Observer Gain Adaptation of Output Feedback Sliding Mode Controller with Support Vector Machine Regression

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Abstract: - The conventional sliding mode controller needs the exact knowledge of system state measurements. In this study, nonlinear second order systems with unmeasured system states and bounded external disturbances are considered. The sliding mode observer based on nonlinear observation error dynamics is considered and the observer gain is adjusted by using a support vector machine based plant model. From the output of the support vector machine model, *k*-step ahead predictions are obtained. Therefore, the value of *k* is first analyzed to search for a proper value. It is also shown with the simulations that the stability conditions are satisfied for the chosen observer gains. Computer simulations are presented to show the effect of the proposed gain adjustment mechanism on the performance of output feedback sliding mode controller. It is seen that the trajectory tracking performance is improved with respect to a conventional output feedback sliding mode control scheme having constant sliding mode observer gains.

Key-Words: - Sliding mode control, Sliding mode observer, Output feedback sliding mode control, Support vector machine regression, Observer gain, Bounded external disturbances.

1 Introduction

Engineers always search for better control methods to attain higher productivity than classical methods and to produce quality products at competitive prices. When unknown but bounded external disturbances are considered, sliding mode control is a promising area of study for both theoretical and application oriented robust control problems [1].

Sliding mode control is based on variable structure systems theory [1]. It is a nonlinear control method with a high-frequency chattering phenomenon that alters dynamics of a nonlinear system. The state-feedback control law switches from one continuous structure to another based on the current position of system trajectory in the state space. The control law must provide the system to move always towards a switching condition. The motion of the system as it slides along these boundaries is called a sliding mode and geometrical locus consisting of the boundaries is called sliding surface [1]. Sliding mode control structures have been applied to various engineering problems in a wide variety of application areas such as electrical motors [2], mobile robots [3], micro-electro-mechanical systems [4], chemical processes, and space systems.

For the implementation of the well-known conventional sliding mode controller (SMC) structure, exact knowledge of system state measurements is needed. However, for most applications, it is either impractical or inappropriate to use sensors for on-line measurement of all state variables considering different reasons as cost, reliability, harsh environment or even induced errors from the sensors [5]. This necessity of completely measuring the states of a system can be regarded as an important drawback of conventional SMCs.

Observers can be used to replace sensors in a control system. Therefore, a considerable amount of work has been done in the field of state estimation of dynamic systems by observers as it is an important requirement for safe and cost-effective operation of industrial units [6]. The observers are first proposed and developed for linear systems [7]. However, all practical systems inherent some degree of nonlinearity and in some cases the linear approximations based on exact linearization or pseudo-linearization may not be accurate enough.

Therefore, observer theory has been extended to include nonlinear process models [8].

The mentioned requirement for state estimation based on nonlinear observation error dynamics, and simple structure and robust stability of SMC prompted the study of sliding mode observers (SMOs) [9]. In SMOs, instead of using an output error feedback between the observer and system linearly, a nonlinear discontinuous term is injected into the observer depending on the output estimation error [10]. SMOs have an inherent robustness in the face of external disturbances and model uncertainties [11]. The equivalent control concept is proposed in [12] for linear systems where the observer states converge to the sliding surface step by step in finite time. Then, the equivalent output injection term that is defined as a counterpart of equivalent control term of SMC is applied to linear systems with unknown inputs [13].

A SMC that uses the state estimates obtained from an observer structure [14] or a SMC that only uses system outputs [15] constitute the concept of output feedback sliding mode controller (OFSMC). State estimation of nonlinear systems in the presence of external disturbances or model uncertainties is an active field of study [16-18]. The idea in [12, 13] is extended in [19] to OFSMC design in which a nonlinear system with unknown disturbances is considered.

The parallel processing capabilities of artificial neural network (ANN) architectures provides a viable means for constructing the states of complex dynamic systems from input output measurements [20]. Therefore, using soft computing methodologies in order to improve the performance of SMOs or SMCs is an active area of research.

For instance, the speed control of an induction motor using a SMC is considered in [2] and a feed-forward ANN architecture is used to estimate the rotor speed. For the SMO case, the modeling error of the ANN observer is compensated by the SMO [21]. Also, a radial basis function ANN and SMO are used in parallel in order to consider different system states or environmental variables [22]. In [23], on the other hand, the ANN observer and SMO are connected in serial and the ANN is used to obtain a nonlinear model of the system. For the SMC case, an ANN based observer is used in order to improve the SMC performance [24].

For the worst case errors, ANNs provide better performance than linear regression techniques [25]. However, ANNs have a local minima problem which is an important drawback for most control problems. Therefore, in this study, a support vector machines (SVMs) based structure is chosen. SVMs were originally created to solve classification problems. In SMC problems, SVMs are used for different purposes. For instance, the design parameters of a time-varying sliding surface for a given initial condition are obtained by using SVMs [26]. Then, the function approximation property of SVMs is used to design a new sliding surface with additional dynamic states [27]. Also, the discontinuous control law of SMC is constructed using the output of SVMs based model in order to eliminate chattering [28].

OFSMCs in the presence of unknown disturbances is examined in [19] by proving the stability under a set of nonrestrictive assumptions and it is shown that the designed controller ensures asymptotic trajectory tracking behavior. To achieve this aim, the gains of the state observer must be properly selected for an acceptable trajectory tracking performance for the observation error to converge towards zero. Therefore, selection of the observer gains is important for the stability and performance of the controller.

An observer that estimates all of the state variables is called a full-order observer. Whereas, an observer that estimates a part of the state variables is referred to be a reduced-order observer [29]. In this study, the OFSMC with a full-order observer presented in [19] is considered and SVM based plant model and controller tuning scheme given in [30] that is developed for tuning PID parameters is extended in order to improve the performance of the SMO and also to compensate the SMC output.

The conventional SMC strategy is originally designed for continuous-time operation and it is more difficult to choose a synthesis for discrete-time case [3]. The discrete-time SMC is quite different from the conventional counterpart and is also called as quasi sliding mode. Discrete-time SMC design is usually based on an approximate sliding mode system evolution due to the non-unique attractivity condition and approximate evolution on sliding surface [3]. In our study, the parameter adaptation with SVM is in discrete-time. However the state-feedback control scheme based on the SMO, SMC and the plant are all in continuous-time.

The structure of the paper is as follows: In the next section, two main components of the OFSMC scheme, the SMC and SMO, are briefly described and SMC law with system state estimates is presented. The SVM based modeling, prediction and Jacobian calculations presented in [30] are given in Section 3 and then SMO based gain adaptation scheme is presented in Section 4. Then, simulations to demonstrate the validity and advantage of the gain adaptation scheme are given in Section 5.

2 Output Feedback Sliding Mode Controller

The OFSMC consists of a SMC to generate the control law and a SMO to obtain the system state estimates from measured system output and control input. These cornerstones of the presented structure are emphasized in this section.

2.1 Continuous-time Sliding Mode Controller

The state space representation of a second order, single-input, nonlinear system in canonical form with state vector $\mathbf{x}=[x_1, x_2]^T$ can be given as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(\mathbf{x}) + b(\mathbf{x})(u(t) + d(t)) \\ y &= x_1 \end{aligned} \quad (1)$$

where $u(t)$ is the control input, $d(t)$ is the external disturbances and $f(\mathbf{x})$, $b(\mathbf{x})$ are nonlinear functions that determine the system characteristics [1, 19].

The SMC scheme involves selection of a sliding surface such that the system trajectory exhibits desirable behavior when confined to this manifold and finding feedback gains so that the system trajectory intersects and stays on the given manifold. Therefore, for system (1), assuming the trajectory tracking problem, the error dynamics for the second order system given in (1) can be written as

$$\begin{aligned} \dot{e}_1 &= x_2 - \dot{x}_{1d} \\ \dot{e}_2 &= f(\mathbf{x}) + b(\mathbf{x})(u(t) + d(t)) - \dot{x}_{2d} \end{aligned} \quad (2)$$

and from this dynamics, the conventional linear sliding surface with constant design parameters can be written as

$$s(\mathbf{e},t) = \mathbf{c}^T \mathbf{e} = c_1 e_1 + e_2 \quad (3)$$

where $\mathbf{e}=[e_1,\ e_2]^T$ is the error state vector and $e_i = x_i - x_{id}$ is the i^{th} error state variable, x_{id} is the desired trajectory of the i^{th} state and $\mathbf{c}=[c_1,\ 1]^T$ is the constant sliding surface parameter that determine the system behavior in the error phase plane. It is necessary and sufficient to differentiate (3) once for $u(t)$ to appear. Thus, this is a first order stabilisation problem based on $s(\mathbf{e},t)$. Lyapunov's direct method could be used to obtain u(t) that would keep $s(\mathbf{e},t)$ at zero. Consider a Lyapunov function candidate as

$$V(s) = \frac{1}{2} s^2(\mathbf{x},t) \quad (4)$$

with $V(0)=0$, $V(s)>0$ for $\forall s(\mathbf{x},t)>0$ [1]. An efficient condition for system stability can be given as

$$\dot{V}(s) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} s^2(\mathbf{x},t) \le -\eta \left| s(\mathbf{x},t) \right| \quad (5)$$

where η is a strictly positive real constant that determines the convergence velocity of the trajectory to the sliding surface. Obtaining the inequality in (5) means that, the distance to the surface decreases along all trajectories and this means that the system is stable. Therefore, (5) is called as the reachability condition for the sliding surface. By substituting (3) into (5) and omitting the arguments of the independent variables one obtains

$$s.(f + b.u + b.d - \dot{x}_{2d} + c_1 \dot{e}_1) \le -\eta |s| \quad (6)$$

Therefore, a control input satisfying the reaching condition can be chosen as

$$u = -b^{-1}(\mathbf{x})(f - \dot{x}_{2d} + c_1 \dot{e}_1) - k_g \operatorname{sign}(s) \hat{=} u_{eq} + u_{dis} \quad (7)$$

where k_g is a strictly positive real constant with a lower bound depending on the bounded external disturbances. The function sign(.) denotes the signum function defined as follows

$$\operatorname{sign}(s) = \begin{cases} -1 & \text{if } s < 0 \\ 0 & \text{if } s = 0 \\ 1 & \text{if } s > 0 \end{cases} \quad (8)$$

The control input in (7) consists of two parts. The first part, u_{eq} is the continuous term that is known as equivalent control based on estimated system parameters and it compensates the estimated undesirable dynamics of the system. The second part with the signum function is the discontinuous control law, u_{dis} that requires infinite switching on the part of the control signal and actuator at the intersection of error state trajectory and sliding surface. In this way, the trajectory is forced to move always towards the sliding surface [1].

2.2 Sliding Mode Observer

The state estimation problem for a system subject to unknown external disturbances under output feedback sliding mode control with an equivalent output injection sliding mode observer is considered in [19]. In this study, the sliding mode observer structure presented in [19] is used.

For the system given in (1), only the system output y is measured. Therefore, the error dynamics (2) could not be obtained. The system states and also the error dynamics can be obtained from y by using an observer of the form given as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \lambda_1 \operatorname{sgn}(y - \hat{x}_1) \\ \dot{\hat{x}}_2 &= f(\hat{\mathbf{x}}) + b(\hat{\mathbf{x}})u + E_1 \lambda_2 \operatorname{sgn}(\tilde{x}_2 - \hat{x}_2) \end{aligned} \quad (9)$$

where $\tilde{x}_2 = \hat{x}_2 + (\lambda_1 \operatorname{sgn}(x_1 - \hat{x}_1))_{eq}$ and the equivalent output injection term $(\lambda_1 \operatorname{sgn}(x_1 - \hat{x}_1))_{eq}$ is obtained by using a low pass filter [5, 19]. The term E_1=0 if $x_1 - \hat{x}_1 \neq 0$ and E_1=1 otherwise [19]. With proper λ_1, λ_2 observer gains, the observer state $\hat{x}_1$ firstly converges to x_1 and then $\hat{x}_2$ converges to x_2.

For the given system (1), finite time convergence of system state estimates to actual plant states is proved in the literature [19]. Therefore, instead of (3) obtained from (2) one can use

$$\hat{s}(\hat{\mathbf{e}},t)=\mathbf{c}^T\hat{\mathbf{e}}=c_1e_1+e_2 \tag{10}$$

where $\hat{\mathbf{e}} = [\hat{e}_1, \hat{e}_2]^T$ is estimated error state vector and $\hat{e}_i=\hat{x}_i-x_{id}$ is i^{th} estimated error state variable. If the system states are not measurable, the conventional form of u_{eq} using state estimates can be rewritten as

$$\hat{u}_{eq}(t)=-b^{-1}(\hat{\mathbf{x}})\left(f(\hat{\mathbf{x}})-\dot{x}_{2d}+c_1\dot{\hat{e}}_1\right) \tag{11}$$

Then, the overall control law based on estimated system states can be designed as

$$\hat{u}(t)=\hat{u}_{eq}(t)+k_g\operatorname{sgn}(\hat{s}) \tag{12}$$

Choosing the Lyapunov function candidate as V=(1/2) $\hat{s}^2$ using estimated state variables and taking the derivative of the Lyapunov function along the trajectories of the estimated system states, the discontinuous control gain k_g must be chosen as [19]

$$k_g=-b^{-1}(\hat{\mathbf{x}})(c_1\lambda_1+\lambda_2+\eta) \tag{13}$$

in order to satisfy the reaching condition.

3 Support Vector Machine based Modeling, Prediction and Jacobian Calculations

Consider a nonlinear system, dynamics of which can be represented by NARX model

$$y_n=f(u_n,\cdots,u_{n-n_u},y_{n-1},\cdots,y_{n-n_y}) \tag{14}$$

where u_n is the control signal applied to the plant at time index n, y_n is the corresponding output of the plant, and n_u and n_y denote the number of past control signals and number of the past outputs involved in the model, respectively. It is assumed that non-linear function f is unknown and that a training data set is obtained in the form given as

$$\begin{aligned} T_{set}&=\{u_k,\cdots,u_{k-n_u},y_{k-1},\cdots,y_{k-n_y}\}_{k=n}^{k=n+N} \\ &= \{\mathbf{x}_k,y_k\}_{k=n}^{k=n+N} \end{aligned} \tag{15}$$

where $\mathbf{x}_k \in X \subseteq \mathbb{R}^{n_u+n_y+1}$ is the k^{th} input data point in input space and $y_k \in Y \subseteq \mathbb{R}$ is the corresponding output value. It is desired to obtain a model that represents the relationship between input and output data points. The training data set T_{set} is to be used to obtain an approximate model of the plant dynamics.

The primal form of an SVM regression model is given by (16), which is linear in a higher-dimensional feature space **F**

$$\hat{y}(\mathbf{x})=\langle\mathbf{w},\mathbf{\Phi}(\mathbf{x})\rangle+bias \tag{16}$$

where **w** is a vector in the feature space **F** , **Φ(x)** is a mapping from the input space to the feature space, *bias* is the bias term and <,> stands for inner product operation in **F** [30].

The SVR algorithms regard the regression problem as an optimization problem in dual space in which the model is given by

$$\hat{y}(\mathbf{x})=\sum_{i=1}^{N}\alpha_iK(\mathbf{x},\mathbf{x}_i)+bias \tag{17}$$

where α_i 's are the coefficients of each training data and $K(\mathbf{x}_i,\mathbf{x}_j)$ is the kernel function given by $K(\mathbf{x}_i,\mathbf{x}_j)=\mathbf{\Phi}(\mathbf{x}_i)^{\mathrm{T}}\mathbf{\Phi}(\mathbf{x}_j)=K_{ij}$ [30]

The kernel function $K(\mathbf{x}_i,\mathbf{x}_j)$ handles inner product in feature space and hence the explicit form of **Φ(x)** does not need to be known. In the model (17), a training point $\mathbf{x}_i$ corresponding to a non-zero α_i value is referred to as the support vector. In [30], ε-SVR algorithm employing Vapnik's ε-insensitive loss function $L(\varepsilon, y, \hat{y})$ given as

$$L(\varepsilon,y,\hat{y})=\begin{cases}0, & y_i-\hat{y}_i\leq\varepsilon \\ y_i-\hat{y}_i, & y_i-\hat{y}_i>\varepsilon\end{cases} \tag{18}$$

is used which formulates the primal form of the regression problem as follows:

$$\min_{w,bias,\xi,\xi^*} P_\varepsilon=\frac{1}{2}\|\mathbf{w}\|^2+C\sum_{i=I}^{N}\left(\xi+\xi_i^*\right) \tag{19}$$

subject to the constraints,

$$y_{\mathrm{i}}-\langle\mathbf{w},\mathbf{\Phi}(\mathbf{x}_i)\rangle-\tau\leq\varepsilon+\xi_i \tag{20}$$

$$\langle\mathbf{w},\mathbf{\Phi}(\mathbf{x}_i)\rangle+\tau-y_i\leq\varepsilon+\xi_i \tag{21}$$

$$\xi_i,\xi_i^*\geq0,\qquad i=1,2,...,N \tag{22}$$

where ε is the upper value of tolerable error, ξ_i , ξ_i^* are slack variables, || . || is the Euclidean norm and C is a regularization parameter that provides a compromise between model complexity and degree of tolerance to the errors larger than ε [30].

Dual form of the optimization problem becomes a QP problem as

$$\min_{\beta,\beta^*} D_\varepsilon = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} K_{ij}\left(\beta_i-\beta_i^*\right)\left(\beta_j-\beta_j^*\right) + \varepsilon\sum_{i=1}^{N}\left(\beta_i+\beta_i^*\right) - \sum_{i=1}^{N} y_i\left(\beta_i-\beta_i^*\right) \quad (23)$$

subject to the constraints,

$$0 \le \beta_i, \beta_i^* \le C, \qquad \sum_{i=1}^{N}\left(\beta_i-\beta_i^*\right)=0, \quad i=1,2,...,N \quad (24)$$

solution of the QP problem (23) and (24) gives the optimum values of β_i and β_i^*'s. The value of bias in the model is determined as follows: the condition $\hat{y}(\mathbf{x}_i) - y_i = -\varepsilon$ is satisfied for each support vector $\mathbf{x}_i$ for which the condition $0 \le \beta_i - \beta_i^* \le C$ holds. If α_j is defined to be the new coefficient of $\mathbf{x}_j$ for j=1, 2,…, N as $\alpha_j = \beta_i - \beta_i^*$, then an SVM model as given by (17) is obtained. Moreover, when only support vectors are considered, the model becomes,

$$\hat{y}(\mathbf{x}) = \sum_{\substack{j=1 \\ j\in SV}}^{\#SV} \alpha_j K\left(\mathbf{x},\mathbf{x}_j\right) + bias \quad (25)$$

where #SV is the number of support vectors. If we follow the procedure given in [30], then we construct the current state vector as,

$$\mathbf{v}_n = [u_n, \cdots, u_{n-n_u}, y_{n-1}, \cdots, y_{n-n_y}]^T \quad (26)$$

then the corresponding output of the SVM model becomes,

$$\hat{y}(\mathbf{x}) = \sum_{\substack{j=1 \\ j\in SV}}^{\#SV} \alpha_j K\left(\mathbf{v}_n,\mathbf{x}_j\right) + bias \quad (27)$$

In SVM-based observer gain adaptation, a radial basis adopted kernel function is used that is given as

$$K_{ij} = K\left(\mathbf{x}_i,\mathbf{x}_j\right) = \exp\left(\frac{(\mathbf{x}_i\text{-}\mathbf{x}_j)^T(\mathbf{x}_i\text{-}\mathbf{x}_j)}{2\sigma^2}\right) \quad (28)$$

where σ is the width parameter [30]. If $D(j,n)$ is defined as Euclidean distance between j^{th} support vector $\mathbf{x}_j$ and current state vector $\mathbf{v}_n$ as

$$\begin{aligned} D(j,n) &= (\mathbf{v}_n - \mathbf{x}_j)^T(\mathbf{v}_n - \mathbf{x}_j) \\ &= \sum_{i=1}^{n_u}(x_{j,i+1} - u_{n-i})^2 + \sum_{i=1}^{n_u}(x_{j,nu+i+1} - y_{n-i})^2 \end{aligned} \quad (29)$$

then the kernel function can be rewritten as,

$$K\left(\mathbf{v}_n,\mathbf{x}_j\right) = \exp\left(-\frac{D(j,n)}{2\sigma^2}\right) \quad (30)$$

and the SVM regression model becomes,

$$\hat{y}_n = \sum_{j=1}^{\#SV} \alpha_j \exp\left(-\frac{D(j,n)}{2\sigma^2}\right) + bias \quad (31)$$

Now, (31) can be used to predict k-step ahead future trajectory of the plant as by

$$\hat{y}_{n+k} = \sum_{j=1}^{\#SV} \alpha_j \exp\left(-\frac{D(j,n+k)}{2\sigma^2}\right) + bias, \quad k=1,2,...,K \quad (32)$$

where

$$\begin{aligned} D(j,n+k) &= \sum_{i=1}^{\min(k,n_y)}(x_{j,nu+i+1} - \hat{y}_{n+k-i})^2 \\ &+ \sum_{i=k+1}^{n_y}(x_{j,nu+i+1} - y_{n+k-i})^2 \\ &+ \sum_{i=k+1}^{n_u}\begin{cases}(x_{j,i+1} - u_{n+k-i})^2, & k-i < 1 \\ (x_{j,i+1} - u_{n+1})^2, & k-i \ge 1\end{cases} \end{aligned} \quad (33)$$

then first order partial derivatives can be written as

$$\frac{\partial \hat{y}_{n+k}}{\partial u_{n+1}} = \sum_{j=1}^{\#SV} \alpha_j \frac{\partial \exp\left(-\frac{D(j,n+k)}{2\sigma^2}\right)}{\partial u_{n+1}} \quad (34)$$

where

$$\begin{aligned} \frac{\partial \exp\left(-\frac{D(j,n+k)}{2\sigma^2}\right)}{\partial u_{n+1}} &= \\ &= \frac{\partial \exp\left(-\frac{D(j,n+k)}{2\sigma^2}\right)}{\partial d_{j(n+k)}} \frac{\partial D(j,n+k)}{\partial u_{(n+1)}} \\ &= -\frac{1}{2\sigma^2}\exp\left(-\frac{D(j,n+k)}{2\sigma^2}\right)\frac{\partial D(j,n+k)}{\partial u_{n+1}} \end{aligned} \quad (35)$$

and

$$\begin{aligned} \frac{\partial D(j,n+k)}{\partial u_{n+1}} = &\sum_{i=1}^{\min(k,n_y)}[(-2)(x_{j,n_u+i+1} - \hat{y}_{n+k-i}) \\ &\text{x}\frac{\partial \hat{y}_{n+k-i}}{\partial u_{n+1}}\delta_1(k-i-1)] \\ &+ \sum_{i=0}^{n_u}[(-2)(x_{j,i+1} - u_{n+1})\delta_1(k-i-1)] \end{aligned} \quad (36)$$

where $\delta_1 = (.)$ stands for unit step function [30]. Now, the first-order terms can be used to calculate the Jacobian matrix (41).

4. Support Vector Machine Based Observer Gain Adaptation

The proposed SVM based sliding mode observer gain adaptation scheme is given in Fig.1. It is adopted from the study proposed in [30] which is first used for tuning PID controller parameters. The idea is mainly based on obtaining the k-step ahead predictions of the plant output by using a SVM model and a Jacobian block for tuning SMO gains.

The SVM model is obtained by applying randomly chosen bounded control signals to the plant. After the training process, k-step ahead predictions $\hat{\mathbf{y}} = [\hat{y}_{n+1}, \hat{y}_{n+2}, ..., \hat{y}_{n+k}]$ are obtained from the output of the SVM model with t_d time durations as shown in Fig.1. Then, in order to minimize the SVM prediction error and to penalize the unwanted rapid changes in the control input, an objective function is chosen as [30]

$$\phi(u_{n+1}) = \frac{1}{2}\left(\sum_{i=1}^{k} \varepsilon_{n+i}^{2} + \rho (u_{n+1} - u_{n})^{2} \right) \quad (37)$$

where $\varepsilon_{n+i} = y_{n+i}^{d} - \hat{y}_{n+i}$ is the prediction error of SVM at i^{th} step, y_{n+i}^{d} is the known desired output at i^{th} step, and ρ determines the amount of penalty on the control deviations. In this study, the proposed idea in [30] is applied to the OFSMC case by choosing the observer gain λ_1 as the updated parameter. In order to have a numerical solution to the problem of minimizing (37), Levenberg-Marquardt learning rule, which interpolate between Gauss-Newton and steepest descent algorithms, can be written as

$$\lambda_1^{new} = \lambda_1^{old} - \left(\mathbf{J}^{T}\mathbf{J} + \kappa \mathbf{I}\right)^{-1} \mathbf{J}^{T} \boldsymbol{\varepsilon} \quad (38)$$

where κ is a blending factor which determines a mixing ratio between gradient-descent and Gauss-Newton algorithms, and $\boldsymbol{\varepsilon}$ is the prediction error vector which is defined as

$$\boldsymbol{\varepsilon} = [\varepsilon_{n+1} \quad \varepsilon_{n+2} \quad \cdots \quad \varepsilon_{n+k} \quad \sqrt{\rho}(u_{n+1} - u_{n})]^{T} \quad (39)$$

The plant's desired output trajectory does not depend on observer gains. Therefore, Jacobian matrix $\mathbf{J}$ in (38) can be obtained from (37) as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \hat{y}_{n+1}}{\partial \lambda_1} & \frac{\partial \hat{y}_{n+2}}{\partial \lambda_1} & \cdots & \frac{\partial \hat{y}_{n+k}}{\partial \lambda_1} & \frac{\sqrt{\rho}\,\partial u_{n+1}}{\partial \lambda_1} \end{bmatrix}^{T} \quad (40)$$

Using the presented scheme, the observer gain λ_1 is updated discretely at every sampling period t_d and used to update the SMO that is in continuous time.

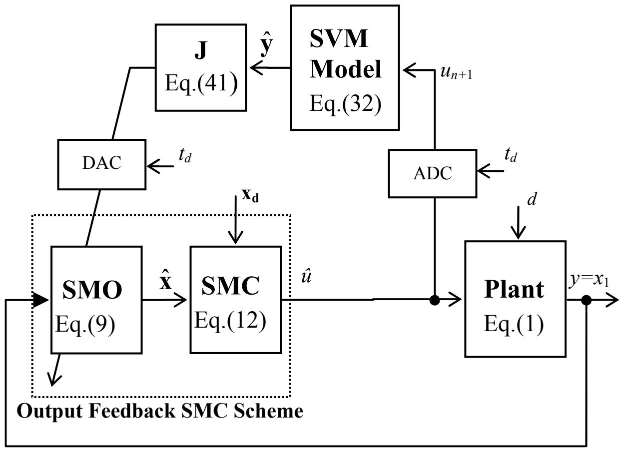

Figure 1. Schematic diagram of the SVM based observer gain adaptation and control law compensation scheme.

The control law compensation mechanism can also be obtained by splitting the Jacobian matrix (40) into two different parts by using second order Taylor approximation of (37) [30]. Thus, applying the given idea to the observer gain adaptation scheme, the Jacobian matrix can be written as follows

$$\mathbf{J} = -\begin{bmatrix} \frac{\partial \hat{y}_{n+1}}{\partial u_{n+1}} & \frac{\partial \hat{y}_{n+2}}{\partial u_{n+1}} & \frac{\partial \hat{y}_{n+k}}{\partial u_{n+1}} & \sqrt{\rho} \end{bmatrix}^{T} \frac{\partial u_{n+1}}{\partial \lambda_1} \quad (41)$$

The partial derivative of u_{n+1} with respect λ_1 could be directly obtained by solving the equations from the SMO and SMC blocks which have both nonlinear structures. This nonlinear structure raises difficulties in obtaining the mathematical solution. Therefore, in this study, the numerical solutions are obtained by using the approximation given as

$$\frac{\partial u_{n+1}}{\partial \lambda_1} \cong \frac{\Delta u_{n+1}}{\Delta \lambda_1} \quad (42)$$

From the stability analysis given in [19], in order to provide the finite time convergence of the estimated states to the actual states the observer gains must satisfy the conditions given as

$$\begin{aligned} &\lambda_1 > \left| x_2 - \hat{x}_2 \right| + \mu_1 \\ &\lambda_2 > \left| f(\mathbf{x}) + b(\mathbf{x})(u+d) - (f(\hat{\mathbf{x}}) + b(\hat{\mathbf{x}})u \right| + \mu_2 \end{aligned} \quad (43)$$

where μ_1, μ_2 are small positive real constants [19]. Therefore, these bounds must be provided when tuning the λ_1 observer gain. Initially, λ_1 is set to acceptable values that provide (43). Proper choice of the gains λ_1 and λ_2 will guarantee that the reduced order dynamics are stable on the sliding surface and this will ensure asymptotic stability of the reference trajectory. To have a better observer performance and thus to provide a better output tracking performance, this preset value should be tuned properly.

5 Simulation Studies

To show the performance of the new tuning scheme, computer simulations are performed on a nonlinear mass-spring-damper system on a horizontal surface under the effect of a horizontal force. The dynamic equations of the system is described as

$$\begin{aligned} &m\ddot{x}+v(\dot{x},t)+k(x,t)=u(t)+d(t)\\ &v(\dot{x},t)=v_0\dot{x}+v_1\dot{x}\left|\dot{x}\right| \\ &k(x,t)=k_0x+k_1x^3 \end{aligned} \tag{44}$$

where m is the mass, $x(t)$ is the displacement, $\dot{x}(t)$ is the velocity, $v(\dot{x},t)$ and $k(x,t)$ are nonlinear terms with respect to the damper and spring, respectively. By taking $x_1 = x$, $x_2 = \dot{x}$ and by rewriting the system equations (44) in the form of (1), one can obtain

$$\begin{aligned} &f(\mathbf{x})=\frac{1}{m}(-v(\dot{x},t)-k(x,t)+u(t)+d(t))\\ &b(\mathbf{x})=1/m \end{aligned} \tag{45}$$

The system parameters in (44) are chosen as, m=1, v_o=v_1=0.35 and k_o=k_1=0.55. The initial state values are chosen as $x_1(0)$=0.5, $x_2(0)$=0. The trajectory tracking problem is considered and the desired state trajectories are chosen as

$$\begin{aligned} &x_{d1}(t)=-0.5\cos(\pi t/5)\\ &x_{d2}(t)=0.1\pi\sin(\pi t/5) \end{aligned} \tag{46}$$

During the simulations, in order to show the robustness against bounded external disturbances, $d(t)$ is modeled with a sinusoidal signal taken as

$$d(t)=0.05+0.25\cos(3\pi t) \tag{47}$$

The SMO for all OFSMCs is taken as (9) and to obtain $\tilde{x}_2$, first order low pass filter with bandwidth w_n=20 rad/s is used. For all of the controllers, the sliding surface parameter is taken as c_1=7.

Simulations have been carried out in Matlab environment and ordinary differential equation solver implementing Runge-Kutta numerical integration method has been selected for simulating the discontinuous nature of sliding mode controller and observer. For the simulation environment, a fixed sampling time of 2e-4s has been applied for simulating the continuous time observer, controller and plant. On the other hand, the SVM block works in a discrete nature by taking observations and calculating the update value at every t_d = 1e-2 s time durations. All simulations are performed in the time interval between [0, 5] s. The system performance is influenced by the selection of the observer initial conditions. Therefore, assuming that the initial values of system states $x_1(0)$ and $x_2(0)$ are at the origin in average $\hat{x}_1(0)=0$ and $\hat{x}_2(0)=0$.

The SVM predicts k-step ahead system behavior and k is a design parameter. To analyze the effect of k on the performance indices and control input magnitude, the system is simulated for different values of k between [2:10] and the results are given in Fig.2-3. As can be seen from Fig.2, the performance has its best values for k=2 and the performance is then similar for $k \geq 4$. However, from Fig.3, it is seen that for k=2, this performance improvement has a trade-off as an increased control input magnitude. Therefore, k=2 and 4 are chosen for comparison. The trajectory tracking and state estimation performances are given in Fig.4 and the control inputs are given in Fig.5 for k=2 and 4, respectively.

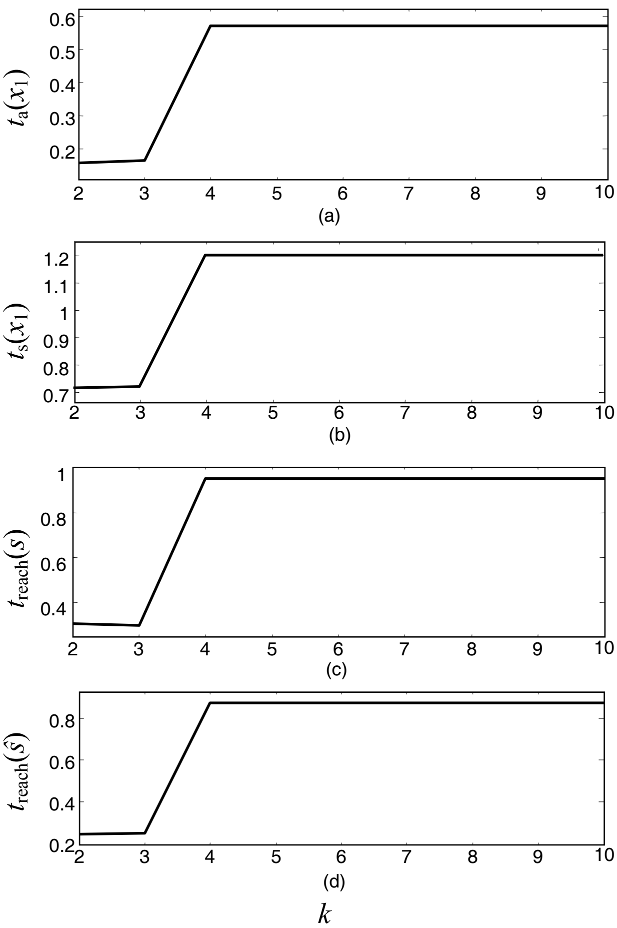

Figure 2. Performance indices for different k values.

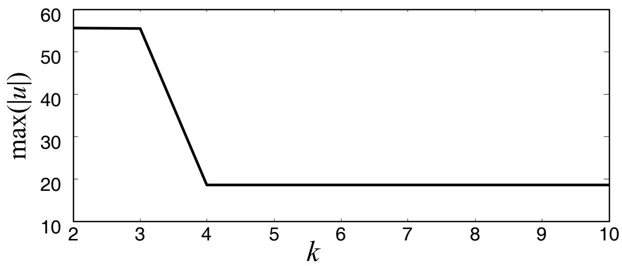

Figure 3. Control input magnitude max(|u|).

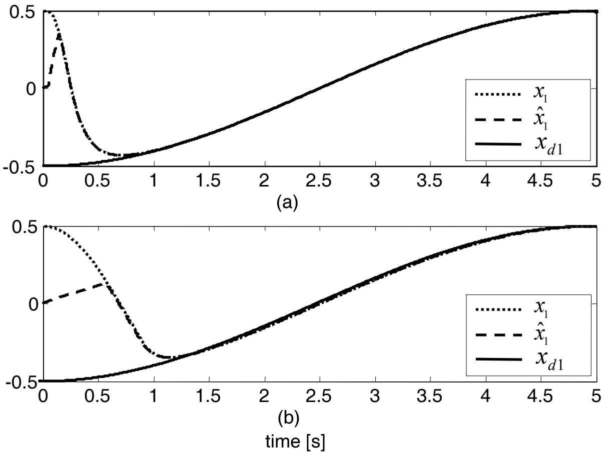

Figure 4. Actual, estimated and reference system output y=x1 for a) k=2 b) k=4.

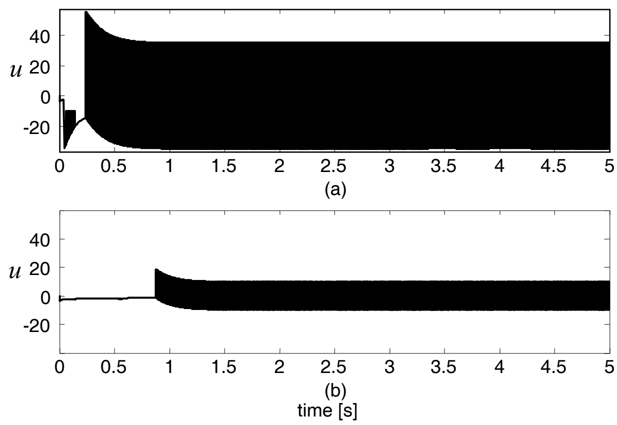

Figure 5. Control inputs for a) k=2, b) k=4.

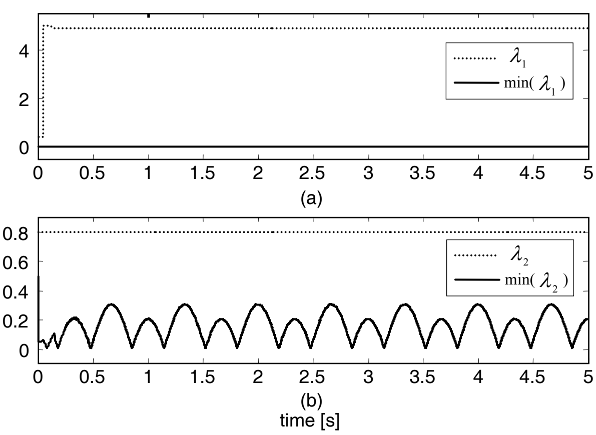

Figure 6. Observer gains and their stability bounds in (43) for k=2: a) λ_1, min(λ_1), b) λ_2, min(λ_2).

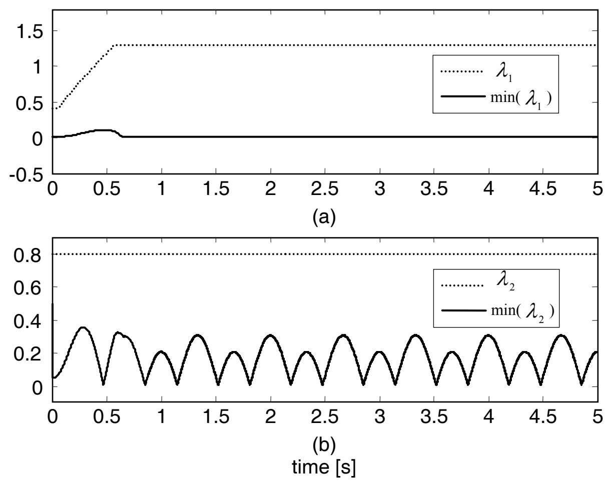

Figure 7. Observer gains and their stability bounds in (43) for k=4: a) λ_1, min(λ_1), b) λ_2, min(λ_2).

The performance for k=2 in Fig.4 is better than the case for k=4. However, the control input magnitude in Fig.5 reaches near 60 for k=2 which is far more than the conventional case. For the stability of the system, the conditions given in (43) must be provided. The λ_1, λ_2 values and their minimum values that are obtained from (43) are plotted in Fig.6 and 7, for k=2 and k=4, respectively. It is seen that the observer gains do not coincide with the given stability conditions.

Considering above analysis on system stability and performance, for detailed comparisons, control input magnitude is also considered and for the k-step ahead prediction output of the SVM model, k=4 is chosen. The simulations are implemented for the proposed OFSMC with SVM based gain adaptation scheme (OFSMC-SVM) and for the conventional OFSMC presented in [19] (OFSMC-C). Bearing in mind the stability conditions in [19], three different cases for OFSMC-C is considered:

OFSMC-C$_1$: $\lambda_1 = 0.4$, $\lambda_2 = 0.8$ and k_g=3.61

OFSMC-C$_2$: $\lambda_1 = 0.4$, $\lambda_2 = 0.8$ and k_g=9.863

OFSMC-C$_3$: $\lambda_1 = 1.293$, $\lambda_2 = 0.8$ and k_g=9.863

The three cases for OFSMC-C are designed in order to show the effect of adjusted values obtained by the SVM scheme. For OFSMC-C$_1$, OFSMC-C$_3$ and OFSMC-SVM, k_g in (12) is calculated from (13) with $\mu = 0.01$. For OFSMC-C$_2$, on the other hand, k_g is chosen as the maximum value obtained with OFSMC-SVM. OFSMC-C$_3$ has constant λ_1 value that is obtained at last with OFSMC-SVM and k_g=9.863 is calculated from (13) for constant $\lambda_1 = 1.293$. The initial value of $\lambda_1(0) = 0.4$ is chosen for OFSMC-SVM.

The boundaries for λ_1 is taken as $\lambda_{1\min} = 0.1$

and $\lambda_{1\max} = 5$. The blending factor in (38) is taken as $\kappa = 0.1$ and the penalty on the control deviations is ρ=0.001.

The time responses of estimated system output and actual system output y=x_1 are given in Fig.8. For all the controllers, there is some transient observation error at the beginning of observation as observer initial conditions are inconsistent with those of the plant. But observer states and thus system output estimate approach to its actual value after a finite time.

The sliding surface s and estimated sliding surface $\hat{s}$ are plotted in Fig.9. The sliding motion in Fig.9 provides an estimate of the system states. For OFSMC-C$_2$, increasing k_g improves $t_{reach}(\hat{s})$. However, this does not improve the observer behavior as can be easily seen from the value of $t_{reach}(s)$. The control inputs are also plotted in Fig.10 and chattering is a result of signum function and can be avoided by using a saturation function.

The time-varying behavior of the updated $\lambda_1(t)$ and calculated $k_g(t)$ for the proposed OFSMC-SVM controller is plotted in Fig.11. The k_g is calculated from (13) by using the time-varying λ_1 value which is calculated with the proposed method at each t_d time intervals. At time t=0.57 s the parameters reach their optimum values and stay constant as λ_1=1.293 and k_g =9.863 after that time instant.

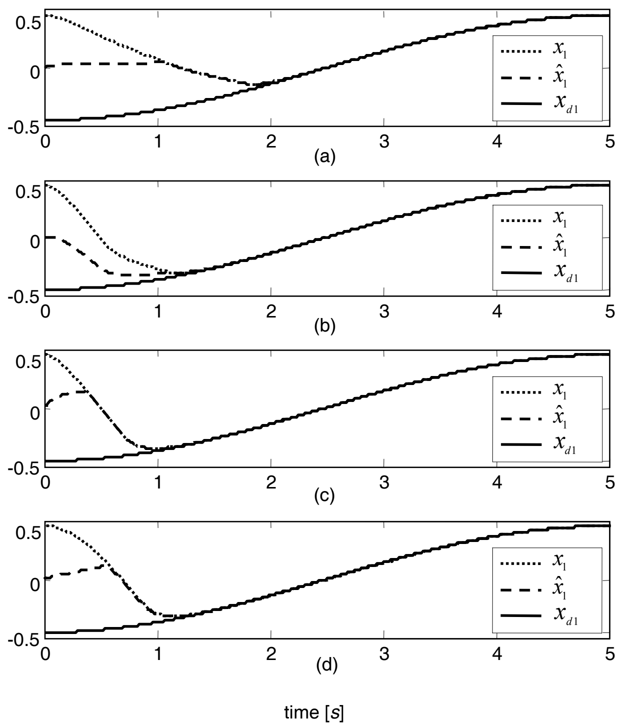

Figure 8. Actual, estimated and reference system output y=x_1: a)OFSMC-C$_1$, b)OFSMC-C$_2$, c)OFSMC-C$_3$, d)OFSMC-SVM.

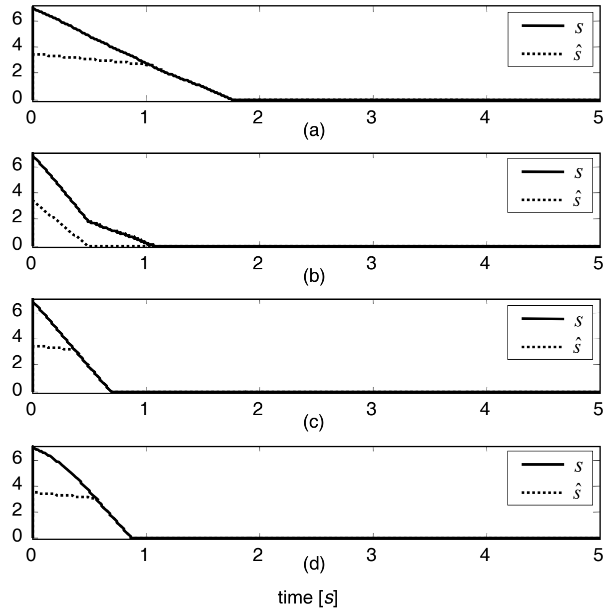

Figure 9. Actual and estimated sliding surface variables: a)OFSMC-C$_1$, b)OFSMC-C$_2$, c)OFSMC-C$_3$, d)OFSMC-SVM.

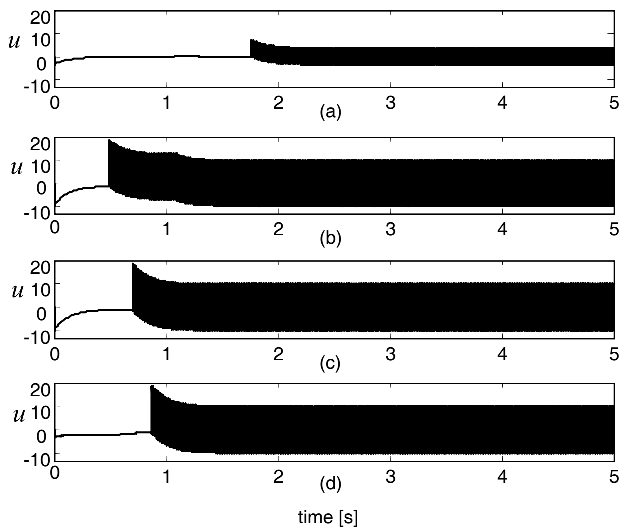

Figure 10. Control inputs: a) OFSMC-C$_1$, b) OFSMC-C$_2$, c) OFSMC-C$_3$, d) OFSMC-SVM.

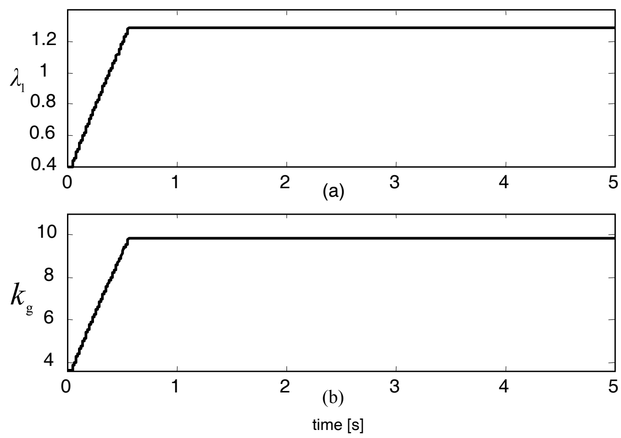

Figure 11. a) λ_1, and b) k_g for OFSMC-SVM

Table 1. Performance indices of the controllers.

	OFSMC-C_1	OFSMC-C_2	OFSMC-C_3 k=4	OFSMC-SVM k=4
$t_a(x_1)$	0,964	0,997	0,344	0,534
$t_a(x_2)$	1,132	1,135	0,395	0,605
$t_s(x_1)$	1,836	1,125	0,904	1,075
$t_s(x_2)$	2,125	1,402	1,190	1,352
$t_{reach}(s)$	1,825	1,085	0,895	1,006
$t_{reach}(\hat{s})$	1,775	0,496	0,700	0,876
max{|u|}	7,234	18,433	18,700	18,632

Finally, the performance indices of the related controllers are given in Table 1. The error bound for the settling time is taken as 5% of the steady state value. In Table 1, $t_a(x_i)$ is the time that estimated state $\hat{x}_i$ approach its actual value x_i, $t_s(x_i)$ is the settling time for state x_i, and t_{reach} is the reaching time of estimated and actual sliding surface variables. It is seen that only increasing k_g does not have a positive effect on t_a. The OFSMC-C_3 represents the obtained values generated by using OFSMC-SVM adaptation scheme. Thus, it has the best observation behavior and settling time performance which shows the constructive tuning strategy of the presented SVM model.

6 Conclusion

In this study, output feedback sliding mode control of a nonlinear second order system subject to bounded external disturbances is considered. The novelty of this study is that the support vector machine regression algorithm is firstly used with the output feedback sliding mode controller structure. In this study, the support vector machine regression algorithm is used to adjust the sliding mode observer gains.

By using computer simulations, it is seen that the number of future data points predicted by the support vector machine based model influence both the performance of the system and the magnitude of the control input. Therefore, a proper value for the number of future data points is selected. From the simulation results, it was concluded that the proposed method improves the system trajectory tracking performance and the observer gains. Also, it is shown with the simulations that the stability conditions for the observer gains are satisfied.

Only one of the observer gains is considered in this study. However, the observer gain adjustment mechanism presented in this study can be applied to both observer gains. Also, in the case of higher order systems in the form of the given structure, proposed method can be extended by considering the stability conditions.

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