# Stabilization of an Inertia Wheel Pendulum using an Implicit controller design

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*Abstract:* - Control design for an underactuated system is complicated, as feedback linearization cannot be applied directly. This issue can be addressed by using partial feedback linearization with appropriate coordinate transformations. Unfortunately, this often results in leaving the core system non-affine. Design of a control law for a non-affine system is quiet difficult due to lack of mathematical tools needed. The Inertia Wheel Pendulum is a benchmark example of such non-affine systems. The under actuation property, posing problems in exact feedback linearization makes design of the control law for this a challenging task. Although Partial feedback linearization reduces a part of the system to linear but leaves the core system non-affine in nature. A novel nonlinear controller design fusing recently introduced Sliding Surface Control technique with Implicit Control of the nonlinear core is presented to tackle the issue. The task of the nonlinear controller is not only to stop the wheel but also to stabilize the pendulum at its unstable upright equilibrium in such a way that the inertial wheel stops rotating. The design procedure is simpler and more intuitive than currently available sliding surfaces, integrator backstepping or energy shaping designs. Stability is analyzed by decomposing the system into a cascade of linear and nonlinear sub-systems. Stability and advantages over existing controller designs are analyzed theoretically and verified using numerical simulations.

*Key-words:* - Inertia Wheel Pendulum, Implicit control, Multiple Sliding Surfaces, Underactuated Mechanical Systems, Backstepping

## 1 Introduction

It is known that a large class of dynamical systems in control theory can be modeled using *Euler Lagrangian* or *Hamiltonian equations* of motion, Nijmeijer [1]. This class of systems is called *Mechanical Nonlinear Control Systems*. Control of mechanical system is one of the longest living dreams of mankind. Mechanical systems can be classified into two broader classes from actuation point of view. A Mechanical system is called fully actuated mechanical system if the number of control inputs is equal to the dimension of the *Configuration Manifold* or the *Degrees Of Freedom*. An underactuated system is one with fewer independent controls than the no of degrees of freedom, Oriolo, G. and Nakamura [2].

These systems are ubiquitous in nature. These systems arise in real life applications, such as space and undersea vehicles and Different robots.

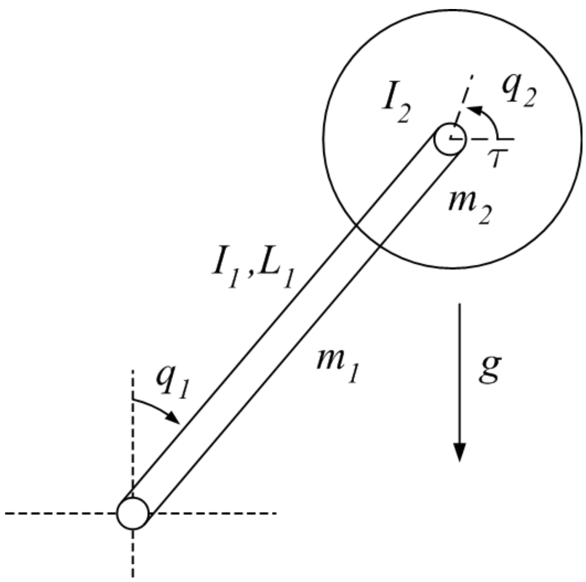


Figure 1 Inetia Wheel Pendulum

These include mobile robots, snake-type and swimming robots, acrobatic robot, flexible robots, walking, brachiating, and gymnastic robots and very recently in Micro Electro mechanical Systems.A mechanical system may be underactuated in several ways. The most obvious reasons can be listed as following

- Natural dynamics of the system (e.g. aircraft, spacecraft, helicopters, underwater vehicles, locomotive systems without wheels), Fantoni, R. Lozano [5].
- By design for reduction of the cost for some practical purposes (e.g. satellites with two thrusters and Flexible-link robots), underactuated systems also arise in mobile robot systems, for example, when a manipulator arm is attached to a mobile platform or a space platform, M. W. Spong [7].
- For achieving efficiency, an interesting example of achieved efficiency is the locomotion of animals. There is considerable experimental evidence that a great part of the swing phase in this locomotion is passive and the leg swings through like a jointed pendulum, T. A. McMahon [8]. This use of inertia and gravity coupled with the elastic energy stored and recovered from tendons, muscles, and bones, helps to account for the efficiency of animal locomotion. The study has been applied to design of passive walking bipeds, T. McGeer [9].
- Actuator failure, especially in space and marine vehicles. It is also interesting to note that certain control problems for fully actuated redundant robots are similar to those for underactuated robots, A. De Luca. [10].
- Imposed artificially to create complex low-order nonlinear systems for the purpose of gaining insight in control of high-order underactuated systems (e.g. the Acrobot, the Pendubot, the Beam-and-Ball system, the Cart-Pole system, the Rotating Pendulum, the TORA system).
- Another way that underactuated systems arise is due to the mathematical model used for control design as, for example, when joint flexibility is included in the model, M.W Spong [11].

Inertia Wheel Pendulum is one such benchmark example. The control theory for fully actuated system is very matured and systematic now. However generally applicable techniques to handle underactuated systems still lag behind. The major issue with such system is their denial to feedback linearization. Partial feedback linearization can be used to tackle the issue but this results in part of the system appearing in non-affine form. In fact, it is impossible to handle the control problem of the non-affine nonlinear system directly because, in general, even if it is known that the inverse of the involved vector field function exists, it is impossible to construct it analytically. A nonlinear controller design for stabilization of benchmark nonlinear underactuated mechanical system: Inertia Wheel Pendulum (IWP) is presented with stability analysis. The system posses non-affine nonlinear cores in addition to the underactuated nature of the system complicating considerably the control law design.

The presented design uses implicit controller design technique after decomposing the system into simpler cascades that reduces the controller design procedure significantly. This decomposition presents a way to solve the issues presented by the underactuated nature of the system. Stability is analyzed using the theory of cascaded systems. To the best of our knowledge it is the first of its type stabilization controller for IWP type Underactuated Mechanical Systems (UMS)

IWP first introduced by Spong et al. [3] is a Benchmark nonlinear UMS, mainly for Energy Shaping and Damping Injection based approaches. In [3] a supervisory hybrid/switching control strategy is applied to asymptotic stabilization of the inertia-wheel pendulum around its upright equilibrium point. First, a passivity-based controller [4] swings up the pendulum. Then, a balancing controller, obtained by Jacobian linearization or (local) exact feedback linearization stabilizes the pendulum around its upright position.

Global stabilization of IWP system using Integrator Backstepping procedure (IBS) is already known. IBS, based on results obtained by Sontag and Sussman 0, is a powerful step-by-step design tool. However it suffers the problem of “explosion of terms” besides putting stringent condition on certain system functions (being $C^n$ at least) 0. The resulting control laws are usually very lengthy and with higher degree terms. Implementation of such law is costly regarding computation efforts. Multiple Sliding Surfaces (MSS) control [12], a procedure similar to IBS, avoids this issue but falls short of integrator backstepping in terms of theoretical rigor, as the need for analytical differentiation is pushed to a numerical one.

Concept of Dynamic Surface Control (DSC), a dynamic extension to MSS, introduced by Swaroop et al. 0 resolves these issues by using low pass filters. A fusion of DSC and Control Lyapunov Method has been used successfully for stabilization of IWP and other under actuated systems successfully by authors [15], [16],[17] and [18]; however this system has another challenge present in its non-affine nonlinear core nature as the involved function is not invertible. We have used Implicit controllers techniques [19] to stabilize IWP and show the design method resulting in a less complicated control law. The designed controller is simpler than the IBS design and doesn't require a supervisory controller like the one by Spong et al [3] and presents a straightforward approach to the problem of non-affine control input present in nonlinear core.

The paper opens formally with Section II, containing the dynamical model of IWP along with necessary coordinate transformations, making the note self contained. Controller design and stability discussion that is the major topic of this note appears in section III. Section IV presents simulation results comparing controller performance to existing designs with study of initial condition effects on the system stability followed by concluding remarks in Section V.

# 2 Dynamical Model

The IWP as depicted in Figure 1is a planar inverted pendulum with a rotating wheel on the end. The joint on the base is unactuated thus; the pendulum is to be controlled only through wheel rotation. The controller task is to stabilize the pendulum in its upright unstable equilibrium position while the wheel stops rotating. The specific angle of rotation of the wheel is not important. Dynamic model of IWP can be obtained easily by Euler Lagrange method [20]. Using configuration variables as shown in Figure 1the Lagrangian for IWP is

$$L(q,\dot{q}) = \frac{1}{2}\dot{q}^T M\dot{q} - V(q_1) \tag{1}$$

the potential energy function is given as

$$V(q_1) = w\cos(q_1)$$
$$where\ w := (m_1 l_1 + m_2 L_1)g$$

IWP is a Flat Underactuated Mechanical Systems with kinetic symmetry thus all the Christoffel Symbols associated with $M$ vanish and the inertial matrix $M$ is constant and $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = Q(q)u$ gives equations of motion as

$$\begin{bmatrix} m_1 l_1^2 + m_2 L_1^2 + I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -w\sin(q_1) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix} \tag{2}$$

Where

$I_1,$ Moment of inertia of the pendulum (Kg m$^2$)
$I_2$ Moment of inertia of the wheel (Kg m$^2$)
$m_1$ Mass of the Pendulum (kg)
$m_2$ Mass of the wheel (kg)
$L_1$ Length of the Pendulum (m)
$l_1$ Distance to the center of the mass (m)
$q_1$ Angle that the Pendulum makes with the vertical
$q_2$ Angle of the wheel
$\tau$ Input Torque applied on the Wheel (Nm)

Underactuated property denies use of feedback linearization so to simplify the model for controller design collocated partial feedback linearization is employed using the following change of control [21]

$$\tau = \alpha u + \beta \tag{3}$$

where

$$\alpha = (m_{22} - m_{21}m_{12} / m_{11})$$
$$\beta = (m_{21}w / m_{11})\ \sin(q_1)$$

The coordinate transformation used is

$$\begin{aligned} z_1 &= m_{11}\dot{q}_1 + m_{12}\dot{q}_2 \\ z_2 &= q_1 \\ z_3 &= \dot{q}_2 \end{aligned} \tag{4}$$

This renders the system as

$$\dot{z}_1 = w\sin(z_2) \tag{5}$$

$$\begin{aligned} \dot{z}_2 &= \frac{1}{m_{11}} z_1 - \frac{m_{12}}{m_{11}} z_3 \\ \dot{z}_3 &= u \end{aligned} \tag{6}$$

$q_2$ doesn't play any important role in dynamics of system thus is not included as a state variable.

## 2.1 Remarks

System after coordinate transformation is a cascade interconnection of a linear double integrator subsystem (5) and a nonlinear core subsystem (6), in strict feedback form. This form is more amenable to several standard controller design techniques like IBS, MSS and DSC. Carefully note that the nonlinear part has a virtual input appearing from linear system in non-affine manner. The function is not invertible making the control law design very difficult. In next section we demonstrate how this problem can be circumvented using implicit control design technique.

# 3 Controller design and satbility analysis

## 3.1 Inner subsystem Controller design

We first design the controller for the inner sub system i.e. the non-affine nonlinear core (5) using the technique of implicit controllers, which in fact is the main emphasis of this paper and then design MSS controller for the lower part(6).

Core dynamics of inertia wheel pendulum after the coordinate transformation are given as following

$$\dot{z}_1 = w\sin(z_2) \tag{7}$$

Assuming $z_2$ as the virtual control input let $z_{2d}$ is the required control law that stabilizes (7). Note it enters in a non-affine fashion with the vector field noninvertible globally. Let us consider the first order non-affine nonlinear systems as

$$\dot{z}_1 = w\sin(z_2) = f(z_1, u) \tag{8}$$

where $z_1$ and $u \in R$ are the state and input respectively. In genral $z_1 = \theta^T \varphi_1(z_1, u) \in R$ is a non-affine function of both $z_1$ and $u$, $\theta$ and $\varphi$ being dimensionally compatible constant parameters and known regressor, respectively. To find a stabilizing control add and subtract $bu$ on the right hand side of equation (8)

$$\dot{z}_1 = \left[\theta^T \varphi_1(z_1, u) - bu\right] + bu \tag{9}$$

Consider the implicit control $u$ given by

$$u = -\frac{1}{b}\left[\theta^T \varphi_1(z_1, u) + Kz_1\right] + u,\ K > 0 \tag{10}$$

Where $b$ is a design constant. From (10), we know that $u$ is actually solved by

$$\theta^T \varphi_1(z_1, u) + Kz_1 = 0 \tag{11}$$

Accordingly, we have $\dot{z}_1 = -kz_1$, which shows that the closed-loop system is stable and $z_1$ will exponentially converge to zero. In theory, the existence of the solution for $u$ is guaranteed as the controllability condition is satisfied, S.S. GE and Spong [19]. The scheme is especially suitable for discrete time controllers where the $u$ and $x$ at right hand side of the equation are available from last clock sample, as shown for IWP.

Design of control law follows directly from results obtained as (10) and the desired virtual control is given by

$$z_{2d}(k+1) = -\frac{1}{b}\left[w\sin(z_{2d}(k)) + Kz_1\right] + z_{2d}(k),\ K > 0 \tag{12}$$

## 3.2 Outer subsystem controller design

To stabilize (5) $z_2$ is required to follow the trajectory given as(12). Applying MSS technique, we design a control law for the linear subsystem that generates the desired trajectory. It's trivial to verify that necessary assumptions [12] for MMS are satisfied by (6) regarding the system and by (12) regarding the trajectory i.e.

— $f$ is a $C^1$ function in it's arguments
— The desired trajectory is bounded and sufficiently smooth
— System has no uncertainties

*Design procedure:*

Take the first sliding surface $S_1$ as the error in generation of stabilization function (12) by $z_2$ then

$$S_1 := z_2 - z_{2d} \tag{13}$$

$$\dot{S}_1 = \dot{z}_2 - \dot{z}_{2d} = \frac{1}{m_{11}} z_1 - \frac{m_{12}}{m_{11}} z_3 - \dot{z}_{2d} \tag{14}$$

Now $z_3$ is chosen as next virtual control to drive $S_1$ to zero i.e.

$$z_{3d} = \frac{m_{11}}{m_{12}}(K_1 S_1 + \frac{1}{m_{11}} z_1 - \dot{z}_{2d}) \tag{15}$$

Similarly defining the second surface $S_2$ as

$$S_2 := z_3 - z_{3d} \tag{16}$$

$$\dot{S}_2 = \dot{z}_3 - \dot{z}_{3d} = u - \dot{z}_{3d} = -K_2 S_2 \tag{17}$$

As the control law chosen to derive $S_2$ to zero

$$u = \dot{z}_{3d} - K_2 S_2 \tag{18}$$

The resulting architecture is shown in 0Controller forces linear part to generate the desired trajectory with the derivatives of the desired trajectory being calculated numerically. Keeping in view the partial feedback linearization and coordinate transformation this control is then used to calculate the required torque for the motor.

Notice that the direct calculation of $\dot{z}_{2d}(t)$ and $\dot{z}_{3d}(t)$ required at this step by the conventional backstepping design procedure leads to complexity due to "explosion of terms". Motivated by MSS technique this problem is dealt by numerical differentiation, i.e.

$$\dot{z}_{xd} = \left(z_{xd}(k+1) - z_{xd}(k)\right)/\Delta T$$

With modern high speed digital electronics the processing speed can be set very high as compared to the slowly evolving dynamics of the mechanical system.

### 3.3 Stability analysis

It is very hard to find a Liapunov function directly for the resulting closed loop system. So for stability analysis we will be decomposing the system to a cascade of driving Linear and driven nonlinear part.

Suppose the following holds

**H1:** *The numerical error in generation of required derivatives can be bounded arbitrarily.*

**H2:** *Assuming zero input the driving system* $\Sigma_L$ *is globally asymptotically stable.*

**H3:** *The driven system* $\Sigma_N$ (the nonlinear non affine part) *with zero input from driving system* $\Sigma_L$ i.e. $S_1 = 0$, *is globally Lipschitz.*

Though we see H1 is a stringent condition but it holds for our system trivially. H2 has already been shown holding, (12). Olfati Saber (2002). Intuitively the numerical error is expected to be bounded if a very small time step for numerical calculation of the derivatives is used. We prove as following that H3 also holds.

**Proof for H1:** Error bounds for numerical differentiations can be calculated using many techniques. Here we use central difference formula. We have already assumed that $z_{2d}$ is sufficiently smooth i.e. $z_{2d} \in C^3$.

Using a second degree Taylor series expansion

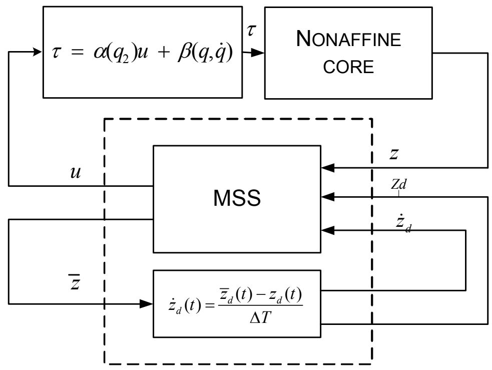


Figure 2 Proposed controller architecture

$$f(x+h) = f(x) + f'(x)h + \frac{f^{(2)}(x)h^2}{2!} + \frac{f^{(3)}(c_1)h^3}{3!} \tag{19}$$

and

$$f(x-h) = f(x) - f'(x)h + \frac{f^{(2)}(x)h^2}{2!} - \frac{f^{(3)}(c_2)h^3}{3!} \tag{20}$$

Subtracting (20) from (19)

$$f(x+h) - f(x-h) = 2f'(x)h + \frac{(f^{(3)}(c_1) + f^{(3)}(c_2))h^3}{3!} \tag{21}$$

and using mean value theorem to obtain

$$f^{(3)}(c) = \frac{f^{(3)}(c_1) + f^{(3)}(c_2)}{2} \tag{22}$$

yields

$$O(h^2) = \frac{f(x+h) - f(x-h)}{2} - f'(x) = \frac{f^{(3)}(c)h^2}{3!} \tag{23}$$

Now if the third derivative doesn't change rapidly then the numerical error goes to zero, as does $h^2$, which is expressed by using the $O(h^2)$

notation and given as $O(h^2) \le M_3 \dfrac{h^2}{6}$ where $M_3 = \max_{a \le x \le b} \left| f^{(3)}(x) \right|$.

With modern high speed digital electronics the processing speed can be set easily very high as compared to the slowly evolving dynamics of the mechanical system. Thus by keeping $h := \Delta T$ sufficiently small the error can practically be made very close to zero.

After justifying the assumptions to hold we show that the driving system $\Sigma_L$ is globally exponentially stable.

Taking the function

$$V_s = \frac{1}{2}(K_1 S_1^2 + K_2 S_2^2)$$

as a Lyapunov function candidate for $\Sigma_L$. Then $\dot{V}_c$ is given as

$$\begin{aligned} \dot{V}_s &= S_1 \dot{S}_1 + S_2 \dot{S}_2 \\ \dot{V}_s &= -K_1 S_1^2 - K_2 S_2^2 \le 0 \end{aligned} \tag{24}$$

Inequality (24) holds for positive values of $K_1$ and $K_2$.

This shows the system (24) is Globally Asymptotically Stable (GAS) but theorem 4.11 Hassan K. Khalil, shows that for linear systems asymptotic stability of the origin is equivalent to exponential stability shows that for linear systems asymptotic stability of the origin is equivalent to exponential stability. It is very important to see that the derivative cancellation of $\dot{z}_{2d}$ in (14) and that of next surface can never be exact due to two major reasons

— Numerical errors

— Parametric uncertainties

the dynamics of (24) are exponentially stable and the boundedness of the numerical error is shown arbitrarily controllable in (23) and can be made to vanish at origin using controller parameters. Application of theorems for vanishing perturbations shows that the perturbed system retains the property of being exponentially stable if $0 < h < \dfrac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)c}$, where $P$ is the solution of the Lyapunov equation. It is trivial to show that composition $\Sigma_N$ - $\Sigma_L$ satisfies all the conditions to avoid peaking as The map $f(z, S_1): R^2 \to R$ is $C^1$, The driven system $\Sigma_N$ is globally Lipschitz and globally asymptotically stable

— The driving system $\Sigma_L$ is globally exponentially stable.

— The map $f(z_1, S_1): R^2 \to R$ is $C^1$, and we have already shown after (11) that the driven system is Globally exponentially Stable and globally Lipschitz.

Thus it is trivially shown that composition satisfies all the conditions to avoid peaking, and the origin of the composition is globally asymptotically stable, Sussman H.J. et al. [24]. □

A comparison to existing designs [3] and [22] reveals the ease of design and simplicity of obtained control law. This simplicity results in implementation ease. The numerical nature of MSS is also very useful for a digital implementation that is pretty obvious with cheap availability of digital controllers with built in ADC and DACs.

# 4 Simulation Results

Numerical simulations were performed to study stability and controller performance. For fair performance comparisons we use same system parameters as R. Olfati [22] and Spong [3] i.e.

$$\begin{aligned} m_{11} &= 4.83 \times 10^{-3} \\ m_{12} &= m_{21} = m_{22} = 32 \times 10^{-6} \\ w &= 379.26 \times 10^{-3} \end{aligned}$$

Following controller parameters were used for simulations $b_1 = 2$, $K = 3$, $K_1 = 4$, $K_2 = 6$ and $\Delta T$=0.001. As obvious from analysis, $K_i$ can be set moderately high for faster convergence rates. Contrary to conventional $K_i$ tuning external layer is not needed to converge faster than the internal necessarily. Filter time constant $\Delta T$ controls boundary layer error hence it must be set as low as possible. However, DAC/ADC sample time and actuator saturation must be kept in mind as smaller As depicted Figure 3 the nonlinear controller aggressively stabilizes pendulum from its downward stable equilibrium point to its upright unstable equilibrium point with negligible transients, while wheel stops rotating, Figure 5 Swing up is faster than the design by R. Olfati [22] that requires more coordinate transformations and IBS exhibiting the phenomenon of explosion of terms. The structure is also simpler than spong's design [3] as it requires different controllers for swing up and local stabilization with a supervisory switching controller. Control effort peaks are around 0.6 Nm. As shown in Figure 4 . The wheel attains higher rotation rates as visible in Figure 5 thus motors with good rpm values are needed. A less aggressive response can be used when higher rpm motors are not available. Phase portrait depicted in Figure 6 Clearly indicates a smooth behavior. The design involves cancellation of undesired terms involving $m_{11}$ and $m_{12}$. When the parameter identification errors exist, this can affect the accuracy and stability of the method. To check these aspects simulations were also done with parametric errors. Although the controller was designed without robustification in mind yet it was able to stabilize system with variations up to 20% in system parameters around nominal values at which controller was designed. However severe effects on transient behavior of the system were observed as shown in Figure 7. This results for the extra gains that have to be added to the sliding part of the control to make it robust against errors in cancellation of derivatives. This also results in higher velocity demands for the wheel and more control effort. Simulations are also done to study the effect of a very small time step. Apparently it reduces error in derivative calculation but as depicted in Figure 8. It tends to increase control peaks in start and error in steady state.

## 5 Conclusions

A novel controller design is presented for stabilization of IWP. The control scheme employs a fusion of Implicit Controller design method with multiple sliding surface technique. The presented architecture demonstrates the potential of implicit controllers handling the non-affine structures appearing in Underactuated Mechanical Systems. Before application of control strategy collocated partial feedback linearization is applied and Model is brought to strict feedback form, more amenable to Multiple sliding surface control. Stability of the system is analyzed theoretically by considering it as a cascaded connection of two exponentially stable systems. Design simplicity is shown and Controller performance is compared to existing designs through both theoretically and simulation studies.

Variations in parameters were also introduced to check robustness and system was found robust with 20% uncertainty in parameter values. System is also studied for introducing a very high sampling rate in derivative calculation and contrary to normal notion it is found that a very small time constant results in noise in control and steady state error.

It is concluded that the presented controller architecture with a simpler design procedure while leading to a less complicated Control Law, achieves faster stabilization and presents a straight forward method for handling non-affine systems.

Further improvements envisioned are establishment of robustification on theoretical grounds.

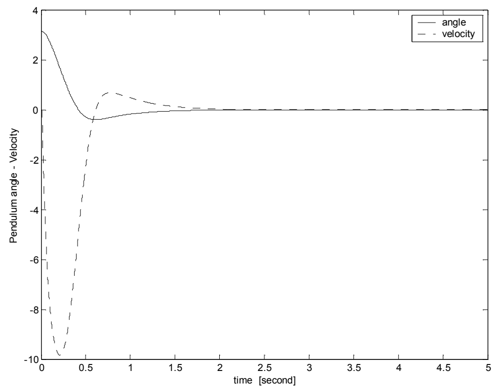


Figure 3 Pendulum angle and Velocity

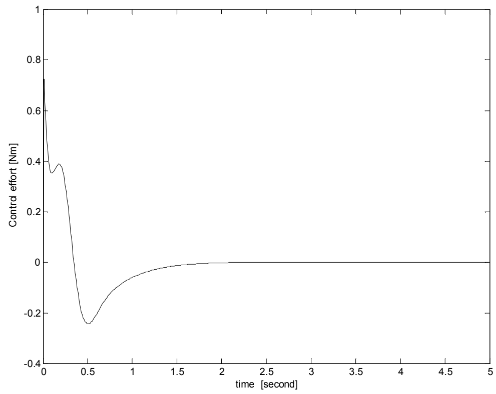


Figure 4 Control effort

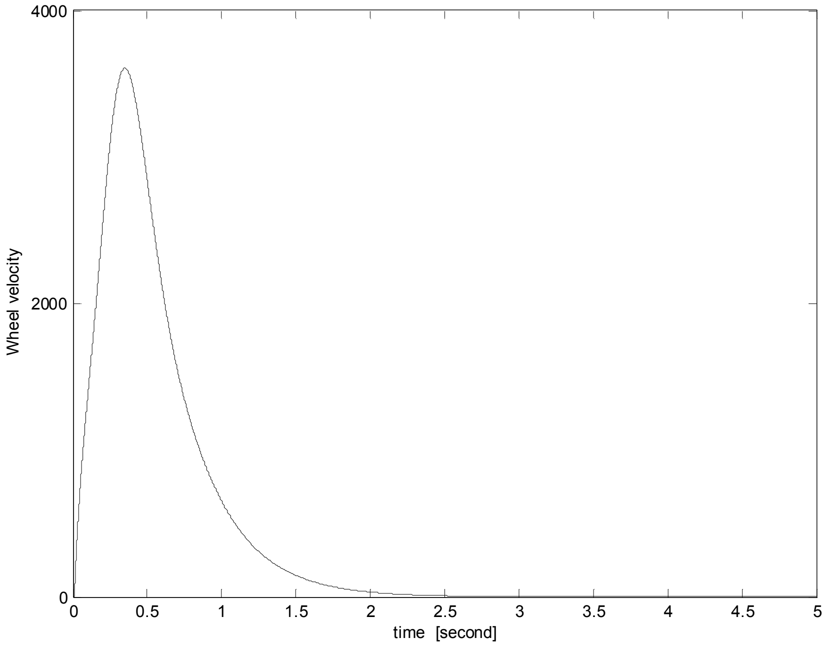


Figure 5 Wheel velocity

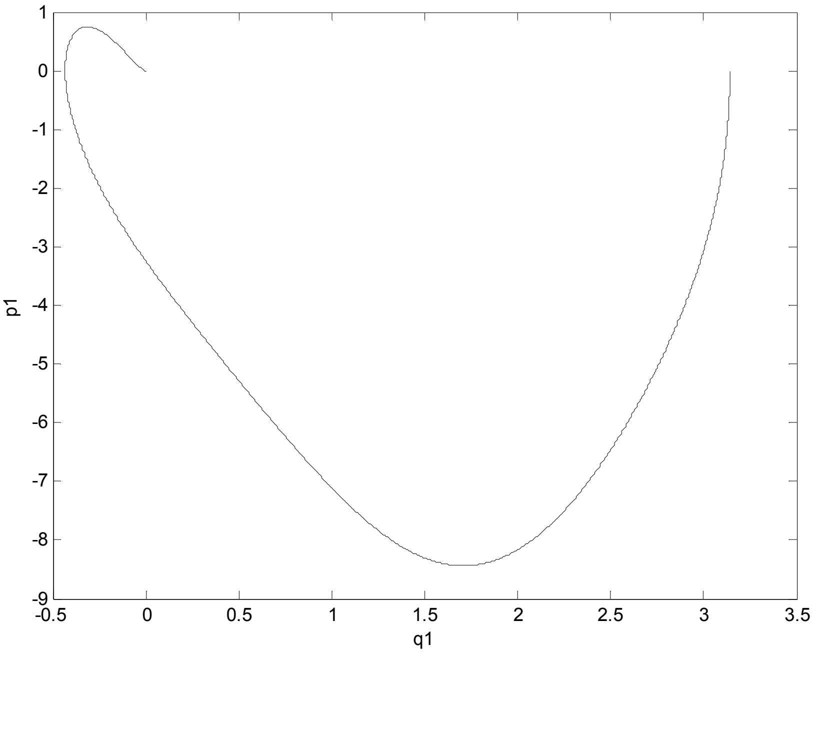


Figure 6 Phase portait

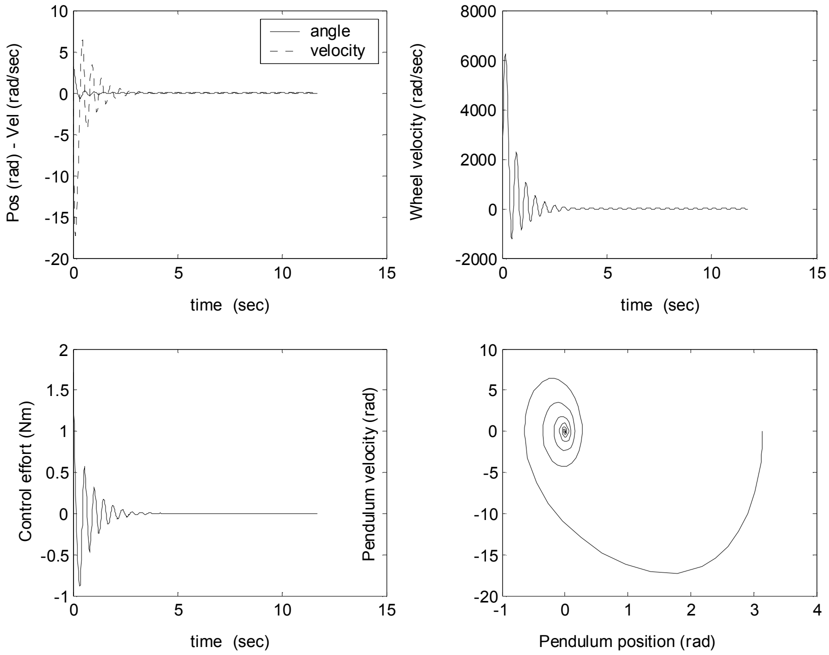


Figure 7 Stabalization with $\pm 20\,\%$ uncertainity in parameters

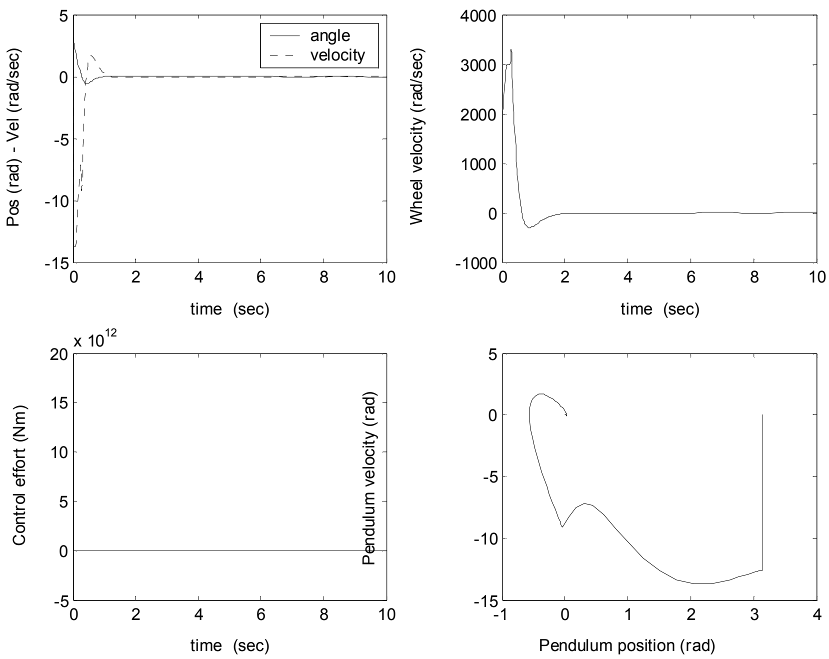


Figure 8 Simulation results for very small time step

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