Takagi-Sugeno vs. Lyapunov-based tracking control for a wheeled mobile robot

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Abstract: In this paper a novel kinematic model is proposed where the transformation between the robot posture and the system state is bijective. Two control approaches are proposed to solve the tracking problem. One approach is based on the Takagi-Sugeno fuzzy model where a parallel distributed compensation control is used. The alternative approach is to use Lyapunov stability analysis to construct a nonlinear controller that achieves asymptotic stability if reference velocities satisfy the condition of persistent excitation.

Key–Words: Mobile robot, Kinematic model, Takagi-Sugeno model, PDC control, Lyapunov stability

1 Introduction

The use of fuzzy techniques has been widely accepted for the control of nonlinear systems [19, 2, 21, 20, 8, 26, 27, 16, 22]. Sector nonlinearity approach represents a very elegant way of constructing a Takagi-Sugeno (TS) fuzzy model of a nonlinear system. When combined with a Takagi-Sugeno based controller with the same antecedent part, the approach is called parallel distributed compensation (PDC) [23]. It is very elegant to treat the stability of the PDC controllers. The stability analysis often results in a set of limitations that have to be met. These limitations are usually represented by a system of linear matrix inequalities (LMI). If the LMIs are solved, the suitable control gains are obtained and therefore this stability analysis can be seen as a constructive one.

The problem of nonholonomic systems control has attracted numerous researches in the past. A thoroughly studied case with great practical significance is wheeled mobile robot with kinematic model similar to a unicycle [13]. A very important class of control approaches tackle the problem of posture control by redefining the problem as the tracking control one [10]. By doing so, the problem is divided into two separate problems. The first one is how to design the desired path. This procedure can be done using optimisation techniques taking into account different limitations, such as obstacle avoidance, shortest path, minimum energy path, minimum travel time, etc., [14, 25, 18, 28, 11, 24, 3].

The area of trajectory tracking control is very rich in literature. Many control algorithms were proposed in this framework such as PID [9], Lyapunov based nonlinear controllers [17], model based predictive controllers [12], the approach with disturbance observer [29], etc. Several attempts also exist when the problem is coped with a fuzzy controller [6, 2, 4].

The problem statement is given in Section 2, the development of the third and the fourth order error model is given in Sections 3 and 4, respectively. The Lyapunov and the TS control approaches are described in Sections 5 and 6, respectively. Both approaches are compared in Section 7. The conclusions are stated in Section 8.

2 Problem statement

Assume a two-wheeled, differentially driven mobile robot as the one depicted in Fig. 1 where (x, y) is the wheel-axis-centre position and θ is the robot orientation. The kinematic motion equations of the mobile robot are equivalent to those of a unicycle. Robots with such an architecture have a nonholonomic constraint of the form:

$$\begin{bmatrix} \sin\theta(t) & \cos\theta(t) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = 0 \qquad (1)$$

resulting from the assumption that the robot cannot slip in the lateral direction. Only the first-order kinematic model of the system will be treated in this paper:

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \qquad (2)$$

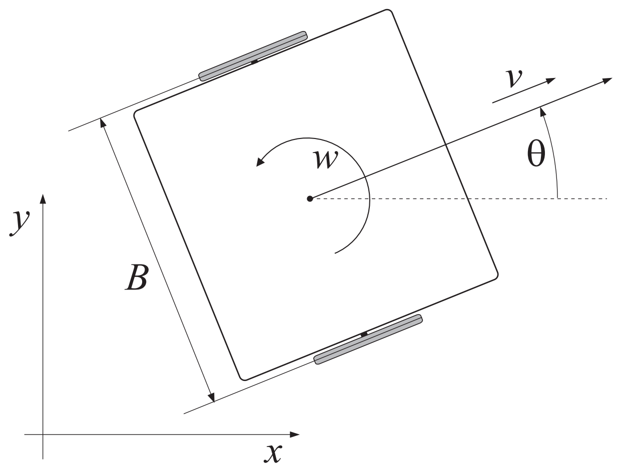

Figure 1: Two-wheeled, differentially driven mobile robot

where v and w are the tangential and the angular velocities, respectively, while $q^T(t) = \begin{bmatrix} x(t) & y(t) & \theta(t) \end{bmatrix}$ is the vector of generalized coordinates. The control design goal is to follow the virtual robot or the reference trajectory (Fig. 2), defined by

$$q_r^T(t) = \begin{bmatrix} x_r(t) & y_r(t) & \theta_r(t) \end{bmatrix} \tag{3}$$

where $q_r(t)$ is a-priori known and smooth. It is very easy to show that the system (2) is flat [5] with flat outputs being x and y. Consequently, (3) can be produced by uniformly continuous control inputs $v_r(t)$ and $w_r(t)$ in the absence of initial conditions, parasitic dynamics and external disturbances. The goal is to design a feedback controller to achieve the tracking and the tracking should be asymptotic under the persistency of excitation (PE) through $v_r(t)$ or $w_r(t)$.

3 Third order error model of the system

The posture error is not given in the global coordinate system, but rather as an error in the local coordinate system of the robot: e_x gives the error in the direction of driving, e_y gives the error in the lateral direction, and e_θ gives the error in the orientation. The posture errors are depicted in Fig. 2. The posture error $e = \begin{bmatrix} e_x & e_y & e_\theta \end{bmatrix}^T$ is determined using the actual posture $q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$ and the reference posture $q_r = \begin{bmatrix} x_r & y_r & \theta_r \end{bmatrix}^T$:

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (q_r - q) \tag{4}$$

From (2) and (4) and assuming that the virtual robot has a kinematic model similar to (2), the posture error model can be written as follows:

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\theta \end{bmatrix} = \begin{bmatrix} \cos e_\theta & 0 \\ \sin e_\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ w_r \end{bmatrix} + \begin{bmatrix} -1 & e_y \\ 0 & -e_x \\ 0 & -1 \end{bmatrix} u \tag{5}$$

The transformation (4) is theoretically imposed by the group operation noting that the model (2) is a system on the Lie group SE(2) [15]. The approach itself was adopted in [10] where the authors also proposed the PID control for the stabilization of the robot at the reference posture. Later, many authors used the error model (5) for the tracking controller design.

Very often [9] the following control u is used to solve the tracking problem:

$$u = \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} v_r \cos e_\theta + v_b \\ w_r + w_b \end{bmatrix} \tag{6}$$

Inserting the control (6) into (5), the resulting model is given by:

$$\begin{aligned} \dot{e}_x &= w_r e_y - v_b + e_y w_b \\ \dot{e}_y &= -w_r e_x + v_r \sin e_\theta - e_x w_b \\ \dot{e}_\theta &= -w_b \end{aligned} \tag{7}$$

where $u_b^T = \begin{bmatrix} v_b & w_b \end{bmatrix}$ is the feedback signal to be determined later.

4 Fourth order error model of the system

The problem of using the third order error model presented in the previous section is that the transformation between the robot posture and the error model is not bijective. This can be observed from the fact that any error-state $\begin{bmatrix} 0 & 0 & 2k\pi \end{bmatrix}^T$ $(k \in \mathbb{Z})$ corresponds to the same robot posture. This should be somehow reflected in the kinematic model of the system and also in the error model of the system. This can be achieved by increasing the order of the system to 4. The variable $\theta(t)$ from the original kinematic model (2) is exchanged by two new periodic variables $s(t) = \sin(\theta(t))$ and $c(t) = \cos(\theta(t))$. Their derivatives are:

$$\begin{aligned} \dot{s}(t) &= \cos(\theta(t))\dot{\theta}(t) = c(t)w(t) \\ \dot{c}(t) &= -\sin(\theta(t))\dot{\theta}(t) = -s(t)w(t) \end{aligned} \tag{8}$$

The new kinematic model is then obtained:

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{s} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & -s \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \tag{9}$$

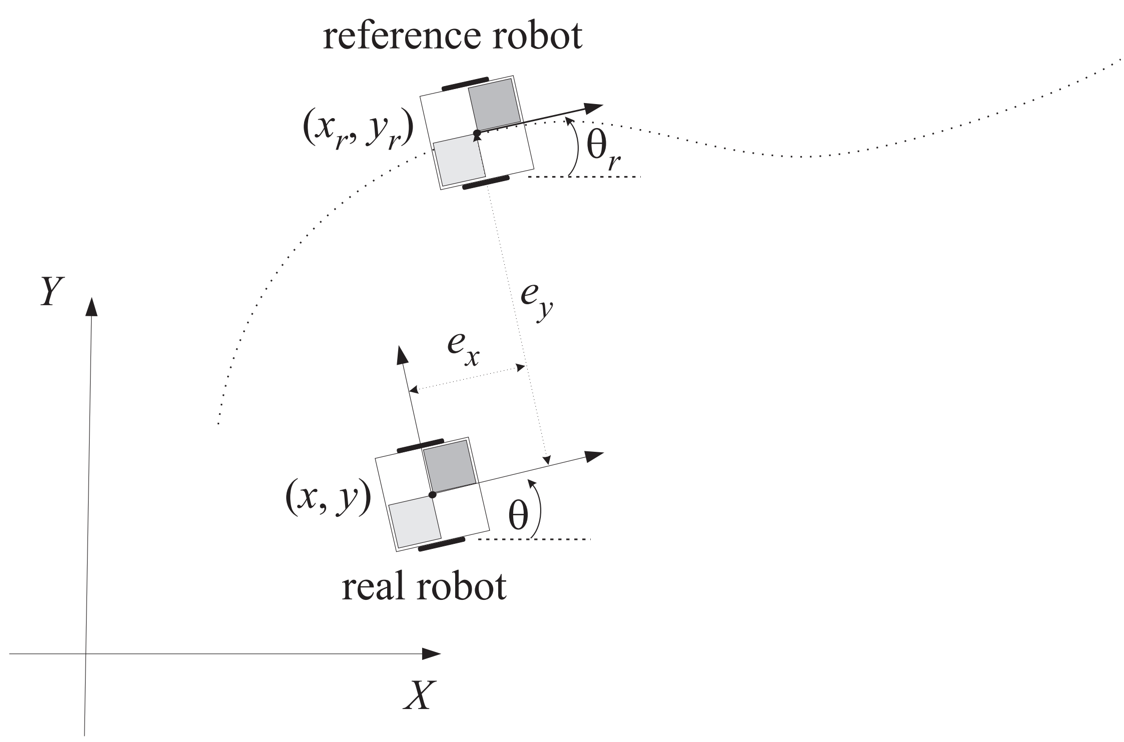

Figure 2: Illustration of the trajectory tracking control

The new error states are defined as:

$$\begin{aligned} e_x &= c(x_r - x) + s(y_r - y) \\ e_y &= -s(x_r - x) + c(y_r - y) \\ e_s &= \sin(\theta_r - \theta) = s_r c - c_r s \\ e_c &= \cos(\theta_r - \theta) = c_r c + s_r s \end{aligned} \quad (10)$$

After derivation of Eq. (10) and some manipulations we obtain the error model of the system:

$$\begin{aligned} \dot{e}_x &= v_r e_c - v + e_y w \\ \dot{e}_y &= v_r e_s - e_x w \\ \dot{e}_s &= w_r e_c - e_c w \\ \dot{e}_c &= -w_r e_s + e_s w \end{aligned} \quad (11)$$

or in the equivalent matrix form

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_s \\ \dot{e}_c \end{bmatrix} = \begin{bmatrix} e_c & 0 \\ e_s & 0 \\ 0 & e_c \\ 0 & -e_s \end{bmatrix} \begin{bmatrix} v_r \\ w_r \end{bmatrix} + \\ + \begin{bmatrix} -1 & e_y \\ 0 & -e_x \\ 0 & -e_c \\ 0 & e_s \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \quad (12)$$

5 Lyapunov based control design

An exponentially stable controller will be developed based on Lyapunov approach. The states e_x, e_y, and e_s should be driven towards 0, while e_c should converge to 1 if we want to achieve perfect tracking. The following Lyapunov function candidate is proposed to achieve this goal:

$$V = \frac{k_x}{2} e_x^2 + \frac{k_y}{2} e_y^2 + \frac{k_s}{2} e_s^2 + \frac{k_c}{2} (e_c - 1)^2 \quad (13)$$

where k_x, k_y, k_s, and k_c are positive constants. Its derivative is:

$$\begin{aligned} \dot{V} = k_x e_x (v_r e_c - v + e_y w) + k_y e_y (v_r e_s - e_x w) + \\ + k_s e_s (w_r e_c - e_c w) + k_c e_c (-w_r e_s + e_s w) - \\ - k_c (-w_r e_s + e_s w) \end{aligned} \quad (14)$$

After simple analysis it is obvious that $k_x = k_y = k$ and $k_s = k_c$ should be used where the latter two constants can be set to 1 without loss of generality. Taking this into account, many terms in Eq. (13) cancel:

$$\dot{V} = k e_x (v_r e_c - v) + e_s (k e_y v_r + w_r - w) \quad (15)$$

Since the values in the parentheses in Eq. (15) should be chosen to make the derivative of the Lyapunov function negative semi-definite, the following control law is proposed:

$$\begin{aligned} v &= v_r e_c + \alpha_x e_x \\ w &= w_r + k e_y v_r + \alpha_s e_s \end{aligned} \tag{16}$$

By taking into account the control law Eq. (16), the function $\dot{V}$ becomes:

$$\dot{V} = -k\alpha_x e_x^2 - \alpha_s e_s^2 \tag{17}$$

Note that the control law (16) is the same as the one proposed by Kanayama et al. [9].

Two very well-known lemmas will be used in the proof of a theorem in this section. The first one is Barbălat's lemma and the other one is a derivation of Barbălat's lemma. Both lemmas are taken from [7] and are given below for the sake of completeness.

Lemma 1 (Barbălat's lemma)**.** *If $\lim_{t\to\infty}\int_0^t f(\tau)d\tau$ exists and is finite, and $f(t)$ is a uniformly continuous function, then $\lim_{t\to\infty} f(t) = 0$.*

Lemma 2. *If $f, \dot{f} \in \mathcal{L}_\infty$ and $f \in \mathcal{L}_p$ for some $p \in [1,\infty)$, then $f(t) \to 0$ as $t \to \infty$.*

Theorem 3. *If the control law (16) is applied to the system where k is a positive constant, α_x and α_s are positive bounded functions, the reference velocities v_r and w_r are bounded, then the tracking errors e_x and e_s converge to 0. The convergence of e_y to 0 is guaranteed provided that at least one of the two conditions is met:*

1. *v_r is uniformly continuous and does not go to 0 as $t \to \infty$ while α_s is uniformly continuous,*
2. *w_r is uniformly continuous and does not go to 0 as $t \to \infty$ while v_r, α_x, and α_s are uniformly continuous.*

Proof:

It follows from (17) that $\dot{V} \le 0$, and therefore the Lyapunov function is non-increasing and thus has the limit $\lim_{t\to\infty} V(t)$. Consequently, the following can be concluded:

$$e_x, e_y, e_s, e_c \in \mathcal{L}_\infty \tag{18}$$

Based on (18), it follows from (16) that the control signals are bounded, and from (11) that the derivatives of the errors are bounded:

$$v, w, \dot{e}_x, \dot{e}_y, \dot{e}_s, \dot{e}_c \in \mathcal{L}_\infty \tag{19}$$

where we also took into account that v_r, w_r, k, α_x, and α_s are bounded. It follows from Eqs. (18) and (19) that e_x, e_y, e_s, and e_c are uniformly continuous (note that the easiest way to check the uniform continuity of $f(t)$ on $[0,\infty)$ is to see if $f, \dot{f} \in \mathcal{L}_\infty$).

In order to show the asymptotic stability of the system, let us first calculate the following integral:

$$\int_0^\infty \dot{V}dt = V(\infty) - V(0) = -\int_0^\infty k\alpha_x e_x^2 dt - \int_0^\infty \alpha_s e_s^2 dt \tag{20}$$

Since V is a positive definite function, the following inequality holds:

$$\begin{aligned} V(0) &\ge \int_0^\infty k\alpha_x e_x^2 dt + \int_0^\infty \alpha_s e_s^2 dt \ge \\ &\ge k\underline{\alpha}_x \int_0^\infty e_x^2 dt + \underline{\alpha}_s \int_0^\infty e_s^2 dt \end{aligned} \tag{21}$$

where the lower bounds of functions $\alpha_x(t)$ and $\alpha_s(t)$ are introduced:

$$\begin{aligned} \alpha_x(t) &\ge \underline{\alpha}_x > 0 \\ \alpha_s(t) &\ge \underline{\alpha}_s > 0 \end{aligned} \tag{22}$$

It follows from (21) that $e_x, e_s \in \mathcal{L}_2$. Applying Lemma 2, the convergence of $e_x(t)$ and $e_s(t)$ to 0 follows immediately. Since e_s and e_c are the sine and the cosine, respectively, of the same argument, e_c^2 converges to 1. Because of $e_s \to 0$, it follows from (11) that $\dot{e}_c \to 0$ and consequently the limit $\lim_{t\to\infty} e_c(t)$ exists and is either 1 or -1. It has been shown that the limit $\lim_{t\to\infty} V(t)$ also exists, and consequently $\lim_{t\to\infty} e_y(t)$ also exists.

Until now we only established the convergence of $e_x(t)$ and $e_s(t)$ to 0, while $e_c(t)$ was shown to converge either to 1 or to -1. To show the convergence of $e_y(t)$ to 0, at least one of the conditions 1 or 2 of Theorem 3 have to be fulfilled. Let us first analyze case 1. Applying Lemma 1 on $\dot{e}_s(t)$ ensures that $\lim_{t\to\infty} \dot{e}_s(t) = 0$ if $\lim_{t\to\infty} e_s(t)$ exists and is finite (which has already been proven) and $\dot{e}_s(t)$ is uniformly continuous. The latter is true (see (11)) if $(w_r - w)e_c$ is uniformly continuous. It has already been shown that e_c is uniformly continuous. The signal $w - w_r$ defined in (16) is uniformly continuous since α_s and v_r are uniformly continuous by the assumption in case 1 of the theorem. The statement $\lim_{t\to\infty} \dot{e}_s(t) = 0$ which is identical to

$$\lim_{t\to\infty} \big((w_r(t) - w(t))\, e_c(t) \big) = 0 \tag{23}$$

has therefore been proven. Since $e_c(t)$ converges to 1 or to -1, the following can be concluded from (16):

$$\lim_{t\to\infty} (w - w_r) = \lim_{t\to\infty} (k e_y v_r + \alpha_s e_s) = 0 \tag{24}$$

The convergence of e_y to 0 follows from (24):

$$\begin{aligned} &ke_yv_r + \alpha_s e_s \to 0, e_s \to 0 \Rightarrow ke_yv_r \to 0 \\ &ke_yv_r \to 0, k_y > 0, v_r \nrightarrow 0 \Rightarrow e_y \to 0 \end{aligned} \tag{25}$$

where it was taken into account that v_r does not diminish as $t \to \infty$.

For the second case we again have to guarantee that $\lim_{t\to\infty}(w - w_r) = 0$. This is true if v_r and α_s are uniformly continuous as shown before. Then Barbălat's lemma (Lemma 1) is applied on $\dot{e}_x$ in Eq. (11) after inserting the control law (16):

$$\begin{aligned} \dot{e}_x &= v_r e_c - v + e_y w = \\ &= -\alpha_x e_x + e_y w_r + k e_y^2 v_r + \alpha_s e_y e_s \end{aligned} \tag{26}$$

It has already been shown that e_x, e_y, and e_s are uniformly continuous, while v_r, w_r, α_x, and α_s are uniformly continuous by the assumption of case 2 of the theorem. It has been proven that $\lim_{t\to\infty} \dot{e}_x(t) = 0$. The first term in Eq. (26) goes to 0 as t goes to infinity. The last two terms also converge to 0 due to $\lim_{t\to\infty}(w - w_r) = 0$. Consequently, the product $w_r e_y$ goes to 0. Since w_r is persistently exciting and does not go to 0, e_y has to go to 0. □

6 Takagi-Sugeno fuzzy control design

In this section the Takagi-Sugeno (TS) model of the mobile robot kinematic model (11) or (12) will be developed. Firstly, the control signals will be separated to the the "feedforward" term and the feedback term to be determined by the TS control law:

$$\begin{aligned} v &= v_r e_c + v_b \\ w &= w_r + w_b \end{aligned} \tag{27}$$

where "feedforward" is not entirely suitable since the linear velocity command depends on the state e_c. But on the other hand, if the control law is chosen properly, e_c should converge to 1, thus meaning that $v_r e_c$ would converge to a feedforward term v_r. Inserting Eq. (27) into Eq. (11) we obtain:

$$\begin{aligned} \dot{e}_x &= -v_b + e_y w_r + e_y w_b \\ \dot{e}_y &= v_r e_s - e_x w_r - e_x w_b \\ \dot{e}_s &= -e_c w_b \\ \dot{e}_c &= e_s w_b \end{aligned} \tag{28}$$

or in the equivalent matrix form

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_s \\ \dot{e}_c \end{bmatrix} = \begin{bmatrix} 0 & w_r & 0 & 0 \\ -w_r & 0 & v_r & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_s \\ e_c \end{bmatrix} + \\ + \begin{bmatrix} -1 & e_y \\ 0 & -e_x \\ 0 & -e_c \\ 0 & e_s \end{bmatrix} \begin{bmatrix} v_b \\ w_b \end{bmatrix} \tag{29}$$

The TS model of (29) can be constructed by the sector nonlinearity approach if v_r, w_r, e_x, e_y, e_s, and e_c are chosen as the antecedent variables with a-priori known upper and lower bounds [1, 6]. A natural way to control such a system is to use parallel distributed compensation (PDC) [23]. It is very easy to determine that the PDC design on the model (29) would result in an infeasible system of linear matrix inequalities. This is due to the fact that the system is not controllable in the linear sense. This problem can be circumvented if the state space description is split into two systems:

$$\dot{e} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_s \end{bmatrix} = \begin{bmatrix} 0 & w_r & 0 \\ -w_r & 0 & v_r \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_s \end{bmatrix} + \\ + \begin{bmatrix} -1 & e_y \\ 0 & -e_x \\ 0 & -e_c \end{bmatrix} \begin{bmatrix} v_b \\ w_b \end{bmatrix} \tag{30}$$

$$\dot{e}_c = e_s w_b \tag{31}$$

First, the TS model of the system (30) will be developed and then the PDC control will be designed for this system. Eq. (31) represents the uncontrollable mode of the system, and it will be important to check if e_c converges to 1 as desired.

6.1 Takagi-Sugeno model of the systen

The TS model is represented through the following polytopic form:

$$\dot{e}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left(A_i e(t) + B_i u(t)\right) \tag{32}$$

In order to construct the TS model the sector nonlinearity approach will be used. This means that the nonlinearities have to be taken from the nonlinear model and used in the premise (or antecedent) vector $z(t)$. The antecedent vector has 5 elements in this case:

$$z(t) = \begin{bmatrix} w_r(t) \\ v_r(t) \\ e_y(t) \\ e_x(t) \\ e_c(t) \end{bmatrix} \tag{33}$$

The system (30) is controllable in the vicinity of the point $\begin{bmatrix} e_x & e_y & e_s \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ if e_c does not approach 0 and either v_r or w_r do not approach 0. This are actually the conditions for the feasibility of LMIs for the determination of control gains as it will be shown later. The lower bounds $\underline{z}_j$ and the upper bounds $\overline{z}_j$ ($j = 1, 2, 3, 4, 5$) of the elements of the antecedent vector are needed for the construction of the PDC control. The bounds on v_r and w_r are obtained from the actual reference trajectory, while the bounds on the tracking error are selected on the basis of any a priori knowledge available.

The matrices A_z and B_z are:

$$\begin{aligned} A_z &= \begin{bmatrix} 0 & z_1 & 0 \\ -z_1 & 0 & z_2 \\ 0 & 0 & 0 \end{bmatrix} \\ B_z &= \begin{bmatrix} -1 & z_3 \\ 0 & -z_4 \\ 0 & -z_5 \end{bmatrix} \end{aligned} \quad (34)$$

The number of fuzzy rules is $r = 2^5 = 32$. The matrices of the linear submodels are:

$$\begin{aligned} A_i &= \begin{bmatrix} 0 & \varepsilon_i^1 & 0 \\ -\varepsilon_i^1 & 0 & \varepsilon_i^2 \\ 0 & 0 & 0 \end{bmatrix} \\ B_i &= \begin{bmatrix} -1 & \varepsilon_i^3 \\ 0 & -\varepsilon_i^4 \\ 0 & -\varepsilon_i^5 \end{bmatrix} \end{aligned} \quad (35)$$

$$i = 1 \dots 2^5$$

where

$$\varepsilon_i^j = \underline{z}_j + i_j \left(\overline{z}_j - \underline{z}_j \right), \quad i = 1 \dots 2^5, \quad j = 1 \dots 5$$

and the mapping from i to i_j is defined so that all the vertices of the hypercube are numbered using binary enumeration:

$$\begin{aligned} i_1 &= \begin{cases} 0 & i \le \frac{r}{2} \\ 1 & \text{else} \end{cases} \\ i_2 &= \begin{cases} 0 & i - \frac{r}{2} i_1 \le \frac{r}{4} \\ 1 & \text{else} \end{cases} \\ i_3 &= \begin{cases} 0 & i - \frac{r}{2} i_1 - \frac{r}{4} i_2 \le \frac{r}{8} \\ 1 & \text{else} \end{cases} \\ i_4 &= \begin{cases} 0 & i - \sum_{l=1}^{3} \frac{r}{2^l} i_l \le \frac{r}{2^4} \\ 1 & \text{else} \end{cases} \\ i_5 &= \begin{cases} 0 & i - \sum_{l=1}^{4} \frac{r}{2^l} i_l \le \frac{r}{2^5} \\ 1 & \text{else} \end{cases} \end{aligned} \quad (36)$$

Finally the membership functions $h_i(z)$ in (32) need to be defined:

$$\begin{aligned} & h_i(z) = \prod_{j=1}^{5} w_{i_j}^i(z_j) \quad i = 1, 2, \dots 32 \\ & w_1^j(z_j) = \frac{z_j - \underline{z}_j}{\overline{z}_j - \underline{z}_j}, \quad w_0^j(z_j) = 1 - w_1^j(z_j) \quad j = 1 \dots 5 \end{aligned}$$

6.2 PDC based tracking control of a mobile robot

In order to stabilize the TS fuzzy model (32), a PDC (Parallel Distributed Compensation) control law is used:

$$u_b(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i e(t) = -F_z e(t) \quad (37)$$

Several results concerning the stability of the TS model with the PDC controllers exist. The problem is often solved within the LMI framework.

Here the solution that tries to optimise the decay rate of the system will be used [23, 6]:

$$\begin{aligned} & \underset{X, M_1 \dots M_r}{\text{minimize}} \, \gamma \text{ subject to} \\ & \Upsilon_{ii} < 0 \qquad i = 1, 2, \dots r \\ & \tfrac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0 \quad i, j = 1, 2, \dots r, i \ne j \end{aligned} \quad (38)$$

with $\Upsilon_{ij} = X A_i^T + A_i X - M_j^T B_i^T - B_i M_j + \gamma X$ where $\gamma > 0$. The solutions of the above generalized eigen-value problem give the control gains:

$$F_i = M_i X^{-1}, \quad i = 1, 2, \dots r \quad (39)$$

7 Comparison of the two control approaches

An extensive simulation study was performed to compare both approaches under the same circumstances. The reference trajectory is the same in all the experiments:

$$\begin{aligned} x_r(t) &= \cos(\omega_0 t) \\ y_r(t) &= \sin(2\omega_0 t) \end{aligned} \quad (40)$$

with $\omega_0 = 0.34$. The experiment always started at $t = 0$ and finished at $t = \frac{2\pi}{\omega_0}$. The control signals v_b and w_b were saturated to ± 10. The experiment was conducted with different initial conditions.

In order to choose the T-S control gains the generalized eigen value problem (38) has to be solved. We have to know the maximum values of the reference velocities a priori. We also have to know lower and upper bound on the errors. In our case ± 0.1 was chosen for e_x and e_y, while e_c was between 0.1 and 1. Note that it is not possible to find a solution if e_c is negative. Thus the orientation error is bounded to

$\pm 90°$. The solution for the first 16 control gains according to Eq. (39) is:

$$
\begin{aligned}
F_1 &= \begin{bmatrix} -37.421 & 2.2619 & 0.89260 \\ 1.1669 & -13.296 & -21.976 \end{bmatrix} \\
F_2 &= \begin{bmatrix} -37.426 & 2.1300 & 0.61430 \\ -0.27874 & -26.206 & -41.995 \end{bmatrix} \\
F_3 &= \begin{bmatrix} -37.427 & 2.2044 & 0.72928 \\ -0.053279 & -23.987 & -37.524 \end{bmatrix} \\
F_4 &= \begin{bmatrix} -37.430 & 2.0940 & 0.57824 \\ -0.20485 & -25.847 & -41.729 \end{bmatrix} \\
F_5 &= \begin{bmatrix} -37.248 & -0.57792 & -3.7756 \\ -0.66309 & -13.294 & -21.971 \end{bmatrix} \\
F_6 &= \begin{bmatrix} -37.404 & 0.66145 & -1.7262 \\ 0.76778 & -26.185 & -41.959 \end{bmatrix} \\
F_7 &= \begin{bmatrix} -37.289 & -0.47157 & -3.5982 \\ 0.79428 & -24.151 & -37.768 \end{bmatrix} \\
F_8 &= \begin{bmatrix} -37.397 & 0.71481 & -1.6811 \\ 0.80535 & -25.824 & -41.692 \end{bmatrix} \\
F_9 &= \begin{bmatrix} -37.440 & 2.5666 & 1.3526 \\ -0.99369 & -28.580 & -45.813 \end{bmatrix} \\
F_{10} &= \begin{bmatrix} -37.432 & 2.1168 & 0.62534 \\ -0.40629 & -27.075 & -43.103 \end{bmatrix} \\
F_{11} &= \begin{bmatrix} -37.443 & 2.3814 & 1.0903 \\ -0.55093 & -27.775 & -45.646 \end{bmatrix} \\
F_{12} &= \begin{bmatrix} -37.435 & 2.1088 & 0.62682 \\ -0.26462 & -26.414 & -42.772 \end{bmatrix} \\
F_{13} &= \begin{bmatrix} -37.339 & -0.47666 & -3.5440 \\ 2.5489 & -28.515 & -45.698 \end{bmatrix} \\
F_{14} &= \begin{bmatrix} -37.408 & 0.70385 & -1.6557 \\ 0.77689 & -27.061 & -43.076 \end{bmatrix} \\
F_{15} &= \begin{bmatrix} -37.357 & 0.15622 & -2.6337 \\ 2.0320 & -27.728 & -45.561 \end{bmatrix} \\
F_{16} &= \begin{bmatrix} -37.403 & 0.77476 & -1.5740 \\ 0.84901 & -26.395 & -42.739 \end{bmatrix}
\end{aligned}
\tag{41}
$$

The remaining control 16 gains are:

$$
\begin{aligned}
F_{17} &= \begin{bmatrix} -37.248 & 0.57792 & 3.7756 \\ 0.66310 & -13.294 & -21.971 \end{bmatrix} \\
F_{18} &= \begin{bmatrix} -37.404 & -0.66145 & 1.7262 \\ -0.76778 & -26.185 & -41.959 \end{bmatrix} \\
F_{19} &= \begin{bmatrix} -37.289 & 0.47157 & 3.5982 \\ -0.79428 & -24.151 & -37.768 \end{bmatrix} \\
F_{20} &= \begin{bmatrix} -37.397 & -0.71481 & 1.6811 \\ -0.80535 & -25.824 & -41.692 \end{bmatrix} \\
F_{21} &= \begin{bmatrix} -37.421 & -2.2619 & -0.89260 \\ -1.1669 & -13.296 & -21.976 \end{bmatrix} \\
F_{22} &= \begin{bmatrix} -37.426 & -2.1300 & -0.61430 \\ 0.27874 & -26.206 & -41.995 \end{bmatrix} \\
F_{23} &= \begin{bmatrix} -37.427 & -2.2044 & -0.72928 \\ 0.053279 & -23.987 & -37.524 \end{bmatrix} \\
F_{24} &= \begin{bmatrix} -37.430 & -2.0940 & -0.57824 \\ 0.20485 & -25.847 & -41.729 \end{bmatrix} \\
F_{25} &= \begin{bmatrix} -37.339 & 0.47666 & 3.5440 \\ -2.5489 & -28.515 & -45.698 \end{bmatrix} \\
F_{26} &= \begin{bmatrix} -37.408 & -0.70386 & 1.6557 \\ -0.77689 & -27.061 & -43.076 \end{bmatrix} \\
F_{27} &= \begin{bmatrix} -37.357 & -0.15622 & 2.6337 \\ -2.0320 & -27.728 & -45.561 \end{bmatrix} \\
F_{28} &= \begin{bmatrix} -37.403 & -0.77476 & 1.5740 \\ -0.84901 & -26.395 & -42.739 \end{bmatrix} \\
F_{29} &= \begin{bmatrix} -37.440 & -2.5667 & -1.3526 \\ 0.99369 & -28.580 & -45.813 \end{bmatrix} \\
F_{30} &= \begin{bmatrix} -37.432 & -2.1168 & -0.62534 \\ 0.40629 & -27.075 & -43.103 \end{bmatrix} \\
F_{31} &= \begin{bmatrix} -37.443 & -2.3814 & -1.0903 \\ 0.55093 & -27.775 & -45.646 \end{bmatrix} \\
F_{32} &= \begin{bmatrix} -37.435 & -2.1088 & -0.62682 \\ 0.26462 & -26.414 & -42.772 \end{bmatrix}
\end{aligned}
\tag{42}
$$

We can observe from the solution that some gains are relatively insensitive to changes of the operating point (e.g. f_{11}), while most of them vary significantly, which again stresses the nonlinear nature of the system and the controller. A simple “average” controller

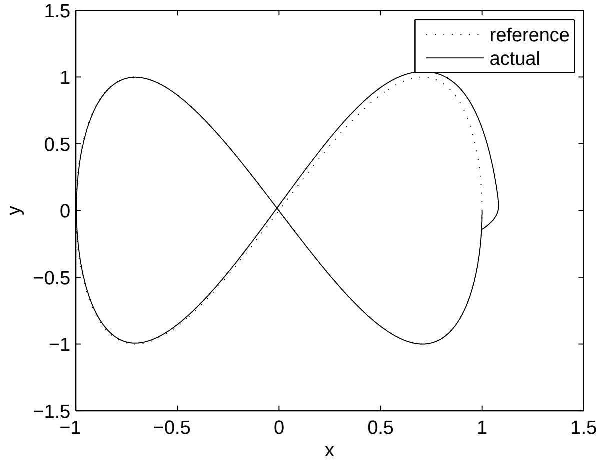

Figure 3: Lyapunov-based control – (x, y) plot

gain is:

$$F_a = \begin{bmatrix} -37.435 & -2.1088 & -0.62682 \\ 0.26462 & -26.414 & -42.772 \end{bmatrix} \quad (43)$$

The average control law is therefore:

$$\begin{bmatrix} v_{ba} \\ w_{ba} \end{bmatrix} = \begin{bmatrix} -37.435 & -2.1088 & -0.62682 \\ 0.26462 & -26.414 & -42.772 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_s \end{bmatrix} \quad (44)$$

Comparing Eq. (44) and Eq. (16) we can see that the control laws are quite different. If trying to make fair comparison, α_x will be set to 37, while α_s will be set to 43. The initial values for the experiment are: $e_x = 0.1$, $e_y = 0.1$, $e_s = \sin(\pi/4)$, $e_c = \cos(\pi/4)$. Figures 3, 4, and 5 show the results of the Lyapunov-based approach. Figures 6, 7, and 8 show the results of the PDC-based approach. These results and also other tests suggest that Lyapunov-based approach is better than PDC-based one.

8 Conclusion

In this paper a novel kinematic model is proposed where the transformation between the robot posture and the system state is bijective. Two control approaches are proposed to solve the tracking problem. One approach is based on the Takagi-Sugeno fuzzy model where a parallel distributed compensation control is used. The alternative approach is to use Lyapunov stability analysis to construct a nonlinear controller that achieves asymptotic stability if reference velocities satisfy the condition of persistent excitation.

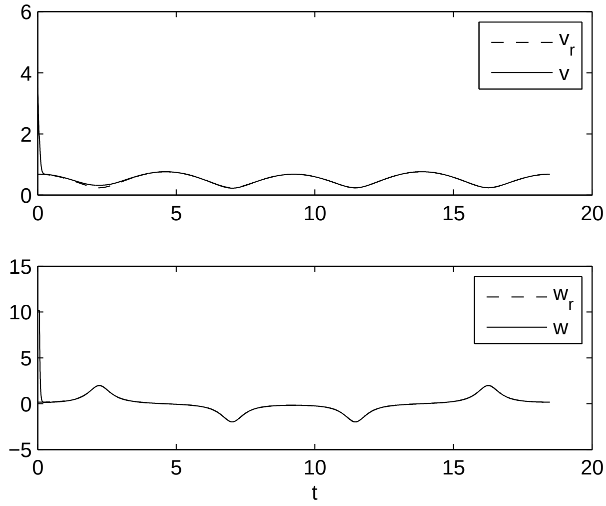

Figure 4: Lyapunov-based control – time plots of the velocities $v(t)$ and $w(t)$

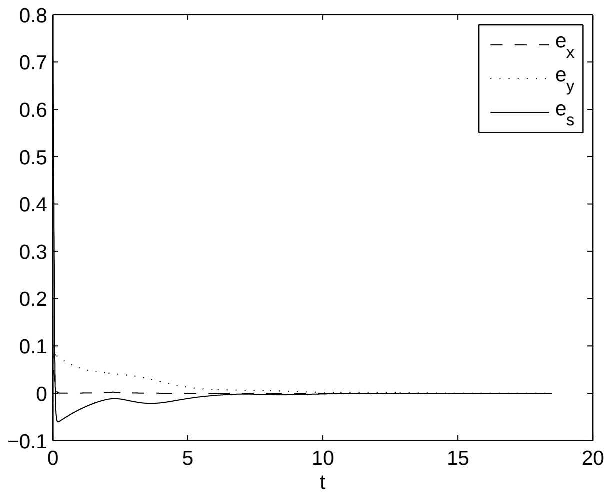

Figure 5: Lyapunov-based control – time plots of the errors

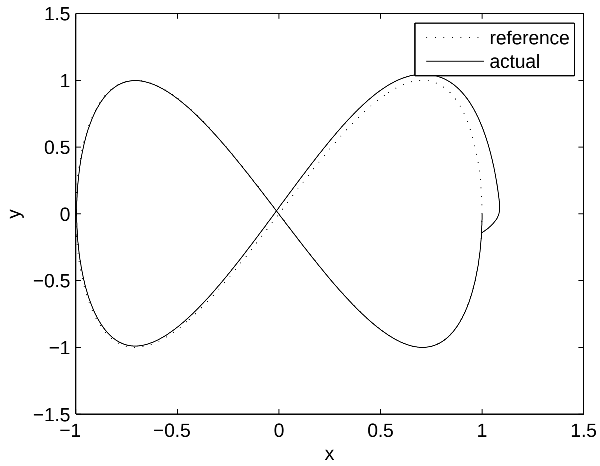

Figure 6: PDC-based control – (x, y) plot

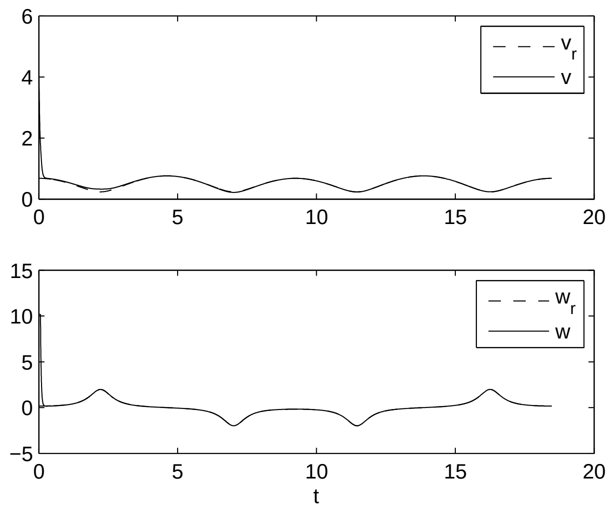

Figure 7: PDC-based control – time plots of the velocities $v(t)$ and $w(t)$

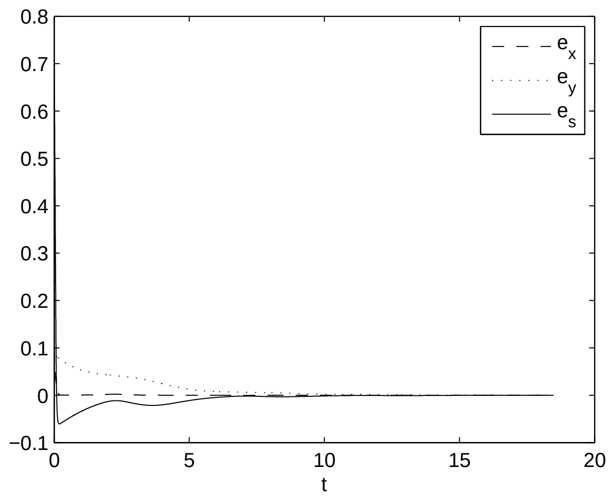

Figure 8: PDC-based control – time plots of the errors

Although the proposed fourth order model is not controllable in the linear sense and the associated PDC control design is not feasible, the Lyapunov based control design easily results in a controller that achieves asymptotic stability under the usual demands of persistently exciting reference velocities. When the uncontrollable node is separated from the controllable states, the PDC control is obtained for the controllable part but the state e_c is restricted to the interval $(0, 1]$ (the orientation error must be within $\pm 90°$) while in the Lyapunov based design it is restricted to $(-1, 1]$ (the orientation error must be within $\pm 180°$).

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