's) with $A_i(k)$, $B_j(k)$ and $C_r(k)$, i=1, 2, ..., n, j=0, 1, ..., m, r=1, 2, ..., h being $p\times p$ matrices whose elements are uniformly bounded away from infinite. It is also assumed that the determinant of $B_0(k)$ is uniformly bounded away from zero. In the CARMA model the signal vectors

$$\begin{aligned}Y'(k)&=\begin{bmatrix}y_1(k+d_1) & y_2(k+d_2) & \cdots & y_p(k+d_p)\end{bmatrix}^{\mathrm{T}}\\U(k)&=\begin{bmatrix}u_1(k) & u_2(k) & \cdots & u_p(k)\end{bmatrix}^{\mathrm{T}}\\W'(k)&=\begin{bmatrix}w_1(k+d_1) & w_2(k+d_2) & \cdots & w_p(k+d_p)\end{bmatrix}^{\mathrm{T}}\end{aligned} \quad (3)$$

are used, where the superscript T denotes matrix transpose, $d_i \geq 1$, i=1, 2, …, p, represent the multiple time delays between the plant inputs and outputs. The above equation shows that the CARMA model allows each output to have a different delay with respect to the inputs. Without losing generality it is assumed that $d_i \leq d_{i+1}$. Letting

$$D(q)=\mathrm{diag}\left(q^{d_1},q^{d_2},\ldots,q^{d_p}\right) \quad (4)$$

be a diagonal matrix for description of the multiple time delays in terms of the one-step-advance operator. The LTV CARMA model can be expressed as

$$\begin{aligned}&A(k,q^{-1})D(q)Y(k)\\&\qquad=B(k,q^{-1})U(k)+C(k,q^{-1})D(q)W(k)\end{aligned} \quad (5)$$

where

$$\begin{aligned}Y(k)&=\begin{bmatrix}y_1(k) & y_2(k) & \cdots & y_p(k)\end{bmatrix}^{\mathrm{T}}\\W(k)&=\begin{bmatrix}w_1(k) & w_2(k) & \cdots & w_p(k)\end{bmatrix}^{\mathrm{T}}\end{aligned} \quad (6)$$

When all the delays are equal the CARMA model (5) reduces to the uniform single delay LTV CARMA model in [13].

The inverse of an LTV MAO is called an LTV autoregressive operator (ARO) and is denoted as $A^{-1}(k,q^{-1})$ whose stability is defined using the stability of the state transition matrix

$$\Phi(k+1,k)=\begin{bmatrix}-A_1(k) & -A_2(k) & -A_3(k) & \cdots & -A_{n-1}(k) & -A_n(k)\\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0}\end{bmatrix} \quad (7)$$

where **I** and **0** are identity and zero matrix of appropriate dimension. The LTV ARO is exponentially stable if and only if there are constants $C > 0$ and $c > 0$ such that

$$\left\|\Phi(k,s)\right\|\leq Ce^{-c(k-s)} \quad (8)$$

for all $k \geq s \geq 0$. The maximum of all possible c is called the rate of exponential stability for the LTV ARO. When the LTV ARO is exponentially stable it leads to the cancellation rules

$$\begin{aligned}A(k,q^{-1})A^{-1}(k,q^{-1})X(k)&=X(k)\\A^{-1}(k,q^{-1})A(k,q^{-1})X(k)&=X(k)+\varepsilon(k)\end{aligned} \quad (9)$$

where $\varepsilon(k)$ satisfies

$$A(k,q^{-1})\varepsilon(k)=0\,. \tag{10}$$

It is a zero input solution to the above autoregressive equation and will decay to zero exponentially when $A^{-1}(k,q^{-1})$ is exponentially stable. In this paper we assume that both $A^{-1}(k,q^{-1})$ and $C^{-1}(k,q^{-1})$ are exponentially stable and all the plant parameters in the MIMO LTV CARMA model are known.

The reference vector for the LTV GMVC to follow has the form

$$\begin{aligned} S'(k) &= \left[s_1(k+d_1)\ \ s_2(k+d_2)\ \cdots\ s_p(k+d_p) \right]^{\mathrm{T}} \\ &= D(q)S(k) \end{aligned} \tag{11}$$

Given the uniformly bounded reference vector the generalised minimum variance control objective is to minimise the cost functional

$$\begin{aligned} J(k) = E\{&[Y'(k)-S'(k)]^{\mathrm{T}} P(k)[Y'(k)-S'(k)] \\ &+U(k)^{\mathrm{T}} R(k)U(k) \mid Data(k)\} \end{aligned} \tag{12}$$

using a sequence of control vectors, $U(k)$, $U(k\text{-}1)$, $U(k\text{-}2)$,…. In the control objective (12) $Data(k)$ is the set of input and output data up to and including time k, both $P(k)$ and $R(k)$ are uniformly positive definite matrix that are uniformly bounded away from infinite.

In comparison with the LTI GMVC the assumption on exponential stability of $A^{-1}(k,q^{-1})$ is additional and the others are simple and natural extensions of those from the LTI case for LTV plants. In addition, the degrees of the LTV MAO's in the LTV CARMA model are time varying because the time varying matrices $A_i(k)$, $B_j(k)$ and $C_r(k)$, i=1, 2, ..., n, j=1, 2, ..., m, r=1, 2, ..., h, are allowed to become zero. Noting (3) we know that this objective includes the generalised minimum variance control cost functional in [13] as a special case by introducing multiple time delays for the plant outputs.

3 Controller

Left dividing (5) using $A(k,q^{-1})$ we have

$$\begin{aligned} D(q)Y(k) &= A^{-1}(k,q^{-1})B(k,q^{-1})U(k) \\ &\quad +A^{-1}(k,q^{-1})C(k,q^{-1})D(q)W(k) \end{aligned} \tag{13}$$

In order to divide the noise term on the far right hand side of the above equation into past and future component left dividing $C(k,q^{-1})D(q)$ by $A(k,q^{-1})$ gives

$$\begin{aligned} A^{-1}(k,q^{-1})C(k,q^{-1})D(q) &= F(k,q) \\ &\quad +A^{-1}(k,q^{-1})G(k,q^{-1}) \end{aligned} \tag{14}$$

where

$$F(k,q)=F_0(k)q^{d_p}+F_1(k)q^{d_p-1}+\cdots+F_{d_p-1}(k)q \tag{15}$$

is the quotient and $G(k,q^{-1})$ is the remainder. The maximum power of q in $G(k,q^{-1})$ is zero. Substituting (14) into (13) we have

$$\begin{aligned} D(q)Y(k) &= A^{-1}(k,q^{-1})B(k,q^{-1})U(k) \\ &\quad +F(k,q)W(k) \\ &\quad +A^{-1}(k,q^{-1})G(k,q^{-1})W(k) \end{aligned} \tag{16}$$

The LTV CARMA model can be rewritten as

$$\begin{aligned} D(q)Y(k) &- F(k,q)W(k) \\ &= A^{-1}(k,q^{-1})B(k,q^{-1})U(k) \\ &\quad +A^{-1}(k,q^{-1})G(k,q^{-1})W(k) \end{aligned} \tag{17}$$

Letting k being the current time we have the plant output and future noise on the left hand side of the above equation because both LTV MAO's, $D(q)$ and $F(k,q)$, have positive power of at least one in the one-step-advance operator q. On the right hand side we have the current and past plant input and noise because $A(k,q^{-1})$ is monic and the maximum powers of both $G(k,q^{-1})$ and $B(k,q^{-1})$ are zero. As a result, if the noise up to and including the current time k can be estimated we can use (17) as a predictor for the plant output because the future noise is of zero mean. Letting

$$\hat{Y}'(k)=Y'(k)-F(k,q)W(k) \tag{18}$$

be the output prediction of the CARMA model and noting (17) and (4) we have

$$\begin{aligned} \hat{Y}'(k) &= A^{-1}(k,q^{-1})B(k,q^{-1})U(k) \\ &\quad +A^{-1}(k,q^{-1})G(k,q^{-1})W(k) \end{aligned} \tag{19}$$

Substituting this predictor into the generalised minimum variance cost functional we have the following GMVC.

GMVC Theorem

If the LTV AROs $A^{-1}(k,q^{-1})$ and $C^{-1}(k,q^{-1})$ are exponentially stable, the LTV GMVC for the MIMO LTV CARMA model is given by

$$\hat{W}(k) = D(q^{-1})C^{-1}(k,q^{-1})\Big[A(k,q^{-1})D(q)Y(k) - B(k,q^{-1})U(k)\Big] \tag{20}$$

$$U(k) = T^{-1}(k,q^{-1})\Big[A(k,q^{-1})D(q)S(k) - G(k,q^{-1})\hat{W}(k)\Big] \tag{21}$$

$$T(k,q^{-1}) = B(k,q^{-1}) + A(k,q^{-1})P^{-1}(k)B_0^{-\mathrm{T}}(k)R(k) \tag{22}$$

where $\hat{W}(k)$ is the estimate of $W(k)$.

Proof

Substituting the minimum variance prediction (19) into the cost functional (12) we have

$$\begin{aligned} J(k) =& [\hat{Y}'(k) - S'(k)]^{\mathrm{T}} P(k)[\hat{Y}'(k) - S'(k)] \\ &+ U^{\mathrm{T}}(k)R(k)U(k) \\ &+ E\left\{[F(k,q)W(k)]^{\mathrm{T}} P(k)[F(k,q)W(k)] \,|\, Data(k)\right\} \end{aligned} \tag{23}$$

From (1), (2), and (18) it can be verified that

$$\frac{\partial \hat{Y}'(k)}{\partial U(k)} = \frac{\partial Y'(k)}{\partial U(k)} = B_0(k). \tag{24}$$

Thus

$$\frac{\partial J(k)}{\partial U(k)} = 2B_0^{\mathrm{T}}(k)P(k)[\hat{Y}'(k) - S'(k)] + 2R(k)U(k) \tag{25}$$

and

$$\frac{\partial^2 J(k)}{\partial U^2(k)} = 2B_0^{\mathrm{T}}(k)P(k)B_0(k) + 2R(k). \tag{26}$$

Because the second derivative is uniformly positive definite there exists optimal control $U(k)$ such that the generalised minimum variance cost functional (12) is minimised. From (25) we know that the optimal control $U(k)$ can be determined from

$$P^{-1}(k)B_0^{-\mathrm{T}}R(k)U(k) = S'(k) - \hat{Y}'(k). \tag{27}$$

It follows from (19) that

$$\begin{aligned} P^{-1}(k)B_0^{-\mathrm{T}}R(k)U(k) =& S'(k) \\ &- A^{-1}(k,q^{-1})[B(k,q^{-1})U(k) \\ &+ G(k,q^{-1})W(k)] \end{aligned} \tag{28}$$

Solving for the control variable we have

$$\begin{aligned} [A(k,q^{-1})P^{-1}(k)B_0^{-\mathrm{T}}R(k) + B(k,q^{-1})]U(k) \\ = A(k,q^{-1})S'(k) - G(k,q^{-1})W(k) \end{aligned} \tag{29}$$

In order to use this equation to determine the optimal control we need the estimate of the noise $W(k)$. The estimator (20) can be rewritten as follows

$$C(k,q^{-1})D(q)\hat{W}(k) = A(k,q^{-1})D(q)Y(k) - B(k,q^{-1})U(k) \tag{30}$$

Comparing the above equation with the CARMA model (5) we have

$$C(k,q^{-1})D(q)\tilde{W}(k) = \mathbf{0}, \tag{31}$$

where

$$\tilde{W}(k) = W(k) - \hat{W}(k) = D(q^{-1})\tilde{W}'(k) \tag{32}$$

is the estimation error. Equation (31) shows that the error will decay exponentially due to exponential stability of $C^{-1}(k,q^{-1})$. Replacing the noise term in (29) using its estimate obtained by (20) and noting (4) we have (21), which can be further expressed as

$$\begin{aligned} T(k,q^{-1})U(k) =& A(k,q^{-1})D(q)S(k) \\ &+ G(k,q^{-1})\tilde{W}(k) - G(k,q^{-1})W(k) \end{aligned} \tag{33}$$

■

Noting (1), (31), (32) and (33) we have the closed-loop system of the LTV GMVC as follows

$$\begin{aligned} &\begin{bmatrix} C(k,q^{-1}) & \mathbf{0} & \mathbf{0} \\ -G(k,q^{-1})D(q^{-1}) & T(k,q^{-1}) & \mathbf{0} \\ \mathbf{0} & -B(k,q^{-1}) & A(k,q^{-1}) \end{bmatrix} \begin{bmatrix} \tilde{W}'(k) \\ U(k) \\ Y'(k) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -G(k,q^{-1}) & A(k,q^{-1})D(q) \\ C(k,q^{-1})D(q) & \mathbf{0} \end{bmatrix} \begin{bmatrix} W(k) \\ S(k) \end{bmatrix} \end{aligned} \tag{34}$$

Remarks:

a) The left most matrix in the above equation is the LTV ARO matrix for the closed-loop system. It is a lower triangle matrix. Its stability is determined by the three diagonal elements, $A^{-1}(k,q^{-1})$, $C^{-1}(k,q^{-1})$ and $T^{-1}(k,q^{-1})$. The exponential stability of $B^{-1}(k,q^{-1})$ is not necessary for stability of the closed-loop system because it is not a diagonal element unless the weighting matrix for the control variable $R(k)$ is zero. Thus, the stable invertability condition of the LTV MVC [8] is removed.

b) Among the three diagonal elements both $A^{-1}(k,q^{-1})$ and $C^{-1}(k,q^{-1})$ are exponentially stable. Therefore, the closed-loop system is exponentially stable if $T^{-1}(k,q^{-1})$ is exponentially stable. From (22) it is known that stability of $T^{-1}(k,q^{-1})$ depends on the choice of $P(k)$ and $R(k)$. Because $A^{-1}(k,q^{-1})$ is exponentially stable we can choose $P^{-1}(k)B_0^{-T}(k)R(k)=\beta\mathbf{I}$ with β being a very large positive number in order for the stability of $A^{-1}(k,q^{-1})$ to dominate the stability of $T^{-1}(k,q^{-1})$.

c) The equations (34) and (22) show that stability of the closed-loop system is independent on the time delay of the system because all the three diagonal elements of the LTV ARO for the closed-loop system do not include the delay operator $D(q)$. This agrees with the stability results for the LTV MVCs and LTI GMVCs.

d) The estimator (20) is introduced for the estimation of the current process noise from the plant input and output. The estimation error depends on the initial conditions used in the estimator. When the initial conditions are accurate the estimator will produce accurate estimate right from the beginning for all the time and the performance of the LTV GMVC will be optimal all the time. When the initial condition is inaccurate there will be an estimation error that decays exponentially to zero according to equation (31). As a result, the performance of the LTV GMVC will converge exponentially to the optimal performance specified by (12). The rate of the exponential convergence is faster than or equal to the rate of the exponential stability of the LTV ARO $C^{-1}(k,q^{-1})$ as is shown by the error equation (31).

e) This GMVC includes the LTV GMVC for MIMO CARMA models with a uniform single delay [13] as a special case. When all diagonal elements in the delay operator $D(q)$ are equal the generalised minimum variance control cost functional (12) becomes the cost functional for the LTV MIMO CARMA models with a uniform single delay and the LTV GMVC becomes the LTV MIMO GMVC of [13].

f) This GMVC includes also the LTV MVC for MIMO CARMA models with multiple delays as a special case. When the time-varying input weighting matrix $R(k)$ is set to zero the generalised minimum variance cost functional (12) becomes the minimum variance cost functional of [14] and the LTV GMVC becomes the LTV MVC for MIMO LTV systems with multiple delays.

g) This GMVC also includes our LTV GMVC for SISO LTV systems [10] as a special case. When the LTV CARMA model (1) reduces to an LTV SISO system the cost functional (12) reduces to that for the SISO LTV CARMA models of [10] and the MIMO LTV GMVC reduces to the SISO LTV GMVC. However, it cannot include our flexible LTV GMVC [11] for LTV SISO CARMA models as a special case. Time-varying filters for the plant input and output were used in the flexible GMVC cost functional in [11]. They were dealt with using the pseudocommutation for the development of the flexible SISO LTV GMVC. However, the pseudocommutation is unavailable for MIMO LTV systems.

4 Examples

We present the design and simulation of our LTV GMVC using a two-input and two-output LTV plant described by the CARMA model

$$\begin{aligned}&\begin{bmatrix} y_1(k+1) \\ y_2(k+2) \end{bmatrix} + A(k)\begin{bmatrix} y_1(k) \\ y_2(k+1) \end{bmatrix} \\ &\quad = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} + B(k)\begin{bmatrix} u_1(k-1) \\ u_2(k-1) \end{bmatrix} \\ &\quad\quad + \begin{bmatrix} w_1(k+1) \\ w_2(k+2) \end{bmatrix} + C(k)\begin{bmatrix} w_1(k) \\ w_2(k+1) \end{bmatrix}\end{aligned} \tag{35}$$

where both noises are independent Gaussian processes with zero mean and unit variance. Specifying the delays using the one-step-advance operator we have

$$D(q)=\mathrm{diag}(q,q^2) \tag{36}$$

The system can then be represented using the LTV CARMA model (5) where

$$\begin{aligned}A(k,q^{-1})&=I+A(k)q^{-1}\\ B(k,q^{-1})&=I+B(k)q^{-1}\\ C(k,q^{-1})&=I+C(k)q^{-1}\end{aligned} \tag{37}$$

Example 1

In this example we study the performance of our LTV GMVC using constant weighting functions. They have the forms

$$P(k)=\begin{bmatrix}6 & 0\\ 0 & 1\end{bmatrix}$$
$$R(k)=\begin{bmatrix}0.7 & 0\\ 0 & 0.03\end{bmatrix} \tag{38}$$

The time-varying parameter matrices are

$$A(k)=\begin{bmatrix}0.7\sin(0.1k) & 0.1\cos(0.3k)\\ 0 & 0.7\text{sign}[\cos(0.2k)]\end{bmatrix}$$
$$B(k)=\begin{bmatrix}1.1 & 2.5\sin(0.5k)\\ 0 & 0.1\text{sign}[\cos(0.6k)]\end{bmatrix} \tag{39}$$
$$C(k)=\begin{bmatrix}0.6\dfrac{k+1}{k+5} & 0.5(1+e^{-0.2(k+1)})\sin(0.6k)\\ 0 & 0.4\sin(0.2k+0.2)\end{bmatrix}$$

Equations (37) and (39) show that $B^{-1}(k,q^{-1})$ is exponentially unstable because $B(k)$ is a triangular matrix and the absolute value of its first diagonal element is uniformly greater than unit. However, both $A^{-1}(k,q^{-1})$ and $C^{-1}(k,q^{-1})$ are exponentially stable because they are upper triangular and the absolute values of their diagonal elements are uniformly less than unit.

Applying the LTV GMVC the estimator (20) has the following form

$$\begin{aligned}&\hat{w}_2(k)+c_{21}(k-2)\hat{w}_1(k-2)+c_{22}(k-2)\hat{w}_2(k-1)\\ &\quad=y_2(k)+a_{21}(k-2)y_1(k-2)\\ &\qquad+a_{22}(k-2)y_2(k-1)\\ &\qquad-u_2(k-2)-b_{21}(k-2)u_1(k-3)\\ &\qquad-b_{22}(k-2)u_2(k-3)\\ &\hat{w}_1(k)+c_{11}(k-1)\hat{w}_1(k-1)+c_{12}(k-1)\hat{w}_2(k)\\ &\quad=y_1(k)+a_{11}(k-1)y_1(k-1)\\ &\qquad+a_{12}(k-1)y_2(k)\\ &\qquad-u_1(k-1)-b_{11}(k-1)u_1(k-2)\\ &\qquad-b_{12}(k-1)u_2(k-2)\end{aligned} \tag{40}$$

where $a_{ij}(k)$ are the elements of $A(k)$ on the ith row and jth column. The same applies to $B(k)$ and $C(k)$. The controller (21) has the form

$$\begin{aligned}T(k,q^{-1})\begin{bmatrix}u_1(k)\\ u_2(k)\end{bmatrix}&=A(k,q^{-1})\begin{bmatrix}s_1(k+1)\\ s_2(k+2)\end{bmatrix}\\ &\quad-G(k,q^{-1})\begin{bmatrix}\hat{w}_1(k)\\ \hat{w}_2(k)\end{bmatrix}\end{aligned} \tag{41}$$

where

$$T(k,q^{-1})=B(k,q^{-1})+A(k,q^{-1})\begin{bmatrix}0.1167 & 0\\ 0 & 0.03\end{bmatrix} \tag{42}$$

and

$$G(k,q^{-1})=C(k,q^{-1})D(q)-A(k,q^{-1})F(k,q^{-1})\,. \tag{43}$$

The reference inputs are square waves. Fig. 1 and Fig. 2 show that the LTV GMVC is able to drive the MIMO CARMA model to follow both square wave references. However, there are steady-state tracking errors in both outputs. This is typical for LTV GMVC as a result of the tradeoff between the output tracking performance and stability by introducing a penalizing term of control variables into the minimum variance cost functional. The plant parameters are shown in Fig. 3 to Fig. 5.

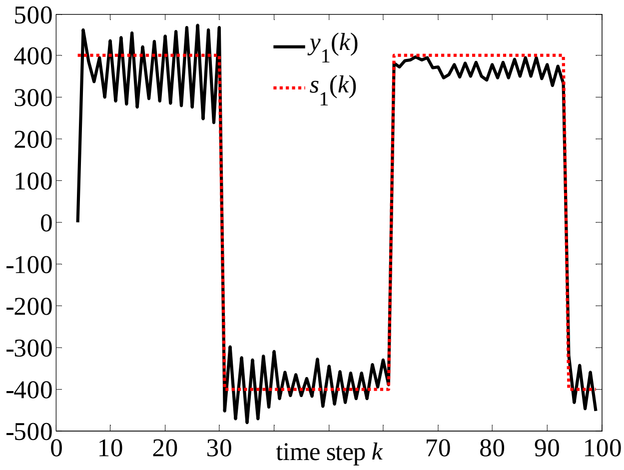

Fig. 1 Example 1 first output.

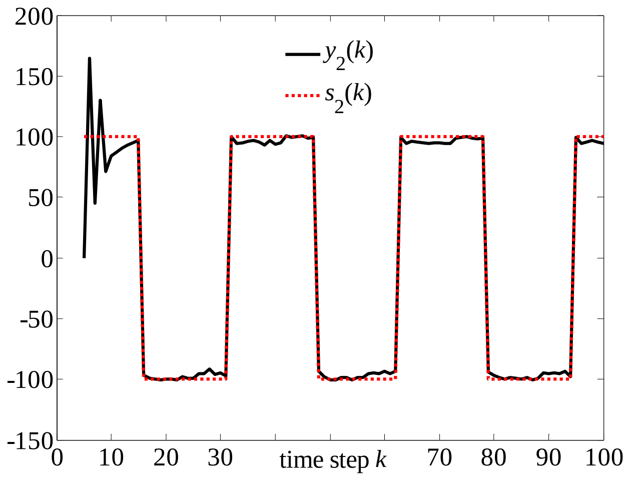

Fig. 2 Example 1 second output.

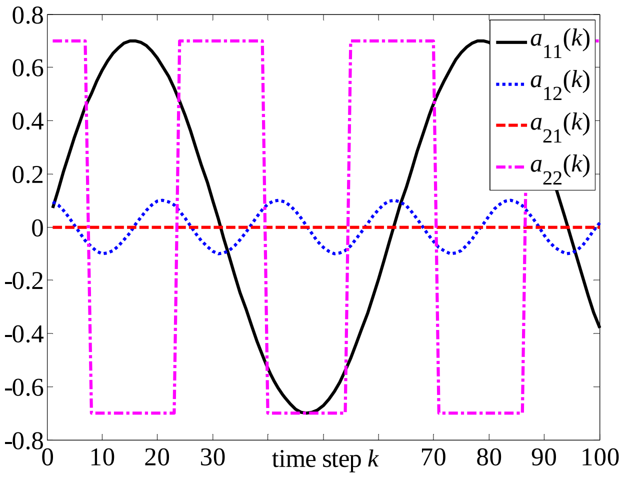

Fig. 3 Example 1 plant parameters of $A(k)$.

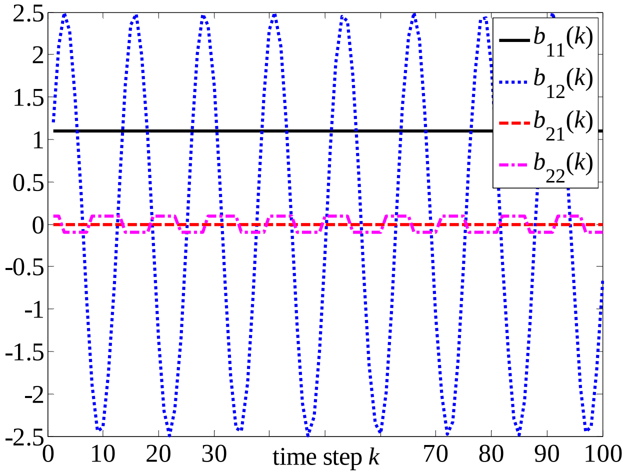

Fig. 4 Example 1 plant parameters of $B(k)$.

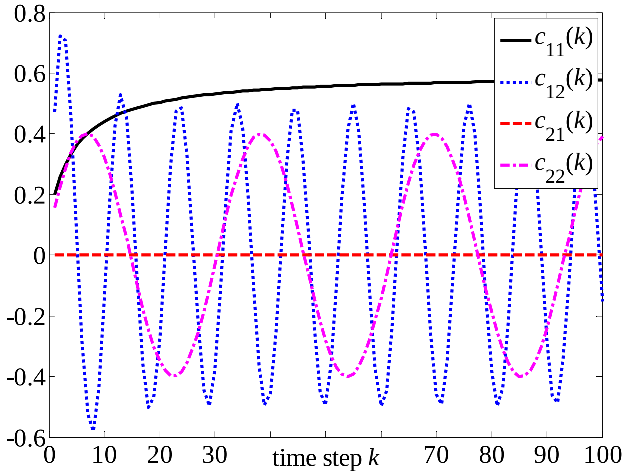

Fig. 5 Example 1 plant parameters of $C(k)$.

Fig. 6 shows the two control variables produced by the LTV GMVC. While the second control variable is quite smooth the first fluctuates a lot over a wide range. As a result, the first output is also more oscillatory than the second as shown in Fig. 1 and Fig. 2. This is partly due to the stably uninvertible nature of the system as is characterised by the top left element in the LTV MAO $B(k,q^{-1})$. It has the form

$$b_{11}(k,q^{-1}) = 1 + 1.1q^{-1} \tag{44}$$

This LTV property is corresponding to the non-minimum phase characteristics in LTI case. When the coefficient of the second term in the above equation is changed from 1.1 to 0.9 the LTV CARMA model becomes stably invertible. The plant outputs and control variables of this modified example are shown in Fig. 7 to Fig. 9.

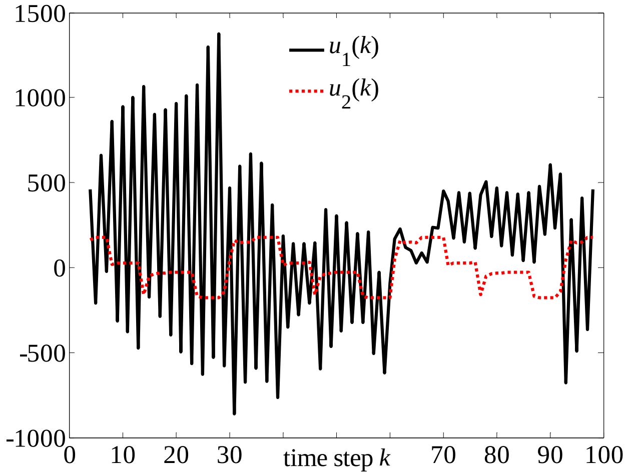

Fig. 6 Example 1 control variables.

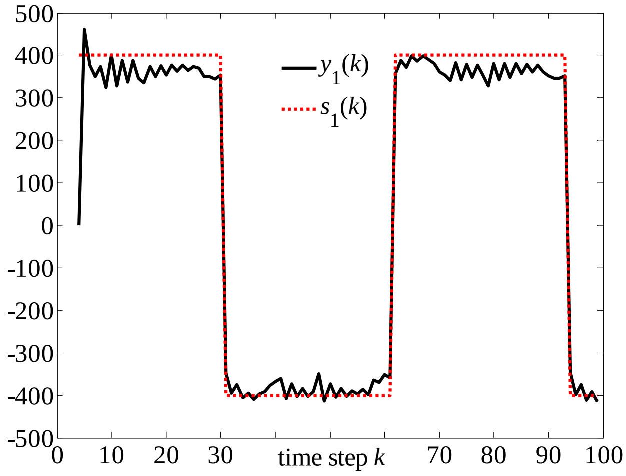

Fig. 7 Modified example 1 first output.

Fig. 9 shows that the oscillation in the first control variable is reduced from a range of -900 to 1400 in Fig. 6 to that of -500 to 700 in Fig. 9 when the system changes from stably uninvertible to stably invertible. Consequently, the oscillation in the first plant output is also reduced significantly as shown in Fig. 7. However, there are very little changes in the second plant output and second control variable. The reasons can be seen from the triangular form of the equations (37) and (39). They show that the second equation in the LTV CARMA model (35) is independent of the first one.

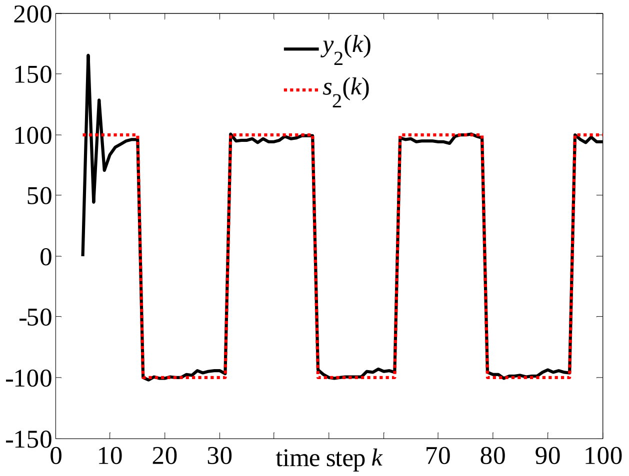

Fig. 8 Modified example 1 second output.

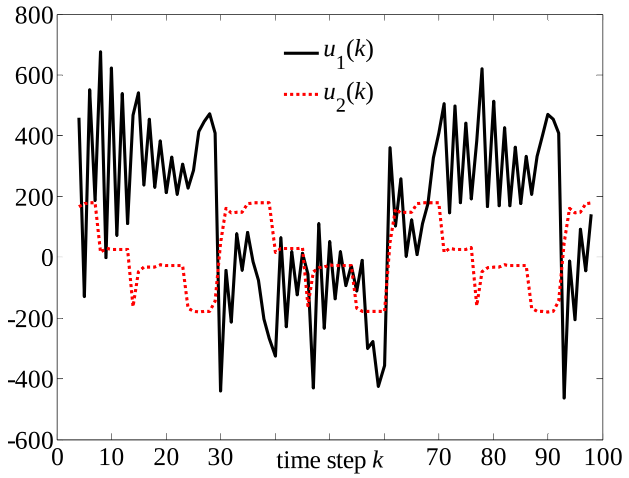

Fig. 9 Modified example 1 control variables.

Example 2

The first example shows that the output of the control system can be quite oscillatory due to the stably uninvertible nature of the system. The choice of the weighting functions can also influence the tradeoffs between the tracking performance and stability. In this example we study the affects of time-varying weighting functions. The structure of the system remains the same as in the first example and the plant parameters are the following.

$$A(k)=\begin{bmatrix} 0.5\cos(0.3k) & 0.2\cos(0.2k) \\ 0.3\text{sign}[\cos(0.5k)] & 0 \end{bmatrix}$$
$$B(k)=\begin{bmatrix} 1.1 & 2.5\sin(0.5k)\cos(0.05k) \\ 0 & 0.1\text{sign}[\cos(0.5k)] \end{bmatrix} \tag{45}$$
$$C(k)=\begin{bmatrix} 0.3\dfrac{k+1}{k+5} & 0.6(1+e^{-0.2(k+1)})\sin(0.5k) \\ 0 & 0.7\sin(0.3k+0.3) \end{bmatrix}$$

The weighting matrices are chosen as

$$P(k)=\begin{bmatrix} 2+2\times\mathbf{1}(k-65) & 0 \\ 0 & 1-0.5\times\mathbf{1}(k-65) \end{bmatrix}$$
$$R(k)=\begin{bmatrix} 0.7 & 0 \\ 0 & 0.5-0.49\times\mathbf{1}(k-35) \end{bmatrix} \tag{46}$$

where

$$\mathbf{1}(t)=\begin{cases} 0 & t<0 \\ 1 & t\geq 0 \end{cases} \tag{47}$$

is the step function.

The nonzero weightings are shown in Fig. 10 and the plant output, input and parameters are shown in Fig. 11 to Fig. 16. Fig. 11 shows oscillatory tracking in the plant first output. The weighting function $P_{11}(k)$ is doubled and $P_{22}(k)$ is halved on k=65 in order to divert more emphasis on the tracking accuracy of the first plant output in the LTV GMVC. However, this causes large oscillation in the first control variable as shown in Fig. 13 and the tracking performance in the first plant output does not improve after the weighting changes. This illustrates the difficulty in accurate tracking for stably uninvertible systems.

Fig. 12 shows a significant tracking error in the second plant output in the early stage. This error jumps at about the same time as the time-varying plant parameters $a_{21}(k)$. The weighting function $R_{22}(k)$ is reduced to 0.1 on k=35 in order to allow large control actions to reduce this error and the performance improvement is significant. After k=65 the tracking accuracy degraded a little due to the changes made in the output weighting matrix $P(k)$.

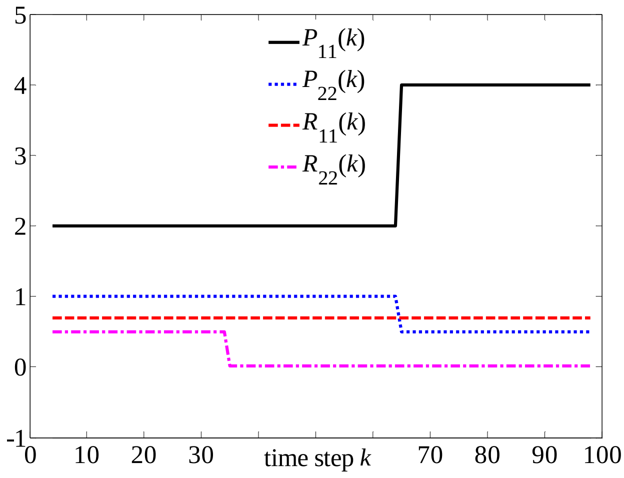

Fig. 10 Example 2 nonzero elements of the weighting functions.

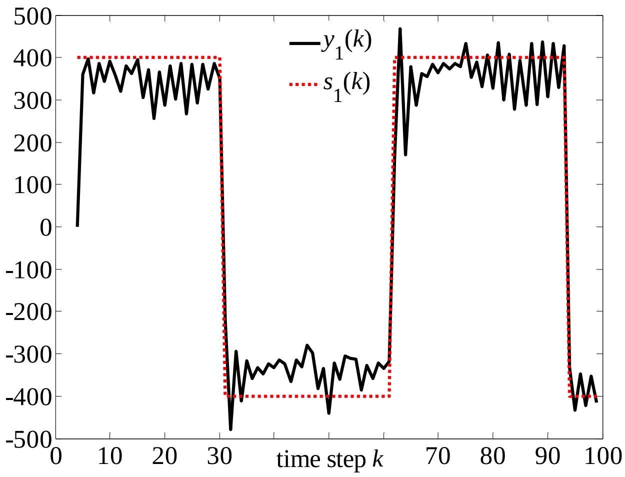

Fig. 11 Example 2 first output.

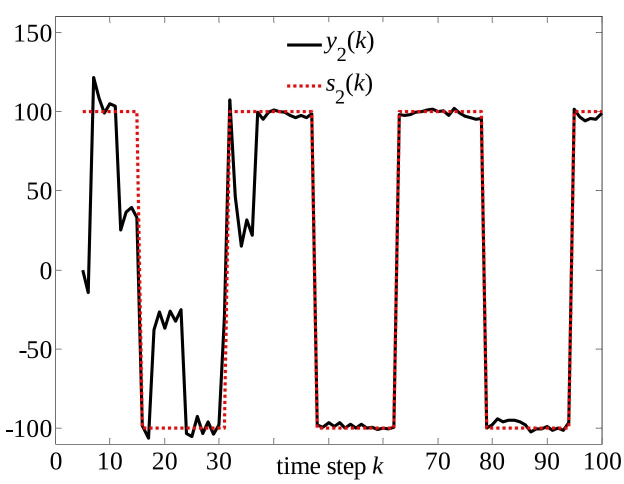

Fig. 12 Example 2 second output.

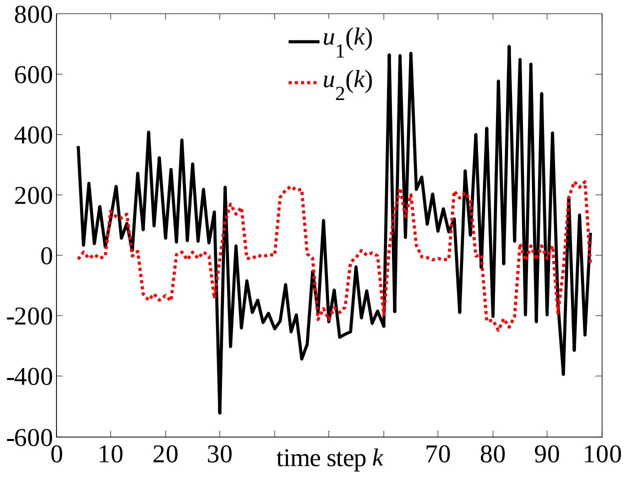

Fig. 13 Example 2 control variables.

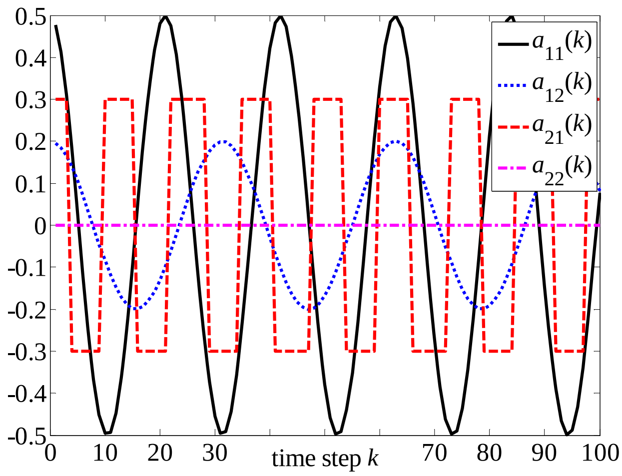

Fig. 14 Example 2 plant parameters of $A(k)$.

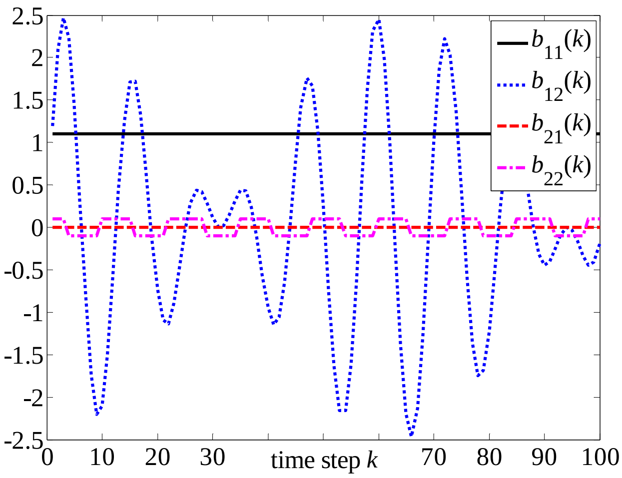

Fig. 15 Example 2 plant parameters of $B(k)$.

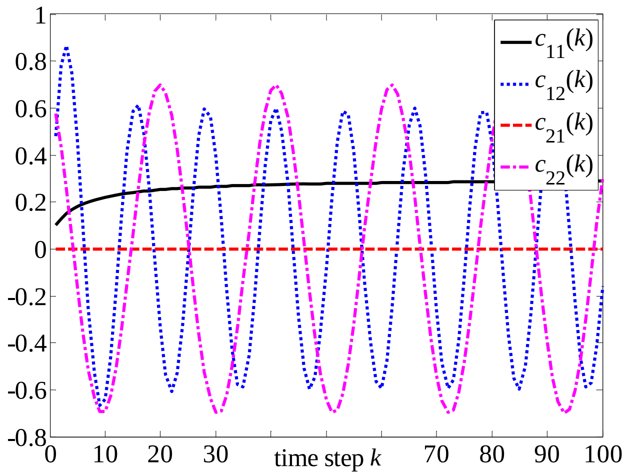

Fig. 16 Example 2 plant parameters of $C(k)$.

5 Conclusion

Generalised minimum variance control has been studied for MIMO LTV systems with multiple delays and an LTV GMVC has been suggested. It extends the previous MIMO LTV GMVC [13] from a uniform single delay case for multiple delay LTV CARMA models. It extends also our previous LTV MVC for MIMO systems [14], [15] with multiple delays by adding a penalised term for the control vector to the minimum variance cost functional and, thus, removes the stable invertability condition for closed-loop stability.

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