Control system of unmanned aerial vehicle used for endurance autonomous monitoring

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Abstract: - The paper purpose is to present some aspects regarding the control system of unmanned aerial vehicle - UAV, used to local observations, surveillance and monitoring interest area. The calculus methodology allows a numerical simulation of UAV evolution in bad atmospheric conditions by using nonlinear model, as well as a linear one for obtaining guidance command. The UAV model which will be presented has six DOF (degrees of freedom), and autonomous control system. This theoretical development allows us to build stability matrix, command matrix and control matrix and finally to analyze the stability of autonomous UAV flight .A robust guidance system, based on Kalman filter will be evaluated for different fly conditions and the results will be presented. The flight parameters and guidance will be analyzed. The paper is inspired by national project SAMO (Autonomous Aerial Monitoring System for Interest Areas of Great Endurance).

Key-Words: - UAV, Simulation, Control, Guidance, Endurance, Surveillance, Monitoring, Kalman filter

NOMENCLATURE

α - Attack angle (tangent definition);
β - Sideslip angle (tangent definition);
δ_a - Aileron deflection;
δ_e - Elevator deflection;
δ_{e0} The balance deflection angle for the elevator;
δ_r - Rudder deflection;
δ_T - Thrust command:
ψ - Azimuth angle;
θ - Inclination angle;
ϕ - Bank angle;
ρ - Air density;
$\mathbf{\Omega}$ - Body angular velocity;
A,B,C,E - Inertia moments;
$C_x^A;C_y^A;C_z^A$ - Aerodynamic coefficients of force in the mobile frame;
$C_l^A;C_m^A;C_n^A$ - Aerodynamic coefficients of momentum in the mobile frame;
$C_x^T;C_y^T;C_z^T$ - Thrust coefficients in the mobile frame;
$C_l^T;C_m^T;C_n^T$ - Thrust momentum coefficients in the mobile frame;
$F_0 = \rho\frac{V^2}{2}S$ - Reference aerodynamic force;
$H_o^A = F_0 l$ - Reference aerodynamic couple;
T_0 - Reference thrust force;
$H_o^T = T_0 l$ -Reference couple thrust;
l - Reference length;
m – Mass;
p,q,r - Angular velocity components along the axes of mobile frame;

S - Reference area;
$\mathbf{T}$ - Thrust vector;
t - Time;
$\mathbf{V}$ - Velocity vector;
u, v, w - Gyroplane velocity components in a mobile frame;
V_x, V_y, V_z -Velocity components in Earth frame;
$OX_0Y_0Z_0$ - Normal Earth-fixed frame;
$Oxyz$ – Body frame (mobile frame);
$x_0y_0z_0$ - Coordinates in Earth-fixed frame.

1 Introduction

The paper aims to evaluate the modelling and simulation of the performances of an UAV with an original design as shown in Fig. 1, in different atmospheric conditions.

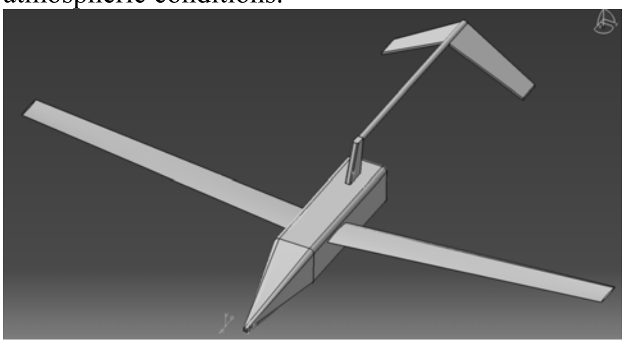

Figure 1 UAV Endurance configuration

The UAV designed will be capable of assuring a great length of video monitoring (8 hours) on an interest area, on a preprogrammed path, or guided, during the mission. The subject approached is a great interest not only in the perspective of commercial and civil applications, such as infrastructure monitoring, search and rescue missions, traffic control, but also in military applications. For achieving this objective, there have been established two major research directions. The first direction consists in designing and achieving of the carrier platform-UAV, of great endurance, capable of transporting equipment required for commanding the aircraft, for communications, data acquisition and data processing. The second major research direction synthesizes and implements the platform's automated command system for tracking the default trajectories. It is taken into account the attainment of a flexible infrastructure for the command system which will test the alternative algorithms used for the guidance and control of the platform.

2 General movement equations

As shown in the papers [2] and [3] the UAV's dynamic equations are the projection equations of the force, that are achieved from the impulse theorem, and the moment equations, which come from the kinetic moment theorem.

In order to obtain the dynamic equation we start by defining the aerodynamic coefficients in the mobile frame:

$$C_x^A = \frac{X^A}{F_0}; C_y^A = \frac{Y^A}{F_0}; C_z^A = \frac{Z^A}{F_0};$$
$$C_l^A = \frac{L^A}{H_0^A}; C_m^A = \frac{M^A}{H_0^A}; C_n^A = \frac{N^A}{H_0^A}. \tag{1}$$

where:

$$F_0 = \rho \frac{V^2}{2} S; \quad H_0^T = F_o l. \tag{2}$$

Similarly, if we consider the thrust T and the nominal thrust as reference T_0, we can define axial thrust coefficient:

$$C_x^T = T / T_0 \tag{3}$$

The force equation can be written in mobile frame:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m} \left\{ F_0 \begin{bmatrix} C_x^A \\ C_y^A \\ C_z^A \end{bmatrix} + T_0 \begin{bmatrix} C_x^T \\ 0 \\ 0 \end{bmatrix} \right\} + $$
$$+ \mathbf{A}_i \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} \tag{4}$$

or in fixed (Earth) frame:

$$\begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ \dot{V}_z \end{bmatrix} = \frac{1}{m} \mathbf{B}_i \left\{ F_0 \begin{bmatrix} C_x^A \\ C_y^A \\ C_z^A \end{bmatrix} + T_0 \begin{bmatrix} C_x^T \\ 0 \\ 0 \end{bmatrix} \right\} + \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}, \tag{5}$$

where the matrix $\mathbf{B}_i$ is defined using the Euler's angles:

$$\mathbf{B}_i = \mathbf{A}_i^T = \left[b_{i,j} \right], \tag{6}$$

with:

$b_{1,1} = \cos\psi \cos\theta$;

$b_{1,2} = \sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi$;

$b_{1,3} = \sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi$;

$b_{2,1} = -\sin\psi \cos\theta$;

$b_{2,2} = -\cos\psi \cos\phi - \sin\psi \sin\theta \sin\phi$;

$b_{2,3} = \cos\psi\sin\phi - \sin\psi\sin\theta\cos\phi$;

$b_{3,1} = \sin\theta; b_{3,2} = -\cos\theta\sin\phi$;

$b_{3,3} = -\cos\phi\cos\theta$.

The moment equation around the centre of the mass of the UAV, written in the mobile frame is:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \mathbf{J}^{-1} H_{\circ}^{A} \begin{bmatrix} C_l^A \\ C_m^A \\ C_n^A \end{bmatrix} + \mathbf{J}^{-1} \begin{bmatrix} (B-C)qr + Epq \\ (C-A)rp + E(r^2 - p^2) \\ (A-B)pq - Eqr \end{bmatrix}, \tag{7}$$

where the inverse matrix for the inertia moment is given by:

$$\mathbf{J}^{-1} = \frac{1}{AC - E^2} \begin{bmatrix} C & 0 & E \\ 0 & (AC - E^2)/B & 0 \\ E & 0 & A \end{bmatrix}. \tag{8}$$

The kinematical equations are additional equations, which allow us to obtain the linear coordinates in the inertial frame. If we use the component of velocity in mobile frame we have:

$$\begin{bmatrix} \dot{x}_0 & \dot{y}_0 & \dot{z}_0 \end{bmatrix}^T = \mathbf{B}_i \begin{bmatrix} u & v & w \end{bmatrix}^T. \tag{9}$$

Equivalent with this equation, if we use the velocity components in fixed frame we can write:

$$\begin{bmatrix} \dot{x}_0 & \dot{y}_0 & \dot{z}_0 \end{bmatrix}^T = \begin{bmatrix} V_x & V_y & V_z \end{bmatrix}^T. \tag{10}$$

For Euler's angle when the rotation velocity components are known we have:

$$\begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T = \mathbf{W}_A \begin{bmatrix} p & q & r \end{bmatrix}^T, \tag{11}$$

where

$$\mathbf{W}_A = \begin{bmatrix} 1 & \sin\phi\,\mathrm{tg}\,\theta & \cos\phi\,\mathrm{tg}\,\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}. \tag{12}$$

Supplementary, we have mass equation which describes mass UAV's modification during the flight:

$$\dot{m} = -C_{sp} T \tag{13}$$

where C_{sp} is specific fuel consumption.

3 Guidance command

Resuming [4], the guidance commands for UAV flight are start from relatives parameters $\tilde{\theta}; \tilde{\psi}; \tilde{\phi}; h_z \tilde{u}$ which are given by:

$$\tilde{\theta} = \theta_d - \theta;\ \tilde{\psi} = \psi_d - \psi;\ \tilde{\phi} = \phi_d - \phi;$$

$$\lambda_x = -V_x;\ \lambda_y = -V_y\ \lambda_z = -V_z;$$

$$h_x = x_{0d} - x_0;\ h_y = y_{0d} - y_0;\ h_z = z_{0d} - z_0;$$

$$\tilde{u} = u_D - u \tag{14}$$

where

$$\phi_D, \theta_d;\ \psi_d;\ x_{0d} y_{0d}\ z_{0d}; u_D; \tag{15}$$

are the input reference values.

Also we use integral terms, defined as:

$$\dot{I}_x = h_x; \dot{I}_y = h_y; \dot{I}_z = h_z. \tag{16}$$

The guidance commands are applied through the actuators which are approximated in the paper [4] by relations:

$$\dot{\delta}_T = -\frac{\delta_T}{\tau_{\delta T}} + \frac{k_{\delta x}^u u_T}{\tau_{\delta T}};\quad \dot{\delta}_a = -\frac{\delta_a}{\tau_{\delta a}} + \frac{k_{\delta a}^u u_a}{\tau_{\delta a}};$$

$$\dot{\delta}_e = -\frac{\delta_e}{\tau_{\delta e}} + \frac{k_{\delta e}^u u_e}{\tau_{\delta e}};\quad \dot{\delta}_r = -\frac{\delta_r}{\tau_{\delta r}} + \frac{k_{\delta r}^u u_r}{\tau_{\delta r}} \tag{17}$$

where $\tau_{\delta T}; \tau_{\delta a}; \tau_{\delta e}; \tau_{\delta r}$ are the time constants and $k_{\delta T}^u; k_{\delta a}^u; k_{\delta e}^u; k_{\delta r}^u$ are the gain constants.

4 Balance movement

The study of flight stability will be made accordingly to Liapunov theory, considering the system of movement equations perturbed around the balanced movement. This involves a disturbance shortly applied on the balance movement, which will produce deviation of the state variables. Developing in series the perturbed movement equations in relation to status variables and taking into account the first order terms of the detention, we will get linear equations which can be used to analyze the stability in the first approximation, as we proceed in most dynamic non linear problems. To determine basic movement parameters in equations (1) and (3) is considered

$$\dot{u} = \dot{v} = \dot{w} = 0; \dot{p} = \dot{q} = \dot{r} = 0 \tag{18}$$

thous:

$$\frac{1}{m} \left\{ F_0 \begin{bmatrix} C_x^A \\ C_y^A \\ C_z^A \end{bmatrix} + T_0 \begin{bmatrix} C_x^T \\ C_y^T \\ C_z^T \end{bmatrix} \right\} + \mathbf{A}_i \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

$$\mathbf{J}^{-1}\left\{H_0\begin{bmatrix}C_l^A\\C_m^A\\C_n^A\end{bmatrix}+U_0\begin{bmatrix}C_l^T\\C_m^T\\C_n^T\end{bmatrix}\right\}+ +\mathbf{J}^{-1}\begin{bmatrix}qr(B-C)+Epq\\rp(C-A)+E(r^2-p^2)\\pq(A-B)-Eqr\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \quad (20)$$

5 Linear form of the general equations

To obtain the general form of linear equations we start from the linear expression between aerodynamic variables and velocity components:

$$\Delta\mathbf{M}=\frac{1}{V}\mathbf{B}_{\beta\alpha}\Delta\mathbf{V}-\frac{1}{a}\frac{\partial a}{\partial z_0}\begin{bmatrix}\Delta z_0 & 0 & 0\end{bmatrix}^T, \quad (21)$$

where:

$$\Delta\mathbf{M}=\begin{bmatrix}\frac{\Delta M}{M} & \Delta\beta & \Delta\alpha\end{bmatrix}^T;$$

$$\Delta\mathbf{V}=\begin{bmatrix}\Delta u & \Delta v & \Delta w\end{bmatrix}^T$$

$$\mathbf{B}_{\beta\alpha}=\begin{bmatrix}\cos\gamma^* & -\mathrm{tg}\beta\cos\gamma^* & \mathrm{tg}\alpha\cos\gamma^*\\ -\frac{\sin 2\beta}{2\cos\gamma^*} & -\frac{\cos^2\beta}{\cos\gamma^*} & 0\\ -\frac{\sin 2\alpha}{2\cos\gamma^*} & 0 & \frac{\cos^2\alpha}{\cos\gamma^*}\end{bmatrix}$$

By definition [8] aerodynamic angles are:

$$\alpha=-\arctan(v/u),\ \beta=\arctan(w/u). \quad (22)$$

and:

$$\gamma^*=\arctan\left(\frac{\sqrt{v^2+w^2}}{u}\right) \quad (23)$$

Similarly, the relation between unstationary variables is:

$$\Delta\hat{\mathbf{M}}=\frac{l}{V^2}\mathbf{B}_{\beta\alpha}\Delta\dot{\mathbf{V}} \quad (24)$$

where:

$$\Delta\hat{\mathbf{M}}=\begin{bmatrix}\frac{\Delta\hat{M}}{M} & \Delta\hat{\beta} & \Delta\hat{\alpha}\end{bmatrix}^T;$$

$$\Delta\dot{\mathbf{V}}=\begin{bmatrix}\Delta\dot{u} & \Delta\dot{v} & \Delta\dot{w}\end{bmatrix}^T$$

The following notations are used for the undimensional angular velocities and non-dimensional aerodynamic sizes:

$$\hat{p}=\dot{p}l/V;\ \hat{q}=\dot{q}l/V;\ \hat{r}=\dot{r}l/V; \quad \hat{\alpha}=\dot{\alpha}l/V;\ \hat{\beta}=\dot{\beta}l/V;\hat{M}=\dot{M}l/V \quad (25)$$

where l is reference length – body length.

Similarly, for un-stationary components we can write:

$$\dot{\mathbf{V}}=\mathbf{a}-\mathbf{A}_\Omega\mathbf{V}, \quad (26)$$

or in linear form:

$$\Delta\dot{\mathbf{V}}=\Delta\mathbf{a}-\mathbf{A}_\Omega\Delta\mathbf{V}-\mathbf{A}_V\Delta\mathbf{\Omega}, \quad (27)$$

where:

$$\mathbf{a}=\begin{bmatrix}a_x & a_y & a_z\end{bmatrix}^T;\quad \mathbf{\Omega}=\begin{bmatrix}p & q & r\end{bmatrix}^T;$$

$$\mathbf{A}_\Omega=\begin{bmatrix}0 & -r & q\\ r & 0 & -p\\ -q & p & 0\end{bmatrix};\ \mathbf{A}_V=\begin{bmatrix}0 & w & -v\\ -w & 0 & u\\ v & -u & 0\end{bmatrix},$$

In this case, the expression of the aerodynamic force becomes:

$$\begin{aligned}\Delta\mathbf{F}=&F_0\mathbf{C}_F\left(2\frac{\Delta M}{M}+2\frac{\Delta a}{a}+\frac{\Delta\rho}{\rho}\right)+\\&+F_0\mathbf{C}_{FM\beta\alpha}\Delta\mathbf{M}+F_0\mathbf{C}_{Fzo}\Delta\hat{z}_0+\\&+F_0\mathbf{C}_{Fpqr}\Delta\hat{\Omega}+\\&+F_0\mathbf{C}_{F\dot{M}\dot{\beta}\dot{\alpha}}\Delta\hat{\mathbf{M}}+F_0\mathbf{C}_{F\delta}\Delta\boldsymbol{\delta}^A,\end{aligned} \quad (28)$$

where:

$$\Delta\mathbf{F}=\begin{bmatrix}\Delta X^A\\ \Delta Y^A\\ \Delta Z^A\end{bmatrix}\ \Delta\boldsymbol{\delta}^A=\begin{bmatrix}\Delta\delta_a\\ \Delta\delta_e\\ \Delta\delta_r\end{bmatrix}\ \mathbf{C}_F=\begin{bmatrix}C_x^A\\ C_y^A\\ C_z^A\end{bmatrix}$$

$$\mathbf{C}_{Fz0}=\begin{bmatrix}C_{x_{z0}}^A\\ C_{y_{z0}}^A\\ C_{z_{z0}}^A\end{bmatrix}\ \mathbf{C}_{FM}=\begin{bmatrix}C_{x_M}^A\\ C_{y_M}^A\\ C_{z_M}^A\end{bmatrix};$$

$$\mathbf{C}_{FM\beta\alpha}=\begin{bmatrix}MC_{xM}^A & C_{x\beta}^A & C_{x\alpha}^A\\ MC_{yM}^A & C_{y\beta}^A & C_{y\alpha}^A\\ MC_{zM}^A & C_{z\beta}^A & C_{z\alpha}^A\end{bmatrix}$$

$$\mathbf{C}_{Fpqr}=\begin{bmatrix}C_{xp}^A & C_{xq}^A & C_{xr}^A\\ C_{yp}^A & C_{yq}^A & C_{yr}^A\\ C_{zp}^A & C_{zq}^A & C_{zr}^A\end{bmatrix}$$

$$\mathbf{C}_{F\dot{M}\dot{\beta}\dot{\alpha}}=\begin{bmatrix}MC_{x\dot{M}}^A & C_{x\dot{\beta}}^A & C_{x\dot{\alpha}}^A\\ MC_{y\dot{M}}^A & C_{y\dot{\beta}}^A & C_{y\dot{\alpha}}^A\\ MC_{z\dot{M}}^A & C_{z\dot{\beta}}^A & C_{z\dot{\alpha}}^A\end{bmatrix}$$

$$\mathbf{C}_{F\delta}=\begin{bmatrix}C^A_{x\delta a} & C^A_{x\delta e} & C^A_{x\delta r}\\ C^A_{y\delta a} & C^A_{y\delta e} & C^A_{y\delta r}\\ C^A_{z\delta a} & C^A_{z\delta e} & C^A_{z\delta r}\end{bmatrix}$$

Similarly, for aerodynamic moment we can write:

$$\begin{aligned}\Delta\mathbf{H}=&H_0\mathbf{C}_H\left(2\frac{\Delta M}{M}+2\frac{\Delta a}{a}+\frac{\Delta\rho}{\rho}\right)+\\&+H_0\mathbf{C}_{HM\beta\alpha}\Delta\mathbf{M}+H_0\mathbf{C}_{Hzp}\Delta\hat{z}_0+\\&+H_0\mathbf{C}_{Hpqr}\Delta\hat{\Omega}+\\&+H_0\mathbf{C}_{H\dot{M}\dot{\beta}\dot{\alpha}}\Delta\hat{\mathbf{M}}+H_0\mathbf{C}_{H\delta}\Delta\boldsymbol{\delta}^A,\end{aligned}\qquad(29)$$

where:

$$\Delta\mathbf{H}=\begin{bmatrix}\Delta L^A\\ \Delta M^A\\ \Delta N^A\end{bmatrix};\Delta\boldsymbol{\delta}^A=\begin{bmatrix}\Delta\delta_a\\ \Delta\delta_e\\ \Delta\delta_r\end{bmatrix};\mathbf{C}_H=\begin{bmatrix}C^A_l\\ C^A_m\\ C^A_n\end{bmatrix};$$

$$\mathbf{C}_{Hz0}=\begin{bmatrix}C^A_{lz0}\\ C^A_{mz0}\\ C^A_{nz0}\end{bmatrix};\mathbf{C}_{HM}=\begin{bmatrix}C^A_{lM}\\ C^A_{mM}\\ C^A_{nM}\end{bmatrix};$$

$$\mathbf{C}_{HM\beta\alpha}=\begin{bmatrix}MC^A_{lM} & C^A_{l\beta} & C^A_{l\alpha}\\ MC^A_{mM} & C^A_{m\beta} & C^A_{m\alpha}\\ MC^A_{nM} & C^A_{n\beta} & C^A_{n\alpha}\end{bmatrix};$$

$$\mathbf{C}_{Hpqr}=\begin{bmatrix}C^A_{lp} & C^A_{lq} & C^A_{lr}\\ C^A_{mp} & C^A_{mq} & C^A_{mr}\\ C^A_{np} & C^A_{nq} & C^A_{nr}\end{bmatrix};$$

$$\mathbf{C}_{H\dot{M}\dot{\beta}\dot{\alpha}}=\begin{bmatrix}MC^A_{l\dot{M}} & C^A_{l\dot{\beta}} & C^A_{l\dot{\alpha}}\\ MC^A_{m\dot{M}} & C^A_{m\dot{\beta}} & C^A_{m\dot{\alpha}}\\ MC^A_{n\dot{M}} & C^A_{n\dot{\beta}} & C^A_{n\dot{\alpha}}\end{bmatrix};$$

$$\mathbf{C}_{H\delta}=\begin{bmatrix}C^A_{l\delta a} & C^A_{l\delta e} & C^A_{l\delta r}\\ C^A_{m\delta a} & C^A_{m\delta e} & C^A_{m\delta r}\\ C^A_{n\delta a} & C^A_{n\delta e} & C^A_{n\delta r}\end{bmatrix}.$$

Considering the relationship between aerodynamic components and velocity components in body-frame the aerodynamic force becomes:

$$\begin{aligned}\Delta\mathbf{F}=&\frac{F_0}{V}\mathbf{C}_{FM\beta\alpha}\mathbf{B}_{\beta\alpha}\Delta\mathbf{V}+\\&+F_0\left(\frac{1}{\rho}\frac{\partial\rho}{\partial z_0}\mathbf{C}_F-\frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{FM}+\frac{1}{l}\mathbf{C}_{Fzp}\right)\Delta z_0+\\&+\frac{F_0 l}{V}\mathbf{C}_{Fpqr}\Delta\boldsymbol{\Omega}+\\&+\frac{F_0 l}{V^2}\mathbf{C}_{F\dot{M}\dot{\beta}\dot{\alpha}}\mathbf{B}_{\beta\alpha}\Delta\dot{\mathbf{V}}+F_0\mathbf{C}_{F\delta}\boldsymbol{\delta}^A\end{aligned}\qquad(30)$$

Similarly, for aerodynamic moment, we can write:

$$\begin{aligned}\Delta\mathbf{H}=&\frac{H_0}{V}\mathbf{C}_{HM\beta\alpha}\mathbf{B}_{\beta\alpha}\Delta\mathbf{V}+\\&+H_0\left(\frac{1}{\rho}\frac{\partial\rho}{\partial z_0}\mathbf{C}_H-\frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{HM}+\frac{1}{l}\mathbf{C}_{Hz0}\right)\Delta z_0+\\&\frac{H_0 l}{V}\mathbf{C}_{Hpqr}\Delta\boldsymbol{\Omega}+\\&+\frac{H_0 l}{V^2}\mathbf{C}_{H\dot{M}\dot{\beta}\dot{\alpha}}\mathbf{B}_{\beta\alpha}\Delta\dot{\mathbf{V}}+H_0\mathbf{C}_{H\delta}\Delta\boldsymbol{\delta}^A\end{aligned}\qquad(31)$$

Regarding thrust, it can be put in linear form as follows:

$$\begin{aligned}\Delta\mathbf{T}=&\frac{T_0}{V}M\mathbf{C}_{TM}\mathbf{b}_{\beta\alpha}\Delta\mathbf{V}+\\&+T_0\left(\frac{1}{l}\mathbf{C}_{Tz0}-\frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{TM}\right)\Delta z_0+,\\&+T_0\mathbf{C}_{T\delta}\Delta\delta_T\end{aligned}\qquad(32)$$

where:

$$\Delta\mathbf{T}=\begin{bmatrix}\Delta X^T\\ \Delta Y^T\\ \Delta Z^T\end{bmatrix}\mathbf{C}_{TM}=\begin{bmatrix}C^T_{xM}\\ C^T_{yM}\\ C^T_{zM}\end{bmatrix}\ \mathbf{b}_{\beta\alpha}=\begin{bmatrix}\cos\gamma^*\\ -\mathrm{tg}\,\beta\cos\gamma^*\\ \mathrm{tg}\,\alpha\cos\gamma^*\end{bmatrix}^T;$$

$$\mathbf{C}_{Tz0}=\begin{bmatrix}C^T_{xz0}\\ C^T_{yz0}\\ C^T_{zz0}\end{bmatrix}\ C_{T\delta}=\begin{bmatrix}C^T_{x\delta T}\\ C^T_{y\delta T}\\ C^T_{z\delta T}\end{bmatrix}$$

Similarly, for thrust moment we can write:

$$\Delta\mathbf{U} = \frac{U_0}{V} M \mathbf{C}_{UM} \mathbf{b}_{\beta\alpha} \Delta\mathbf{V} + U_0 \left(\frac{l}{l}\mathbf{C}_{Uz0} - \frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{UM} \right)\Delta z_0 + \frac{U_0 l}{V}\mathbf{C}_{Upqr}\Delta\mathbf{\Omega} + U_0 \mathbf{C}_{U\delta}\Delta\delta_T \qquad (33)$$

where:

$$\Delta\mathbf{U} = \begin{bmatrix} \Delta L^T \\ \Delta M^T \\ \Delta N^T \end{bmatrix}; \mathbf{C}_{UM} = \begin{bmatrix} C_{lM}^T \\ C_{mM}^T \\ C_{nM}^T \end{bmatrix}; \mathbf{C}_{Uzp} = \begin{bmatrix} C_{lzp}^T \\ C_{mzp}^T \\ C_{nzp}^T \end{bmatrix};$$

$$\mathbf{C}_{Upqr} = \begin{bmatrix} C_{lp}^T & 0 & 0 \\ 0 & C_{mq}^T & 0 \\ 0 & 0 & C_{nr}^T \end{bmatrix}; \mathbf{C}_{U\delta} = \begin{bmatrix} C_{l\delta T}^T \\ C_{m\delta T}^T \\ C_{n\delta T}^T \end{bmatrix}$$

Finally, if we know the expression of the components of the gravity along the mobile frame:

$$\mathbf{g} = \mathbf{A}_i \mathbf{g}_0 \qquad (34)$$

we express the variation of the gravity along mobile frame axis:

$$\Delta\mathbf{g} = \mathbf{A}_{gR}\Delta\mathbf{r} \qquad (35)$$

where:

$$\Delta\mathbf{r} = \begin{bmatrix} \Delta\phi & \Delta\theta & \Delta\psi \end{bmatrix}^T$$

$$\mathbf{A}_{gR} = \begin{bmatrix} \dfrac{\partial\mathbf{A}_i}{\partial\phi} & \dfrac{\partial\mathbf{A}_i}{\partial\theta} & \dfrac{\partial\mathbf{A}_i}{\partial\psi} \end{bmatrix} \begin{bmatrix} \mathbf{g}_0 & 0 & 0 \\ 0 & \mathbf{g}_0 & 0 \\ 0 & 0 & \mathbf{g}_0 \end{bmatrix}$$

Starting from force equation (4) in mobile frame, we obtain:

$$\Delta\dot{\mathbf{V}} = \frac{1}{m}\Delta\mathbf{F} + \frac{1}{m}\Delta\mathbf{T} + \Delta\mathbf{g} - \mathbf{A}_\Omega\Delta\mathbf{V} - \mathbf{A}_V\Delta\mathbf{\Omega} + \Delta\mathbf{f}_p, \qquad (36)$$

where:

$$\Delta\mathbf{f}_p = \frac{1}{m}\begin{bmatrix} \Delta X_p & \Delta Y_p & \Delta Z_p \end{bmatrix}^T \qquad (37)$$

means perturbation force.

Similarly, for momentum equation (7) we can write:

$$\Delta\dot{\mathbf{\Omega}} = \mathbf{J}^{-1}\Delta\mathbf{H} + \mathbf{J}^{-1}\Delta\mathbf{U} + \mathbf{J}^{-1}\mathbf{A}_K\Delta\mathbf{\Omega} + \Delta\mathbf{m}_p, \qquad (38)$$

where:

$$\mathbf{A}_K = \begin{bmatrix} B-C & 0 & 0 \\ 0 & C-A & 0 \\ 0 & 0 & A-B \end{bmatrix} \begin{bmatrix} 0 & r & q \\ r & 0 & p \\ q & p & 0 \end{bmatrix} + E \begin{bmatrix} q & p & 0 \\ -2p & 0 & 2r \\ 0 & -r & -q \end{bmatrix}$$

and

$$\Delta\mathbf{m}_p = \mathbf{J}^{-1}\begin{bmatrix} \Delta L_p & \Delta M_p & \Delta N_p \end{bmatrix}^T \qquad (39)$$

means perturbation moment.

Starting from these relations we can write:

$$\Delta\dot{\mathbf{V}} = \mathbf{F}_V\Delta\mathbf{V} + \mathbf{F}_\Omega\Delta\mathbf{\Omega} + \mathbf{f}_{z_0}\Delta z_0 + \mathbf{G}_R\Delta\mathbf{r} + \mathbf{F}_{\dot{V}}\Delta\dot{\mathbf{V}} + \mathbf{F}_\delta^A\Delta\boldsymbol{\delta}^A + \mathbf{f}_\delta^T\Delta\delta_T + \Delta\mathbf{f}_p \qquad (40)$$

where:

$$\mathbf{F}_V = \frac{F_0}{mV}\mathbf{C}_{FM\beta\alpha}\mathbf{B}_{\beta\alpha} + \frac{T_0}{mV}M\mathbf{C}_{TM}\mathbf{b}_{\beta\alpha} - \mathbf{A}_\Omega$$

$$\mathbf{F}_\Omega = \frac{F_0 l}{mV}\mathbf{C}_{Fpqr} - \mathbf{A}_V ; \mathbf{G}_R = \mathbf{A}_{gR} ;$$

$$\mathbf{F}_{\dot{V}} = \frac{F_0 l}{mV^2}\mathbf{C}_{F\dot{M}\dot{\beta}\dot{\alpha}}\mathbf{B}_{\beta\alpha} ; \mathbf{F}_\delta^A = \frac{F_0}{m}\mathbf{C}_{F\delta}$$

$$\mathbf{f}_{z0} = \frac{F_0}{m}\left(\frac{1}{\rho}\frac{\partial\rho}{\partial z_0}\mathbf{C}_F - \frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{FM} + \frac{1}{l}\mathbf{C}_{Fz0} \right) + \frac{T_0}{m}\left(\frac{1}{l}\mathbf{C}_{Tz0} - \frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{TM} \right)$$

$$\mathbf{f}_\delta^T = \frac{T_0}{m}\mathbf{C}_{T\delta} ;$$

Starting from moment equation (3) we obtain:

$$\Delta\dot{\mathbf{\Omega}} = \mathbf{M}_V\Delta\mathbf{V} + \mathbf{M}_\Omega\Delta\mathbf{\Omega} + \mathbf{m}_{z_0}\Delta z_0 + \mathbf{M}_{\dot{V}}\Delta\dot{\mathbf{V}} + \mathbf{M}_\delta^A\Delta\boldsymbol{\delta}^A + \mathbf{m}_\delta^T\Delta\delta_T + \Delta\mathbf{m}_p \qquad (41)$$

where:

$$\mathbf{M}_V = \frac{H_0}{V}\mathbf{J}^{-1}\mathbf{C}_{HM\beta\alpha}\mathbf{B}_{\beta\alpha} + \frac{U_0}{V}M\mathbf{J}^{-1}\mathbf{C}_{UM}\mathbf{b}_{\beta\alpha}$$

$$\mathbf{M}_\Omega = \frac{H_0 l}{V}\mathbf{J}^{-1}\mathbf{C}_{Hpqr} + \frac{U_0 l}{V}\mathbf{J}^{-1}\mathbf{C}_{Upqr} + \mathbf{J}^{-1}\mathbf{A}_K$$

$$\mathbf{M}_{\dot{V}} = \frac{H_0 l}{V^2}\mathbf{J}^{-1}\mathbf{C}_{F\dot{M}\dot{\beta}\dot{\alpha}}\mathbf{B}_{\beta\alpha} ; \mathbf{M}_\delta^A = H_0\mathbf{J}^{-1}\mathbf{C}_{H\delta} ;$$

$$\mathbf{m}_{z0} = H_0\mathbf{J}^{-1}\left(\frac{1}{\rho}\frac{\partial\rho}{\partial z_0}\mathbf{C}_H - \frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{HM} + \frac{1}{l}\mathbf{C}_{Hz0} \right) + U_0\mathbf{J}^{-1}\left(\frac{l}{l}\mathbf{C}_{Uzp} - \frac{M}{a}\frac{\partial a}{\partial z_0}\mathbf{C}_{UM} \right)$$

$$\mathbf{m}_\delta^T = U_0\mathbf{J}^{-1}\mathbf{C}_{U\delta} ;$$

Starting from cinematic equation (9) we obtain:

$$\Delta\dot{\mathbf{p}} = \mathbf{P}_V\Delta\mathbf{V} + \mathbf{P}_R\Delta\mathbf{r} \qquad (42)$$

where:

$$\Delta\mathbf{p} = \begin{bmatrix} \Delta x_0 & \Delta y_0 & \Delta z_0 \end{bmatrix}^T$$

$$\mathbf{P}_R = \begin{bmatrix} \dfrac{\partial\mathbf{B}_i}{\partial\boldsymbol{\varphi}} & \dfrac{\partial\mathbf{B}_i}{\partial\boldsymbol{\theta}} & \dfrac{\partial\mathbf{B}_i}{\partial\boldsymbol{\psi}} \end{bmatrix} \begin{bmatrix} \mathbf{V} & 0 & 0 \\ 0 & \mathbf{V} & 0 \\ 0 & 0 & \mathbf{V} \end{bmatrix}$$

$$\mathbf{P}_V = \mathbf{B}_i$$

and from equation (11):

$$\Delta\dot{\mathbf{r}} = \mathbf{R}_\Omega\Delta\mathbf{\Omega} + \mathbf{R}_R\Delta\mathbf{r} \qquad (43)$$

where:

$$\mathbf{R}_R = \left[\frac{\partial \mathbf{W}_A}{\partial \phi} \;\vdots\; \frac{\partial \mathbf{W}_A}{\partial \theta} \;\vdots\; \frac{\partial \mathbf{W}_A}{\partial \psi} \right] \begin{bmatrix} \boldsymbol{\Omega} & 0 & 0 \\ 0 & \boldsymbol{\Omega} & 0 \\ 0 & 0 & \boldsymbol{\Omega} \end{bmatrix}$$

$$\mathbf{R}_\Omega = \mathbf{W}_A$$

Using linear equations (40)…(43) we can put the system in regular form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad (44)$$

where:

$$\mathbf{A} = \left[\mathbf{I} - \mathbf{A}_1\right]^{-1} \mathbf{A}_0 \; ; \; \mathbf{B} = \left[\mathbf{I} - \mathbf{A}_1\right]^{-1} \mathbf{B}_0$$

We can also highlight the stability and control matrixes as:

Table 1 The stability matrix with stationary variables $\mathbf{A}_0$

	$\mathbf{V}$	$\boldsymbol{\Omega}$	$\mathbf{p}$	$\mathbf{r}$
$\mathbf{V}$	$\mathbf{F}_V$	$\mathbf{F}_\Omega$	$\begin{matrix} 0 & 0 & \\ 0 & 0 & \mathbf{f}_{z_0} \\ 0 & 0 & \end{matrix}$	$\mathbf{G}_R$
$\boldsymbol{\Omega}$	$\mathbf{M}_V$	$\mathbf{M}_\Omega$	$\begin{matrix} 0 & 0 & \\ 0 & 0 & \mathbf{m}_{z_0} \\ 0 & 0 & \end{matrix}$	
$\mathbf{p}$	$\mathbf{P}_V$			$\mathbf{P}_R$
$\mathbf{r}$		$\mathbf{R}_\Omega$		$\mathbf{R}_R$

Table 2 The stability matrix with nonstationary variables $\mathbf{A}_1$

	$\dot{\mathbf{V}}$	$\dot{\boldsymbol{\Omega}}$	$\dot{\mathbf{p}}$	$\dot{\mathbf{r}}$
$\mathbf{V}$	$\mathbf{F}_{\dot{V}}$			
$\boldsymbol{\Omega}$	$\mathbf{M}_{\dot{V}}$			
$\mathbf{p}$				
$\mathbf{r}$				

Table 3 Control matrix $\mathbf{B}_0$

	$\boldsymbol{\delta}^A$	δ^T
$\mathbf{V}$	$\mathbf{F}_\delta^A$	$\mathbf{f}_\delta^T$
$\boldsymbol{\Omega}$	$\mathbf{M}_\delta^A$	$\mathbf{m}_\delta^T$
$\mathbf{p}$		
$\mathbf{r}$		

Finally, we express the perturbation vector:

Table 4 The perturbation vector $\mathbf{p}_0$

$\mathbf{V}$	$\Delta \mathbf{f}_p$
$\boldsymbol{\Omega}$	$\Delta \mathbf{m}_p$
$\mathbf{p}$	
$\mathbf{r}$	

4. Extended stability and control matrixes

Besides the general motion equations in linear form as outlined above, UAVs needs other relationships to be added. Among them, the most important and which can not be neglected are the actuator equations and the guidance equations. For the autonomous flight, as is case of UAV's, the guidance equation is necessary to introduce integrated terms specific to PID-type controllers.

Starting from (17) linear form of the actuator equation became:

$$\begin{aligned}&\left[\Delta\dot{\delta}_a \quad \Delta\dot{\delta}_e \quad \Delta\dot{\delta}_r \quad \Delta\dot{\delta}_T\right]^T = \\ &= \mathbf{D}_\delta\left[\Delta\delta_a \quad \Delta\delta_e \quad \Delta\delta_r \quad \Delta\delta_T\right]^T + \mathbf{D}_u\Delta\mathbf{u}\end{aligned} \tag{45}$$

where:

$$\mathbf{D}_\delta = \begin{bmatrix} -1/\tau_{\delta a} & 0 & 0 & 0 \\ 0 & -1/\tau_{\delta e} & 0 & 0 \\ 0 & 0 & -1/\tau_{\delta r} & 0 \\ 0 & 0 & 0 & -1/\tau_{\delta T} \end{bmatrix};$$

$$\mathbf{D}_u = \begin{bmatrix} k^u_{\delta a}/\tau_{\delta a} & 0 & 0 & 0 \\ 0 & k^u_{\delta e}/\tau_{\delta e} & 0 & 0 \\ 0 & 0 & k^u_{\delta r}/\tau_{\delta r} & 0 \\ 0 & 0 & 0 & k^u_{\delta T}/\tau_{\delta T} \end{bmatrix},$$

Similarly, linear form of auxiliary equation (16) became:

$$\left[\Delta\dot{I}_x \quad \Delta\dot{I}_y \quad \Delta\dot{I}_z\right]^T = -\left[\Delta x_p \quad \Delta y_p \quad \Delta z_p\right]^T + \Delta\mathbf{f} \tag{46}$$

where $\Delta\mathbf{f}$ means reference values as input function. Using linear relation (45 and (46) we can build extended stability and control matrixes.

Table 5 Stability extended matrix $\mathbf{A}$

	$\mathbf{V}$	$\mathbf{\Omega}$	$\mathbf{p}$	$\mathbf{r}$	$\boldsymbol{\delta}$	$\mathbf{I}$
$\mathbf{V}$	$\mathbf{F}_V$	$\mathbf{F}_\Omega$	$\begin{matrix}0 & 0 & \\ 0 & 0 & \mathbf{f}_{z_0} \\ 0 & 0 & \end{matrix}$	$\mathbf{G}_R$	$\mathbf{F}^A_\delta \quad \mathbf{f}^T_\delta$	
$\mathbf{\Omega}$	$\mathbf{M}_V$	$\mathbf{M}_\Omega$	$\begin{matrix}0 & 0 & \\ 0 & 0 & \mathbf{m}_{z_0} \\ 0 & 0 & \end{matrix}$		$\mathbf{M}^A_\delta \quad \mathbf{m}^T_\delta$	
$\mathbf{p}$	$\mathbf{P}_V$			$\mathbf{P}_R$		
$\mathbf{r}$		$\mathbf{R}_\Omega$		$\mathbf{R}_R$		
$\boldsymbol{\delta}$						
$\mathbf{I}$			$-\mathbf{I}$			

.

Table 6 Control extended matrix $\mathbf{B}$

	$u_a \quad u_e \quad u_r \quad u_T$
$\mathbf{V}$	
$\mathbf{\Omega}$	
$\mathbf{p}$	
$\mathbf{r}$	
$\boldsymbol{\delta}$	$\mathbf{D}_u$
$\mathbf{I}$	

5 Optimal control using state vector

Supposing to have access to extend state vector $\mathbf{x}$, we can obtain directly the controller $\mathbf{K}$ for optimal command:

$$\mathbf{u} = -\mathbf{K}\mathbf{x} \tag{47}$$

In order to satisfy the linear quadratic performance index (cost function):

$$\min J = \int_0^\infty (\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})\,\mathrm{d}t, \tag{48}$$

where the extended pair $(\mathbf{A},\mathbf{B})$ is controllable and the state weighting matrix $\mathbf{Q}$ is symmetric and quasi positive:

$$\mathbf{Q} \geq 0;\ \mathbf{Q} = \mathbf{Q}^T. \tag{49}$$

while the control weighting matrix $\mathbf{R}$ is symmetric and positive:

$$\mathbf{R} > 0;\ \mathbf{R} = \mathbf{R}^T; \tag{50}$$

In this case, the following relation gives the optimal controller

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{51}$$

where the matrix $\mathbf{P}$ is the solution of the algebraic Riccati equation:

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0} \qquad (52)$$

6 Optimal control using Kalman filter

Using the optimal controller designed above requires access to all system states, very difficult in view of the limited number of sensors. In this case, for a complete description of the system we use a linear state estimator constructed as a Kalman filter. For this purpose we start from the regular relations:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{G}\mathbf{w} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{v}\end{aligned} \qquad (53)$$

where $\mathbf{w}$ is the external noise and $\mathbf{v}$ is the internal noise introduced by the sensors, where the matrixes $\mathbf{G}, \mathbf{C}, \mathbf{D}$ are considerate corrected with the stability matrix with non-stationary variables $\mathbf{A}_1$

$$\mathbf{G} = [\mathbf{I} - \mathbf{A}_1]^{-1}\mathbf{G}_0\,;\mathbf{C} = [\mathbf{I} - \mathbf{A}_1]^{-1}\mathbf{C}_0\,;\mathbf{D} = [\mathbf{I} - \mathbf{A}_1]^{-1}\mathbf{D}_0\,, \qquad (54)$$

The idea of estimator operation is if that the deliver system $\Sigma_1 : (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ with state $\mathbf{x}$, can be "predicted" by system $\Sigma_2 : (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ that uses state $\mathbf{z}$, which is accessible in this case to be controlled. In order that the system Σ_2 follows the system Σ_1 we calculate a regulator $\mathbf{L}$ which brings the difference between actual read states $\mathbf{y_1}$ and estimated states $\mathbf{y_2}$ as a correction into the system Σ_2. In this case we can write:

$$\Sigma_1 : \begin{cases}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{x}_0\delta + \mathbf{G}\mathbf{w} \\ \mathbf{y}_1 = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{v}\end{cases} \qquad (55)$$

$$\Sigma_2 : \begin{cases}\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u} + \mathbf{z}_0\delta + \mathbf{L}(\mathbf{y}_1 - \mathbf{y}_2) \\ \mathbf{y}_2 = \mathbf{C}\mathbf{z} + \mathbf{D}\mathbf{u}\end{cases} \qquad (56)$$

where initial conditions are introduced by $\mathbf{x}_0$, respectively $\mathbf{z}_0$. Tracking error, including the initial conditions, is given by:

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{z}\,; \quad \tilde{\mathbf{x}}_0 = \mathbf{x}_0 - \mathbf{z}_0 \qquad (57)$$

If we decrease Σ_2 from Σ_1 and neglect the noise is obtained:

$$\tilde{\mathbf{x}} = he^{(\mathbf{A}-\mathbf{L}\mathbf{C})t}\tilde{\mathbf{x}}_0\,. \qquad (58)$$

Hence if $\mathbf{L}$ is dimensioned such that **A-LC** have eigenvalues with negative real part, the estimation error tends to zero. Since $\mathbf{z}$ is provided by the estimator, we have access to all states to make control of the form:

$$\mathbf{u} = -\mathbf{K}\mathbf{z} \qquad (59)$$

In this case the system Σ_1 is described by the equation:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\mathbf{z} + \mathbf{x}_0\delta = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}\mathbf{K}\tilde{\mathbf{x}} + \mathbf{x}_0\delta \qquad (60)$$

which has the solution:

$$\mathbf{x} = he^{(\mathbf{A}-\mathbf{B}\mathbf{K})t}(\mathbf{x}_0\delta + h\mathbf{B}\mathbf{K}e^{(\mathbf{A}-\mathbf{L}\mathbf{C})t}\tilde{\mathbf{x}}_0) \qquad (61)$$

The process of calculating the estimator is similar to that described above for the optimal regulator. This is based on the dual system:

$$\dot{\bar{\mathbf{x}}} = \mathbf{A}^T\bar{\mathbf{x}} + \mathbf{C}^T\bar{\mathbf{u}} \qquad (62)$$

for which is considered performance index:

$$\min J = \int_0^\infty [\bar{\mathbf{x}}'(\mathbf{G}\overline{\mathbf{Q}}\mathbf{G}^T)\bar{\mathbf{x}} + \bar{\mathbf{u}}^T\overline{\mathbf{P}}\bar{\mathbf{u}}]\,\mathrm{d}t \qquad (63)$$

By solving the matrix Riccati equation:

$$\mathbf{A}\mathbf{R} + \mathbf{R}\mathbf{A}^T - \mathbf{R}\mathbf{C}^T\overline{\mathbf{P}}^{-1}\mathbf{C}\mathbf{R} + \mathbf{G}\overline{\mathbf{Q}}\mathbf{G}^T = \mathbf{0} \qquad (64)$$

matrix estimator is obtained:

$$\mathbf{L} = \mathbf{R}\mathbf{C}^T\overline{\mathbf{P}}^{-1} \qquad (65)$$

where $\mathbf{R}$ is the solution of Riccati equation.

7 Input data, calculus algorithm and results

7.1 Input data for the model

7.1.1 Geometrical data

As input data we use the geometrical elements of the UAV from Fig. 2.

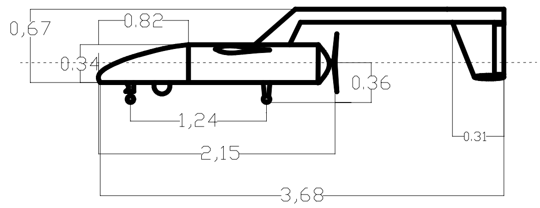

Figure 2 a UAV geometry

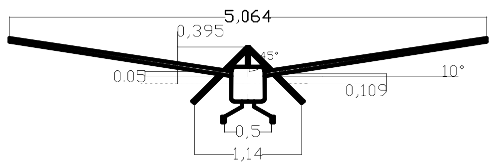

Figure 2 b UAV geometry

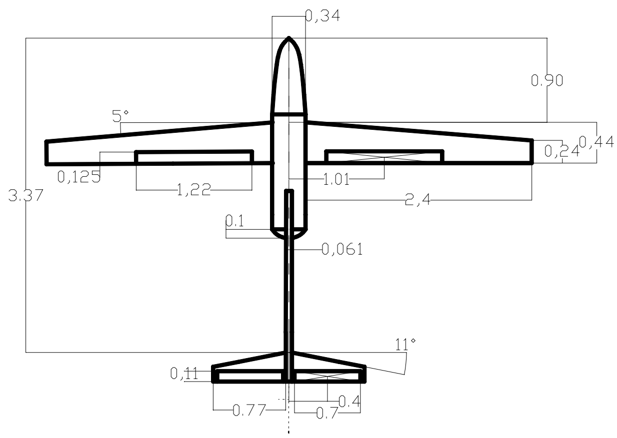

Figure 2 c UAV geometry

Geometrical characteristics for the model are: Reference length – body length: $l = 2.15\,m$; Reference area – cross body area: $S = 0.116\,m^2$;.

7.1.2 Mechanical data

Mass characteristics of the model are: $m_i = 70\,kg$;
Corresponding to initial mass, we have:
Centre of mass position: $x_{cm} = 1.3\ m$.
Inertial moments: $A = 10\,kgm^2$; $B = 20\,kgm^2$;
$C = 30\,kgm^2$ $E = 0.5\,kgm^2$

7.1.3 Aerodynamic data

For the configuration from Figure 2, considering a Taylor series expanding around the origin, taking into account the parity of the terms, we obtain the following polynomial form of the aerodynamic coefficients in a body frame:

$$C_x^A = a_1 + a_{21}\alpha^2 + a_{22}\beta^2 + a_6\delta_e^2 + a_7\delta_a^2 + a_8\delta_r^2 +$$
$$+ a_9\alpha + a_{10}\hat{q}\delta_e + a_{13}\alpha^3 + a_{14}\alpha^4$$
$$C_y^A = b_{12}\beta + b_{42}\hat{r} + b_{52}\delta_r + b_6\delta_a + b_{92}\hat{\beta} + b_{10}\hat{p}$$
$$C_z^A = b_0 + b_{11}\alpha + b_{41}\hat{q} + b_{51}\delta_e + b_{91}\hat{\alpha} + b_{13}\alpha^2 + b_{14}\alpha^3$$
$$C_l^A = c_3\hat{p} + c_5\hat{r} + c_6\delta_a + c_7\delta_r + c_{13}\beta$$
$$C_m^A = d_0 + d_{11}\alpha + d_{41}\hat{q} + d_{51}\delta_e + d_{91}\hat{\alpha}$$
$$C_n^A = d_{12}\beta + d_{42}\hat{r} + d_{52}\delta_r + d_6\delta_a + d_{92}\hat{\beta} + d_{10}\hat{p}\,, \tag{66}$$

where the coefficients a_1, a_{21} … generally are depending on Mach number.

In our case, for low subsonic flow, the coefficients a_1, a_{21} … d_{10} are practically constant, having the following values:

$a_1 = -0.53$; $a_{21} = 4.7$; $a_{22} = -0.83$; $a_6 = -6.5$; $a_7 = -19.6$; $a_8 = -6.5$; $a_9 = -2.5$; $a_{10} = 6.7$; $a_{13} = -32.1$; $a_{14} = -963.5$ $b_{12} = 14.1$; $b_{42} = 4.5$; $b_{52} = -6.5$; $b_6 = 1.83$; $b_{92} = 0.$; $b_{10} = -7.57$; $b_0 = -1.34$; $b_{11} = -99.6$; $b_{41} = -1.8$; $b_{51} = 6.47$; $b_{91} = 0.0$; $b_{13} = 35.2$ $b_{14} = 952.0$ $c_3 = -26.43$; $c_5 = -0.2$; $c_6 = 1.62$; $c_{13} = 0.054$; $c_7 = -0.21$; $d_0 = 0.179$; $d_{11} = -5.29$; $d_{41} = -11.6$; $d_{51} = 6.63$; $d_{91} = 0.0$; $d_{12} = -3.63$; $d_{42} = -5.5$; $d_{52} = 6.63$; $d_6 = -0.7$;
$d_{92} = 0.$; $d_{10} = 6.3$. (67)

7.1.4 Thrust

The propeller thrust is determined by the relation:

$$T = T_0 C_x^T \tag{68}$$

Where T_0 is the nominal value at ground, a fix point, and C_x^T and axial gas -dynamic coefficient. Fashioning experimental results indicated in work [9] we obtain the following approximate relation:

$$C_x^T = f_1(M) f_2(z_p) f_3(\delta_T) \tag{69}$$

where the influence of the main parameters ware separated:

Mach number M:

$$f_1(M) = 1 - 3.0M\,; \tag{70}$$

Altitude z_p:

$$f_2(z_p) = 1 - 9.1 \cdot 10^{-5} z_p\,; \tag{71}$$

Thrust command δ_T:

$$f_3(\delta_T) = -0.677 + 1.677\delta_T\,. \tag{72}$$

Thrust command is limited between: $0.5 < \delta_T < 1$. It is obvious that for the null velocity (fixed point flight) at ground level with maximum command the thrust takes the nominal value: $T_0 = 563[N]$

Similarly, we can obtain specific fuel consumption, by using relation:

$$C_{sp} = C_{sp0} g_1(M) g_2(z_p) g_3(\delta_T), \quad (73)$$

where the Mach dependence is:

$$g_1(M) = 1.026 - 0.133M + 0.966M^2; \quad (74)$$

Altitude dependence:

$$g_2(z_p) = 1 - 8.\cdot 10^{-6} z_p; \quad (75)$$

Thrust command influence:

$$g_3(\delta_T) = 1 + 0.3(1-\delta_T)^2. \quad (76)$$

The nominal specific consume at ground level with maximum command, corresponding with T_0 has the value:

$$C_{sp0} = 1.59\times 10^{-7}\,[Kg/N/s]. \quad (77)$$

Using specific fuel consumption, we can evaluate fuel debit that coincides with mass variation, as we see in the equation (8).

6.1.5 Guidance parameters

For flight control system applying Kalman filter relations, we obtain the following guidance gains:

- Controller $\mathbf{K}$ values:

$$k_{1,1} = 0;\ k_{1,2} = 1.21\ldots.k_{4,19} = -0.181 \quad (78)$$

- Estimator $\mathbf{L}$ values:

$$l_{1,1} = 12.4;\ l_{1,2} = 0.\ldots l_{19,19} = 10.05 \quad (79)$$

7.2 Calculus algorithm

The calculus algorithm consists in multi-step method Adams' predictor-corrector with variable step integration method: [1] [11]. Absolute numerical error was 1.e-12, and relative error was 1.e-10.

7.3 Calculus test case

We will consider as a calculus test the situation when the UAV takes-off, makes a rectangular path with four turns maintaining velocity and altitude flight, followed by a descending phase. During the flight, after the second turn, it attends a turbulent zone. Crossing the turbulent zone, UAV uses the guidance command system, in order to maintain flight parameters. Turbulence zone was designed accordingly with work [5]. Flight parameters are typical for surveillance activity: altitude $z_{pd} = 200\,m$, and axial velocity $u_d = 44\,m/s$.

7.4 Results

In Figure 3 we showed the flight-path diagram, the test situation when the UAV made a rectangular path with four turns. Fig. 4 shows the velocity diagram during the test flight described above.

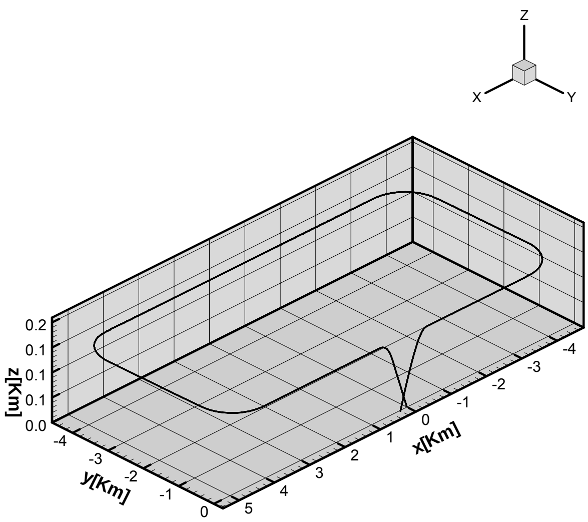

Figure 3 UAV flight-path diagram

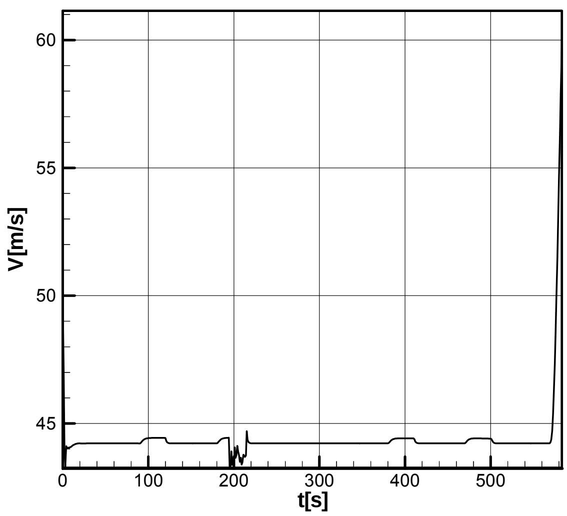

Figure 4 Velocity diagram

Fig. 5 and Fig. 6 show the ruder deflection and elevator deflection necessary to obtain desired trajectory.

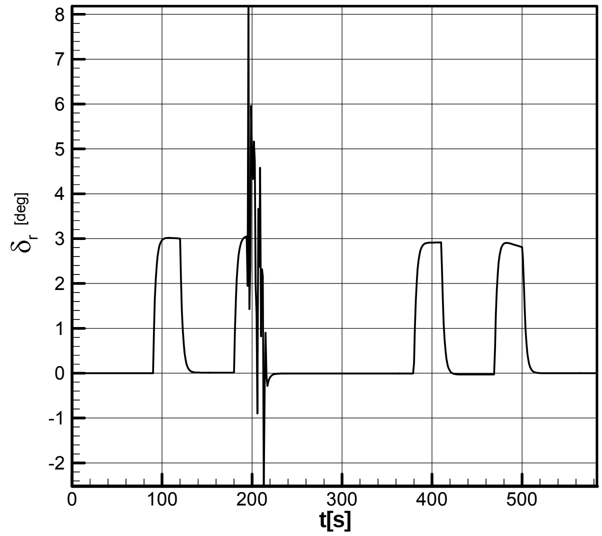

Figure 5 Ruder deflection diagram

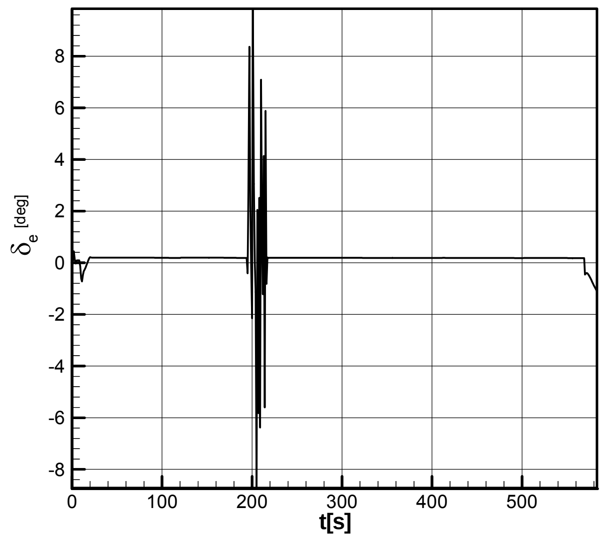

Figure 6 Elevator deflection diagram

In Fig. 7 and 8 are shown the attitude angles: bank angle and azimuth angel, during the flight.

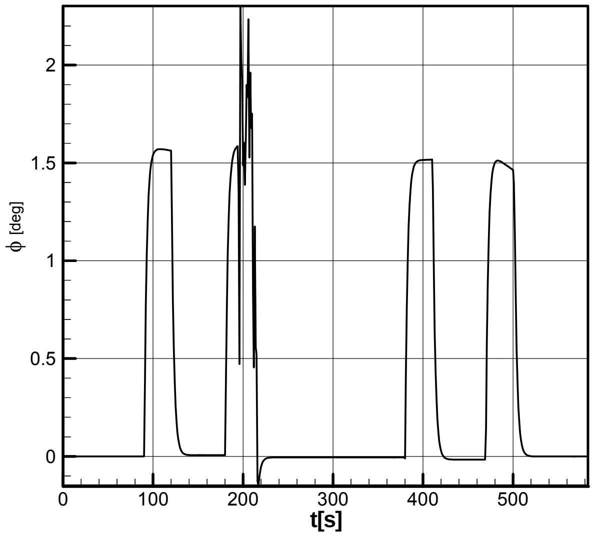

Figure 7 Bank angle diagram

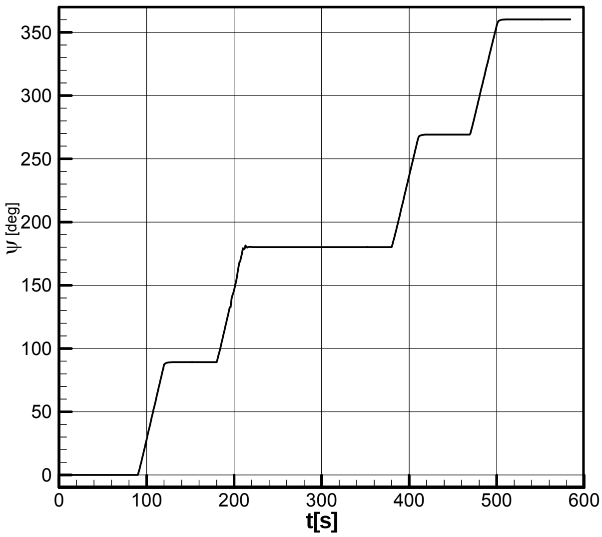

Figure 8 Azimuth angle diagram

In all diagrams, except flight –path diagram and azimuth diagram on can observe the influence of turbulence zone on flight parameters.

7 Conclusions

The conclusions are structured in two points as the following.

Guidance scheme: A first conclusion regarding the guiding scheme consists in the fact that the UAV will have robust command structure, based on Kalman filter which is capable to lead the UAV on desired flight-path in different atmospheric conditions. From the diagram, previously presented, one can observe that the flight parameters after the turbulence zone came back to the normal values.

From this point of view we must define a number of evolutions that will make several guiding structures, which will be dedicated to each kind of evolutions. This part of the command structure will be able to evolve in the same time with the development of the project, when the experimental results will be available.

Technical solution: Regarding the adopted solution, using unusual tail having two consoles arranged in the shape of the letter "V" inverted, instead of regular vertical and horizontal tail, we can obtain a better stability and at the same time a better control without increasing the deflection attach angle due to upstream wing.

References:

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