A Fuzzy Controller with Various T-norms Applied in Robot Navigation

ION IANCU[1], MIHAELA COLHON[1], MIHAI DUPAC[2]
[1] Department of Computer Science
University of Craiova
13 A. I. Cuza Street 200585
ROMANIA
[2] Department of Design, Engineering and Computing
Bournemouth University
Fern Barrow, BH12 5BB Poole
UNITED KINGDOM

i_iancu@yahoo.com http://inf.ucv.ro/~iancu/
mcolhon@inf.ucv.ro http://inf.ucv.ro/~ghindeanu/
mdupac@bournemouth.ac.uk http://dec.bournemouth.ac.uk/people.html

Abstract: - Fuzzy set theory and fuzzy logic are the convenient tools for handling uncertain, imprecise, or unmodeled data in intelligent decision-making systems. The utility of fuzzy logic in system controls domain is presented in the context of a mobile robot navigation control application. The Takagi-Sugeno controller is a fuzzy model capable of approximating a wide class of nonlinear systems by decomposing the input space into several partial fuzzy subspaces and representing the output space with a linear equation. The output control action is obtained from the rule-base and a set of crisp inputs. A Takagi-Sugeno type Fuzzy Logic Controller (FLC), to work with crisp data, intervals and fuzzy sets inputs, is proposed in connection with a mobile robot navigation model. The model also works with a set of t-norms, and for any t-norm an output value is obtained. Finally, these outputs are combined to obtain the overall output of the system.

Key-Words: - Fuzzy set, Fuzzy logic control, Zero order system, Mobile robot, Repulsive angle, T-norm.

1 Introduction

Applications in the field of autonomous robots are generally based on navigating from a start point to goal point in a known or unknown environment. In daily life, autonomous robots are used in many missions like planetary exploration and space applications. In these applications the points that should be visited are unknown. But, for an autonomous robot, consuming less energy is very important and it is obvious that the more autonomous robot travels more energy and time is consumed. To fulfil these constraints, a shorter path is preferred rather than a longer path. Therefore an intelligent path planning algorithm is always required. Since a robot works in a real and dynamic environment, the path planning algorithm should construct the path in real time. Research on robots has attracted attention in the last years [1] and was mainly directed to the use of kinematic models of the mobile robots to achieve and accomplish the motion control [2 - 6]. Later on, the research has been focused on robots with additional sensory system to develop autonomous guidance path planning systems [7]. Sophisticated sensory systems has been used in [8], helping the software to learn about the operating environment and to evaluate path constraints for a good path planning programming. A virtual reality modeling of autonomous searching robots has been described in [9] and [10]. A fuzzy logic controller is a collection of fuzzy *if-then* rules that accompanied by suitable membership functions form the rule-base core of the system. A FLC is well suited for controlling a robot since it is capable of making inferences even under uncertainty [11, 12, 13]. Handling uncertain and imprecise data for decision-making systems was discussed in [14]. From the class of classical FLC including Mamdani, Takagi-Sugeno, Tsukamoto, Larsen, the Takagi-Sugeno model is an engineering tool for modeling and controlling complex systems. It is described by fuzzy IF-THEN rules which can give local linear representation of a nonlinear system by decomposing the whole input space into several partial fuzzy subspaces and representing each output space with a linear equation. Such a model is capable of approximating a wide class of nonlinear systems. Because it uses the linear model in the consequent part, conventional linear system theory can be applied for the analysis and synthesis accordingly. The model proposed by Takagi and Sugeno is one of the most efficient fuzzy models that can represent nonlinear systems or functions. The consequent part in Takagi-Sugeno inference system is a linear equation or constant coefficient. In the case of linear equation, the system is of ”first order” and the constant type has ”zero order”. In this paper is proposed a version of Takagi-Sugeno FLC with the following characteristics:

- the linguistic terms (or values) are represented by triangular fuzzy numbers
- the system accepts crisp data, intervals and fuzzy sets as inputs
- a set of t-norms are used to compute the firing level of the rules
- for every t-norm, the output of the system is calculated using the discrete Center Of Gravity (COG), to aggregate the outputs of the rules
- an Ordered Weighted Averaging (OWA) operator is used to aggregate the previous outputs in order to obtain the overall control action of the system.

The proposed newly fuzzy inference mechanism is structured on the same set of rules and very close values of the membership functions as the ones used in [15].

2 Mathematical Model of the Robot

A robot has to move on a time and collision-free path and navigating among obstacles while satisfying the kinematic and dynamic constraints. Therefore, the major main work for path planning for autonomous mobile robot is to search a collision free path. Many works on this topic have been carried out for the path planning of autonomous mobile robot [16]. In the next two sub-sections, the kinematic and dynamic constraints of an autonomous robot are presented.

2.1 Robot Kinematics

An autonomous mobile robot has to move on an optimal path (time and collision-free optimal path [17]) on which it must avoid various obstacles and satisfy the kinematic constraints.

For this study the robot is modeled as a rigid body with two wheels, as shown in Fig. 1. The robot motion is defined on a plane surface, with the 2 wheels making contact point with the planar ground surface. The rigid body coordinate system of the robot has the origin at the midpoint of the axle of the robot, the longitudinal axis x_r points toward the front of the robot and the transversal axis y_r points toward the left wheel. The robot is moving in a defined environment navigating among obstacles to reach a target. For the presented case, only one obstacle is considered as it is shown in Fig. 1. It is also considered that the designed target represents an attractive force and the obstacle represents a repulsive force. The attractive vector force v_A (due to the attractive target force), the repulsive vector force v_R (due to the repulsive obstacle acting on the robot), and the robot trajectory to the target are depicted in Fig. 1. The robot direction is denoted by v_D.

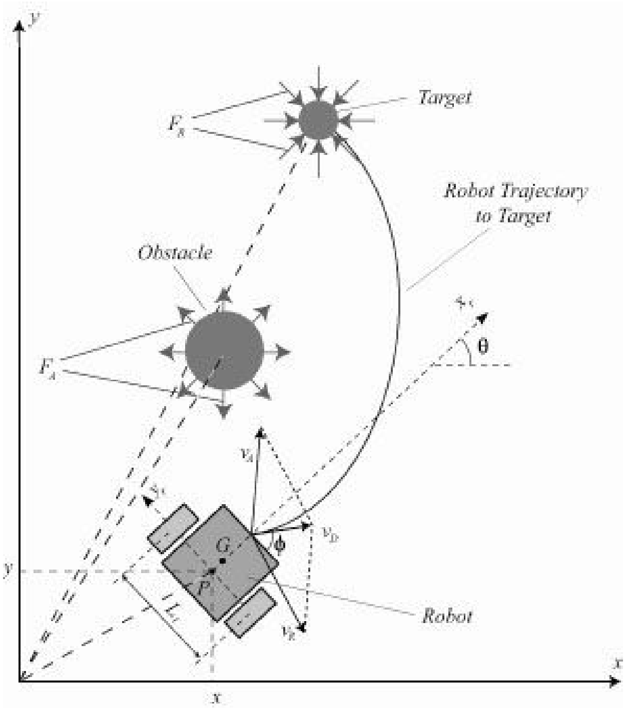


Fig. 1: The robot model

For the study of the robot kinematics the configuration of the moving robot at each time instant is defined by the triple $[x, y, \theta]$, where (x, y) represents the global position of the robot, i.e., the x-coordinate and y-coordinate of the midpoint of the robot axle (relative to the origin of a Cartesian reference frame xOy), and θ represents the robot orientation, i.e., the angle between the robot's longitudinal axis (the main axis of the robot) and the Ox axis.

In order to obtain the kinematic model of the robot, the differential relations between the configuration variables and the driving inputs, has to be derived. For the presented kinematic model the no-slip condition is assumed, i.e., the robot acceleration is such that the exercised forces (in the longitudinal and lateral directions) on the robot's tires do not exceed the maximum static friction between the tires and the ground. Based on the no-slip model, it can be observed that the velocities of the center of the robot's wheels do not have any lateral components, is parallel with the wheel planes and can be related to the rotational velocity of the wheels.

The lateral no-slip condition states that the lateral velocity of the robot is zero at all times. The no-slip condition can be expressed as

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0,$$

and so

$$\theta(t) = \arctan\frac{\dot{y}(t)}{\dot{x}(t)}.$$

Assuming that the point (x, y) of the robot moves with a linear speed v, while the robot has an angular velocity u, one can write the velocity components of the point (x, y) in the inertial frame as

$$\dot{x} = u\cos\theta,\ \dot{y} = u\sin\theta. \qquad (1)$$

The rate of change of robot orientation can be expressed

$$\dot{\theta} = \omega. \qquad (2)$$

Using Eqs. (1) and (2) the kinematic equations of the robot motion can be written as

$$\dot{x} = ucox\theta,$$
$$\dot{y} = u\sin\theta,$$
$$\dot{\theta} = \omega,$$

or equivalent in a matrix form

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u\\ \omega\end{bmatrix}. \qquad (3)$$

The motion of the robot can be predicted by specifying the driving inputs u and ω. Assuming that

$$\omega = (u_r - u_l)/L_t,$$

Eq. 3 can be rewritten as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}cos\,\theta & 0\\ sin\,\theta & 0\\ 0 & 1/L_t\end{bmatrix}\begin{bmatrix}u\\ u_r - u_l\end{bmatrix},$$

where L_t is the distance between the wheels.

2.2 Robot Dynamics

To study the robot dynamics it was considered that the robot wheels can be controlled independently. While in motion, the robot is subjected to various dynamic constraints, such as wheels sliding or wheels torque constraint. Since the dynamics of the robot changes depending on whether the wheels are rolling or sliding, there are three different cases to be studied,

- both wheels are rolling
- both wheels are sliding
- one wheel is rolling and another wheel is sliding

The motion of the robot can be described as follows

- the robot moves in a straight line if both wheels are rotating with the same angular velocity in the same direction
- the robot moves on a curved trajectory if the wheels are rotating with different angular velocity in the same direction
- the robot rotate (pivot) about its vertical axis if the wheels are rotating in opposite directions, with the same angular velocity (the same magnitude)

As mentioned before, the robot is described by the position of the center of mass (x and y), by its orientation θ, and by the angles of rotation of the wheels (the robot has two independently actuated wheels, with the inputs the torques u_l and u_r). Based on the no-slip model, the velocity of the global position P of the robot (midpoint of the robot axle) with respect to the right and left wheel can be written as,

$$v_r = \dot{\psi}_r d + \frac{L_t}{2}\omega,$$
$$v_l = \dot{\psi}_l d + \frac{L_t}{2}\omega,$$

where d is the wheel radius, $\dot{\psi}_l$ and $\dot{\psi}_r$ are the rotational velocities of the left and right wheel respectively, ω is the rotational velocity of the body of the robot, and the velocity constraint equation is

$$L_t\omega = (\psi_l - \psi_r)d.$$

The velocity components of the midpoint of the robot axle can now be written as

$$\dot{x}_P = \left(\dot{\psi}_r d + \frac{L_t}{2}\omega\right)\cos\theta,$$

$$\dot{y}_P = \left(\dot{\psi}_r d + \frac{L_t}{2}\omega \right) \sin\theta .$$

The acceleration of the mass center G of the robot can be calculated using the first derivative of the velocity, with robot acceleration such as that the no-slip condition is assumed, i.e., acting forces on the tires not exceed the maximum static force.

Applying the equations of the dynamics [18], the robot equations of motion can be written as in [19]

$$\left(I_v + ms^2\right)\ddot{\theta} - mds\omega\dot{\psi}_r + m\frac{L_t s}{2}\omega^2 + \frac{b}{d}\frac{L_t^2}{2}\omega = F\left(h - s\right)\sin\phi,$$

$$md\ddot{\psi}_r - m\frac{L_t}{2}\ddot{\theta} + 2b\dot{\psi}_r - \frac{b}{d}\frac{L_t^2}{2}\omega + \mu mg + ms^2\ddot{\theta} = F\cos\phi,$$

where m is the mass of the robot, I_v is the moment of inertia of the robot around the vertical axis through its center of mass, F is the resultant force acting on the robot due to the attractive and repulsive forces, ϕ is the angle between the robot longitudinal axis x_r and the resultant force, μ is the friction coefficient, g is the gravity, and s is the distance between the robot center of the mass and the midpoint of the robot axle ($s = 0$ if the location of the robot center of the mass and the midpoint of the robot axle are the same).

The attractive and the repulsive forces are considered to be functions depending on of both the position and velocity of the robot, defined as the negative gradient of the attractive and repulsive potential respectively, as described in [20].

3 Sensor Data Processing

The mobile robot is equipped with a sensorial system. The sensors installed on the robot (vision sensors such as CCD cameras, laser scanning system, or ultrasonic sensors) allow the robot to navigate in the environment with obstacle avoidance (collision free).

Given such sensor modalities, the usual procedure for a fuzzy logic controller consists of first defining linguistic terminology for the input and output variables, partitioning the sensor space using appropriate fuzzy sets (membership functions), and formulating fuzzy rules that can give the desired response to the robot in its navigation problems [14].

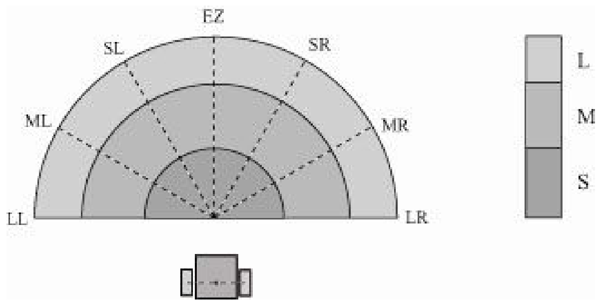


Fig. 2: Scan area radial sectors of the robot

For practical purposes, the robot's sensors area is divided in seven radial sectors labelled as:

- *Large Left* (*LL*), *Medium Left* (*LM*) and *Small Left* (*SL*) for the left areas,
- EZ for the straight area, and
- *Large Right* (*LR*) *Medium Right* (*MR*) and *Small Right* (*SR*) for the right area respectively.

Each radial sector is further divided in other three regions like

$$Small\ (S),\ Medium\ (M)\ and\ Large(L)$$

as shown in Fig. 2. It was considered that the sensors range 30 meters, and that the robot can identify an obstacle anywhere inside the interval $\left[-90^o, 90^o\right]$.

All the considered radial sectors are used to construct the Takagi-Sugeno type FLC for robot navigation to a goal point (discussed in the next Section) and to calculate the corresponding *repulsive angle*, noted with θ_R. The controller has two inputs, the *direction angle*, noted with θ, and the *distance* d towards to the target point (see Fig. 3).

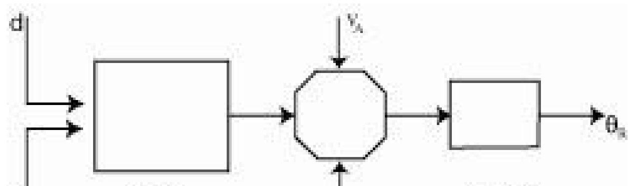


Fig. 3: A Takagi-Sugeno FLC for a Mobile Robot Navigation

4 Takagi-Sugeno FLC

4.1 The Structure of a Fuzzy Controller

Conventional controllers are derived from control theory techniques based on mathematical models of the open-loop process, called system, to be controlled. Fuzzy control provides a formal methodology for representing, manipulating and implementing human's heuristic knowledge about how to control a system. Fuzzy logic control is the result of converting the linguistic control strategy based on expert knowledge into control rules and of combining fuzzy logic theory with inference processes. This fuzzy logic control is very useful when the needed models are not known or when they are too complex for analysis with conventional quantitative techniques. In fuzzy logic controller the dynamic behavior of a fuzzy system is characterized by a set of linguistic description rules based on expert knowledge. The expert knowledge is usually of the form

IF (*a set of conditions are satisfied*)
THEN (*a set of consequences can be inferred*).

Because the antecedents and the conditions are associated with fuzzy concepts such a rule is called fuzzy conditional statement. A fuzzy control rule is a fuzzy conditional statement in which the antecedent is a condition in its application domain and the consequent is a control action for the system under control. The task of a FLC system is to find a crisp control action from the fuzzy rule-base and from the actual crisp inputs. Because the inputs and the outputs of fuzzy rule-based systems are fuzzy sets, we have to fuzzify the crisp inputs and to defuzzify the fuzzy outputs. A standard FLC system [21] consists from four parts, as it results from the Fig. 4.

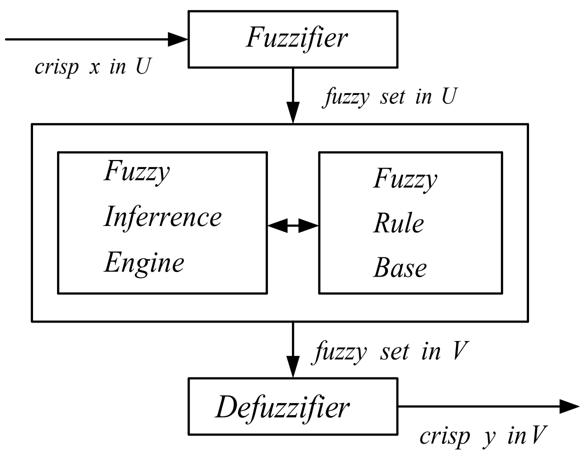


Fig. 4: Fuzzy Logic Controller

A fuzzification operator transforms crisp data into fuzzy sets. For instance, the crisp data x_0 is fuzzified into $\bar{x}_0$ (according to Fig. 5).

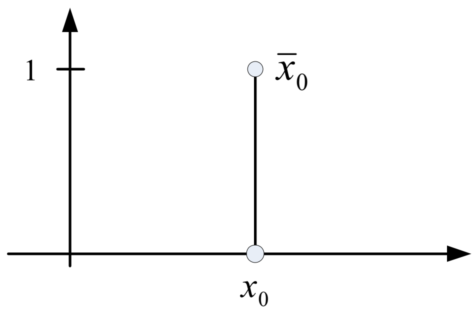


Fig. 5: Fuzzy singleton as fuzzifier

The procedure used by Fuzzy Inference Engine in order to obtain a fuzzy output consists of the following steps:

1) find the firing level of each rule,
2) find the output of each rule,
3)aggregate the individual rules outputs, corresponding to an implication, in order to obtain the overall output of this implication,
4) combining the previous values to obtain the overall action of control system.

The fuzzy control action C inferred from the fuzzy control system is transformed into a crisp control action:

$$z_0 = defuzzifier(C)$$

where *defuzzifier* is a defuzzification operator. The most used defuzzification operators, for a discrete fuzzy set C having the universe of discourse V are:

● Center-of-Gravity:

$$z_0 = \frac{\sum_{j=1}^{N} z_j \mu_C(z_j)}{\sum_{j=1}^{N} \mu_C(z_j)}$$

● Middle-of-Maxima: the defuzzified value is defined as mean of all values of the universe of discourse, having maximal membership grades

$$z_0 = \frac{\sum_{j=1}^{N} z_j}{N}$$

● Max-Criterion: this method chooses an arbitrary value, from the set of maximizing elements of

$$z_0 \in \left\{ z / \mu_C(z) = \max_{v \in V} \mu_C(v) \right\}$$

where where μ_C is the membership function of C and $Z = \{z_1, \cdots, z_N\}$ is a set of elements from the universe V.
The most known FLC systems are: Mamdani, Tsukamoto, Sugeno and Larsen which work with crisp data as inputs. An extension of the Mamdani model in order to work with interval inputs is presented in [22], where the fuzzy sets are represented by triangular fuzzy numbers and the firing level of the conclusion is computed as the product of firing levels from the antecedent. In [23] and [24] this model is extended in order to work simultaneously with crisp data, intervals and linguistic values as input. In this paper we extend the standard Takagi-Sugeno controller to work with the same data and inputs and propose to use this in a problem concerning the navigation of a mobile robot.

4.2 The Proposed Model

The Takagi-Sugeno FLC consists of a set of rules defined in the following form:

$R_{(A_1, A_2, \cdots, A_m)}$:
If x_1 *is* A_1 *and* x_2 *is* A_2 *and* $\cdots$ *and* x_m *is* A_m
then $y = a_0 + a_1 x_1 + \cdots + a_m x_m$

where x_i is an input, A_j is a linguistic label characterized by its membership function $\mu_{A_i}, i \in \{1, 2, \cdots, m\}$ and y is the output of the fuzzy rule. The firing level of each fuzzy rule is:

$$\alpha = \mu_{A_1}(x_1) \wedge \cdots \wedge \mu_{A_m}(x_m),$$

where $\wedge$ is, usually, the *min* operator. The output of the fuzzy system is the weighted average of the outputs of all fuzzy rules.
Usually, controllers applied in robotics use asymmetrical triangular and trapezoidal membership functions which allows a fast computation, essential under real-time conditions [25].
Membership functions represented by triangular fuzzy numbers have been selected for the considered computation. Such numbers $N = (m, \alpha, \beta)$ are defined by

$$\mu_N(x) = \begin{cases} 0, & for\ x < m - \alpha \\ 1 - \dfrac{m - x}{\alpha}, & for\ x \in [m - \alpha,\ m] \\ 1 - \dfrac{x - m}{\beta}, & for\ x \in [m\ \ m + \beta] \\ 0, & for\ x > m + \beta \end{cases}$$

Definition 1 *A function* $T : [0,1]^2 \to [0,1]$ *is a t-norm iff it is commutative, associative, nondecreasing and* $T(x,1) = 1,\ \forall x \in [0,1]$.

T-norms are used to compute the firing levels of the rules.

Definition 2 *An OWA operator of dimension* n *is a mapping* $F : R^n \to R$ *that has an associated* n *vector* $w = (w_1, \cdots, w_n)^t$ *such as*

$$w_i \in [0,1]\ , 1 \le i \le n, \sum_{i=1}^{n} w_i = 1$$

The aggregation operator of the values $\{a_1, \cdots, a_n\}$

$$F(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} w_j b_j$$

where b_j *is the* j*-th largest element from* $\{a_1, \cdots, a_n\}$.

4.3 Fuzzy inference

The linguistic description of the robot sensorial area is made of some linguistic variable [26]. Indeed, a rule in the proposed FLC is characterized by a set of linguistic variables A having as domain the interval $I_A = [a_A, b_A]$(as in [24]). Each linguistic variable has:

- n_A linguistic values $A_1, A_2, \cdots, A_{n_A}$ for each linguistic variable A
- membership function for each value A_i is $\mu^0_{A_i}(x)$, where $i \in \{1, 2, \cdots, n_A\}$ and $x \in I_A$.

The following steps (A, B, C and D) are necessary to work with the system.

A. *Compute the firing level for each linguistic value*

The firing level of the linguistic value A_i generated by a crisp value x_0 is computed as

$$\mu_{A_i} = \mu^0_{A_i}(x_0),\ i \in \{1, 2, \cdots, n_A\}.$$

If the input value for the variable x_i is an interval $[a, b]$ with $a_A \le a < b \le b_A$ then the intersection with the linguistic value A_i is

$$\mu_{A_i}(x) = min\left(\mu^0_{A_i}(x), \mu_{[a,b]}(x)\right), \forall x \in I_A \quad (4)$$

where

$$\mu_{[a,b]}(x) = \begin{cases} 1 & if \ \ x \in [a,b] \\ 0 & otherwise \end{cases}$$

The firing level, generated by the interval input $[a,b]$ corresponding to the linguistic value A_i is computed as the ratio,

$$\mu_{A_i} = \frac{\int_a^b \mu_{A_i}(x)dx}{\int_a^b \mu_{A_i}^0(x)dx}, \quad i \in \{1, 2, \cdots, n_A\} \quad (5)$$

which is the area defined by μ_{A_i} divided by area defined by $\mu_{A_i}^0$.

If the input value for the variable x_i is a fuzzy set F then the firing level corresponding to the linguistic value is computed using the formulas (4)+(5), where the membership $\mu_F(x)$ is used instead of $\mu_{[a,b]}(x)$.

B. *Compute the firing level of a rule*

For a rule $R_{(A_1, A_2, \cdots, A_m)}$ defined as in subsection 4.2 the firing level α is computed by

$$\alpha = T(\alpha_1, \cdots, \alpha_m)$$

where T is a t-norm and α_j is the firing level for A_j, $j \in \{1, 2, \cdots, m\}$ generated by the corresponding data input.

C. *Compute the output of a rule*

The output of a rule in a "zero order" system is a constant number. For a first order inference system the output of a system rule is computed as

$$z = a_0 + a_1\overline{x}_1 + \cdots + a_m\overline{x}_m$$

where $\overline{x}_j$ is obtained from the observation x_j *is* $\overline{A}_j$ in the following manner:

$i)$ if $\overline{A}_j$ is a crisp data then $\overline{x}_j = \overline{A}_j$

$ii)$ if $\overline{A}_j$ is an interval or a linguistic value then $\overline{x}_j$ is derived from the firing level α_j. Considering u_j and v_j as two numbers with the property

$$\mu_{A_j}(u_j) = \mu_{A_j}(v_j) = \alpha_j$$

then $\overline{x}_j = x_0$, where x^0 is computed as following:

$ii_1)$ if $\overline{A}_j = I_j = [a_j, b_j]$

and $X_{G_j} = \frac{a_j + b_j}{2}$

then $x^0 \in \{u_j, v_j\}$ and has the property

$$\left|x^0 - X_{G_j}\right| = min\left\{\left|u_j - X_{G_j}\right|, \left|v_j - X_{G_j}\right|\right\} \quad (6)$$

$ii_2)$ if $\overline{A}_j$ is a linguistic value one consider the gravity center of coordinates (X_{G_j}, Y_{G_j}) of the area between the membership functions μ_{A_i} and $\mu_{\overline{A}_j}$; in this case we compute x_0 using Eq. 6.

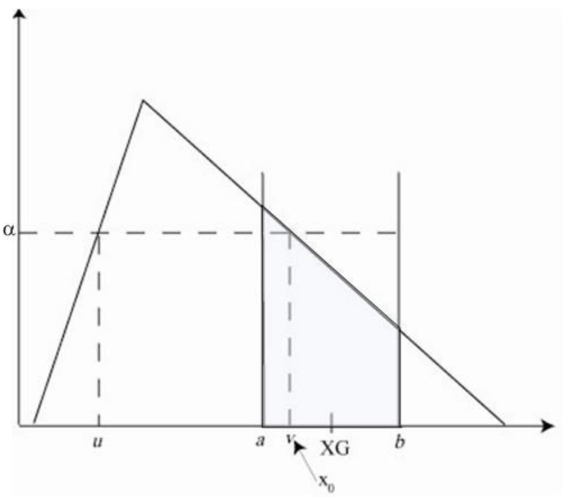


Fig. 6: x_0 computed using $ii_1)$ case

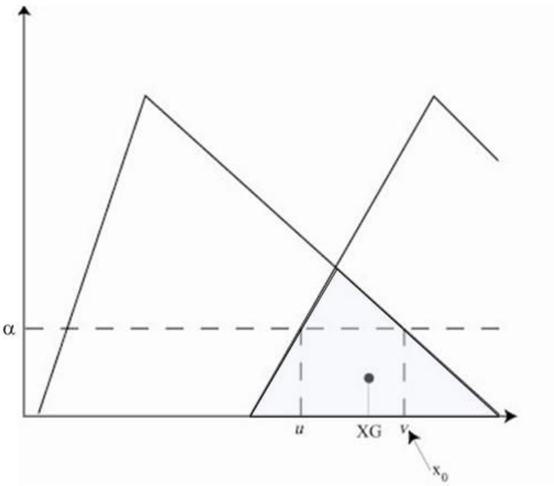


Fig. 7: x_0 computed using $ii_2)$ case

D. *Compute the output of the system*

The number of the fuzzy rules in the Takagi-Sugeno FLC are denoted by n. For any i-th rule of the system, $R_{(A_1, A_2, \cdots, A_m)}$ with the firing level α_i computed with a certain t-norm, one obtain an output z_i, $i \in \{1, 2, \cdots, n\}$. The output of the system corresponding to a t-norm is:

$$z_0 = \frac{\sum_{i=1}^{n} \alpha_i z_i}{\sum_{i=1}^{n} \alpha_i}.$$

The value

$$\alpha^0 = \frac{1}{n} \sum_{i=1}^{n} \alpha_i$$

will be named *the firing level of the system* corresponding to a t-norm. For a set of r t-norms $\{T_1, \cdots, T_r\}$, the system gives the set of outputs $\{z_j^0\}$ corresponding to the firing levels $\{\alpha_j^0\}$, $j \in \{1, 2, \cdots, r\}$. The overall output of the system is computed using the OWA operator with

$$w_j = \frac{\alpha_j^0}{\sum_{i=1}^{r} \alpha_i^0}, \; j \in \{1, 2, \cdots, r\}.$$

4.4 A case study

In order to show how the proposed system works, we consider a system having rules with two inputs and one output. The input variables are *distance* (d) and *direction angle* (θ); the output variable is *repulsive angle* (θ_R).

The values of input variable d are defined with three membership functions (Fig. 4 (a)):

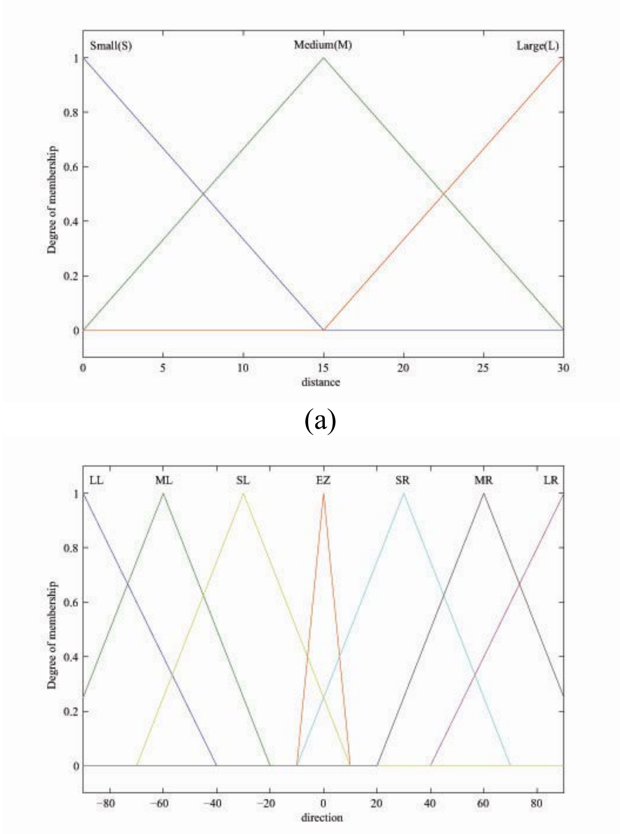
Small (*S*), *Medium* (*M*), *Large* (*L*),

$$S = (0, 0, 15),\; M(15, 15, 15),\; L = (30, 15, 0).$$

Corresponding values for the *direction angle* θ, are represented using seven membership function distributions (Fig. 4 (b)):

Large Left (*LL*), *Medium Left* (*ML*), *Small Left* (*SL*), *Straight* (*EZ*), *Small Right* (*SR*), *Medium Right* (*MR*), *Large Right* (*LR*),

$$LL = (-90, 0, 50),\; ML = (-60, 40, 40),$$
$$SL = (-30, 40, 40),\; EZ = (0, 10, 10),$$
$$SR = (30, 40, 40),\; MR = (60, 40, 40),$$
$$LR = (90, 40, 0)$$



(a)

(b)

Fig. 4: Membership functions for the input variables, (a) *distance*, (b) *direction angle*

The membership functions (having values very close to the ones used in [15]) of the output variable θ_R are given in Fig. 5.

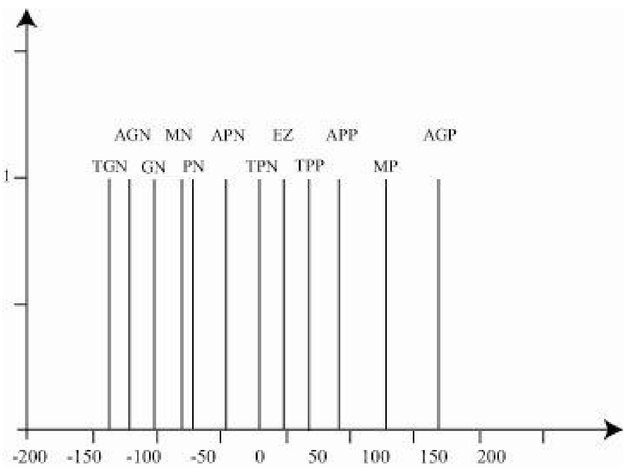


Fig. 5 Membership functions for the output variable

The rules set of the proposed Takagi-Sugeno FLC are those given below

	LL	ML	SL	EZ	SR	MR	LR
S	APP	MP	AGP	TGN	AGN	MN	APN
M	TPP	APP	MP	GN	MN	APN	TPN
L	EZ	TPP	APP	PN	APN	TPN	EZ

Table 1 The rules of the FLC for determining the *repulsive angle*

The crisp values corresponding to the notations used in Table 1 are the following:

$APP = 47$, $ML = 83$, $AGP = 123$, $TPP = 2$, $EZ = 0$, $TGN = -133$, $AGN = -120$, $MN = -80$, $APN = -43$, $GN = -100$, $TPN = -17$, $PN = -67$.

For a particular case in which the variable *distance* has as value the interval [11, 13] and the variable *direction angle* is given by the *ML* linguistic value, six rules are fired:

$R_{(S, LL)}$: IF d is S and θ is LL THEN θ_R is APP

$R_{(S, ML)}$: IF d is S and θ is ML THEN θ_R is MP

$R_{(S, SL)}$: IF d is S and θ is SL THEN θ_R is AGP

$R_{(M, LL)}$: IF d is M and θ is LL THEN θ_R is TPP

$R_{(M, ML)}$: IF d is M and θ is ML THEN θ_R is APP

$R_{(M, SL)}$: IF d is M and θ is SL THEN θ_R is MP

Consider the following t-norms denoted by T_1, T_2, T_3, T_4:

(Product) $T_1 = T_P(x, y) = xy$
(Minim) $T_2 = T_M(x, y) = \min(x, y)$
(Lukasiewicz) $T_3 = T_L(x, y) = \max(0, x + y - 1)$
(Dubois-Prade) $T_4 = T_{DP}(x, y) = \dfrac{xy}{\max(x, y, 0.5)}$.

The following outputs for each of the considered t-norms are:

$$z_1^0 = 56, z_2^0 = 62.11, z_3^0 = 58.96, z_4^0 = 59.4$$

The corresponding firing levels of the system are

$$\alpha_1^0 = 0.0163, \alpha_2^0 = 0.0228, \alpha_3^0 = 0.0076, \alpha_4^0 = 0.0212$$

where α_i^0 is the firing level generated by t-norm T_i. The overall output of the system computed using the OWA operator with the associated vector $w = (0.24, 0.34, 0.11, 0.31)^t$ is

$$z^0 = 59.97 .$$

From the considered case study it was observed an important difference between the four results given by the used t-norms. It is obviously that different t-norms will give different results if they will be used separately. The proposed method offers a possibility to avoid this drawback because the aggregation operation achieves 'mediation' between the results given by various t-norms.

4 Conclusion and Future Works

This paper presents a fuzzy reasoning method of Takagi-Sugeno type controller applied in two wheels autonomous robot navigation. Because it can work not only with crisp data as input but also with intervals and/or linguistic terms, its area of applications is very large. In order to obtain more accurate results for the proposed system different t-norms are used in the aggregation of results process. The method can be improved by using various matching techniques in order to compute the firing levels of the linguistic values from the premises. Further theoretical and experimental work is needed to validate the proposed model.

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