





workstation cannot process other components because the buffer is a full collection of products and then stops. Hence they have analyzed the main performance measures used in the literature for the evaluation of kanban systems, including:

- Average number of WIP;
- Demand;
- Average waiting time of kanban in queue;
- Average production time;
- Average stocks number;
- Average setup time;
- Number of orders not fulfilled.

They have finally analyzed the various modelling approaches proposed in the literature, namely:

- Mathematical models;
- Queuing models;
- Markovian models;
- Simulation;
- Cost minimization model;
- Variability.

Richard P. Marek, Debra A. Elkins and Donald R. Smith (2001) [8] have introduced a simulation model of a series system controlled by kanban and CONWIP, respectively, using the software Arena. The system considered consists of 4 stations and produces 2 types of products. The costs of changeover are taken early and of low magnitude and then the setup cost is set equal to zero, assuming similar products. Assigned all the model parameters, the authors first established a model using the language of SIMAN Arena.

Consequently they have conducted experiments on the model and for each station have determined: the average time for crossing into the system, the average number of WIP, utilization of the stations and number of completed parts.

Georg N. Krieg [5, 6, 7] proposes a classification of kanban systems in:

- Single Stage Single Product Systems (fig.2);
- Single Stage Multi Product Systems (fig.3);
- Multi Stage Multi Product Systems (fig.4).

A single-stage single-product system [13] is characterized by a single production phase and only one product. In addition to the production plant, the system contains a collection box (the scheduling board, otherwise known as kanban board), an output storage for finished products, the containers for storing and handling the finished products, and a series of kanban for each product in the system (Fig. 2).

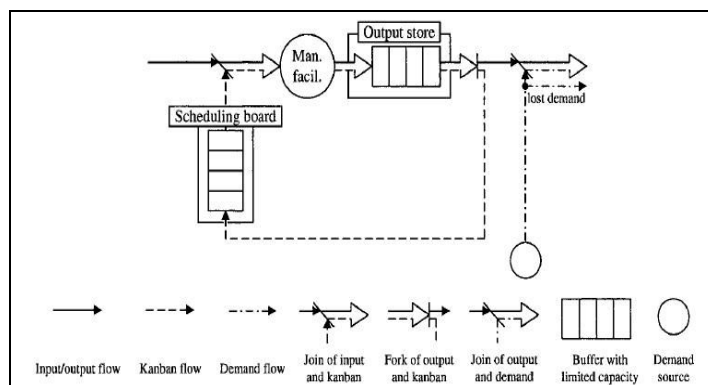


Fig. 2 – Example of a single – stage single – product Kanban system

The single-stage multi-product kanban system is always characterized by a single stage production but by several different product codes. In kanban systems the customers whose request cannot be satisfied just in time (through the inventory stocked at warehouse) or they withdraw their request and apply to another supplier offering the same product or, if they have no alternative, they wait until their request is not satisfied.

This is the standard situation for the production stages taking raw materials or parts from one supplier (the supplier may be a previous production phase, or an external supplier).

In these systems, the maximum number of backorders depends on customer number generating the request. The model of a single stage multi-product kanban system with limited number of backorders and lost demand is illustrated in fig. 3.

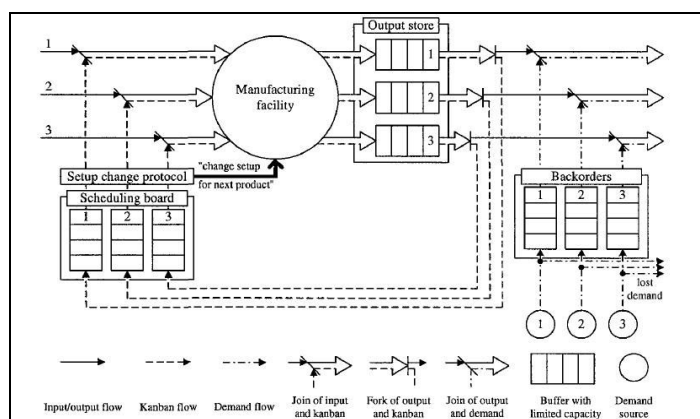


Fig. 3 – Example of a single-stage multi-product kanban system

The multi-stage multi-product kanban system is the one more generally characterized by two or more production phases and by two or more products.

In this case the products processed by a phase are input materials of the following phases (fig. 4).

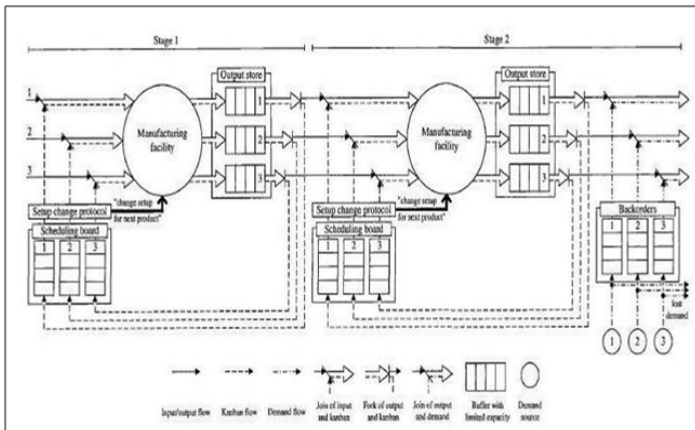


Fig. 4 – Example of a multi – stage multi – product kanban system

Mohammad D. Al - Tahat and Adnan M. Mukattash (2006) [14] have studied a synchronization mechanism for a kanban system, single - single stage - product, considering as main parameters of modelling:

- the WIP level in the system;
- the capacity at the station;
- the processing speed;
- the utilization factor of the system;
- the number server in the system;
- the arrival rate of raw materials.

The workstation line was modelled with a queuing system of type  $M / M / S: GD / \infty / \infty$  and here [it] was calculated timing parameters of the station and operating costs of the system (as the sum of the costs of raw materials, WIP in the system, the elements of production and transport).

Finally, the authors have diagrammed the influence of the number of servers on the average number of WIP in the system, the influence of utilization factor on the average number of WIP, and the influence of system parameters of total costs.

#### 4 Single – stage multi - product model

The system considered consists of a production shop and an assembly shop acting as a demand generator, so this system is a single – multi-stage – product (fig.6).

This system produces three types of components C1, C2 and C3, which are produced in the upstream manufacturing stage and are sent to the downstream assembly stage, that will mount the finished products P1, P2 e P3 (from C1, C2 and C3 respectively).

The coefficient of employment of the three products is a unit, which means that the demand for P1, P2 and P3 will be automatically translated into the demand of C1, C2 and C3 components. The two shops are decoupled from a supermarket managed by a kanban board where production orders coming from the assembly shop are collected.

This system was modelled on the basis of a series of assumptions:

- Production shop consists of a single machine, and then all products share the production plant.
- The annual demand of 540,000 pieces P1, 360,000 pieces P2 and 720,000 pieces P3 is perfectly levelled on daily basis with deterministic distribution.
- The production time is the same for all products.
- The values of demand required are already net of any waste.
- Demand arrivals are mutually independent.
- The failure rate is assumed to be zero.
- The setup time is negligible.
- The transportation time is negligible.
- The materials handling time is negligible.
- There is infinite availability of raw materials in the production shop.
- The demand is generated at the beginning of each day and the customer requires that his request be immediately satisfied. If not, the customer is willing to tolerate a delay that would not be extended beyond the day in question. If the day is spent, the request is to be considered withdrawn. Therefore in the system the backorders are not allowed and just only a temporary backlog situations is allowed.

This model was built by several sub models in Arena 8.0 [4], in particular, the block diagram depicted in Fig. 6 is structured in:

1. final customer demand sub model;
2. the kanban board logic sub model;
3. the production shop sub model;
4. the supermarket statement sub model.

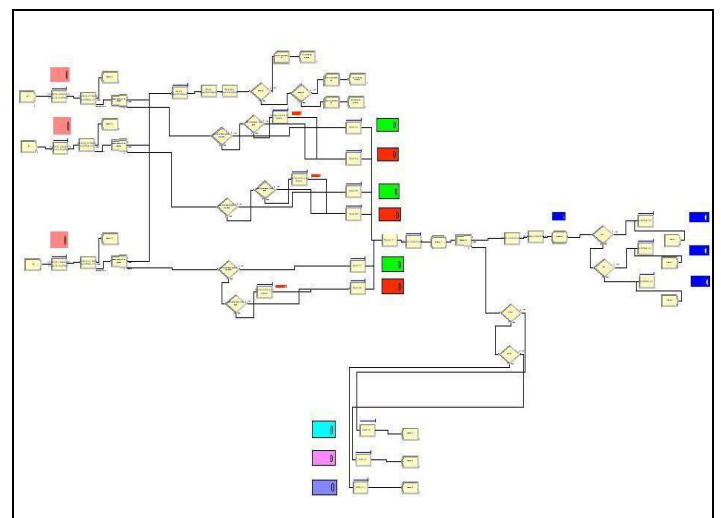


Fig. 6 - Model implemented in the Arena environment

#### 4.1 Part I: final customer demand

The demand of products P1, P2 e P3 been modelled through three create modules, having constant arrival distribution and daily arrival mode (all request arrives in the first hours of the day). The entities are retained in three modules (one for each product), and are released only when there is at least one container of that product present in the appropriate shelf of the supermarket.

A counter for each product marks the average number of retained entities in every hold, so providing a measure of the units that will be late delivered to the customer (backlog).

As soon as the entities are released, it was thought to split the entity into two distinct parts: a physical part, which represents the component which is delivered to assembly, and a signal representing the request of the recovery container consumed by the downstream shop (assembly).

#### 4.2 Part II: the kanban board

The kanban board columns are formed by two areas: the green zone – the white one and the red zone.

Each one of these zones has been modelled with a hold module. Particularly, within each hold the following priority rule of issuing cards has been set: the cards in the 3 green areas may be issued only if the 3 red zones are empty. So the red kanban have processing priority and they will be issued as soon as the machine will be free.

Since the machine can process only one container at time, the first card to be issued will be the one that belongs to the red queue once it has reached its maximum size. If no queue is saturated, the precedence is given to the red zone, which has the largest number of elements in queue.

This until all red areas has at least one element on queue. As soon as the red zone empties, we can pass to the green – white ones.

The card choosing mode to be processed is similar to the one established for the red zone, with the only difference that, once a card from the zone with the highest number of elements in queue has been taken, we will keep on processing cards of that zone until the zone will be empty or until at least one card is present in the red zone.

In this way the machine will work, when possible, the same product, reducing the setup number to process different products.

#### 4.3 Part III: the production shop

Each card issued by a hold is sent to the production shop. This shop consists of a single block process formed by size, delay, and release modules.

#### 4.4 Part IV: the supermarket

Once processed, the entity is sent to the supermarket and, particularly, through the modules, it decides on the product container, which is then sent to the appropriate shelf. The shelves are modelled with hold blocks known as “infinite hold”, holding the entities until one selecting condition through a remove module is set (using in part I to collect containers from the shelf).

### 5 Optimization and parametric analysis

The next step is to estimate the maximum size (control variable), which must have every zone of the Kanban board in order to optimize a specific chosen objective function [9, 15]. Afterwards, we will proceed to the parametric analysis of system main factors and to the study of their interactions. The optimizations are carried out with Optquest, an ARENA tool, where the objective function consists of minimizing the Total Cost ( $C_T$ ), given by the sum of the backlog cost (CB) and the holding cost (CM).

So the problem is:

$$\text{Min! } C_T = C_B + C$$

$$Z_{r,i} \geq 0$$

$$Z_{v-b,i} \geq 0$$

Where:

$Z_{r,i}$  is the maximum size of red zone of the product “i”;

$Z_{v-b,i}$  is the maximum size of green plus white zone of the product “i”;

$Z_{r,i}$  and  $Z_{v-b,i}$  are the control variables of the system.

Particularly:

- $C_M$  is the cost that the company holds up in order to holding at stock the codes of the product stored on the supermarket shelves, and it is equal to

$$C_M = \sum_{i=1,3} (G_{m,i} * C_{u,i}) \quad (1)$$

where:  $G_{m,i}$  = average stock of the product “i” over the considered period

$C_{u,i}$  = unit holding cost of maintaining to stock

- CB is the cost that the company holds up when it is not able to immediately satisfy customer requirements, so that it will have accumulated work to be done, and it is equal to

$$C_B = \sum_{i=1,3} (N_{m,r,i} * C_{u,i}) \quad (2)$$

where:  $N_{m,r,i}$  = average number of units in delay for the “i” product

$C_{u,i}$  = unit cost of backlog for the “i” product ([euro/unit]).

Another statistic for the optimization purpose is the **Service Level**, defined for each product as follows, has been considered:

$$L_{S,i} = 1 - (N_{m,r,i} / N_{m,e,i}) \quad (3)$$

where:  $N_{mR,i}$  = average number of units in delay for the “i” product  
 $N_{mE,i}$  = total number of processed orders for the “i” product

The technique of planning the used tests is of type OFAT (one factor at a time). We start the optimization of a baseline scenario (basic setting) and make one change at a time the various factors considered.

The basic setting is:

- $Z_{r,1} = 1$
- $Z_{r,2} = 1$
- $Z_{r,3} = 1$
- $Z_{v-b,1} = 3$
- $Z_{v-b,2} = 2$
- $Z_{v-b,3} = 3$ .

In order to test the validity of the solution, the initial conditions were changed several times and the result has always been the same.

Thus the optimizations were run in different scenarios, by varying some parameters such as CB, CM, the demand, the production time, the number of products, and evaluating the influence of each parameter on size zones of the Kanban board (control variable), on  $C_T$ , and on the Service Level.

**5.1 Scenario I: Backlog cost variability (CB)**

The first analysed scenario concerns the backlog cost CB. Since we started from a situation where the backlog cost and the holding cost are both unit costs, (it is thought from us) we think that it is interesting to vary, iteratively, the value of only one factor, keeping the other cost variable fixed to unit value. In particular varying this parameter we obtained the following curves.

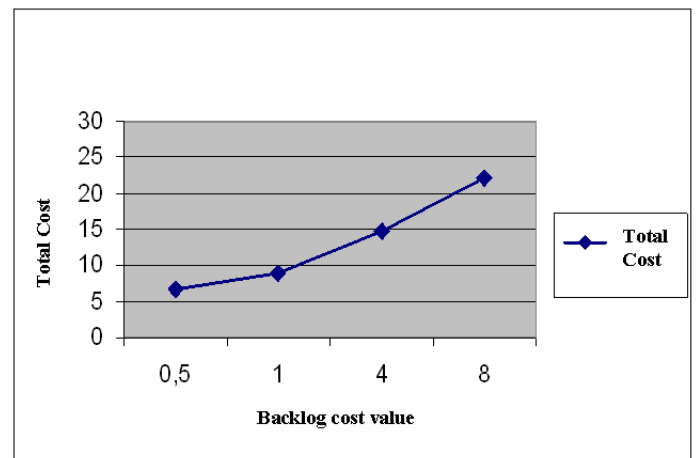
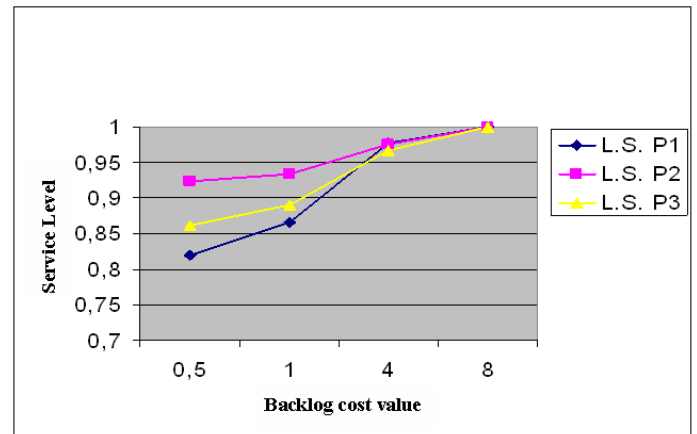
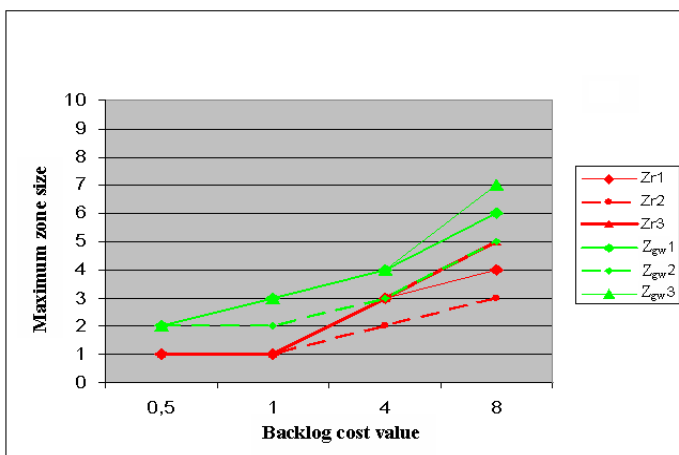


Fig. 7 - Influence of backlog cost value on the maximum kanban board size, service level and total cost

These graphs highlight an increase of size zones by level with increase backlog cost value. That is because the customer satisfaction is preferred and the delivery timeliness rather than warehouse stock reduction. Subsequently a service level increase is highlighted. From data analysis it also appears that the optimal zones find through OptQuest are always consistent with starting model data. As matter of fact the zone size of P3 product is the highest value compared to other products, since P3 is the product most required.

The consistency with expectations has also been strengthened from the analysis of total cost, already having an increasing trend to the backlog cost value increase.

**5.2 Scenario II: holding cost variability**

In this case the tendency is opposed to the previous case, as shown in fig. 8. In order to stock the least number of components, the firm must necessarily reduce the zones, following the service level reduction (since the number of pieces delivered on delay is greater); while the total cost is higher since the holding cost value increases.

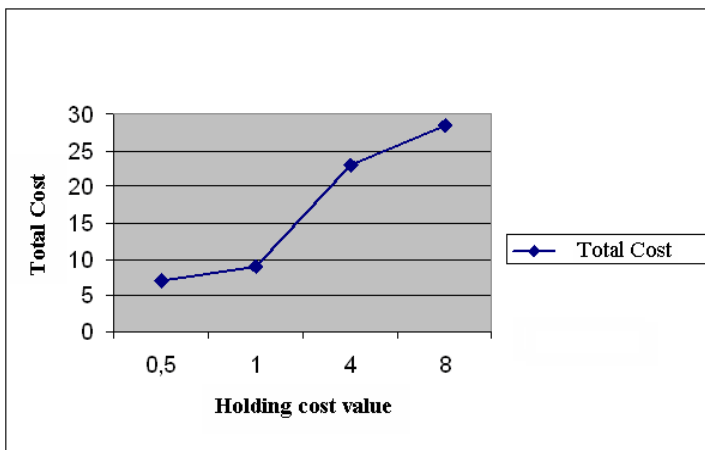
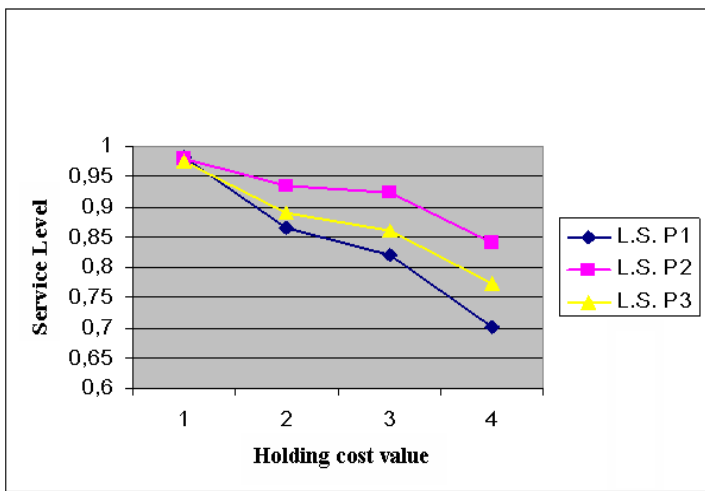
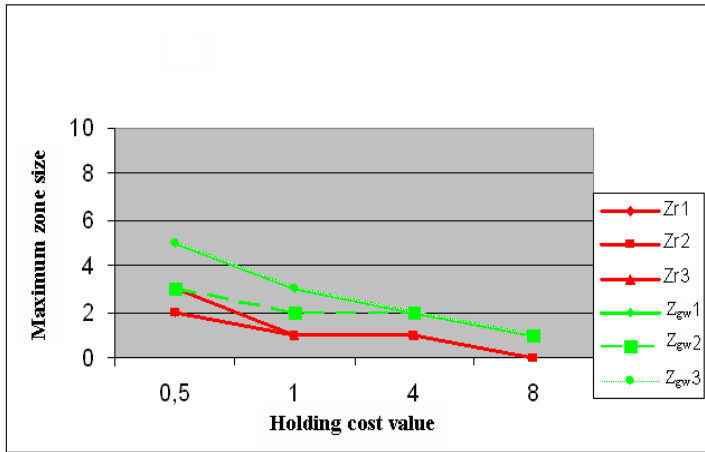


Fig. 8 - Influence of holding cost value on the maximum kanban board size, service level and total cost

**5.3 Scenario III: demand variability**

This considered scenario is related to the demand. At demand increasing the containers' number to produce increase as well. Then it implies a greater zone sizes in the kanban board in order to avoid a meaningful number of

units in delay. Besides this increase involves an overall one of service level and total cost (Fig. 9).

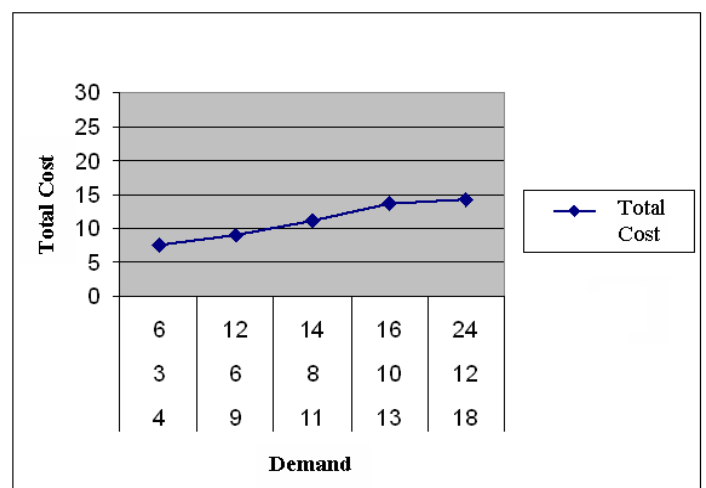
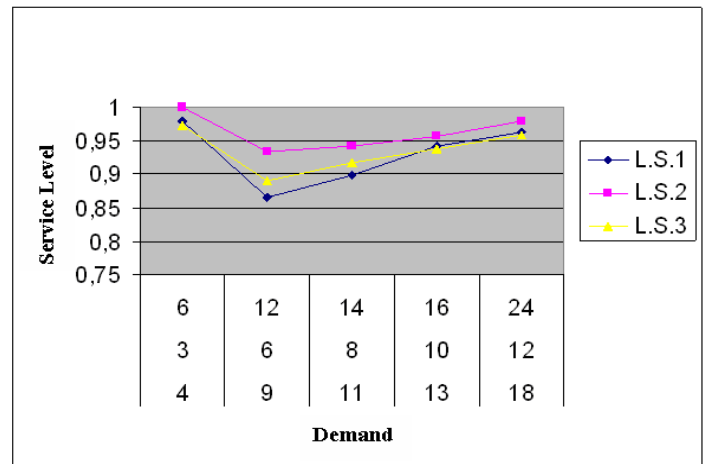
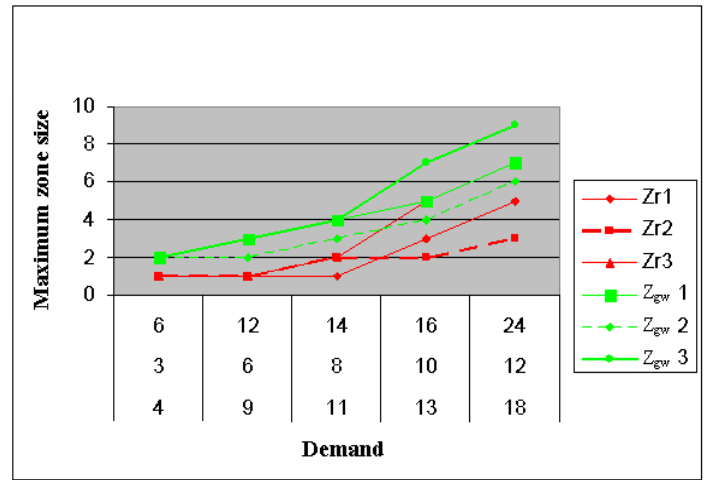


Fig. 9 - Influence of customer demand on the maximum kanban board size, service level and total cost

**5.4 Scenario IV: production time variability**

In this scenario we show the relationship between kanban size board and production time. Increasing production time, an increase of kanban board has been found. Since to process a container longer implies reintroducing codes slower, then a bigger size in the kanban board able to accept a bigger number of requests is needed. All results are shown in fig. 10.

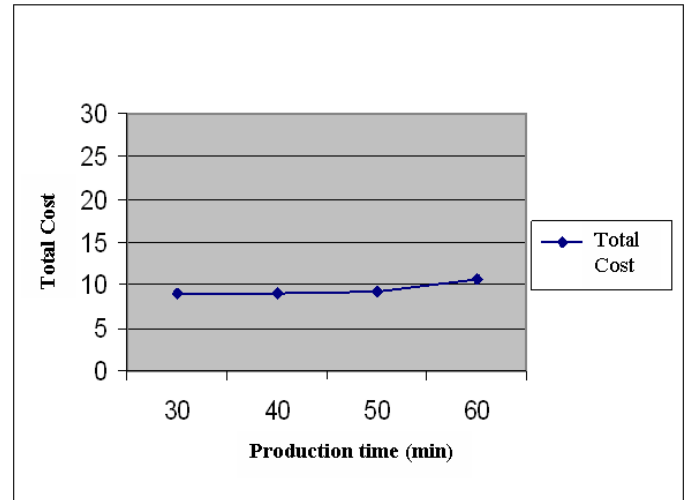
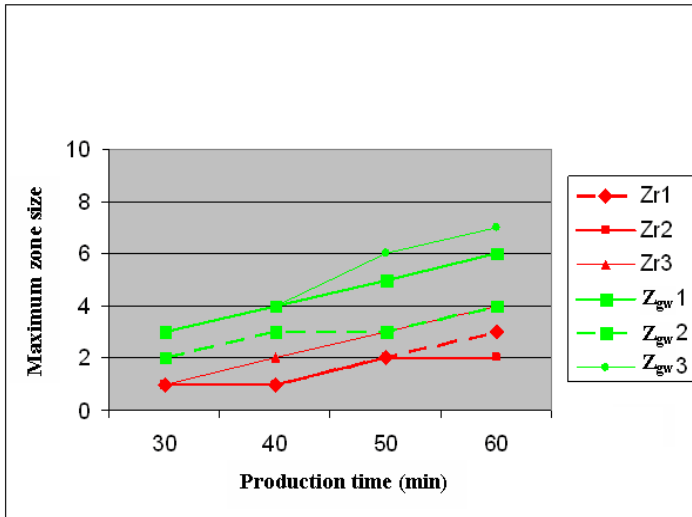
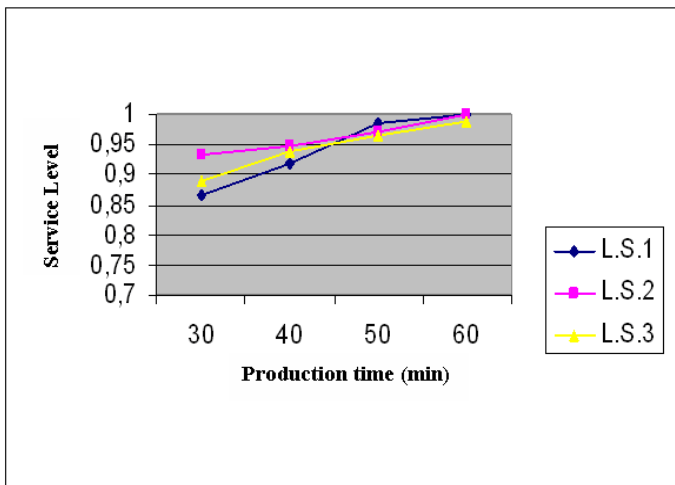
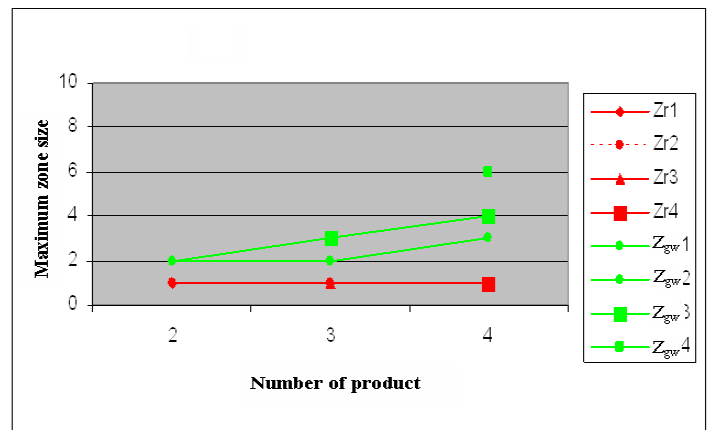


Fig. 10 - Influence of production time on the maximum kanban board size, service level and total cost.



### 5.5 Scenario V: number of products variability

The increase in the number of products has underlined an increase of kanban board zone since the management of a product or more, having only one available machine, implies production slow-downs reducing the number of delivered-on-time units to the customers. Therefore, the increasing product number increases the total cost. Instead meaningful variations in the service level are not highlighted.





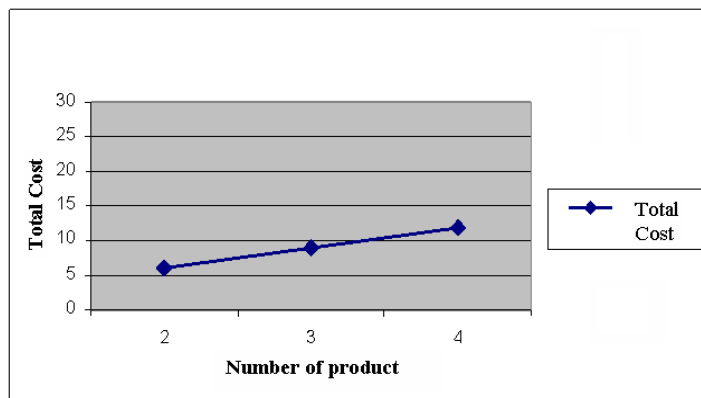


Fig. 11 - Influence of the number of product on the maximum zone size and total cost.

## 6 Conclusion

The experimental results obtained showed that the model constructed can be used as a valuable tool for strategic decision support for firms that want to design the supermarket managed with pull kanban logic: optimal size of the kanban board.

A future perspective would be to perform a DOE analysis on experimental data in order to understand the critical factors and the impact of their interactions on the scale of the regions of table (buffer size), total costs and service level.

It is also possible to stress the current model, whereby stochastic and random distribution data, then the assumption will be removed on setup time, considered to be in force on the relevant item of total cost incurred by the company.

The project is the study of multi-product with more than one phase (i.e. the system with various production departments are working like products using multiple machines in parallel) and therefore, will address the problem of multiple tables kanban sizing, more supermarket and the synchronization of the various production cells operating in logic and operated by a supermarket pull pull kanban.

The idea is to make the model more adaptable to many corporations, but more flexible in responding to the random fluctuations that characterize the production so as to constitute a valid support for the strategic management of any kanban pull system.

### References:

[1] R.G. Askin, Shravan Krishnan - Defining inventory control points in multiproduct, stochastic pull systems, *International Journal of Production Economics*, Vol.120, No.2, 2009, pp. 418-429.

[2] S. Deva Prasad, C. Rajendran, O.V. Krishnaiah Chetty - A genetic algorithmic approach to multi-objective scheduling in a Kanban-controlled flowshop with intermediate buffer and transport constraints, *The*

*International Journal of Advanced Manufacturing Technology*, Vol. 29, No. 5, 2006 , pp. 564-576(13).

[3] F. Karaesmen, Y. Dallery, A performance comparison of pull type control mechanisms for multi-stage manufacturing, *International Journal of Production Economics*, Vol.68, No1, 2000, pp. 59-71.

[4] Kelton, W. D., R. P. Sadowski, and D. T. Sturrock. 2004. *Simulation with Arena*. 3rd ed. New York: McGraw-Hill.

[5] G. N. Krieg, Heinrich Kuhn, A decomposition method for multi-product kanban systems with setup times and lost sales, *IIE Transactions*, Vol.34, No.7, pp. 613-625.

[6] G. N. Krieg, Kanban controlled manufacturing systems, Springer, 2003.

[7] G.N Krieg, Kuhn, Heinrich - Performance evaluation of two-stage multi-product kanban systems, *IIE TRANSACTIONS*, Vol.40; No.3, 2008, pages 265-283

[8] R. P. Marek, D. A. Elkins, and D. R. Smith, Understanding the fundamentals of KANBAN and CONWIP pull systems using simulation, in *Proc. Winter Simulation Conf.*, vol. 2, 2001, pp. 921-929.

[9] Romano E., Santillo L.C., Zoppoli P., Transformation of a production/assembly washing machine lines into a Lean Manufacturing System, *WSEAS TRANSACTIONS on SYSTEMS and CONTROL*, Issue 2, Vol. 4, February 2009, Pages 65-76.

[10] C. Sendil Kumar, R. Panneerselvam, Literature review of JIT-Kanban system, *The International Journal of Advanced Manufacturing Technology*, Vol. 32, No. 3-4, 2007 , pp. 393-408(16).

[11] M. Sreenivasa Rao, K. V. Sharma, A. Seshu Kumar, G. Krishna Mohana Rao, Buffer stocks in kanban controlled (traditional) unsaturated multi-stage production system, *Journal of Engineering and Applied Sciences*, Vol. 3, No. 1, 2008, pp.51-58.

[12] Treadwell, M. and Herrmann, J., A Kanban Module for Simulating Pull Production in Arena, *Proceedings of the Winter Simulation Conference*, Orlando, Fla. Dec. 4-7, 2005.

[13] W. E. Wilhelm, Pradip Som, Analysis of a single stage, single product, stochastic, MRP – controlled assembly system, *European journal of operational research*, Vol.108, No.1, 1998, pp. 74-93.

[14] M. D. Al-Tahata, A. M. Mukattashb, Design and analysis of production control scheme for Kanban-based JIT environment, *Journal of the Franklin Institute*, Vol. 343, Issues 4-5, July-August 2006, Pages 521-531.

[15] A.J. S. Santiago, A.J. Yuste, J.E. Munoz Exposito, S. G. Galan, J.M.M. Marin, S. Bruque, A dynamic-balanced scheduler for Genetic Algorithms for Grid Computing. *WSEAS TRANSACTIONS on COMPUTERS*, Vol. 8, Issues 1, January 2009,