The heuristic and analytical methodologies applied in a computer assisted exercise

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Abstract: - This very brief example of military simulation shows usefulness as a practical basis for education, training, analysis, and decision-support. As an analytically challenging field of pursuit, military simulation has the potential to make a significant improvement to the effectiveness of military personnel.

Key-Words: - Modeling and Simulation, Analytic Model, HERO heuristics Helmbold model, Asymmetric Conflict, War Gaming, Computer Assisted Exercises (CAXs)

1 Introduction

The heuristic and analytical methodologies are very useful to evaluate the course of a conflict. They are used in modelling and simulation, in war gaming, computer assisted exercises, etc. More details regarding the heuristic HERO and analytical methodologies and about the computing algorithms that correspond to these methodologies can be found in [3], [4], [5].

The aim of this paper is to apply the heuristic and analytical methodologies in a computer assisted exercise and to compute some parameters of the attacker's forces and the defender's forces in case of a military conflict. This is an example of a military simulation that shows the usefulness as a practical basis for education, training, analysis and decision-support. Also, the exercise has the potential to improve the effectiveness of military personnel.

In the first part of the paper, the operational scenario and the hypothesis about scenario of a conflict are presented. In the second part, the heuristic HERO methodology and the analytic model (Helmbold model) for symmetric and asymmetric conflict are used to compute the ratio of forces between the sides, the losses, the forwarding speed and some others parameters. Finally, based on the obtained numerical results, some conclusions about the evolution of the conflict are presented.

2 Operation scenario

Operation time: autumn of 1976;
Operation area: Fulda breach on German interior border;
Operation theme: blue forces defence against a red forces attack;
Attacker's objective: to destroy the blue forces and to occupy some objectives at 100 km depth;
Battle fighting: according to the doctrines of the two forces.

3 Hypothesis about scenario

Participating forces:
- attacker (red forces (side Y)):
  - echelon 1: Tank Division + Motorised Infantry Division + Motorised Infantry Regiment;
  - echelon 2: Artillery Brigade + 2 Tank Division;
- defender (blue forces (side X)): 1/2 Mechanised Division + 1/3 Artillery Company.

Attack planning:
- main direction;
- making use of surprise element.

Defence planning:
- comprises a security area of 30 km in depth between the state border and the main battle area

where a delaying fight is to be fought by the cover forces;
- missile arrangements to organise on successive fighting positions;
- laying mine fields.
Terrain: uneven, with wooded areas, still allowing armoured vehicle operations;
Additional scenario:
- air support provided to land forces of both sides is somewhat balanced and mutually compensated, having no impact on the force ratio;
- equal fighting efficacies of both land forces;
- the chance to take the attacker by surprise is low;
- no replacements throughout battle fighting are forecast for either of the forces.
Combat "environment" parameters:
- uneven, mixed terrain;
- humid weather, clouds, temperature;
- autumn, mild;
- defence in a hurry;
- low surprise;
- road quality: roads in good shape;
- road density: European standard;
- rivers or channels can be crossed through ford;
- mines density is of 10 per km of front.

The forces' armament inventories are presented in tables 1 and 2.

4 The heuristic HERO methodology
4.1 The ratio of forces between the sides
4.1.1 The calculating of the sides' forces
The calculating of the operational lethal indexes (OLI) for the side Y is presented in the following.

OLI index of the little weapons W_s (LW rows in table 1) is:

$$W_s = 5871.89 + 121.36 + 1449 = 7442.25 . \quad (1)$$

OLI index of the machine-guns W_{mg} (MG row in table 1) is:

$$W_{mg} = 1496 + 640.80 = 2136.80 . \quad (2)$$

OLI index of the light armament W_{hw} (GT - grenade thrower - light rows in table 1) is:

$$W_{hw} = 3600 + 46.67 = 3646.67 . \quad (3)$$

OLI index of the antitank armament W'_{gi} (AT row in table 1) is:

$$W'_{gi} = \begin{cases} W_{gi}, \text{ if } W_{gi} < W_i, \text{ for the enemy} \\ W_i + \frac{1}{2}\left(W_{gi} - W_i\right), \text{ otherwise,} \end{cases} \quad (4)$$

$$W_{gi} = 5973.10+1585.47+2558.33+5698+ \\ +4095.36+4647.35\ 24557.61 < 144531 , \quad (5)$$

$$W'_{gi} = W_{gi} = 24557.61 . \quad (6)$$

OLI index of the armoured armament W_i (Arm. rows in table 1) is:

$$W_i = 38990 + 23353.23 + 1870 + 7899.21 + \\ +10715.76 + 6348.69 + 440004.89 + \\ +4800 = 533981.78 . \quad (7)$$

OLI index of the artillery W_g (AR rows in table 1) is:

$$W_g = 8568 + 15652 + 24516 + 3776 = 52512.00 . \quad (8)$$

OLI index of the air defence armament W'_{gy} (AD row in table 1) is:

$$W'_{gy} = \begin{cases} W_{gy}, \text{ if } W_{gy} < W_y, \text{ for the enemy} \\ W_y + \frac{1}{2}\left(W_{gy} - W_y\right), \text{ otherwise,} \end{cases} \quad (9)$$

$$W_{gy} = 26730.07 + 5376 + 5372 + 5320 = \\ = 42798.07 > 8712 , \quad (10)$$

$$W'_{gy} = 8712 + 0.5 \cdot \left(42798.07 - 8712\right) = \\ =25755.03 . \quad (11)$$

OLI index of the armament of ground forces Σ is:

$$\Sigma = W_s + W_{mg} + W_{hw} + W_{gi} + W_g + W_{gy} + W_i = \\ = 7442.25 + 2136.80 + 3646.67 + 24557.61 + \\ +52512 + 42798.07 + 533981.78 = 667075.18 . \quad (12)$$

OLI index of the allied forces' air defence support armament W'_y (AE row in table 1) is:

$$W'_y = \begin{cases} W_y, \text{ if } W_y < \Sigma \\ \Sigma + \frac{1}{2}\left(W_y - \Sigma\right), \text{ otherwise,} \end{cases} \quad (13)$$

$$W_y = 317.56 < 667075.18 , \quad (14)$$

$$W'_y = W_y = 317.56 . \quad (15)$$

The Y's force S_a is:

$$S_a = \left[\left(W_s + W_{mg} + W_{hw}\right) r_n\right] + \left(W'_{gi}\, r_n\right) + \\ +\left[\left(W_g + W'_{gy}\right)\left(r_{wg} h_{wg} z_{wg} w_{mg}\right)\right] + \\ +\left[W_i\left(r_{wi} h_{wi}\right)\right] + \left[W'_y\left(r_{wy} h_{wy} z_{wy} w_{yy}\right)\right], \quad (16)$$

$$\left(W_s + W_{mg} + W_{hw}\right) r_n = \\ \left(7442.25 + 2136.80 + 3646.67\right) \cdot 0.9 = \\ = 11903.148 , \quad (17)$$

where $r_n = 0.9$ is terrain factor,

$$W'_{gi}\, r_n = 24557.61 \cdot 0.9 = 22101.849 , \quad (18)$$

$$\left(W_g + W'_{gy}\right)\left(r_{wg} h_{wg} z_{wg} w_{mg}\right) = \\ = \left(52512 + 25755.03\right) \cdot \left(0.9 \cdot 0.9 \cdot 1 \cdot 1\right) = \\ = 63396.2943 , \quad (19)$$

where $r_{wg} = 0.9$ is terrain factor, $h_{wg} = 0.9$ weather factor, $z_{wg} = 1$ is season factor and $w_{mg} = 1$ is air superiority factor. All these factors are relative to the artillery armament.

$$W_i\left(r_{wi}h_{wi}\right) = 533981.78 \cdot \left(0.8 \cdot 0.5\right) = 213592.712 \quad (20)$$

where $r_{wi} = 0.8$ is terrain factor and $h_{wi} = 0.5$ is weather factor, both relative to the armoured armament.

$$W'_y\left(r_{wy}h_{wy}z_{wy}w_{yy}\right) = = 317.56 \cdot \left(0.95 \cdot 0.2 \cdot 0.9 \cdot 1\right) = 54.30276, \quad (21)$$

Table 1

Armament inventory for the red forces (Y)

Category	Type	Echelon 1		Echelon 2	Total	OLI/item	OLI/type
		M.I. Div.	M.I.Reg. (1/3M.I.Div.)	Tk. Div.			
LW	Guns	8951	2983.67	6415	18349.67	0.32	5871.89
	Pistols	1534	511.33	2000	4045.33	0.03	121.36
MG	Light MG	1160	386.67	720	2266.67	0.66	1496.00
	Heavy MG	360	120.00	240	720.00	0.89	640.80
GT Light	GT 82mm	54	18.00	-	72.00	50	3600.00
GT AR	GT 120mm	54	18.00	54	126.00	68	8568.00
Arm.	BTR 50	300	100.00	157	557.00	70	38990.00
GT Light	Flame thrower	20	6.67	20	46.67	1	46.67
Arm.	RPG-7	400	133.33	220	753.33	31	23353.23
AT	Sagger, AT3	40	13.33	32	85.33	70	5973.10
	RR 82mm	20	6.67	12	38.67	41	1585.47
	RR 107mm	10	3.33	12	25.33	101	2558.33
	SPG-9	30	10.00	34	74.00	77	5698.00
	AT cannon 85mm	16	5.33	-	21.33	192	4095.36
	AT cannon 100mm	8	2.67	12	22.67	205	4647.35
AR	MRL 122mm	12	4.00	12	28.00	559	15652.00
	Mortar 122mm	54	18.00	36	108.00	227	24516.00
	Mortar 152mm	12	4.00	-	16.00	236	3776.00
LW	Bayonet carrier	9	3.00	9	21.00	69	1449.00
AD	ZSU-23-4	64	21.33	64	149.33	179	26730.07
	AA-S-60	12	4.00	16	32.00	168	5376.00
	SA 7	81	27.00	50	158.00	34	5372.00
	SA 6	6	2.00	12	20.00	266	5320.00
Arm.	BRDM 1	120	40.00	10	170.00	11	1870.00
	BRDM 2	16	5.33	12	33.33	237	7899.21
	BMP	14	4.67	14	32.67	328	10715.76
	BMP-Sagger	14	4.67	12	30.67	207	6348.69
	Tank T-62	188	62.67	323	573.67	767	440004.89
	Tank PT-76	-	-	30	30.00	160	4800.00
AE	Helicopters	2	0.67	2	4.67	68	317.56
N2	Trucks	1300	433.33	1300	3033.33	-	-
	AFV	250	83.33	400	733.33	-	-
P	Personnel	10485	433.33	8415	19333.33	-	-
Total OLI							667392.74

Attacker's armament inventory (Y)

Table 2

Armament inventory for the blue forces (X)							
Category		**Type**	**1/2Mech Div.**	**1/3Art. Body**	**Total**	**OLI/item**	**OLI/type**
LW		Guns	8200	2733.33	10933.33	0.35	3826.67
		Pistols	1900	633.33	2533.33	0.02	50.67
MG		Light machine-guns	1817	100.00	1917.00	0.82	1571.94
		Machine-gun cal. 30	335	16.66	351.66	1.04	365.73
GT	Light	Mortars 81mm	27	3.33	30.33	50	1516.50
	AR	Mortars 4.2”	32.50	10	42.50	90	3825.00
Arm.		APC 113	366	-	366.00	27	9882.00
		APC 114	53	10	63.00	75	4725.00
GT	Light	Flame-thrower	27	6.66	33.66	1.0	33.66
		Grenade thrower 40mm	534.50	20	554.50	8.0	4436.00
	AR	LAW	500	133.33	633.33	18	11399.94
AT		RR 90mm	60.50	-	60.50	74	4477.00
		RR 106mm	24	-	24.00	133	3192.00
		ATGM TOW	15	-	15.00	176	2640.00
AR		Mortar 105mm	9	18	27.00	160	4320.00
		Mortar 155mm	27	12	39.00	235	9165.00
		Cannon 155mm	-	18	18.00	303	5454.00
		Cannon 175mm	-	18	18.00	356	6408.00
		Mortar 8”	6	8	14.00	212	2968.00
		Ra. Honest John	2	-	2.00	107	214.00
		Ra. Pershing	-	2.66	2.66	300	798.00
GT	Light	AAMG cal. 0.5	225	200	425.00	1.7	722.50
AD		Light Aag	12	8	20.00	86	1720.00
		Medium SAM	12	8	20.00	158	3160.00
		SAM HAWK	-	8	8.00	300	2400.00
Arm.		ARV	12	-	12.00	81	972.00
		Medium tank M60	162	-	162.00	796	128952.00
AE		Helicopters	49	50	99.00	88	8712.00
N2		Trucks	2025	333.33	2358.33	-	-
		AFV	550	66.66	616.66	-	-
P		Personnel	10100	3366.66	13466.66	-	-
Total OLI							227907.61

Attacker’s armament inventory (Y)

where $r_{wy} = 0.95$ is terrain factor, $h_{wy} = 0.2$ weather factor, $z_{wy} = 0.9$ is season factor and $w_{yy} = 1$ is air superiority factor. All these factors are relative to the air support.

Consequently, the Y’s force S_a is:

$$S_a = 11903.148 + 22101.849 + 63396.2943 + \\ +213592.712 + 54.30276 = 311048.30606. \quad (22)$$

Using the same relations, the operational lethal indexes (OLI) for side X are presented below.

OLI index of the little weapons W_s (LW row in table 2) is:

$$W_s = 3826.67 + 50.67 = 3877.34. \quad (23)$$

OLI index of the machine-guns W_{mg} (MG row in table 2) is:

$$W_{mg} = 1571.94 + 365.73 = 1937.67. \quad (24)$$

OLI index of the light armament W_{hw} (GT - grenade thrower - light rows in table 2) is:

$$W_{hw} = 1516.50 + 33.66 + +4436 + 722.50 = 6708.66 . \quad (25)$$

OLI index of the antitank armament W'_{gi} (AT row in table 2) is:

$$W_{gi} = 4477 + 3192 + +2640 = 10309 < 533981.78 , \quad (26)$$

$$W'_{gi} = W_{gi} = 10309 . \quad (27)$$

OLI index of the armoured armament W_i (Arm. row in table 2) is:

$$W_i = 9882 + 4725 + 972 + 128952 = 144531 . \quad (28)$$

OLI index of the artillery W_g (AR rows in table 2) is:

$$W_g = 3825 + 11399.94 + 4320 + 9165 + 5454 + 6408 + 2968 + 214 + 798 = 44551.94 . \quad (29)$$

OLI index of the air defence armament W'_{gy} (AD row in table 2) is:

$$W_{gy} = 1720 + 3160 + 2400 = 7280 > 317.56 , \quad (30)$$

$$W'_{gy} = 181.56 + 0.5 \cdot (7280 - 181.56) = = 3730.78 . \quad (31)$$

OLI index of the armament of ground forces Σ is:

$$\Sigma = 3877.34 + 1937.67 + 6708.66 + 10309 + +44551.94 + 7280 + 144531 = 219195.61 . \quad (32)$$

OLI index of the allied forces' air defence support armament W'_y (AE row in table 2) is:

$$W_y = 8712 , \quad (33)$$

$$W'_y = W_y = 8712 . \quad (34)$$

The X's force S_d is computed with the same relation (16):

$$(W_s + W_{mg} + W_{hw}) r_n = (3877.34 + 1937.67 + 6708.66) \cdot 0.9 = = 11271.303 , \quad (35)$$

$$W'_{gi} r_n = 10309 \cdot 0.9 = 9278.1 , \quad (36)$$

$$(W_g + W'_{gy})(r_{wg} h_{wg} z_{wg} w_{mg}) = = (44551.94 + 3730.78) \cdot (0.9 \cdot 0.9 \cdot 1 \cdot 1) = = 39109.0032 , \quad (37)$$

$$W_i (r_{wi} h_{wi}) = 144531 \cdot (0.8 \cdot 0.5) = = 57812.4 , \quad (38)$$

$$W'_y (r_{wy} h_{wy} z_{wy} w_{yy}) = = 8712 \cdot (0.95 \cdot 0.2 \cdot 0.9 \cdot 1) = 1489.752 , \quad (39)$$

$$S_d = 11271.303 + 9278.1 + 39109.0032 + +57812.4 + 1489.752 = 118960.5582 . \quad (40)$$

4.1.2 The calculating of the mobility

The operational mobility factor m is:

$$m = 1 - r_m h_m (1 - M) , \quad (41)$$

where r_m is mobility factor related to terrain, h_m is mobility factor related to weather and M is the mobility characteristic:

$$M = \sqrt{\frac{(N_a + 20J_a + W_{ia}) m_{ya}}{(N_a + 20J_d + W_{id}) m_{yd}} \cdot \frac{N_d}{N_a}} , \quad (42)$$

where N_a is the attacker's effective (personnel row in table 1), N_d is the defender's effective (personnel row in table 2), W_{ia} is the attacker's OLI index of the armoured armament from relation (7), W_{id} is defender's OLI index of the armoured armament from relation (28), m_{ya} is the influence factor of air situation on the mobility for the attacker and m_{yd} for defender.

$$J_a = N_{1a} + 2N_{2a} + 10N_{3a} , \quad (43)$$

$$J_d = N_{1d} + 2N_{2d} + 10N_{3d} , \quad (44)$$

where N_{1a} is the number of the attacker's armoured armament (Arm. rows in table 1) and N_{1d} for defender (Arm. rows in table 2), N_{2a} is the number of the attacker's armoured vehicles that not fighting (N2 row in table 1) and N_{2d} for defender (N2 row in table 2), N_{3a} is the number of the attacker's airplanes and helicopters (AE row in table 1) and N_{3d} for defender (AE row in table 2).

The calculating of the mobility for the side Y is presented below.

$$J_a = (557 + 753.33 + 170 + 33.33 + 32.67 + +30.67 + 573.67 + 30) + +2 \cdot (3033.33 + 733.33) + 10 \cdot 4.67 = 9760.69 , \quad (45)$$

$$J_d = (366 + 63 + 12 + 162) + +2 \cdot (2358.33 + 616.66) + +10 \cdot 99) = 7542.98 , \quad (46)$$

$$M = \sqrt{\frac{(19333.33 + 20 \cdot 9760.69 + 533981.78) \cdot 1}{(19333.33 + 20 \cdot 7542.98 + 144531) \cdot 1}} \cdot \sqrt{\frac{13466.66}{19333.33}} = 1.28711 . \quad (47)$$

The operational mobility factor for the attacker m_a is:

$$m_a = 1 - (0.8 \cdot 0.6 \cdot (1 - 1.28711)) = 1.13781 . \quad (48)$$

For the defender, the obtained operational mobility factor m_d is:

$$m_d = 1 . \quad (49)$$

4.1.3 The calculating of the vulnerability

The vulnerability factor has the formula:

$$v = \begin{cases} 1 - \frac{V}{S} \cdot \frac{D_i}{3000}, \text{ if } \frac{V}{S} \cdot \frac{D_i}{3000} \le 0.3 \\ 0.73 - 0.1 \cdot \frac{V}{S} \cdot \frac{D_i}{3000}, \text{ if } \frac{V}{S} \cdot \frac{D_i}{3000} > 0.3, \end{cases} \quad (50)$$

where D_i is the variance factor, S is the side's force and V is the vulnerability characteristic which is computed with the next formula for the defender:

$$V_d = N_d \cdot c_d \cdot \sqrt{\frac{S_a}{S_d}} \cdot v_{yd} \cdot v_{rd}, \quad (51)$$

where N_d is the defender's effective (personnel row in table 2), S_a is the attacker's force, S_d is the defender's force, v_{yd} is the defender's air superiority factor, v_{rd} is the beach vulnerability factor and c_d is:

$$c_d = \frac{u_{vd}}{r_{ud}}, \quad (52)$$

where u_{vd} is the defender's vulnerability coefficient and r_{ud} is the terrain factor.

The numerical values obtained for attacker are:

$$V_a = 19333.33 \cdot \frac{1}{1} \cdot \sqrt{\frac{311048.30606}{118960.5582}} \cdot 1 \cdot 1 = 31262.16763, \quad (53)$$

$$\frac{V_a}{S_a} \cdot \frac{D_i}{3000} = \frac{31262.16763}{311048.30606} \cdot \frac{4000}{3000} = 0.13400 < 0.3, \quad (54)$$

$$v_a = 1 - 0.134 = 0.86600. \quad (55)$$

The numerical values obtained for defender are:

$$V_d = 13466.66 \cdot \frac{0.7}{1.3} \cdot \sqrt{\frac{311048.30606}{118960.5582}} \cdot 1 \cdot 1 = 11725.38216, \quad (56)$$

$$\frac{V_d}{S_d} \cdot \frac{D_i}{3000} = \frac{11725.38216}{118960.5582} \cdot \frac{4000}{3000} = 0.13142 < 0.3, \quad (57)$$

$$v_d = 1 - 0.13142 = 0.86858. \quad (58)$$

4.1.4 The calculating of the surprise element

The taking by surprise is generally performed by the attacker and this affects the mobility and the vulnerability of both sides. Consequently, the characteristics M and V change into the characteristics M' and V':

$$M' = M_{sur} \cdot M, \quad (59)$$

$$V' = V_{sur} \cdot V, \quad (60)$$

where M_{sur} and V_{sur} are parameters from the standard tables [4].

Thus, for the side Y we have:

$$M'_a = M_{sur} \cdot M_a = 1.3 \cdot 1.28711 = 1.67324, \quad (61)$$

$$V'_a = V_{sur} \cdot V_a = 0.9 \cdot 31262.16763 = 28135.95086. \quad (62)$$

Recalculating the mobility and the vulnerability, we obtain:

$$m'_a = 1 - \left(0.8 \cdot 0.6 \cdot (1 - 1.67324)\right) = 1.32315, \quad (63)$$

$$\frac{V'_a}{S_a} \cdot \frac{D_i}{3000} = \frac{28135.95086}{311048.30606} \cdot \frac{4000}{3000} = 0.12060 < 0.3, \quad (64)$$

$$v'_a = 1 - 0.12060 = 0.87940. \quad (65)$$

For the side X we have:

$$M'_d = M_{sur} \cdot M_d = 1 \cdot 1 = 1, \quad (66)$$

$$V'_d = V_{sur} \cdot V_d = 1.2 \cdot 11725.38216 = 14070.45859, \quad (67)$$

$$m'_d = 1, \quad (68)$$

$$\frac{V'_d}{S_d} \cdot \frac{D_i}{3000} = \frac{14070.45859}{118960.5582} \cdot \frac{4000}{3000} = 0.15770 < 0.3, \quad (69)$$

$$v'_d = 1 - 0.15770 = 0.84230. \quad (70)$$

4.1.5 The calculating of the potential of fighting capacity

The attacker's potential of fighting capacity is:

$$P'_y = S_a \cdot m'_a \cdot v'_a \cdot l_e \cdot t \cdot o \cdot b \cdot u_s \cdot r_u \cdot h_u \cdot z_u = 311048.30606 \cdot 1.32315 \cdot 0.87940 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 0.7 \cdot 1.1 = 278685.33006, \quad (71)$$

where all terms in relation (71) are explained in [3].

The defender's potential of fighting capacity is:

$$P'_x = S_d \cdot m'_d \cdot v'_d \cdot l_e \cdot t \cdot o \cdot b \cdot u_s \cdot r_u \cdot h_u \cdot z_u = 118960.5582 \cdot 1 \cdot 0.84230 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1.3 \cdot 1.3 \cdot 1 \cdot 1 = 169338.80811. \quad (72)$$

4.1.6 The calculating of the force ratio

The force ratio is:

$$\frac{P'_y}{P'_x} = \frac{278685.33006}{169338.80811} = 1.64572. \quad (73)$$

4.2 The calculating of the losses and forwarding speeds

4.2.1 The calculating of the losses in armoured vehicles

We use the following relations [3]:

$$\frac{1}{P}\frac{dP}{dt} = k_P f(\rho) g(P) + k_0, \quad (74)$$

$$\frac{1}{B}\frac{dB}{dt} = k_B \left(\frac{1}{P}\frac{dP}{dt}\right) h(B), \quad (75)$$

where $f(\rho)$ is the opposition factor, $g(P)$ is the capacity-size factor in personnel, $h(B)$ is the capacity-size factor in armoured armament, k_0 is the personnel operational losses. The coefficient of the personnel losses k_P and the coefficient of the armoured armament losses k_B are:

$$k_P = K_{SP} k_{MP}, \quad (76)$$

$$k_B = k_{SB} k_{MB}, \quad (77)$$

where K_{SP} is the standard losses in personnel, k_{MP} is the mission coefficient for personnel, k_{SB} is the standard losses in armoured armament and k_{MB} is the mission coefficient for armoured armament.

In safety strip, for side Y we obtain the values:

$$k_P = 2.8 \cdot 1.0 = 2.8, \quad (78)$$

$$\frac{1}{P}\frac{dP}{dt} = (2.8 \cdot 0.7 \cdot 1.0) + 0.2 = 2.16\%/\text{day}, \quad (79)$$

$$k_B = 5.4 \cdot 1.0 = 5.4, \quad (80)$$

$$\frac{1}{B}\frac{dB}{dt} = 5.4 \cdot 2.16 \cdot 0.8 = 9.33\%/\text{day}. \quad (81)$$

In central strip, for side Y we obtain the following values:

$$k_P = 2.8 \cdot 1.0 = 2.8, \quad (82)$$

$$\frac{1}{P}\frac{dP}{dt} = (2.8 \cdot 0.9 \cdot 1.0) + 0.2 = 2.72\%/\text{day}, \quad (83)$$

$$k_B = 5.4 \cdot 1.0 = 5.4, \quad (84)$$

$$\frac{1}{B}\frac{dB}{dt} = 5.4 \cdot 2.72 \cdot 0.8 = 11.75\%/\text{day}. \quad (85)$$

In safety strip, for side X we obtain the values:

$$k_P = 1.5 \cdot 1.0 = 1.5, \quad (86)$$

$$\frac{1}{P}\frac{dP}{dt} = (1.5 \cdot 0.7 \cdot 1.0) + 0.2 = 1.25\%/\text{day}, \quad (87)$$

$$k_B = 5.4 \cdot 1.0 = 5.4, \quad (88)$$

$$\frac{1}{B}\frac{dB}{dt} = 5.4 \cdot 1.25 \cdot 0.8 = 5.4\%/\text{day}. \quad (89)$$

In central strip, for side X we obtain the following values:

$$k_P = 1.5 \cdot 1.0 = 1.5, \quad (90)$$

$$\frac{1}{P}\frac{dP}{dt} = (1.5 \cdot 0.9 \cdot 1.0) + 0.2 = 1.55\%/\text{day}, \quad (91)$$

$$k_B = 5.4 \cdot 1.0 = 5.4, \quad (92)$$

$$\frac{1}{B}\frac{dB}{dt} = 5.4 \cdot 1.55 \cdot 0.9 = 7.53\%/\text{day}. \quad (93)$$

These data are synthetically presented in the table 3.

Table 3

	Losses in personnel (%/day)	Losses in armoured vehicles (%/day)
Attacker		
Safety strip	2.16	9.33
Central strip	2.72	11.75
Defender		
Safety strip	1.25	5.4
Central strip	1.55	7.53

The attacker's and the defender's losses

4.2.2 The calculating of the forwarding speed

The forwarding speed is computed by the relation:

$$v = v(\rho) * k_T * k'_D * k''_D * k'_0 * k''_0 = \\ = 9.0 \cdot 0.8 \cdot 1 \cdot 1 \cdot 0.9 \cdot 0.9 = 5.83 \text{ km/day}, \quad (94)$$

where all terms from the above relation are explained in [3].

5 The Helmbold model for the symmetric conflict

In the case of Fulda operation, the information parameters α and β are equal to 1/4 for the symmetric conflict. From the computing with the heuristic HERO methodology we have some data.

The fighting capacity potentials are:

- defender: $X_0 = 169{,}338.80811$;
- attacker: $Y_0 = 278{,}685.33006$;

The original force ratio is $\rho_0 = \frac{X_0}{Y_0} = 0.607$;

The equivalent in tanks is:

- defender: $S_x = 287$;
- attacker: $S_y = 772$;

The number of staff is:

- defender: $P_x = 13{,}466$;
- attacker: $P_y = 19{,}334$.

5.1 Calculus of the lethal constants (relative average daily losses in armoured vehicles)

The X's staff losses k_{1P} are:

$$k_{1P} = K_{1SP} g_{1P} k_{1MP} = 2.8 \cdot 1.0 \cdot 1.0 = 2.8\%/\text{day}. \quad (95)$$

The X's armoured vehicles losses k_{1B} are:

$$k_{1B} = k_{1SB} k_{1P} h_{1B} k_{1MB} = \\ = 5.4 \cdot 2.8 \cdot 0.8 \cdot 1.0 = 12.1\% / \text{day}. \quad (96)$$

The Y's staff losses k_{2P} are:

$$k_{2P} = k_{2SP} g_{2P} k_{2MP} = 1.5 \cdot 1.0 \cdot 1.0 = 1.5\% / \text{day}. \quad (97)$$

The Y's armoured vehicles losses k_{2B} are:

$$k_{2B} = k_{2SB} k_{2P} h_{2B} k_{2MB} = \\ = 5.4 \cdot 1.5 \cdot 1.0 \cdot 1.0 = 8.1\% / \text{day}. \quad (98)$$

The relative average daily losses Λ_1 caused by the defender to the attacker is:

$$\Lambda_1 = \prod_{1B} k_{1B} = 1.0 \cdot 12.1 = 12.1\% / \text{day} = 0.121. \quad (99)$$

The relative average daily losses Λ_2 caused by the attacker to the defender is:

$$\Lambda_2 = \prod_{2B} k_{2B} = 1.0 \cdot 8.1 = 8.1\% / \text{day} = 0.081. \quad (100)$$

There are some notations:

- $k_{iSP(B)}$ is standard losses in staff (armoured vehicles);
- $g_{iP(B)}$ is capacity-size parameter for staff (armoured vehicles);
- $k_{iMP(B)}$ is mission parameter for staff (armoured vehicles);
- $\prod_{iB}$ is armoured vehicles weights;
- $i = 1$ (for the defender) or $i = 2$ (for the attacker).

5.2 Calculus of the general superiority coefficient

The general superiority coefficient Q is:

$$Q = \frac{\Lambda_2}{\Lambda_1} \cdot \rho_0^{-2\alpha} = \left(\frac{0.081}{0.121}\right) \cdot 1.283 = \\ = 0.858 < 1. \quad (101)$$

Consequently, the X's forces are in advantage and the Y's forces are cancelled first.

5.3 Calculus of the cancellation moment of side Y

The cancellation moment t_Y of side Y is:

$$t_Y = \frac{1}{2\alpha\sqrt{\Lambda_1 \Lambda_2}} \ln \frac{1+\sqrt{Q}}{1-\sqrt{Q}} = \\ = \left[\frac{1}{2 \cdot 0.25 \cdot \sqrt{0.081 \cdot 0.136}}\right] \ln \frac{1+\sqrt{0.858}}{1-\sqrt{0.858}} = \quad (102) \\ = 51.94 \text{ days}.$$

5.4 Calculus of the X's potential at the cancellation moment of side Y

The X's potential at the cancellation moment of side Y is computed below.

$$Y(t_Y) = 0, \quad (103)$$

$$X(t_Y) = X_0 (1-Q)^{1/(2\alpha)}, \quad (104)$$

$$X(t_Y) = 169{,}338.80811 \cdot (1-0.858)^2 = \\ = 3{,}414.54772 \; (2.01\%). \quad (105)$$

5.5 Calculus of the moment of force ratio reversal

$$\Delta(\rho = 1) = \frac{1}{2\alpha\sqrt{\Lambda_1 \Lambda_2}} \ln \left(\frac{1+\sqrt{Q}}{1-\sqrt{Q}} \cdot \frac{\sqrt{\Lambda_2/\Lambda_1} - 1}{\sqrt{\Lambda_2/\Lambda_1} + 1} \right) = \\ = 19.26 \text{ days}. \quad (106)$$

5.6 Calculus of the battle cease moment

The battle cease parameter for Y's forces is $d = 2/3$. The battle cease parameter for X's forces is:

$$c = \left[1 + Q\left(d^{2\alpha} - 1\right)\right]^{1/2\alpha} = \\ = \left[1 + 0.858 \cdot (0.816 - 1)\right] = 0.709; \quad (107)$$

The battle cease moment t_d is:

$$t_d = t_0 + \Delta(Y/Y_0 = d) = t_0 + \Delta(X/X_0 = c) = \\ = t_0 + \Delta(X/X_0 = 0.709) = \\ = \frac{1}{2\alpha\sqrt{\Lambda_1 \Lambda_2}} \ln \frac{1+\sqrt{Q}}{1-\sqrt{Q}} \frac{1 - \sqrt{1 + \dfrac{Q-1}{1+Q\left(d^{2\alpha}-1\right)}}}{1 + \sqrt{1 + \dfrac{Q-1}{1-Q\left(d^{2\alpha}-1\right)}}} = \quad (108) \\ = 4.12 \text{ days}.$$

5.7 Calculus of the forces ratio at battle cease moment

The force ratio at battle cease moment is:

$$\frac{X(t_d)}{Y(t_d)} = \frac{c}{d}\rho_0 = \frac{0.709}{0.667} \cdot 0.607 = 0.645. \quad (109)$$

5.8 Calculus of the forces ratio and of the forwarding speed per day

The forces ratio is:

$$\rho(t)=\rho_0\left(\frac{1-\sqrt{Q}\cdot\text{th}\,\tau}{1-\dfrac{\text{th}\,\tau}{\sqrt{Q}}}\right)^{1/\alpha}, \quad (110)$$

where:

$$\tau=\alpha\sqrt{\Lambda_1\Lambda_2}\,(t-t_0)=0.0247(t-t_0). \quad (111)$$

The forwarding speed is:

$$V(\rho)=V_0(\rho)k_T k'_D k''_D k'_O k''_O = \\ =0.8\cdot1.0\cdot1.0\cdot0.9\cdot0.9=0.648V_0(\rho), \quad (112)$$

where k_T is the general terrain parameter, k'_D is the road quality parameter, k''_D is the road density parameter, k'_O is the natural obstacles parameter and k'' is the mine fields parameter.

5.9 Calculus of the penetration

The length of penetration $\Delta(t)$ through the defender's forces is:

$$\Delta(t)=\sum_{i=0}^{4}V(\rho_i), \quad (113)$$

where the daily forwarding speed $V(\rho_i)$ depend on the forces ratio:

$$V(\rho_i)=V\left(\rho_{(t_{i-1})}\right), \quad (114)$$

$$t_i=t_0+i\Delta t, \quad (115)$$

with $\Delta t=1$ day, $i=\overline{1,4}$.

6 The Helmbold model for the asymmetric conflict

In case of the asymmetric conflict, the informational parameters are $\alpha=0.4$ for the attacker and $\beta=0.2$ for the defender, the informing level being higher for the attacker and lower for the defender than the central European conflict.

From the computing with the heuristic methodology HERO we have some data.

The equivalent in tanks is:

- defender: $X_0=287$, $X_F=198$;
- attacker: $Y_0=772$, $Y_F=390$.

6.1 Calculus of the lethal constants

For the assumed conflict scenario, the functions $F_1(x)$ and $F_2(x)$ are:

$$F_1\left(\frac{1}{4}\right)=\Lambda_1^*=0.121, \quad (116)$$

$$F_2\left(\frac{1}{4}\right)=\Lambda_2^*=0.081 \quad (117)$$

The coefficients q_1 and q_2 are:

$$\frac{1}{2}\left(2\cdot\frac{1}{4}\right)^{q_1}=0.121\Rightarrow q_1=\frac{\ln(0.242)}{\ln\left(\frac{1}{2}\right)}=2.04; \quad (118)$$

$$\frac{1}{2}\left(2\cdot\frac{1}{4}\right)^{q_2}=0.081\Rightarrow q_2=\frac{\ln(0.162)}{\ln\left(\frac{1}{2}\right)}=2.62; \quad (119)$$

Consequently, the lethal parameters Λ_1 and Λ_2 are:

$$\Lambda_1=F_1(\beta)=\frac{1}{2}(2\beta)^{q_1}= \\ =\frac{1}{2}(2\cdot0.2)^{2.04}=0.07712; \quad (120)$$

$$\Lambda_2=F_2(\alpha)=\frac{1}{2}(2\alpha)^{q_2}= \\ =\frac{1}{2}(2\cdot0.4)^{2.62}=0.27865; \quad (121)$$

6.2 Calculus of the general superiority coefficient

The used relation is:

$$Q=\frac{\Lambda_2}{\Lambda_1}\rho_0^{\alpha+\beta}, \quad (122)$$

where ρ_0 is the initial forces ratio.

$$Q=\frac{0.27865}{0.07712}\cdot\left(1.3\cdot\frac{772}{287}\right)^{0.6}=7.65>1. \quad (123)$$

Thus, the Y's forces are in advantage and the X's forces are cancelled first.

6.3 The first version of the asymmetric conflict

The critical value of stopping the combat for the defender is:

$$c^*=\frac{2}{3}=0.667 \quad . \quad (124)$$

The critical value of stopping the combat related to the losses for the attacker is:

$$d=\left\{1+\frac{1}{Q}\left[\left(c^*\right)^{\alpha+\beta}-1\right]\right\}^{\frac{1}{\alpha+\beta}}, \quad (125)$$

$$d=\left[1+\frac{1}{7.65}\cdot(0.784\text{-}1)\right]^{1.66}=0.953. \quad (126)$$

For the validation of the Helmbold model in this case we compute the next value:

$$\frac{\left|\frac{Y_F}{Y_0}-d\right|}{d}=\frac{0.953\text{-}\frac{390}{772}}{0.953}=0.469\cong 47\% . \quad (127)$$

The critical value of stopping the combat with regards to the combat duration is:

$$\Delta\left(\frac{X}{X_0}=c\right)=\Delta\left(\frac{X}{X_0}=\frac{2}{3}\right)= =K\left[H(A)-H(B)\right], \quad (128)$$

where $H(T)$ is a function corresponding to the polynomial:

$$\frac{T^{m-1}}{T^{m+n}-1}, \quad (129)$$

and:

$$\alpha=0.4=\frac{n}{p}=\frac{2}{5}, \quad (130)$$

$$\beta=0.2=\frac{m}{p}=\frac{1}{5}, \quad (131)$$

$$\Rightarrow n=2;\ \mathrm{m}=1. \quad (132)$$

But:

$$\frac{1}{T^3-1}=\frac{1}{3}\left(\frac{1}{T-1}-\frac{T+2}{T^2+T+1}\right); \quad (133)$$

$$\Rightarrow 3H(T)=\frac{\ln\left(T^2+T+1\right)^{\frac{1}{2}}}{T\text{-}1}+\operatorname{arctg}\frac{2T+1}{\sqrt{3}} . \quad (134)$$

The integration limits are:

$$A=Q^{\frac{1}{(\alpha+\beta)p}}=7.65^{\frac{1}{\left(\frac{2}{5}+\frac{1}{5}\right)\cdot 5}}=1.95708, \quad (135)$$

$$B=\left[1+(Q-1)\cdot\left(c^*\right)^{-(\alpha+\beta)}\right]^{\frac{1}{(\alpha+\beta)p}}, \quad (136)$$

$$B=\left[1+(7.65\text{-}1)\cdot 0.667^{-0.6}\right]^{\frac{1}{3}}=2.10054; \quad (137)$$

$$k=\frac{p}{\Lambda_1^{\frac{\alpha}{\alpha+\beta}}\Lambda_2^{\frac{\beta}{\alpha+\beta}}}, \quad (138)$$

$$k=\frac{5}{(0.07712)^{2/3}\cdot(0.27865)^{1/3}}=3\cdot 13.78635; \quad (139)$$

$$3H(A)=\ln\frac{\left(A^2+A+1\right)^{1/2}}{A-1}+\sqrt{3}\cdot\operatorname{arctg}\frac{2A+1}{\sqrt{3}}, \quad (140)$$

$$3H(A)=1.001+1.732\cdot 1.23=3.135109; \quad (141)$$

$$3H(B)=\ln\frac{\left(B^2+B+1\right)^{1/2}}{B-1}+\sqrt{3}\cdot\operatorname{arctg}\frac{2B+1}{\sqrt{3}}, \quad (142)$$

$$3H(B)=0.912+1.732\cdot 1.24=3.076358 . \quad (143)$$

Thus, the critical value of stopping the combat with regards to the combat duration is:

$$\Delta=\Delta\left(X/X_0=c^*\right)=k\left[H(A)-H(B)\right], \quad (144)$$

$$\Delta=13.78635\cdot(3.135109\text{-}3.076358)=0.809 \text{ days}. \quad (145)$$

The value $H(\infty)$ is:

$$H(\infty)=\lim_{B\to\infty}\mathrm{H}(\mathrm{B}), \quad (146)$$

$$H(\infty)=\sqrt{3}\cdot\frac{\pi}{2}=1.732\cdot(3.14159/2)=2.720616; \quad (147)$$

The critical value of stopping the combat when the defender loses all its forces is:

$$\Delta\left(X/X_0=0\right)=k\left[H(A)-H(\infty)\right], \quad (148)$$

$$\Delta\left(X/X_0=0\right)= =13.78635\cdot(3.135109\text{-}2.720616)= =5.714 \text{ days}. \quad (149)$$

For the validation of the Helmbold model related to the combat duration we compute the next value:

$$\frac{|\Delta-5|}{\Delta}=(5\text{-}0.809)/0.809=5.180\cong 518\% . \quad (150)$$

If not considering the day of fighting in the safety streak, we have:

$$\frac{|\Delta-4|}{\Delta}=(4\text{-}0.809)/0.809=3.944\cong 394\% . \quad (151)$$

The final results for the first version of asymmetric conflict are presented in table 4.

The attacker's advantage materialises in the fact that the critical level for the defender's fighting potential is reached in the first day. The cancellation of the defender's fighting potential is achieved in the 6th day. In the case of combat stopping criterion with respect to losses, the validation error is 47%, and with respect to duration, the error is 394%.

Table 4

α	β	Λ_1	Λ_2	ρ_0^{-1}	Q	c^*	d
0.4	0.2	0.077	0.278	3.49	7.65	2/3	0.953

$\frac{Y_F}{Y_0}$	$\frac{\left\|\frac{Y_F}{Y_0}-d\right\|}{d}$	$\Delta(\frac{X}{X_0}=c^*)$	$\frac{\|\Delta-4\|}{\Delta}$	$\Delta(\frac{X}{X_0}=0)$	Conclusions
0.505	0.469	0.809	3.944	5.714	Non-validation

Validating the results for the first version of asymmetric conflict

6.4 The second version of the asymmetric conflict

The critical value of stopping the combat is:

$$c^*=\frac{1}{3}=0.334 . \quad (152)$$

The critical value of stopping the combat with regards to losses is obtained from (125):

$$d=\left[1+\frac{1}{7.65}\cdot\left(0.517\text{-}1\right)\right]^{1.66}=0.897\,, \qquad (153)$$

$$\frac{\left|\frac{\text{Y}_\text{F}}{\text{Y}_0}-d\right|}{d}=\frac{0.897\text{-}\frac{390}{772}}{0.897}=0.436\cong\ 44\%\,. \qquad (154)$$

From (135) it is obtained $A=1.95708$.

From (136):

$$B=\left[1+\left(7.65\text{-}1\right)\cdot 0.334^{-0.6}\right]^{\frac{1}{3}}=2.38000\,. \qquad (155)$$

From (138), $k=3\cdot 13.78635$.

From (140), $3H\left(A\right)=3.135109$.

From (142):

$$3H(B)=0.778+1.732\cdot 1.27=2.993701\,. \qquad (156)$$

From (144):

$$\Delta=13.78635\cdot\left(3.135109\text{-}2.993701\right)= =1.949 \text{ days.} \qquad (157)$$

From (146), $H(\infty)=2.720616$.

From (148), $\Delta\left(X/X_0=0\right)=$ 5.714 days and $\frac{|\Delta-5|}{\Delta}=$ (5-1.949)/1.949 = 1.565 $\cong$ 156% .

If not considering the day of fighting in the safety streak, it is obtained:

$$\frac{|\Delta-4|}{\Delta}=(4\text{-}1.949)/1.949=1.052\cong 105\%\,. \qquad (158)$$

The final results for the second version of asymmetric conflict are presented in table 5.

The attacker's advantage materialises in the fact that the critical level for the defender's fighting potential is reached in the second day. The cancellation of the defender's fighting potential is achieved in the 6th day. In the case of the combat stopping criterion with respect to losses, the validation error is 44%, and with respect to duration, the error is 105%.

Table 5

α	β	Λ_1	Λ_2	ρ_0^{-1}	Q	c*	d
0.4	0.2	0.077	0.278	3.49	7.65	1/3	0.897

$\frac{\text{Y}_\text{F}}{\text{Y}_0}$	$\frac{\left\|\frac{Y_F}{Y_0}-d\right\|}{d}$	$\Delta(\frac{X}{X_0}=c^*)$	$\frac{\|\Delta-4\|}{\Delta}$	$\Delta(\frac{X}{X_0}=0)$	Conclusions
0.505	0.436	1.949	1.052	5.714	Non-validation

Validating the results for the second version of asymmetric conflict

6.5 The third version of the asymmetric conflict

The critical value of stopping the combat is:

$$c^*=\frac{1}{6}=0.167\ . \qquad (159)$$

The critical value of stopping the combat with regards to losses is obtained from (125):

$$d=\left[1+\frac{1}{7.65}\cdot\left(0.341\text{-}1\right)\right]^{1.66}=0.861\,, \qquad (160)$$

$$\frac{\left|\frac{\text{Y}_\text{F}}{\text{Y}_0}-d\right|}{d}=\frac{0.861\text{-}\frac{390}{772}}{0.861}=0.413\cong 41\%\,. \qquad (161)$$

From (135) it is obtained $A=1.95708$;

From (136):

$$B=\left[1+\left(7.65\text{-}1\right)\cdot 0.167^{-0.6}\right]^{\frac{1}{3}}=2.70779\,. \qquad (162)$$

From (138), $k=3\cdot 13.78635$.

From (140), $3H(A)=3.135109$.

From (142):

$$3H(B)=0.665+1.732*1.307=2.929481\,. \qquad (163)$$

From (144):

$$\Delta=13.78635\cdot\left(3.135109\text{-}2.929481\right)= =\ 2.834 \text{ days.} \qquad (164)$$

From (146), $H(\infty)=2.720616$.

From (148), $\Delta\left(X/X_0=0\right)=$ 5.714 days and $\frac{|\Delta-5|}{\Delta}=$ (5-2.834)/2.834 = 0.764 $\cong$ 76% .

If not considering the day of fighting in the safety streak, it is obtained:

$$\frac{|\Delta-4|}{\Delta}=(4\text{-}2.834)/2.834=0.411\cong 41\%\,. \qquad (165)$$

The final results for the third version of asymmetric conflict are presented in table 6.

Table 6

α	β	Λ_1	Λ_2	ρ_0^{-1}	Q	c*	d
0.4	0.2	0.077	0.278	3.49	7.65	1/6	0.861

$\frac{\text{Y}_\text{F}}{\text{Y}_0}$	$\frac{\left\|\frac{Y_F}{Y_0}-d\right\|}{d}$	$\Delta(\frac{X}{X_0}=c^*)$	$\frac{\|\Delta-4\|}{\Delta}$	$\Delta(\frac{X}{X_0}=0)$	Conclusions
0.505	0.413	2.834	0.411	5.714	Weak Validation

Validating the results for the third version of asymmetric conflict

The attacker's advantage materialises in the fact that the critical level for the defender's fighting potential is reached in the third day. The cancellation of the defender's fighting potential is achieved in the 6th day. In the case of the combat stopping criterion with respect to losses, the validation error is 41%, and with respect to duration, the error is 41%.

7 Conclusions

From the data obtained with the heuristic HERO methodology, the conclusion is the following. The penetration achieved inside the defender's territory is of 54 km (30 km in the safety strip and 24 km in the central strip), insufficient to accomplish the mission. This is in the case of 5 days long main attack on one direction (one day in the safety strip) and with the present forces.

The final results obtained with the Helmbold model for the symmetric conflict are presented in the table 7. The evolution of the forces ratio proves the defender's advantage. The losses decrease for the defender and they increase for the attacker. Taking into consideration the battle in the area of security as well, the total battle timeframe is of 5 days and the penetration achieved by the attacker is $30 + 23.32 \cong 54$ km.

Table 7

T	0	1	2	3	4	Days
τ	0	0.0247	0.0495	0.0742	0.0990	–
th τ	0	0.0246	0.0494	0.0740	0.0986	–
$\rho(t)$	0.607	0.616	0.626	0.637	0.649	–
$\Lambda_2\rho^{-\alpha}(t)$	9.17	9.14	9.10	9.06	9.02	%
$\Lambda_1\rho^{\alpha}(t)$	10.68	10.71	10.76	10.80	10.86	%
$\rho^{-1}(t)$	1.645	1.623	1.597	1.569	1.540	–
$V_0(\rho^{-1}(t))$	9	9	9	9	9	km/day
$V(\rho(t))$	5.83	5.83	5.83	5.83	5.83	km/day
$\Delta(t)$	–	5.83	11.66	17.49	23.32	km

Final results from the Helmbold model for the symmetric conflict

The comparative results from the Helmbold model for the three asymmetric conflict versions are presented in table 8.

The validation is performed for the asymmetric conflict model in which the defender reaches the 6th part of its original fighting potential on the third day, when the combat stops.

The heuristic HERO methodology and the analytic model (Helmbold model) for symmetric conflict lead to the similar results.

Table 8

α	β	Λ_1	Λ_2	ρ_0^{-1}	Q	c^*	d
0.4	0.2	0.077	0.278	3.49	7.65	2/3	0.953
0.4	0.2	0.077	0.278	3.49	7.65	1/3	0.897
0.4	0.2	0.077	0.278	3.49	7.65	1/6	0.861

$\frac{Y_F}{Y_0}$	$\frac{\left\lvert \frac{Y_F}{Y_0} - d \right\rvert}{d}$	$\Delta(\frac{X}{X_0} = c^*)$	$\frac{\lvert \Delta - 4 \rvert}{\Delta}$	$\Delta(\frac{X}{X_0} = 0)$	Conclusions
0.505	0.469	0.809	3.944	5.714	Non-validation
0.505	0.436	1.949	1.052	5.714	Non-validation
0.505	0.413	2.834	0.411	5.714	Weak Validation

Comparative results from the Helmbold model for the three asymmetric conflict versions

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