Convex hull manipulation based control performance optimisation

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Abstract: - This paper deals with the topic of qLPV state-space model based control design in which LMIs are used to optimize the multi-objective control performance. In this paper we investigate how the manipulation convex hull of the polytopic model influences the control performance which is derived by LMIs. We examine these influences through the control design of the two dimensional aeroelastic system's example. First we define various TP type polytopic model representations of a wing section whose vertices define different convex hulls. In the second step we investigate how these models lead to different control performances.

Key-Words: - polytopic model, TP model, LMI, convex hull

1 Introduction

In the past few years various studies of aeroelastic systems have emerged. Regarding their properties one can find the studies of free-play nonlinearity by Price at al. in[2] and by Lee and LeBlanc [3] as well as a complete study of a class of nonlinearities [4]. O'Neil and Straganac[5] examined the continuous structural nonlinearity of these systems. Recent analysis is given in [6]. These papers conclude that an aeroelastic system may exhibit nonlinear phenomena such as limit cycle oscillation, flutter and even chaotic vibrations.

Control strategies have also been derived for aeroelastic systems. Block and Straganac[7] show that in the case of large amplitude limit cycle oscillation behavior the linear-control methodologies do not stabilize these system consistently. At the NASA Langley research center a benchmark active control technique (BACT) wind tunnel model has been designed and control algorithms for flutter suspension have been developed by Waszak[8], Mukhopadhyay[9] and Keller and Joshi[10]. For an aeroelastic apparatus, tests have been performed in a wind tunnel to examine the effect of nonlinear structural stiffness, and control systems have been designed using linear control theory, feedback linearization techniques and adaptive control strategies.[11-12]

One can find studies focusing attention on the two dimensional prototypical aeroelastic wing section. Block and Straganac[8] and Ko et et al.[13] proposed nonlinear feedback control methodologies for a class of non linear structural effects of the prototypical aeroelastic wing section. In this regard Ko et al. [11] developed a controller via partial-feedback linearization. It has been shown that that global stabilization can be achieved by applying an additional control surface (e.g., in [15]). Adaptive-feedback linearization and global-feedback-linearization techniques were introduced for two control actuators in [11] and the Ricatti-equation based method was used in [18]. Neural network based design was also discussed in [17].

Time delays are inevitable in control loops [18]. Time delay effects were introduced by Marzocca [19] and Yu et al. [20] focused on time delay feedback control of supersonic lifting surface on flutter boundary. Zhao [18] presented a systematic study on aeroelastic stability with single or multiple time delays in the feedback control loop.

Tensor Product (TP) type polytopic model based state feedback, output feedback and LMI based controller design was proposed in [21, 22]. In [25] the effect of the convex hull manipulation of the TP models on the control performance is discussed. We proved that in modern LMI based multi objective control design the optimization of the control performance must include the manipulation of the convex hull beside constructing LMIs, and the TP model transformation offers a systematic solution. We can see some examples in [26-29, 31 33] where TP model transformation is applied.

In this paper we show that the manipulation of the convex hull is necessary for the the LMI based controller and observer design.

2 Nomenclature

a = nondimensional distance from the midchord to the elastic axis;

b = semichord of the wing;
C_h = plunge structural damping coefficient;
c_{l_α} = lift coefficients per angle of attack;
c_{l_β} = lift coefficients per control surface deflection;
c_{m_α} = moment coefficients per angle of attack;
c_{m_β} = moment coefficients per control surface deflection;
c_α = pitch-structural damping coefficient;
h = plunging displacement;
I_α = mass moment of inertia;
k_h = plunge structural spring constant;
$k_\alpha(\alpha)$ = nonlinear stiffness contribution;
L = aerodynamic force;
M = aerodynamic moment;
m = mass of the wing;
U = free stream velocity;
x_α = nondimensional distance between the elastic axis and the center of mass;
α = pitching displacement;
β = control surface deflection;
ρ = the density of the air;
$a,b,...$ = scalar values;
$\mathbf{a},\mathbf{b},...$ = vectors;
$\mathbf{A},\mathbf{B},...$ = matrices;
$A,B...$ = tensors
$A \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{U}_n$ = multiple product as $A\times_1\mathbf{U}_1\times_2\mathbf{U}_2\times_3\ldots\times_N \mathbf{U}_N$;
$(\cdot)_{i,j,n}$ = indices;
$(\cdot)_{I,J,N}$ = index upper bound: for example: i=1…I;

3 Tensor Product model transformation

3.1 Concept of TP type polytopic model

The following definition are used in this paper:

Definition 1 (qLPV model): Consider the Linear Parameter Varying State Space model:

$$\begin{aligned}\dot{x}(t) &= \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t)+\mathbf{B}(\mathbf{p}(t))\mathbf{u}(t)\\ y(t) &= \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t)+\mathbf{D}(\mathbf{p}(t))\mathbf{u}(t)\end{aligned} \tag{1}$$

where u(t) is the input, y(t) is the output. The system matrix applying the state space model:

$$\mathbf{S}(\mathbf{p}(t)) = \begin{pmatrix}\mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t))\\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t))\end{pmatrix} \tag{2}$$

where $\mathbf{S}(\mathbf{p}(t))$ is a parameter varying object, and $\mathbf{p}(t)\in\Omega$ is time varying N dimensional parameter vector, where $\Omega=[a_1 \quad b_1]\times[a_2 \quad b_2]\times...\times[a_N \quad b_N]\subset\Re^N$ is a closed hypercube. Parameter $\mathbf{p}(t)$ can also include some elements of , in this case (2) is termed as quasi LPV (qLPV) model. Therefore this type of model is considered to belong to the class of non-linear models. Let us assume, that the size of $\mathbf{S}(\mathbf{p}(t))$ is O times I.

Definition 2 (Finite element polytopic model):

$$\mathrm{S}(\mathrm{p}(\mathrm{t})) = \sum_{\mathrm{r}=1}^{\mathrm{R}} \mathrm{w}_{\mathrm{r}}(\mathrm{p}(\mathrm{t}))\cdot\mathrm{S}_{\mathrm{r}} \tag{3}$$

where $\mathbf{p}(t)\in\Omega$. $\mathbf{S}(\mathbf{p}(t))$ is given for any parameter vector $\mathbf{p}(t)$ as the parameter varying combinations of linear time invariant (LTI) system matrices $\mathbf{S}_r\in R^{(m+k)\times(m+l)}$ also called LTI vertex systems. The combination is defined by the weighting functions $w_r(p(t))\in[0,1]$. By finite we mean that R is bounded.

The TP model belongs to the class of polytopic models. In case of the TP model the multi variable weighting functions $w_r(p)$ are decomposed to the product of one variable weighing functions $w_n(p_n)$.

Definition 3 (Finite element TP type polytopic model):

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{i_1=1}^{I_1}\sum_{i_2=1}^{I_2}...\sum_{i_N=1}^{I_N} w_{n,i_n}(p_n(t))\cdot\mathbf{S}_{i_1,i_2...i_N} \tag{4}$$

applying the compact notation based on tensor algebra (Lathauwer's work [1]) we have:

$$\mathbf{S}(\mathbf{p}(t)) = S \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{w}_n(p_n(t)), \tag{5}$$

where the (N+2) dimensional coefficient tensor $S\in R^{I_1\times I_2\times..\times I_N\times(m+k)\times(m+l)}$ is constructed from the LTI vertex systems $\mathbf{S}_{i_1,i_2...i_N}$ (4) and the row vector $\mathbf{w}_n(p_n(t))\in[0,1]$ contains one variable and continuous weighting functions $w_{n,i_n}(p_n(t))\in[0,1]$, $(i_n=1...I_N)$.(4)

Remark 1: TP model (5) is a special class of polytopic models (2), where the weighting functions are decomposed to the Tensor Product of one variable functions.

For Linear Matrix Inequality based design, the convexity of the TP model is required. Therefore let us define the following types of TP models:

Definition 4 (Convex type TP model): The TP model is convex if the weighting functions satisfy the following criteria:

$$\forall_n, p_n(t): \sum_{i_n=1}^{I_n} w_{n,i_n}(p_n(t))=1, \tag{6}$$

$$\forall_n, i_n, p_n(t): 0 \le w_{n,i_n}(p_n(t)) \le 1 . \tag{7}$$

We can define various types of convex TP models. These types can readily be determined via constraints defined for the weighting functions. Let us define two types of TP models which we use in this paper; the other possible types of TP models are discussed in [21]

Definition 5 (NO/CNO, NOrmal type TP model): The convex TP model is a NO (normal) type model, if its w(p) weighting functions are Normal, that is, if it satisfies (5, 6) , and the largest value of all weighting functions is 1. Also, it is CNO (close to normal), if it is satisfies (5, 6) and the largest value of all weighting functions is 1 or close to 1.

Definition 6 (IRNO, Inverted and Relaxed NOrmal type TP model): The TP model is IRNO type, if the smallest values of all weighting functions are 0, and the largest values of all weighting functions are the same.

3.2 Execution

Step 1**:** Discretisation

The goal of this step is to represent the given parameter dependent system matrix by tensor that is ready to find the tensor product structure in the model. First of all we define the transformation space Ω in which we expect the TP model be relevant, then we discretise the qLPV model in M points.

Definition 7 (Transformation space Ω). Ω is a bounded hyper rectangular space where the parameter vector of the system matrix varies: p(t) $\Omega:[a_1 \quad b_1]\times[a_2 \quad b_2]\times\ldots\times[a_N \quad b_N]$ Practically should be defined according to the working space of **p** that is determined based on the physical behavior of the model.

Definition 8 (Discretisation grid M). M denotes a hyper rectangular discretisation grid defined in Ω. M_n (n = 1… N) denotes the number of grid on the n-th dimension.

Step 2: Extracting the TP structure

The goal of this step is to reveal the TP structure of the given qLPV model and find the minimal number of LTI components .We use Higher Order Singular Value Decomposition (HOSVD) to find the TP structure of the model. In this paper we generate the exact minimized form, this means that we eliminate only the zero singular values. In [32] a detailed description of HOSVD form can be found.

Step 3**:** Determination of the weighting function

The weighting functions can be determined in discretised and also in continuous form. While we apply TP toolbox, we generate the weighting functions in descretised form.

Remark 2: To get convex TP model of the proper nonlinear system transform the weighting functions to complete (6-7) criteria.

These steps can be easily executes by TPtoolbox for Matlab.[23]

4 The qLPV model

We recall the qLPV model of the system presented in [21]:

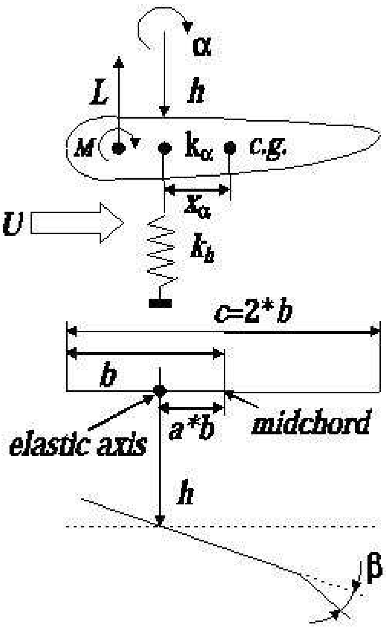

Figure 1. The aeroelastic wing section

$$\dot{x}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) = \mathbf{S}(\mathbf{p}(t))\begin{pmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{pmatrix} \tag{7}$$

where

$$\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_1(t) \\ x_1(t) \\ x_1(t) \end{pmatrix} = \begin{pmatrix} h \\ \alpha \\ \dot{h} \\ \dot{\alpha} \end{pmatrix}, \tag{8}$$

and $u(t) = \beta$.

$$\mathbf{A}(\mathbf{p}(t)) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1 & -\left(k_2U^2 + p(x_2(t))\right) & -c_1(U) & -c_2(U) \\ -k_3 & -\left(k_4U^2 + q(x_2(t))\right) & -c_3(U) & -c_4(U) \end{pmatrix}, \tag{9}$$

where:

$$k_1 = \frac{I_\alpha k_h}{m(I_\alpha - m x_\alpha^2 b^2)}, \tag{10}$$

$$k_2 = \frac{I_\alpha \rho b c_{l_\alpha} + m x_\alpha b^3 \rho c_{m_\alpha}}{m(I_\alpha - m x_\alpha^2 b^2)}, \quad (11)$$

$$k_3 = \frac{-m x_\varepsilon b k_h}{m(I_\alpha - m x_{\alpha\wedge}^2 b^2)}, \quad (12)$$

$$k_4 = \frac{-m x_\alpha b^2 \rho c_{l_\alpha} - m\rho r^2 c_{m_\alpha}}{m(I_\alpha - m x_\alpha^2 b^2)}, \quad (13)$$

$$c_1 = \frac{I_\alpha (c_h + \rho U b c_{l_\alpha}) + mUm x_\alpha b^3 \rho c_{m_\alpha}}{m(I_\alpha - m x_\alpha^2 b^2)}, \quad (14)$$

$$c_2 = \frac{I_\alpha \rho U b c_{l_\alpha}(1/2\text{-}a) - m x_\alpha b c_\alpha a + m x_\alpha \rho U b^4 c_{m_\alpha}(1/2\text{-}a)}{m(I_\alpha - m x_\alpha^2 b^2)}, \quad (15)$$

$$c_3 = \frac{-m x_\alpha b c_h - m x_\alpha \rho U b^2 c_{l_\alpha} - m\rho m \rho^2 c_{l_\alpha}}{m(I_\alpha - m x_\alpha^2 b^2)}, \quad (16)$$

$$c_4 = \frac{m c_\alpha - m x_\alpha \rho U b 3 c_{m_\alpha}(1/2\text{-}a) - m\rho a)\ {}^3 c_{m_\alpha}(1/2\text{-}a)}{m(I_\alpha - m x_\alpha^2 b^2)}. \quad (17)$$

$$\mathbf{B}(\mathbf{p}(t)) = \begin{pmatrix} 0 \\ 0 \\ g_3 U^2 \\ g_4 U^2 \end{pmatrix}, \quad (18)$$

where

$$g_3 = \frac{-I_\alpha \rho b c_{l_\beta} - m^* x_\alpha b^3 \rho c_{l_\beta}}{m(I_\alpha - m x_\alpha^2 b^2)}, \quad (19)$$

$$g_4 = \frac{m x_\alpha b^2 \rho c_{l_\beta} + m b^2 \rho c_{m_\beta}}{m(I_\alpha - m x_\alpha^2 b^2)}. \quad (20)$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad (21)$$

where $\mathbf{p}(t) \in \Re^{N=2}$ contains values $x_2(t) = \alpha$ and U. Note that the equations of motion are also dependent upon the elastic axis location a. In the present case we assume that a is a constant.

5 Convex TP models

We create two convex TP models of the aeroelastic system, the IRNO and the CNO type. In order to generate them, we utilize the TP model transformation. For a detailed description see [21, 24, 25]. For this purpose we TP toolbox MatLab [23] According to the 3.2 subsection of this paper, first let we define the transformation space $\mathbf{\Omega}$. We are interested in the interval $U \in [14 \quad 25]$m/s and $\alpha \in [-0.1 \quad 0.1]$.This has practical significance, because the prototypical aeroelastic model is accurate for low speeds. Therefore, let $\mathbf{\Omega} : [14 \quad 25] \times [-0.1 \quad 0.1]$ in the present example. Let he grid density be defined as $M_1 \times M_2$; let M1 =101 and M2=101. After the execution of the TP model transformation (see section 3.2), we can observe that in the first dimension the rank is 3, and the rank of the second dimension is 2. The basis functions are: $w_{1,i}(U(t))$, i=1…3 and $w_{2,j}(x_2(t))$, j=1…2. At this point we formulated the TP model type polytopic convex model of the system:

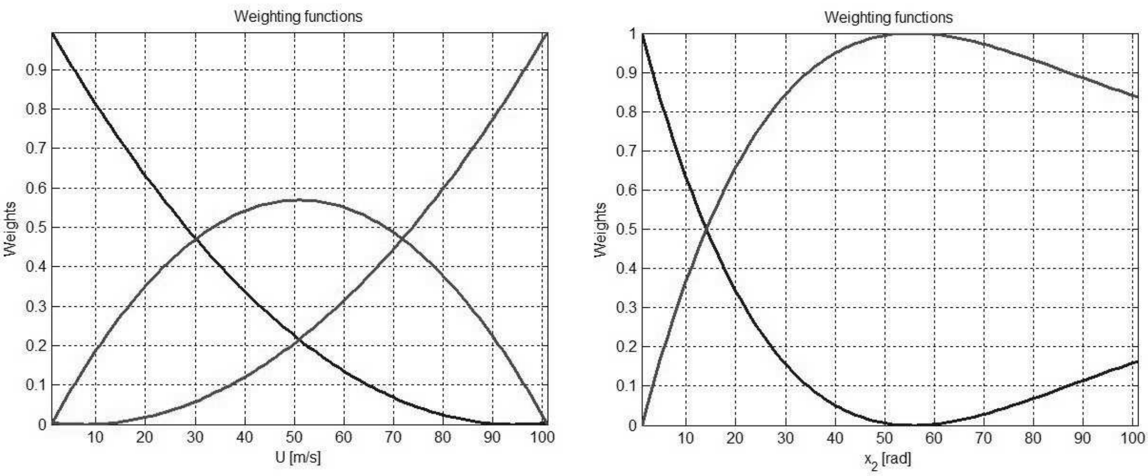

Figure 2. CNO type weighting functions

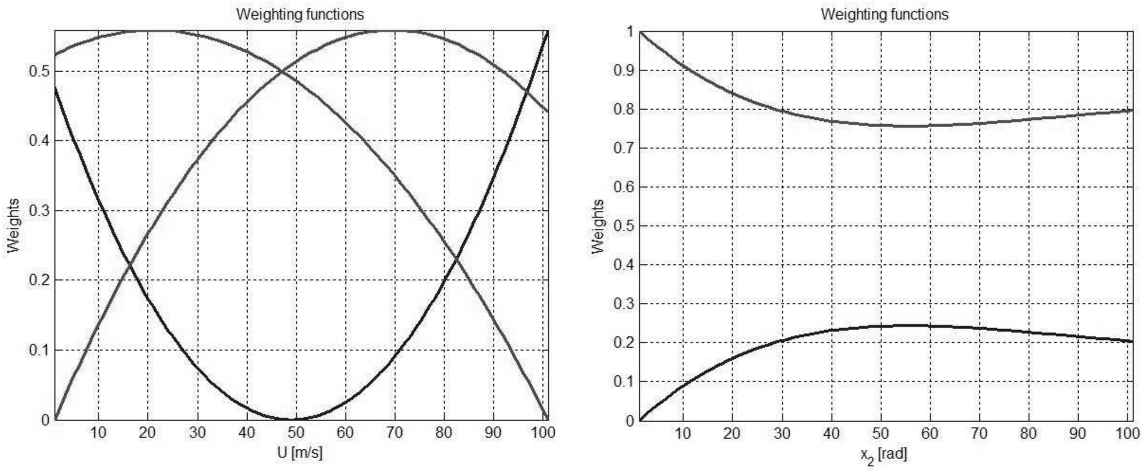

Figure 3. IRNO type weighting functions

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{i=1}^{3} \sum_{j=1}^{2} w_{1,i}(U(t)) w_{2,j}(x_2(t)) \cdot \left(\mathbf{A}_{i,j}\mathbf{x}(t) + \mathbf{B}_{i,j}(u(t))\right) \tag{22}$$

that is

$$\mathbf{S}(\mathbf{p}(t)) = S \mathop{\boxtimes}_{n=1}^{2} \mathbf{w}_n(p_n(t)), \tag{23}$$

where $\mathbf{w}_n(p_n(t))$ contains the elements of the weighting functions.

The solution of the TP model transformation dives us the convex type weighting functions, see Definition 4.

5.1 CNO type TP model

The weighting functions ($w_{1,i}$ and $w_{2,j}$), of this TP model can be seen in Figure 2. This is a tight convex hull.

5.2 IRNO type TP model

The weighting functions of this TP model can be seen in Figure 3. This is a large convex hull.

5.3 Manipulation of the TP models

We can manipulate the TP type polytopic models via convex hull manipulation. We make a transition from IRNO to CNO type hull, and investigate the trajectory of the vertex systems.

The advantage of the TP type polytopic model is that the convex hull can be manipulated in each dimension by the weighting functions. The manipulation can be executed in two steps:

Step 1: We generate the new weighting functions between IRNO and CNO type applying simple linear approximation:

$$\mathbf{w}_z(\lambda, p_n(t)) = \lambda \cdot \mathbf{w}_{CNO}(p_n(t)) + (1-\lambda) \cdot \mathbf{w}_{IRNO}(p_n(t)) \tag{24}$$

where λ is a coefficient and its value goes from 0 to1. Note, that the linear interpolation conserves the convexity. We discretize λ in 30 equidistant points (z=1…Z; Z=30). So as we generate Z number of different TP model representations for further investigation.

Step 2: We compute the LTI vertex systems according to the new weighting functions. We do this by executing the 3rd step of the TP model transformation [21].

$$\mathbf{w}_z(\lambda, p_n(t)) \Rightarrow S_z(\lambda)$$

Finally we obtain 30 different TP model representing the same system, $\mathbf{S}(\mathbf{p}(t))$:

$$\mathbf{S}(\mathbf{p}(t)) = S_z \mathop{\boxtimes}_{n=1}^{2} \mathbf{w}_{n,z}(p_n(t)). \tag{25}$$

5.3 The geometry of the convex hulls

Let us investigate the geometrical location of the vertex systems. The number of elements in $\mathbf{S}(\mathbf{p}(t))$ is 20, so we may need a 20 dimensional space for drawing. For the sake of simplicity, let us consider only the nonlinear elements of $\mathbf{S}(\mathbf{p}(t))$:

$$\mathrm{S}_z = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\mathrm{k}_1 & \mathrm{S}_z(\mathrm{x}_2,\mathrm{U})(3,2) & \mathrm{S}_z(\mathrm{U})(3,3) & \mathrm{S}_z(\mathrm{U})(3,4) & \mathrm{S}_z(\mathrm{U})(3,5) \\ -\mathrm{k}_2 & \mathrm{S}_z(\mathrm{x}_2,\mathrm{U})(4,2) & \mathrm{S}_z(\mathrm{U})(4,3) & \mathrm{S}_z(\mathrm{U})(4,4) & \mathrm{S}_z(\mathrm{U})(4,5) \end{pmatrix} \tag{18}$$

Since the number of nonlinear elements is 8, we use 2 dimensional section of this 8 dimensional space..In order to investigate the convex hull defined by the vertices we represent some elements $S_z(3,2)$ and $S_z(4,2)$ in a two dimensional coordinate system, (see Figure 4). In this coordinate system we draw the elements of $\mathbf{S}(\mathbf{p}(t))$ for all possible $\mathbf{p}(t)\in\Omega$, and it is drawn by the bold line in the Figure 4. Actually this line is a “space” (if we draw the 20 elements it is a 20 dimensional space), where $\mathbf{S}(\mathbf{p}(t))$ is varying. The vertices of the IRNO (Z=1) type TP model are drawn by dots. Connecting these dots, the two dimensional section of the convex hull can be seen. The two dimensional section of the CNO (Z=30) type convex hull is represented by the connected stars. We can find out that the IRNO type is a wide convex hull, and the CNO type is very tight; significantly tighter than the classical convex hull defined by connected squares.

6 LMI based multi-objective control

6.1 LMI based controller design

After convex hull manipulation (described in the previous section) we found that we need the tightest hull to obtain the best control performance. For details see [25].Therefore we applied CNO type TP model to generate the feedback gains via LMI, for simulation purposes.

We seek the control value *u*. The controller takes the same TP type polytop form and the weighting coefficients as the system has. Thus the control value is formulated as:

$$u=\sum_{i=1}^{3}\sum_{j=1}^{2} w_{1,i}(U(t))w_{2,j}(x_2(t))\cdot\mathbf{F}_r\,, \tag{26}$$

We can generate the control feedback gains by the execution of the following steps:

Step 1: Deriving the polytopic model of the system. In the present case we use a finite element convex TP type polytopic models.

Step 2: Selecting the LMI for the desired multi-objective control performances; in this case asymptotic stability and decay rate control.

To obtain different control performances, we define the following LMIs:

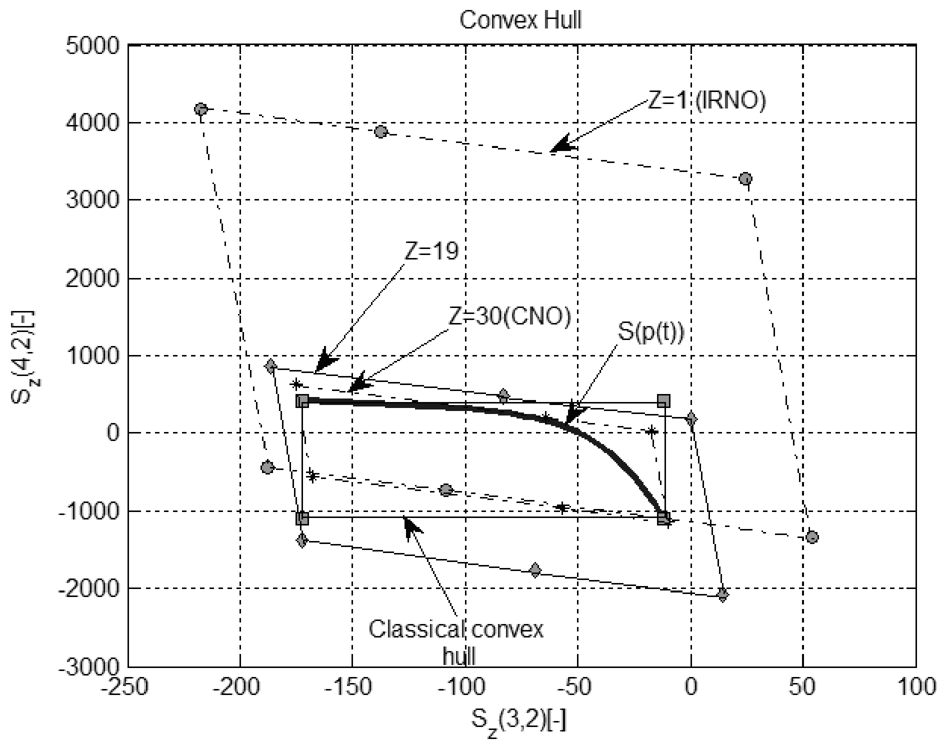

Figure 4. Representation of different convex hulls

Theorem 1 (Asymptotic stability design for continuous convex polytopic models). Polytopic model (22) with control value (26) is asymptotically stable if there exist $\mathbf{X}>0$ and M_r satisfying equations

$$-\mathbf{X}\mathbf{A}_\mathrm{r}^T+\mathbf{M}_\mathrm{r}^T\mathbf{B}_\mathrm{r}^\mathrm{T}-\mathbf{A}_\mathrm{r}\mathbf{X}+\mathbf{B}_\mathrm{r}\mathbf{M}_\mathrm{r}\ >\ 0,$$

for all *r* and

$$-\mathbf{X}\mathbf{A}_r^T + \mathbf{M}_s^T\mathbf{B}_r^T - \mathbf{A}_r\mathbf{X} + \mathbf{B}_r\mathbf{M}_s - \mathbf{X}\mathbf{A}_s^T + \mathbf{M}_r^T\mathbf{B}_s^T - \mathbf{A}_s\mathbf{X} + \mathbf{B}_s\mathbf{M}_1 \geq 0,$$

for $r < s \leq R$, except the pairs (r; s) such that $\forall \mathbf{p}(t): w_r\mathbf{p}(t)w_s\mathbf{p}(t) = 0$, and where the feedback gains are determined form the solutions X and Mr as

$$\mathbf{F}_r = \mathbf{M}_r\mathbf{X}_1^{-1} \tag{27}$$

The speed of the response of the controlled system is related to decay rate, that is, the largest Lyapunov exponent. Based on this fact define the following theorem:

Theorem 2 (Decay rate control). Assume the polytopic model (22) with controller (26). The largest lower bound on the decay rate by quadratic Lyapunov function is guaranteed by the solution of the following generalized eigenvalue minimizations problem (GEVP):

$$\mathbf{X} > 0,$$

$$-\mathbf{X}\mathbf{A}_r^T + \mathbf{M}_r^T\mathbf{B}_r^T - \mathbf{A}_r\mathbf{X} + \mathbf{B}_r\mathbf{M}_r - 2\alpha\mathbf{X} > 0,$$

$$-\mathbf{X}\mathbf{A}_r^T + \mathbf{M}_s^T\mathbf{B}_r^T - \mathbf{A}_r\mathbf{X} + \mathbf{B}_r\mathbf{M}_s - \mathbf{X}\mathbf{A}_s^T + \mathbf{M}_r^T\mathbf{B}_s^T - \mathbf{A}_s\mathbf{X} + \mathbf{B}_s\mathbf{M}_1 - 2\alpha\mathbf{X} \geq 0,$$

for $r < s \leq R$, except the pairs (r; s) such that $\forall \mathbf{p}(t): w_r\mathbf{p}(t)w_s\mathbf{p}(t) = 0$, and where the feedback gains are determined form the solutions by ()

In practical control designs we have to deal with the physical constraints of the system. In order to overcome such difficulties we may guaranty such constraints via the following LMIs:

Theorem 3 (Constraint on the control value). Assume that $\|x(0)\| \leq \phi$,where x(0) is unknown, but the upper bound ϕ is known. The constraint $\|u(t)\|_2 \leq \mu$ is enforced at all times $t \geq 0$ if the LMIs

$$\phi^2\mathbf{I} \leq \mathbf{X}$$

$$\begin{pmatrix} \mathbf{X} & \mathbf{M}_r^T \\ \mathbf{M}_r & \mu^2\mathbf{I} \end{pmatrix} \geq 0$$

hold.

Theorem 4 (Constraint on the output). Assume that $\|x(0)\| \leq \phi$, where x(0) is unknown, but the upper bound ϕ is known. The constraint $\|y(t)\|_2 \leq \lambda$ is enforced at all times $t \geq 0$ if the LMIs

$$\phi^2\mathbf{I} \leq \mathbf{X}$$

$$\begin{pmatrix} \mathbf{X} & \mathbf{X}\mathbf{C}_r^T \\ \mathbf{C}_r\mathbf{X} & \lambda^2\mathbf{I} \end{pmatrix} \geq 0$$

In order to ensure the above condition for a large set of initial states, we can set ϕ to be a large quantity even if x(0) is unknown. However, one should note that a large ϕ could lead to conservative designs .Note that the LMIs of the above Theorems 4 and 5 must be simultaneously solved with the LMIs of the selected stability theorem. These derivations and further LMIs developed for multi-objective control design of discrete systems are detailed in [30]

Step 3: Substituting the vertices of the polytopic model into the LMIs

Step 4: Solving the LMIs. We are capable of determining the feedback vertices $\mathbf{F}_r$ of the controller. We substitute respectively z=1…Z, different vertex systems into the selected LMI we find out in the cases of Z=1…18 the LMIs are not feasible, however in case of Z=19…30, the solution is feasible. This means that only those TP model type polytopic representations give feasible solutions for the LMI, where Z≥19. We refer to Figure 6 where we can see a convex hull defined by connected diamonds, whenever the system is surely controllable.

6.2 LMI based observer design

In this section we investigate the polytop model LMI based observer design. The typical steps of this observer design are the same four steps what we applied during the controller feedback design. In the fourth step we get the observer feedback gains, $\mathbf{K}_r$.

First of all, let us define the polytop observer structure we are going to deal with. Note that, there are various alternative ways for output feedback and observer design (in this regard we refer to (Scherer and Weiland 2000; Tanaka and Wang 2001)). The observers are required to satisfy

$$x(t) - \hat{\mathbf{x}}(t) \to 0 \text{ as } t \to \infty,$$

where $\hat{\mathrm{x}}(t)$ denotes the state-vector estimated by the observer. This condition guarantees that the steady-state error between x(t) and $\hat{\mathrm{x}}(t)$ converges to 0. In order to achieve this goal we introduce the following observer polytopic structure.

The state values can be estimated as

$$\hat{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(t))\hat{\mathbf{x}}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) + \left(\sum_{i=1}^{3} \sum_{j=1}^{2} w_{1,i}(U(t))w_{2,j}(x_2(t)) \cdot K_r \right) \cdot (\mathbf{y}(t) - \hat{\mathbf{y}}(t)), \quad (28)$$

where $\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t)$ and $\hat{\mathbf{y}}(t) = \mathbf{C} \cdot \hat{\mathbf{x}}(t)$.

Theorem 5 (Globally and asymptotically stable observer): Assume the polytopic model (22) with controller (26) and observer structure (27). This output-feedback control structure is globally and asymptotically stable if there exists such $\mathrm{X} > 0$ and; $\mathrm{N_r}$ (r = 1,…., R) satisfying equations

$$-\mathbf{A}_\mathrm{r}^T \mathbf{X} + \mathbf{C}_\mathrm{r}^T\ \mathbf{N}_\mathrm{r}^T - \mathbf{X}\mathbf{A}_\mathrm{r} + \mathbf{N}_\mathrm{r}\mathbf{C}_\mathrm{r} > 0,$$

$$\mathbf{A}_\mathrm{r}^T \mathbf{X} - \mathbf{C}_\mathrm{s}^T\ \mathbf{N}_\mathrm{r}^T + \mathbf{X}\mathbf{A}_\mathrm{r} - \mathbf{N}_\mathrm{r}\mathbf{C}_s + \mathbf{A}_\mathrm{s}^T \mathbf{X} - \mathbf{C}_\mathrm{r}^T\ \mathbf{N}_\mathrm{s}^T + \mathbf{X}\mathbf{A}_\mathrm{s} - \mathbf{N}_\mathrm{s}\mathbf{C}_\mathrm{r} \geq 0.$$

for $r < s \leq R$, except the pairs (r; s) such that $\forall \mathbf{p}(t) : w_r \mathbf{p}(t) w_s \mathbf{p}(t) = 0$, and $\mathbf{N}_\mathrm{r} = \mathbf{X}\mathbf{K}_r$. The feedback gains and the observer gains can then be obtained from the solution of the above LMIs as $\mathbf{K}_r = \mathbf{X}^{-1}\mathbf{N}_\mathrm{r}$.

We use the same four steps to obtain the observer feedback gains what we have applied in the previous subsection for the controller design process. We can select also the LMIs according to the desired performance. In this paper we select asymptotic stability criteria. See Theorem 5.

When we substitute in order z=1…Z, that means 30 different vertex systems into the selected LMI we find out in all the cases of Z=1…30 the LMIs are feasible, this means we can apply also a large convex hull, and IRNO type TP model gives good solution for observer design. We check the performance of the observer with simulation.

7 Simulation results

7.1 Closed loop simulation

In the followings we investigate how the control performance varies when we change the value of Z in the "feasible" domain (Z=19…30).

We apply constraint on the control value. We applied the LMIs defined in Theorem 5. In the case of controllers we searched the minimal bound of the present control value while the LMIs are feasible. The response of the resulting controllers (Z=19, Z=25, Z=30) is presented on Fig. 8. We can see the control value (torque) is 7 Nm, if Z=30 (CNO) and it is significantly smaller than the other cases.

7.2 Open loop simulation

Our goal is to estimate precisely plunge (h) and pitch (α) parameters. We check the performance of the observer by open loop simulation. We set the free stream velocity U=20 m/s. We select 4 different observer gain system out of 30: Z=1, it is for the IRNO type TP model; Z=10, Z=20, Z=30 is for the CNO type TP model. We can see that we can obtain the best observer performance if we apply CNO type TP model. See Figure 6. In the figure the dotted line signs the trajectory of the observer, continuous line is the real one.

8 Conclusion

In this paper we demonstrated that the LMIs are very sensitive on the convex hull defined by the selected polytopic model. While changing the convex hull, the performance of the controller and also the observer is changing.

The paper shows that the manipulation of the convex hull is just important not only for the controller but also for the observer design and optimization.

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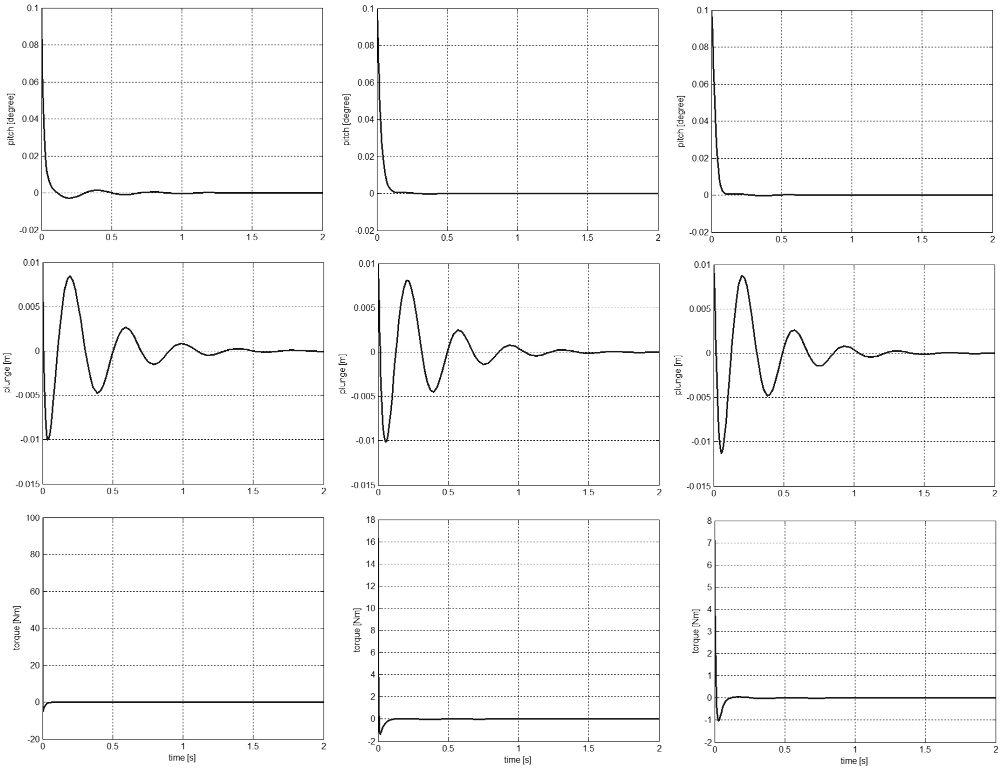

Figure 5. Time response of controller for U = 20 m/s and a = −0.4, while Z=19, Z=25 and Z=30.

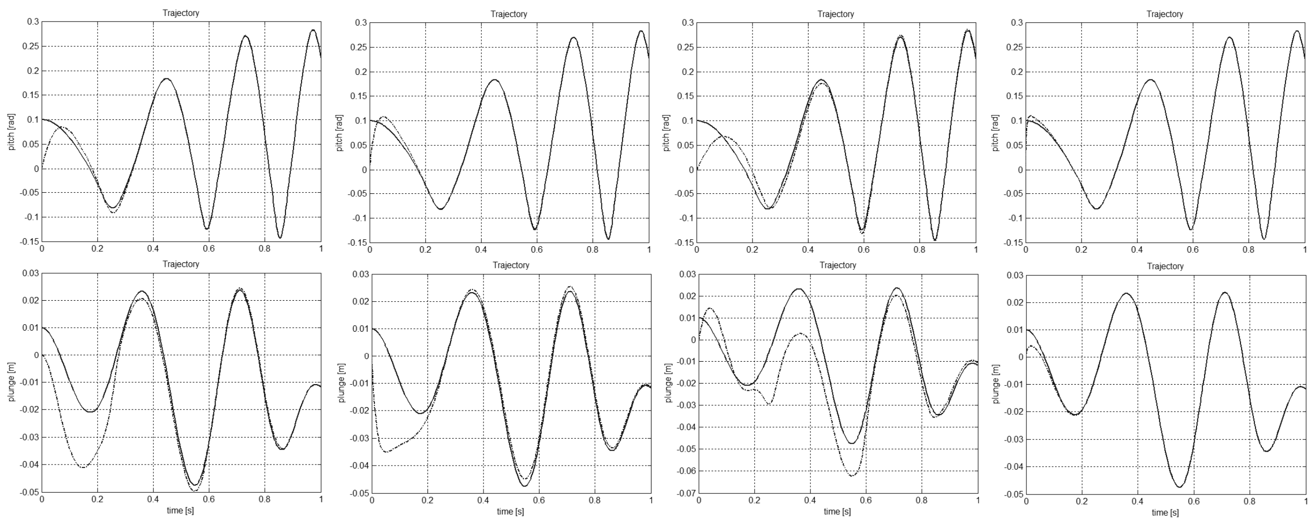

Figure 6.. Open loop simulation, while U=20 m/s; Z=1(IRNO); Z=10, Z=20, Z=30 (CNO)

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