A survey on the application of the Elliptical Trigonometry in industrial electronic systems using controlled waveforms with modeling and simulating of two functions *Elliptic Mar* and *Elliptic Jes-x*

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Abstract: - In industrial electronic systems, power converters with power components are used. Each controlled component has its own control circuit. In this paper, the author proposes an original control circuit for each function in order to replace the different existing circuits. The proposed circuit is the representation of an elliptical trigonometric function as "*Elliptic Mar*" and "*Elliptic Jes-x*" that are particular cases of the elliptical trigonometry. Thus, with one function, by varying the values of its parameters, the output waveform will change and can describe more than 12 different waveforms. Finally for each function, a block diagram, a model of the circuit and a programming part are treated using Matlab/Simulink. The results of the studied circuit are presented and discussed.

Key-words: - Power electronics, power converters, mathematics, trigonometry, multi form signal.

1 Introduction

In motor drives, robotics, or other industrial electronic applications, the use of power converters is essential to improve the control and, therefore, the efficiency of the studied system [17],[18]. Power converters and power electronics circuits are generally composed of power components with different characteristics [17]. These components are divided in two categories: the controlled components and the uncontrolled components [17], [19],[20]. The controlled components that are based on semi-conductors like thyristors, Triac and transistors need controlled signals with specified waveforms in order to be applied on their controlled terminals (base or gate) [17],[19]. Thus, each component has its own control source. This paper underlines the importance of the elliptical trigonometric functions in generating different waveforms by varying some parameters of a single function. In a particular case, the *Elliptic Mar* and the *Elliptic Jes-x* functions are chosen to be treated.

In fact, the elliptical trigonometry is an original study introduced with new concepts [1],[2]. The existed trigonometry (*Circular trigonometry*) is a particular case of the elliptical trigonometry [6],[7]. The traditional trigonometry has an enormous variety of applications in all scientific domains [6],[7],[8],[9]. It can be considered as the basis and

foundation of many domains as electronics, signal theory, astronomy, navigation, propagation of signals and many others… [10],[11],[12],[13]. Particularly, the mathematical topics of Fourier series and Fourier transforms rely heavily on knowledge of trigonometric functions [10],[11] and find application in a number of areas, including statistics [12],[13].

In this paper, the new concept of the elliptical trigonometry is introduced and few examples are shown and discussed briefly. Figures and results are drawn and simulated using Matlab/Simulink and AutoCAD. A survey on the applications of this trigonometry in the power electronics domains presented in section 2. In the third section, the angular functions are defined, these functions have enormous applications in all domains, and it can be considered as the basis of this trigonometry [1],[2]. The definition of the Elliptical trigonometry is presented and discussed briefly in section 4. In the fifth section, a survey on the Elliptical Trigonometric functions is discussed and two principal functions are presented. In the sections 6 and 7, two functions are studied and discussed briefly with simulation on Simulink/Matlab, their block diagrams are presented, their programming parts and so their modeling circuits. Finally, a conclusion about the elliptical trigonometry is presented in the section 8.

2 A survey on the application of the elliptical trigonometry in engineering domain

The used controlled circuits for power transistors (MOSFET, IGBT, etc) differ from those used for Thyristors (GTO, Triac, etc). Designing and modeling circuits for all these controlled components taking into account their different characteristics, take time, and realizing them practically, take time and money. In this paper, for each elliptical trigonometric function, one electronic circuit is proposed to be used in simulation (Labview, Matlab, Simulink etc). For a particular case, in order to control the different existed power components for the function *Elliptic Mar* (figure 1), two parameters $'b'$ and $'i'$ are used as variable inputs. The output will be the studied elliptical trigonometry function.

Thus, the main goal of the Elliptical Trigonometry is to produce a huge number of multi form signals using a single function and by varying some parameters of this function. For a particular case, more than 12 different output signals can be obtained by varying two parameters of the Elliptic *Mar* function.
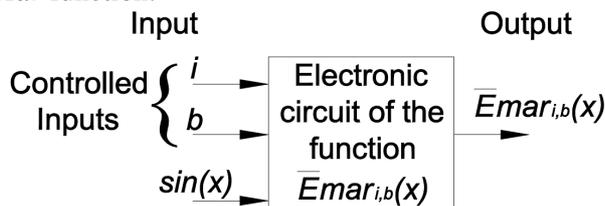
Fig. 1: Electronic circuit of the function $\bar{E}mar_{i,b}(x)$ with its inputs and output.

3 The angular functions

In order to make a review on the elliptical trigonometry, it is necessary to introduce the definition of the angular functions. In fact, angular functions are new mathematical functions that produce a rectangular signal, in which period is function of angles. Similar to trigonometric functions, the angular functions have the same properties as the precedent, but the difference is that a rectangular signal is obtained instead of a sinusoidal signal [14],[15],[16] and moreover, one can change the width of each positive and negative alternate in the same period. This is not the case of any other trigonometric function. In other hand, one can change the frequency, the amplitude and the width of any period of the signal by using the general form of the angular function.

In this section three types of angular functions are presented, they are used in this trigonometry; of course there are more than three types, but in this paper the study is limited to three functions.

3.1 Angular function $ang_x(x)$

The expression of the angular function related to the (ox) axis is defined, for $K \in \mathbb{Z}$, as:

$ang_x(\beta(x + \gamma)) =$
$$\begin{cases} +1 \ for \ (4K - 1)\frac{\pi}{2\beta} - \gamma \le x \le (4K + 1)\frac{\pi}{2\beta} - \gamma \\ -1 \ for \ (4K + 1)\frac{\pi}{2\beta} - \gamma < x < (4K + 3)\frac{\pi}{2\beta} - \gamma \end{cases} \quad (1)$$
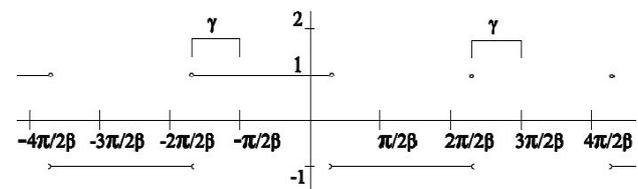
Fig. 2: The $ang_x(\beta(x + \gamma))$ waveform.

For $\beta = 1$ and $\gamma = 0$, the expression of the angular function becomes:

$$ang_x(x) = \begin{cases} +1 \ for \ \cos(x) \ge 0 \\ -1 \ for \ \cos(x) < 0 \end{cases}$$

3.2 Angular function $ang_y(x)$

The expression of the angular function related to the (oy) axis is defined, for $K \in \mathbb{Z}$, as:

$ang_y(\beta(x + \gamma)) =$
$$\begin{cases} +1 \ for \ 2K\pi/\beta - \gamma \le x \le (2K + 1)\pi/\beta - \gamma \\ -1 \ for \ (2K + 1)\pi/\beta - \gamma < x < (2K + 2)\pi/\beta - \gamma \end{cases}$$
$$(2)$$
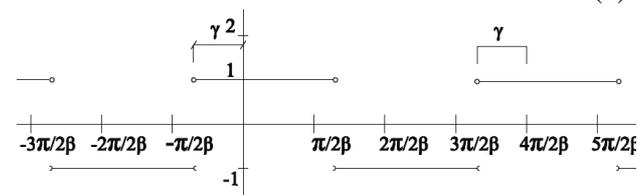
Fig. 3: The $ang_y(\beta(x + \gamma))$ waveform.

For $\beta = 1$ and $\gamma = 0$, the expression of the angular function becomes:

$$ang_y(x) = \begin{cases} +1 \ for \ \sin(x) \ge 0 \\ -1 \ for \ \sin(x) < 0 \end{cases}$$

3.3 Angular function $ang_\alpha(x)$

α (called firing angle) represents the angle width of the positive part of the function in a period. In this case, we can vary the width of the positive and the

negative part by varying only α. The firing angle must be positive.

$$ang_\alpha\big(\beta(x+\gamma)\big) =$$
$$\begin{cases} +1 \; for \\ (2K\pi - \alpha)/\beta - \gamma \le x \le (2K\pi + \alpha)/\beta - \gamma \\ \\ -1 \; for \\ (2K\pi + \alpha)/\beta - \gamma < x < (2(K+1)\pi - \alpha)/\beta - \gamma \end{cases}$$
$$(3)$$
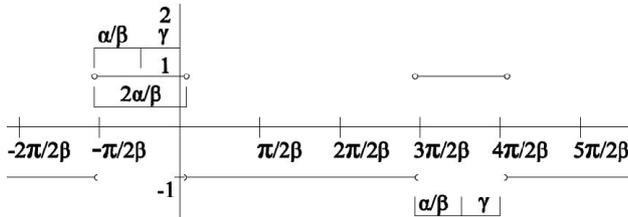
Fig. 4: The $ang_\alpha(\beta(x+\gamma))$ waveform.

4 Definition of the Elliptical Trigonometry

In order to study the proposed circuits, their block diagrams and their simulations, it is necessary to introduce the mathematical definition of the elliptical trigonometry and its functions.

4.1 The Elliptical Trigonometry unit

The Elliptical Trigonometry unit is an ellipse with a center O (x = 0, y = 0) and has the equation form:
$$(x/a)^2 + (y/b)^2 = 1 \qquad (4)$$
With:
'a' is the radius of the ellipse on the (x'ox) axis,
'b' is the radius of the ellipse on the (y'oy) axis.
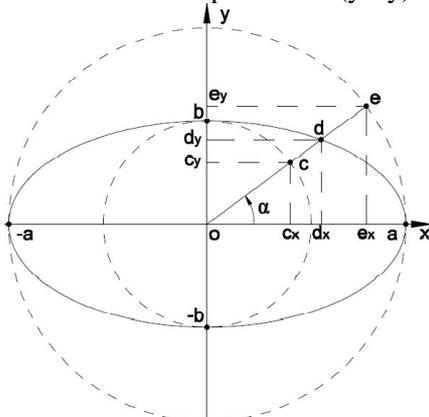
Fig. 5: The elliptical trigonometry unit.

It is essential to note that 'a' and 'b' must be positive. In this paper, 'a' is fixed to 1. One is interested to vary only a single parameter which is 'b'.

4.2 Intersections and projections of different elements of the Elliptical Trigonometry on the relative axes

From the intersections of the ellipse with the positive parts of the axes (ox) and (oy), define respectively two circles of radii $[oa]$ and $[ob]$. These radii can be variable or constant according to the form of the ellipse.

The points of the intersection of the half-line $[od)$ (figure 5) with the internal and external circles and with the ellipse and their projections on the axes (ox) and (oy) can be described by many functions that have an extremely importance in creating plenty of signals and forms that are very difficult to be created in the traditional trigonometry.

Definition of the letters in the Figure 5:
a: Is the intersection of the ellipse with the positive part of the axe (ox) that gives the relative circle of radius "a". It can be variable.
b: Is the intersection of the ellipse with the positive part of the axe (oy) that gives the relative circle of radius "b". It can be variable.
c: Is the intersection of the half-line $[od)$ with the circle of radius b.
d: Is the intersection of the half-line $[od)$ with the ellipse.
e: Is the intersection of the half-line $[od)$ with the circle of radius a.
c_x: Is the projection of the point c on the ox axis.
d_x: Is the projection of the point d on the ox axis.
e_x: Is the projection of the point e on the ox axis.
c_y: Is the projection of the point c on the oy axis.
d_y: Is the projection of the point d on the oy axis.
e_y: Is the projection of the point e on the oy axis.
α: Is the angle between the (ox) axis and the half-line $[od)$.
o: Is the center (0, 0).

4.3 Definition of the Elliptical Trigonometric functions $Efun(\alpha)$

The traditional trigonometry contains only 6 principal functions: Cosine, Sine, Tangent, Cosec, Sec, Cotan. [15],[16]. But in the Elliptical Trigonometry, there are 32 principal functions and each function has its own characteristics. These

functions give a new vision of the world and will be used in all scientific domains and make a new challenge in the reconstruction of the science especially when working on the economical side of the power of electrical circuits, the electrical transmission, the signal theory and many other domains [15],[17].

The functions $Cjes(\alpha), Cmar(\alpha), Cter(\alpha)$ and $Cjes_y(\alpha)$, which are respectively equivalent to cosine, sine, tangent and cotangent. These functions are particular cases of the "*Circular Trigonometry*". The names of the cosine, sine, tangent and cotangent are replaced respectively by *Circular Jes*, *Circular Mar, Circular Ter* and *Circular Jes-y*.

$Cjes(\alpha) \Leftrightarrow cos(\alpha);$　　$Cmar(\alpha) \Leftrightarrow sin(\alpha)$
$Cter(\alpha) \Leftrightarrow tan(\alpha);$　　$Cjes_y(\alpha) \Leftrightarrow cotan(\alpha).$

The Elliptical Trigonometric functions are denoted using the following abbreviation "$Efun(\alpha)$":
-the first letter "E" is related to the Elliptical trigonometry.
-the word "$fun(\alpha)$" represents the specific function name that is defined hereafter: (refer to Figure 5).

• **Elliptical *Jes* functions:**

El. Jes: $Ejes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe}$　　　　　　(5)

El. Jes-x: $Ejes_x(\alpha) = \frac{od_x}{oe_x} = \frac{Ejes(\alpha)}{Cjes(\alpha)}$　　　(6)

El. Jes-y: $Ejes_y(\alpha) = \frac{od_x}{oe_y} = \frac{Ejes(\alpha)}{Cmar(\alpha)}$　　(7)

• **Elliptical *Mar* functions:**

El. Mar: $Emar(\alpha) = \frac{od_y}{ob} = \frac{od_y}{oc}$　　　　　(8)

El. Mar-x: $Emar_x(\alpha) = \frac{od_y}{oc_x} = \frac{Emar(\alpha)}{Cjes(\alpha)}$　　(9)

El. Mar-y: $Emar_y(\alpha) = \frac{od_y}{oc_y} = \frac{Emar(\alpha)}{Cmar(\alpha)}$　(10)

• **Elliptical *Ter* functions:**

El. Ter: $Eter(\alpha) = \frac{Emar(\alpha)}{Ejes(\alpha)}$　　　　　(11)

El. Ter-x:
$Eter_x(\alpha) = \frac{Emar_x(\alpha)}{Ejes_y(\alpha)} = Eter(\alpha) \cdot Cter(\alpha)$　(12)

El. Ter-y: $Eter_y(\alpha) = \frac{Emar_y(\alpha)}{Ejes_x(\alpha)} = \frac{Eter(\alpha)}{Cter(\alpha)}$　(13)

• **Elliptical *Rit* functions:**

El. Rit: $Erit(\alpha) = \frac{od_x}{ob} = \frac{od_x}{oc} = \frac{Emar(\alpha)}{Cter(\alpha)}$　(14)

El. Rit-y: $Erit_y(\alpha) = \frac{od_x}{oc_y} = \frac{Erit(\alpha)}{Cmar(\alpha)}$　　(15)

• **Elliptical *Raf* functions:**

El. Raf: $Eraf(\alpha) = \frac{od_y}{oa} = Cter(\alpha).Ejes(\alpha)$　(16)

El. Raf-x: $Eraf_x(\alpha) = \frac{od_y}{oe_x} = \frac{Eraf(\alpha)}{Cjes(\alpha)}$　　(17)

• **Elliptical *Ber* functions:**

El. Ber: $Eber(\alpha) = \frac{Eraf(\alpha)}{Erit(\alpha)}$　　　　　(18)

El. Ber-x:
$Eber_x(\alpha) = \frac{Eraf_x(\alpha)}{Erit_y(\alpha)} = Eber(\alpha) \cdot Cter(\alpha)$　(19)

El. Ber-y: $Eber_y(\alpha) = \frac{Eraf_y(\alpha)}{Erit_x(\alpha)} = \frac{Eber(\alpha)}{Cter(\alpha)}$　(20)

4.4 The reciprocal of the Elliptical Trigonometric function

$Efun^{-1}(\alpha)$ is defined as the inverse function of $Efun(\alpha)$. $(Efun^{-1}(\alpha) = 1/Efun(\alpha))$. In this way the reduced number of functions is equal to 32 principal functions.

E.g.: $Ejes^{-1}(\alpha) = \frac{1}{Ejes(\alpha)}$

4.5 Definition of the Absolute Elliptical Trigonometric functions $\overline{E}fun(\alpha)$

The *Absolute Elliptical Trigonometry* is introduced to create the absolute value of a function by varying only one parameter without using the absolute value "| |". The advantage is that one can change and control the sign of an Elliptical Trigonometric function without using the absolute value in an expression. Some functions are treated to get an idea about the importance of this new definition. To obtain the Absolute Elliptical Trigonometry for a specified function (e.g.: $Ejes(\alpha)$) we must multiply it by the corresponding Angular Function (e.g.: $\left(ang_x(\alpha)\right)^i$ with $i \in \mathbb{N}$) in a way to obtain the original function if i is even, and to obtain the absolute value of the function if i is odd (e.g.: $|Ejes(\alpha)|$).

If the function doesn't have a negative part (not alternative) it will be multiplied by $\left(ang_x(\beta(\alpha - \gamma))\right)^i$ to obtain an alternating signal which form depends on the value of the frequency "β" and the translation value " γ ". By varying the last parameters, one can get a multi form signals.

• $\bar{E}jes_i(\alpha) = \left(ang_x(\alpha)\right)^i \cdot Ejes(\alpha)$ (21)

$= \begin{cases} \left(ang_x(\alpha)\right)^1 \cdot Ejes(\alpha) = |Ejes(\alpha)| & if\ i = 1 \\ \left(ang_x(\alpha)\right)^2 \cdot Ejes(\alpha) = Ejes(\alpha) & if\ i = 2 \end{cases}$

• $\bar{E}jes_{i,x}(\alpha) = \left(ang_x(\alpha - \gamma)\right)^i \cdot Ejes_x(\alpha)$ (22)

$= \begin{cases} ang_x(\alpha - \gamma) \cdot Ejes_x(\alpha) & if\ i = 1 \\ Ejes_x(\alpha) & if\ i = 2 \end{cases}$

• $\bar{E}jes_{i,y}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Ejes_y(\alpha)$ (23)

$= \begin{cases} ang_y(2\alpha) \cdot Ejes_y(\alpha) = |Ejes_y(\alpha)| & if\ i = 1 \\ Ejes_y(\alpha) & if\ i = 2 \end{cases}$

• $\bar{E}mar_i(\alpha) = \left(ang_y(\alpha)\right)^i \cdot Emar(\alpha)$ (24)

• $\bar{E}mar_{i,x}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Emar_x(\alpha)$ (25)

• $\bar{E}mar_{i,y}(\alpha) = \left(ang_x(\alpha - \gamma)\right)^i \cdot Emar_y(\alpha)$ (26)

• $\bar{E}rit_i(\alpha) = \left(ang_x(\alpha)\right)^i \cdot Erit(\alpha)$ (27)

And so on…

5 A survey on the Elliptical Trigonometric functions

As previous sections, a brief study on the Elliptical Trigonometry is given. Two functions of 32 are treated; the others functions can be easily interpreted using formulae from (5) to (20).

Elliptic cosine and Elliptic sine that appear in the previous articles [1] and [2], are particular cases of the Elliptic *Jes* and Elliptic *Mar* respectively.

For this study the following conditions are taken:
- $a = 1$
- $b > 0$ the radius of the ellipse on the $y'oy$ axis.
- $i \in \mathbb{N}$

5.1 Determination of the Elliptic *Jes* function

The Elliptical form in the figure 5 is written as the equation (4). Thus, given (5), the Elliptic *Jes* function can be determined using the following method. In fact:

$Cter(\alpha) = \frac{y}{x} = \frac{oc_y}{oc_x}$, it is significant to replace the equation $y = Cter(\alpha).x$ in that defined in (4).

$\left(\frac{x}{a}\right)^2 + \left(Cter(\alpha)\frac{x}{b}\right)^2 = \left(\frac{x}{a}\right)^2 \left(1 + \left(Cter(\alpha)\frac{x}{b}\right)^2\right) = 1 \Rightarrow$

$Ejes_b(\alpha) = \frac{\pm 1}{\sqrt{1 + \left(\frac{a}{b}Cter(\alpha)\right)^2}}$

Therefore:

• $Ejes_b(\alpha) = \frac{+1}{\sqrt{1 + \left(\frac{a}{b}Cter(\alpha)\right)^2}}$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2};\ \frac{x}{a} \ge 0$

• $Ejes_b(\alpha) = \frac{-1}{\sqrt{1 + \left(\frac{a}{b}Cter(\alpha)\right)^2}}$ for $\frac{\pi}{2} < x < \frac{3\pi}{2};\ \frac{x}{a} < 0$

Thus, the expression of the Elliptic *Jes* can be unified by using the angular function expression (1), therefore the expression becomes:

$Ejes_b(\alpha) = \frac{ang_x(\alpha)}{\sqrt{1 + \left(\frac{a}{b}Cter(\alpha)\right)^2}} \Rightarrow$

$Ejes_b(x) = \frac{ang_x(x)}{\sqrt{1 + \left(\frac{a}{b}Cter(x)\right)^2}}$ (28)

• Expression of the Absolute Elliptic *Jes*:

$\bar{E}jes_{i,b}(x) = \frac{ang_x(x)}{\sqrt{1 + \left(\frac{a}{b}Cter(x)\right)^2}} \cdot \left(ang_x(x)\right)^i$ (29)

The Absolute Elliptic *Jes* is a powerful function that can produce more than 12 different signals by varying only two parameters i and b. Similar to the cosine function in the traditional trigonometry, the Absolute Elliptic *Jes* is more general than the precedent.

5.2 Determination of the Elliptic *Mar* function

The elliptical form in the figure 5 is written as the equation (4). Thus, given (8), the Elliptic *Mar* function can be determined using the following method. In fact:

$Cter(\alpha) = \frac{y}{x} = \frac{oc_y}{oc_x} \Rightarrow x = \frac{y}{Cter(\alpha)}$, it is significant to replace the equation $x = \frac{y}{Cter(\alpha)}$ in that defined in (4). Thus, $\left(\frac{y}{a \cdot Cter(\alpha)}\right)^2 + (y/b)^2 = 1 \Rightarrow$

$Emar_b(\alpha) = \frac{y}{b} = \frac{\pm \frac{a}{b}Cter(\alpha)}{\sqrt{1 + \left(\frac{a}{b}Cter(\alpha)\right)^2}}$

$\Rightarrow Emar_b(x) = \frac{y}{b} = \frac{\pm \frac{a}{b}Cter(x)}{\sqrt{1 + \left(\frac{a}{b}Cter(x)\right)^2}}$

Therefore:

• $Emar_b(x) = \frac{+\frac{a}{b}Cter(x)}{\sqrt{1 + \left(\frac{a}{b}Cter(x)\right)^2}}$ for $0 \le x < \frac{\pi}{2}$

- $Emar_b(x) = \dfrac{-\frac{a}{b}\text{Cter}(x)}{\sqrt{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}}$ for $\frac{\pi}{2} < x \le \pi$

- $Emar_b(x) = \dfrac{-\frac{a}{b}\text{Cter}(x)}{\sqrt{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}}$ for $\pi \le x < 3\frac{\pi}{2}$

- $Emar_b(x) = \dfrac{+\frac{a}{b}\text{Cter}(x)}{\sqrt{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}}$ for $3\frac{\pi}{2} \le x \le 2\pi$

Thus, the expression of the elliptic *Mar* can be unified by using the angular function expression (1), therefore the expression becomes:

$$Emar_b(x) = \frac{a}{b}\frac{\text{Cter}(x)ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}} \qquad (30)$$

- Expression of the Absolute Elliptic *Mar*:

$$\bar{E}mar_{i,b}(x) = Emar_b(x) \cdot \left(ang_y(x)\right)^i \qquad (31)$$

The Absolute Elliptic *Mar* is a powerful function that can produce more than 12 different signals by varying only two parameters i and b. Similar to the sine function in the traditional trigonometry, the Absolute Elliptic *Mar* is more general than the precedent.

5.3 Original formulae of the Elliptical Trigonometry

In this sub-section, a brief review on some remarkable formulae formed using the elliptical trigonometric functions.

- $\left(Ejes_b(x)\right)^2 + \left(Emar_b(x)\right)^2 = 1 \qquad (32)$

In fact: $\left(Ejes_b(x)\right)^2 + \left(Emar_b(x)\right)^2 =$

$$\left(\frac{ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}}\right)^2 + \left(\frac{a}{b}\frac{\text{Cter}(x)ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}}\right)^2 =$$

$$\frac{1}{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2} + \left(\frac{a}{b}\right)^2 \frac{\text{Cter}(x)^2}{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2} =$$

$$\frac{1+\left(\frac{a}{b}\right)^2\text{Cter}(x)^2}{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2} = \frac{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2} = 1$$

- $\dfrac{1}{Ejes_x^2(x)+Emar_x^2(x)} + \dfrac{1}{Ejes_y^2(x)+Emar_y^2(x)} = 1$ (33)

In fact:

$$Ejes_x^2(x) + Emar_x^2(x) = \left(\frac{Ejes(x)}{Cjes(x)}\right)^2 + \left(\frac{Emar(x)}{Cjes(x)}\right)^2$$

$$= \frac{1}{(Cjes(x))^2} \Rightarrow \frac{1}{Ejes_x^2(x)+Emar_x^2(x)} = (Cjes(x))^2$$

And

$$Ejes_y^2(x) + Emar_y^2(x) = \left(\frac{Ejes(x)}{Cmar(x)}\right)^2 + \left(\frac{Emar(x)}{Cmar(x)}\right)^2$$

$$= \frac{1}{(Cmar(x))^2} \Rightarrow \frac{1}{Ejes_y^2(x)+Emar_y^2(x)} = (Cmar(x))^2$$

Therefore: $\dfrac{1}{Ejes_x^2(x)+Emar_x^2(x)} + \dfrac{1}{Ejes_y^2(x)+Emar_y^2(x)} =$

$Cjes^2(x) + Cmar^2(x) = \cos^2(x) + \sin^2(x) = 1$

6 Studying the function $\bar{E}mar_{i,b}(x)$

In this section, the Absolute Elliptic *Mar*, which is defined in equation (31), is chosen to be treated. The main goal of this study is to model and simulate the function using Matlab and Simulink.

6.1 The block diagram of $\bar{E}mar_{i,b}(x)$

The block diagram of the function $\bar{E}mar_{i,b}(x)$ is illustrated in figure 6. There are three inputs connected to this diagram, two variable parameters "b" and "i", and one sinusoidal waveform (Circular *Mar* or "sine"). "a" is chosen to be a constant with $a = 1$, so it can't be considered as an input.
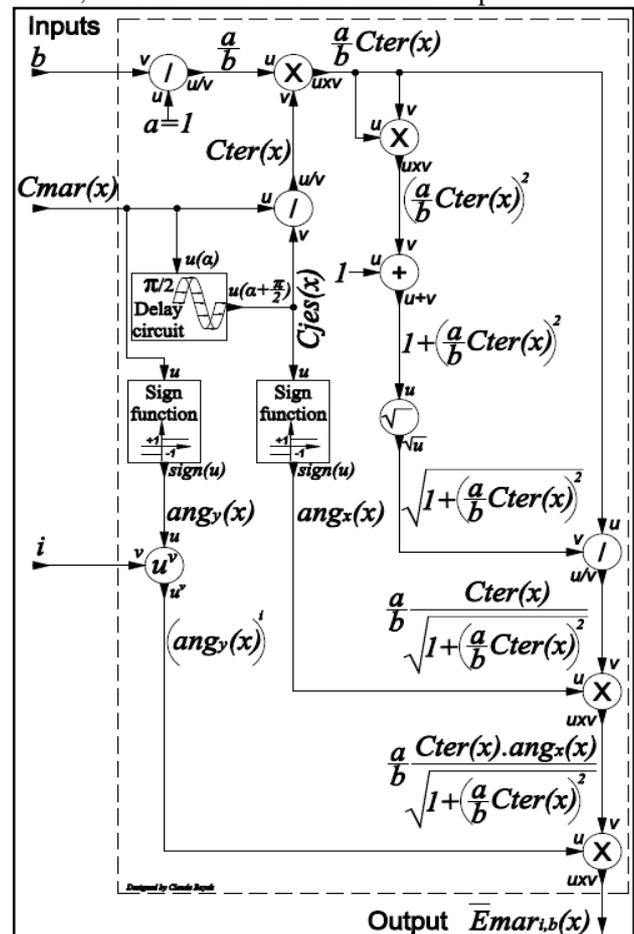
Fig. 6: Block diagram of the function $\bar{E}mar_{i,b}(x)$

6.2 Modeling and simulating the function $\bar{E}mar_{i,b}(x)$ using Matlab-Simulink

Consider $a = 1$, "b" and "i" are the controlled inputs, The Absolute Elliptic *Mar* function is obtained as an output. With three inputs, one can obtain a single output, which can produce more than 12 different signals.
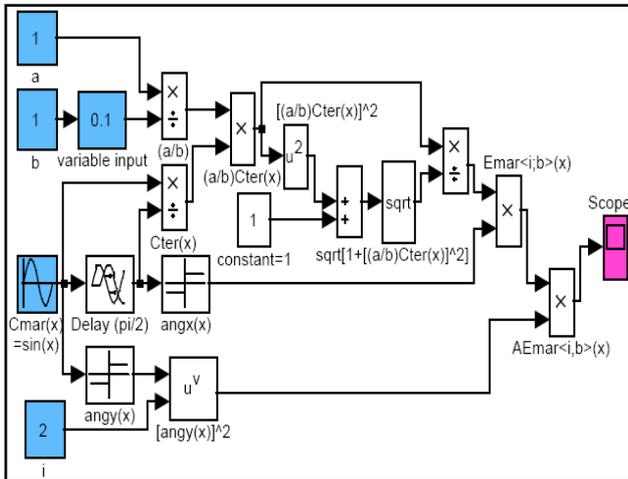
Fig. 7: the circuit model of the function $\bar{E}mar_{i,b}(x)$

6.2.1 First case $(b \ll 1)$

From equation (30), two configurations are studied:
1. Consider $x \neq k\pi$ and $b \ll a \cdot Cter(x)$.
In this case, $Cter(x) \neq 0$. As "b" is too small number but non zero, therefore:

$\left(\frac{a}{b}Cter(x)\right)^2 \gg 1$. Thus the equation (30) becomes:

$$Emar_b(x) = \frac{a}{b}\frac{Cter\,(x)ang_x(x)}{\sqrt{1+\left(\frac{a}{b}Cter\,(x)\right)^2}} \approx \frac{a}{b}\frac{Cter\,(x)ang_x(x)}{\sqrt{\left(\frac{a}{b}Cter\,(x)\right)^2}}$$

$$\approx \pm\frac{a}{b}\frac{Cter\,(x)ang_x(x)}{\frac{a}{b}Cter\,(x)} \approx \pm ang_x(x) = \pm 1$$

Therefore: $Emar_b(x) =$
$$\begin{cases} +1\ for & 2k\pi < x < (2k+1)\pi \\ -1\ for & (2k+1)\pi < x < (2k+2)\pi \end{cases}$$

2. Consider $x = k\pi$ and $b \gg a \cdot Cter(x)$.
In this case, $Cter(x) = 0$. As "b" is too small number but non zero, therefore:

$\left(\frac{a}{b}Cter(x)\right)^2 = 0$. Thus the equation (30) becomes:

$$Emar_b(x) = \frac{a}{b}\frac{Cter\,(x)ang_x(x)}{\sqrt{1+\left(\frac{a}{b}Cter\,(x)\right)^2}} = 0$$

Consequently: $Emar_b(x) =$

$$\begin{cases} 0\ for & x = k\pi \\ +1\ for & 2k\pi < x < (2k+1)\pi; x \neq k\pi \\ -1\ for & (2k+1)\pi < x < (2k+2)\pi; x \neq k\pi \end{cases}$$

Thus, a rectangular signal is obtained.

Figures 8.a and 8.b represent the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = 0.001 \ll 1$, (rectangular waveform for $i = 2$ and continuous signal for $i = 1$).
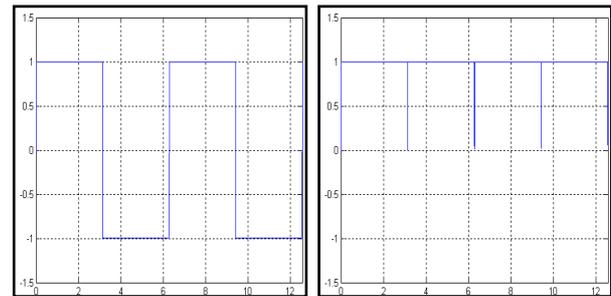
a) $i = 2; b = 0.001$ b) $i = 1; b = 0.001$
Fig. 8: the waveforms of the function $\bar{E}mar_{i,b}(x)$
for $b = 0.001 \ll 1$

6.2.2 Second case $(b < 1)$

In this case, an elliptic swollen form is obtained for $i = 2$ (figure 9.a), and a rectified elliptic swollen form is obtained for $i = 1$ (figure 9.b). The importance of this signal is by obtaining an average value greater than that of an absolute value of the sinusoidal signal. Hence, the average of the signal can be increased by varying only the value of one parameter "b".
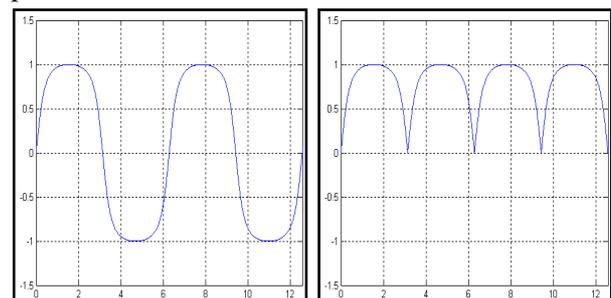
a) $i = 2; b = 0.4$ b) $i = 1; b = 0.4$
Fig. 9: the waveforms of the function $\bar{E}mar_{i,b}(x)$
for $b = 0.4 < 1$

6.2.3 Third case $(b = 1)$

When $a = b = 1$, the ellipse equation defined in (4) becomes:

$$(x)^2 + (y)^2 = 1 \tag{34}$$

This is the equation of a circle of radius $r = 1$.

• The Elliptic *Mar* function defined in (8) becomes:
$$Emar(\alpha) = \frac{od_y}{ob} = \frac{oc_y}{oc} = Cmar(\alpha) = \sin(\alpha)$$
• The Elliptic *Jes* function defined in (5) becomes:
$$Ejes(\alpha) = \frac{od_x}{oa} = \frac{oc_x}{oc} = Cjes(\alpha) = \cos(\alpha)$$
Therefore, $\bar{E}mar_{i,b}(x) = \sin(x) \cdot \left(ang_y(x)\right)^i$ (35)

Figures 10.a and 10.b represent the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = 1$, (sinusoidal signal for $i = 2$ and rectified sinusoidal signal for $i = 1$).
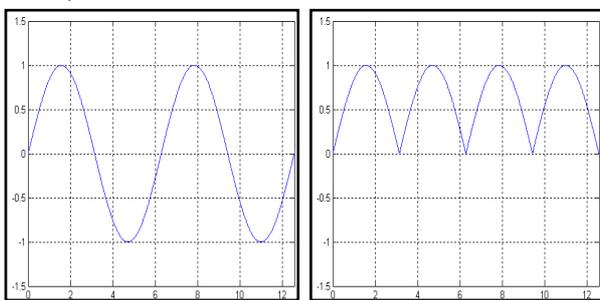
a) $i = 2; b = 1$ b) $i = 1; b = 1$

Fig. 10: the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = 1$

6.2.4 Fourth case ($b > 1$)

In this case, an elliptic deflated form is obtained for $i = 2$ (figure 11.a), and a rectified elliptic deflated form is obtained for $i = 1$ (figure 11.b). The importance of this signal is by obtaining an average value smaller than that of an absolute value of the sinusoidal signal. Hence, the average of the signal can be decreased by varying only the value of one parameter "b".
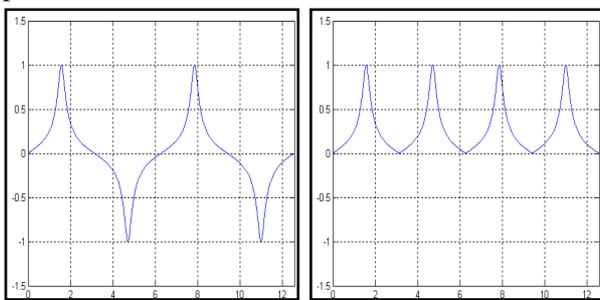
a) $i = 2; b = 6$ b) $i = 1; b = 6$

Fig. 11: the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = 6 > 1$

6.2.5 Fifth case ($b \gg 1$)

From equation (30), two configurations are studied:

1. Consider $x \neq \frac{(2k+1)}{2}\pi$ and $b \gg a \cdot Cter(x)$.

In this case, $Cter(x) \neq \pm\infty$. As "b" is too large number but non infinite, therefore:

$\left(\frac{a}{b}Cter(x)\right)^2 = \varepsilon \ll 1$. It is much smaller than the unit. Thus the equation (30) becomes:

$$Emar_b(x) = \frac{a}{b}\frac{Cter(x)ang_x(x)}{\sqrt{1+\left(\frac{a}{b}Cter(x)\right)^2}} \approx \frac{\sqrt{\varepsilon}\cdot ang_x(x)}{\sqrt{1+\varepsilon}} \quad, \quad \text{by}$$

using Taylor development for the first degree

$$Emar_b(x) = \sqrt{\varepsilon} \cdot ang_x(x)\left(1 - \frac{\varepsilon}{2}\right) \approx \sqrt{\varepsilon} \cdot ang_x(x)$$

Therefore:

$$Emar_b(x) = \begin{cases} 0^+ \text{ for } & 2k\pi < x < (2k+1)\pi \\ 0^- \text{ for } (2k+1)\pi < x < (2k+2)\pi \end{cases}$$

2. Consider $x = \frac{(2k+1)}{2}\pi$ and $b \ll a \cdot Cter(x)$.

In this case, $Cter(x) = \pm\infty$. As "b" is too large number but non infinite, therefore:

$\left(\frac{a}{b}Cter(x)\right)^2 \gg 1$. Thus the equation (30) becomes:

$$Emar_b(x) = \frac{a}{b}\frac{Cter(x)ang_x(x)}{\sqrt{1+\left(\frac{a}{b}Cter(x)\right)^2}} \approx \frac{a}{b}\frac{Cter(x)ang_x(x)}{\sqrt{\left(\frac{a}{b}Cter(x)\right)^2}}$$

$$\approx \pm\frac{a}{b}\frac{Cter(x)ang_x(x)}{\frac{a}{b}Cter(x)} \approx \pm ang_x(x) = \pm1$$

Consequently: $Emar_b(x) =$

$$\begin{cases} 0^+ \text{ for } & 2k\pi < x < (2k+1)\pi; x \neq \frac{(2k+1)\pi}{2} \\ 0^- \text{ for } (2k+1)\pi < x < (2k+2)\pi; x \neq \frac{(2k+1)\pi}{2} \\ +1 \text{ for } & 2k\pi < x < (2k+1)\pi; x = \frac{(2k+1)\pi}{2} \\ -1 \text{ for } (2k+1)\pi < x < (2k+2)\pi; x = \frac{(2k+1)\pi}{2} \end{cases}$$

Thus, an impulse signal is obtained.

Figures 12.a and 12.b represent the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = 100 \gg 1$, (impulse train with positive and negative part for $i = 2$ and impulse train with positive part only for $i = 1$).
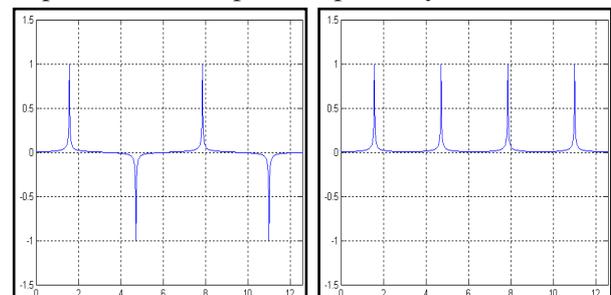
a) $i = 2; b = 100$ b) $i = 1; b = 100$

Fig. 12: the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = 100 \gg 1$

6.2.6 Determining the value of "b" for a quasi-triangular signal

The main objective of this part is to obtain a signal closed to a triangular one. Therefore, the following method is proposed in order to calculate the value of "b" for which the error between the desired signal and the obtained one is minimal. This study is limited to the interval $\left[0; \frac{\pi}{2}\right]$.

1. consider the equation of a straight line:

$$y = cx + d \qquad (36)$$

This line is supposed to contain the following two points $(x = 0; y = 0)$ and $(x = \frac{\pi}{2}; y = 1)$.

Therefore, the expression (36) becomes: $y = \frac{2}{\pi}x$, it is represented in the red color in the figure 13.

2. For the same interval $\left[0; \frac{\pi}{2}\right]$, the angular function $ang_x(x)$ is equal to one, therefore the function (30) becomes: $Emar_b(x) = \frac{a}{b}\dfrac{Cter(x)}{\sqrt{1+\left(\frac{a}{b}Cter(x)\right)^2}}$, it is represented in the blue color in the figure 13.
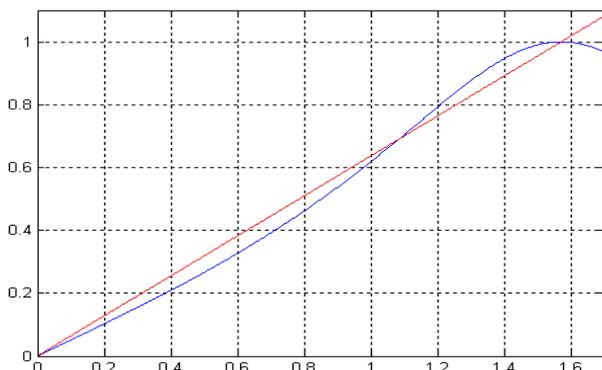
Fig. 13: represents the function $Emar_b(x)$ (in blue color) and the straight line y (in red color) in the considered interval.
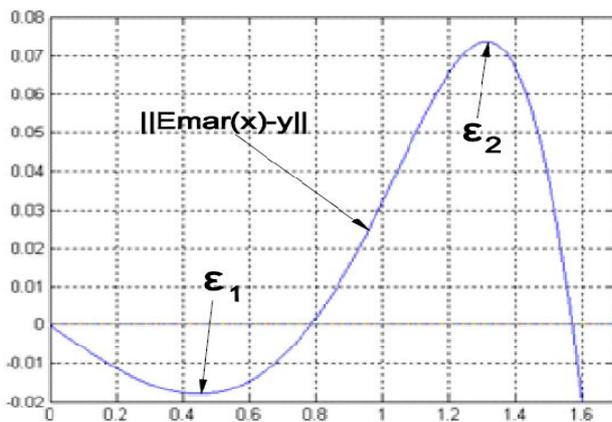
Fig. 14: represents the expression $\left\|Emar_b(x) - y\right\|$ and two maximal errors to determine.

To obtain the minimal error, the difference between the two functions must be smaller than a certain value "ε_{max}", thus $\left\|Emar_b(x) - y\right\| = \varepsilon \le \varepsilon_{max}$.

It is considered that for $x = \frac{\pi}{4}$ (the center of the studied interval) the error ε is equal to zero. Thus, for $x = \frac{\pi}{4}$, the error $\varepsilon = 0$,

$$\Rightarrow \left\|Emar_b\left(\frac{\pi}{4}\right) - \frac{2}{\pi}\cdot\frac{\pi}{4}\right\| = 0$$

$$\Rightarrow \frac{a}{b}\frac{Cter\left(\frac{\pi}{4}\right)}{\sqrt{1+\left(\frac{a}{b}Cter\left(\frac{\pi}{4}\right)\right)^2}} - \frac{1}{2} = 0 \Rightarrow b^2 = 3a^2 \Rightarrow b = \sqrt{3}$$

To calculate errors ε_1 and ε_2 presented in the figure 14, the derivative of ε must be equal to zero,

$$\varepsilon' = \left\|Emar_b(x) - y\right\|' = 0$$

$$\Rightarrow \begin{cases} \varepsilon_1 = 1.784\% \ \ for \ \ x = 0.45085 \\ \varepsilon_2 = 7.375\% \ \ for \ \ x = 1.31307 \end{cases}$$

And the average error $\varepsilon_{av} = 2.5062\%$

• The average error can be reduced by using another method, for example for $b = \sqrt{3} + 0.24782$, the average error is reduced as the following:

$$\begin{cases} \varepsilon_1 = 5.55\% \ \ for \ \ x = 0.6241 \\ \varepsilon_2 = 5.55\% \ \ for \ \ x = 1.381 \end{cases} \Rightarrow \varepsilon_{av} = 2.157\%$$

Figures 15.a and 15.b represent the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = \sqrt{3}$, (quasi-triangular signal for $i = 2$ and saw signal for $i = 1$).
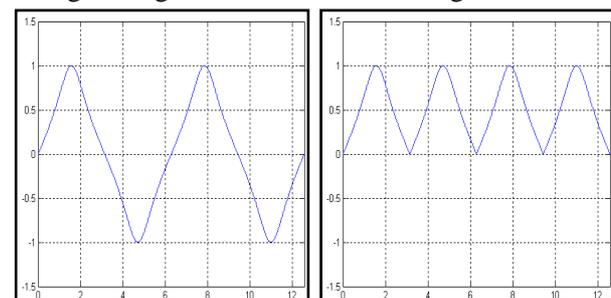
a) $i = 2; b = \sqrt{3}$ b) $i = 1; b = \sqrt{3}$

Fig. 15: the waveforms of the function $\bar{E}mar_{i,b}(x)$ for $b = \sqrt{3}$

Practically, the value of the average error $\varepsilon_{av} = 2.5\%$ is insignificant in some applications, therefore the obtained signal can be considered as a pure triangular signal for $i = 2$ and a pure saw signal for $i = 1$.

Tables 1 and 2 represent a summary of different waveforms obtained using the Elliptic functions $\bar{E}mar_{i,b}(x)$ and $\bar{E}jes_{i,b}(x)$.

b	Absolute Elliptic *Mar* " $\bar{E}mar_{i,b}(x)$"	
	$i = 2$	$i = 1$
b<<1	Rectangle signal	Continuous signal
b<1	Elliptical swollen signal	Rectified elliptical swollen signal
b=1	Sinusoidal signal (sine wave form)	Rectified sinusoidal signal
b=$\sqrt{3}$	Quasi-triangluar signal	Saw signal
b>1	Elliptical deflated signal	Rectified elliptical deflated signal
b>>1	Impulse train with positive and negative pulses	Impulse train with positive part only

Table 1: summary of multi form signals obtained using the Absolute Elliptic *Mar* function.

b	Absolute Elliptic *Jes* " $\bar{E}jes_{i,b}(x)$"	
	$i = 2$	$i = 1$
b<<1	Impulse train with positive part only	Impulse train with positive and negative pulses
b<1	Elliptical deflated signal	Rectified elliptical deflated signal
b=$\sqrt{3}$/3	Quasi-triangluar signal	Saw signal
b=1	Sinusoidal signal (cosine wave form)	Rectified sinusoidal signal
b>1	Elliptical swollen signal	Rectified elliptical swollen signal
b>>1	Rectangle signal	Continuous signal

Table 2: summary of multi form signals obtained using the Absolute Elliptic *Jes* function.

These types of signals are widely used in power electronics, electrical generators and in transmission of analog signals [17].

6.3 First conclusion

As presented previously, the Elliptic *Mar* function takes different waveforms by varying the parameter "b". The same analysis can be treated using the parameter "a". Therefore, the same waveforms can be obtained. Practically, instead of varying the value of "b" from 0 to $+\infty$ in a goal to obtain all waveforms, by introducing "a", one can change the values of "b" or "a" form 0 to 1 in a way to obtain the desired waveform. E.g.: $\frac{a}{b} = \frac{1}{20} = \frac{0.05}{1}$

6.4 programming the Elliptic *Mar* function in Matlab

As presented and analyzed in the previous section, the *Elliptic Mar* function can be also programmed and written in the Matlab software. Thus, the elliptical trigonometry functions can be used in any industrial applications.

The following program represents the detailed steps in writing the *Elliptic Mar* function in Matlab.

```
%------------------------------------------------------------
%Programming the Elliptic Mar function in Matlab
%Introduced by Claude Ziad Bayeh
a=1; x=-15:0.0001:15; clc
fprintf('Absolute Elliptic Mar "AEmar"\n');
repeat='y';
while repeat=='y'
   b=input('determine the form of the Elliptic
       trigonometry: b=');
   fprintf('b is a variable can be changed to obtain
       different signals \n'); %b is the intersection of the
       Ellipse and the axe y'oy in the positive part.
   if b<0,
      b
      error('ATTENTION: ERROR b must be greater than
      Zero');
   end;
   fprintf('AEmar=Emar*(angy(x))^i\n');
   i=input('for Absolute Elliptic Mar put 1, for Elliptic
       Mary put 2: i=');
   if i<0,
      i
      error('ATTENTION: ERROR i must be greater or
      equal to Zero');
   end;
   Ejes=(1./(sqrt(1.+((a/b).*tan(x)).^2))).*angx(x);
   Emar=(1./(sqrt(1.+((a/b).*tan(x)).^2))).*angx(x).*tan(x
       ).*a/b; % the Elliptic Mar "Emar"
   AEmar=Emar.*(angy(x)).^i; % Absolute Elliptic Mar
   plot(x,AEmar); axis([0 4*pi -1.5 1.5]); grid on;
   fprintf('Do you want to repeat ?\nPress y for "Yes" or
       any key for "No"\n');
   repeat=input('Y/N=','s'); %string input
   clc; close all
end; %End while
%------------------------------------------------------------
```

7 Studying the function $\bar{E}jes_{x_b^i}(x)$

In this section, a brief study on the Absolute Elliptic *Jes-x*. The main goal of this study is to model and simulate the function using Matlab and Simulink.

7.1 The Elliptic *Jes-x* function

The elliptical form in the figure 5 is written as the equation (4). Thus, given (6), the Elliptical *Jes-x* function can be determined. In fact:

$$Ejes_{x_b}(x) = \frac{Ejes_b(x)}{Cjes(x)} = \frac{ang_x(x)}{Cjes(x)\cdot\sqrt{1+\left(\frac{a}{b}\text{Cter}(x)\right)^2}} \quad (37)$$

• Expression of the Absolute Elliptic *Jes-x*

$$\bar{E}jes_{x_b^i}(x) = Ejes_{x_b}(x) \cdot \left(ang_x(x-\gamma)\right)^i \quad (38)$$

7.2 The block diagram of $\bar{E}jes_{x_b^i}(x)$

The block diagram of the function $\bar{E}jes_{x_b^i}(x)$ is illustrated in figure 16. There are four inputs connected to this diagram, three variable parameters "b", "i" and "γ", and one sinusoidal waveform (Circular *Mar* or "sine"). "a" is chosen to be a constant with $a = 1$, so it can't be considered as an input.
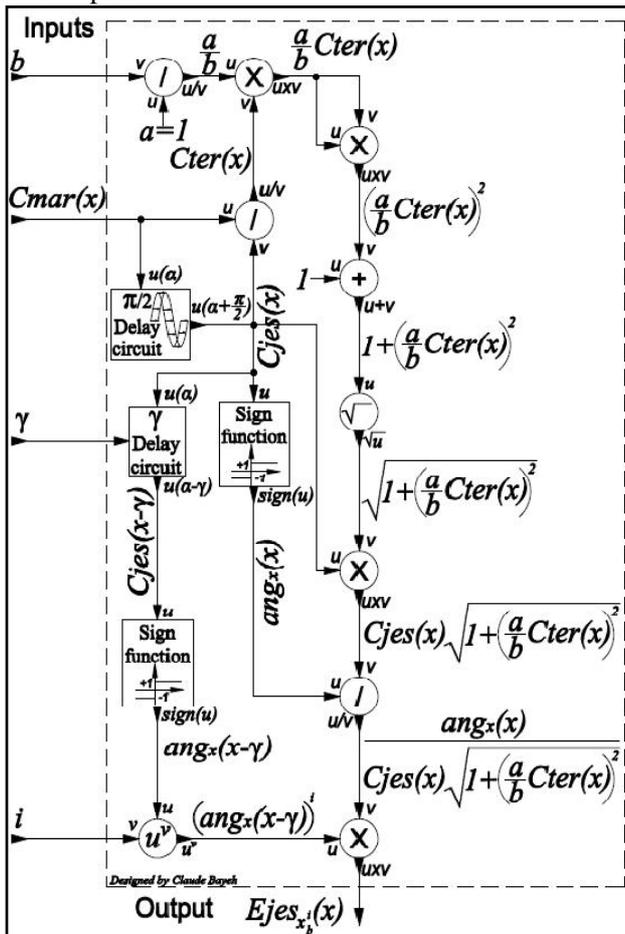
Fig. 16: the block diagram of the function $\bar{E}jes_{x_b^i}(x)$

7.3 Modeling and simulating the function $\bar{E}jes_{x_b^i}(x)$ using Matlab-Simulink

Consider $a = 1$, "b", "i" and "γ" are the controlled inputs, The Absolute Elliptic *Jes-x* function is obtained as an output. With four inputs, one can obtain a single output, which can produce more than 12 different signals.
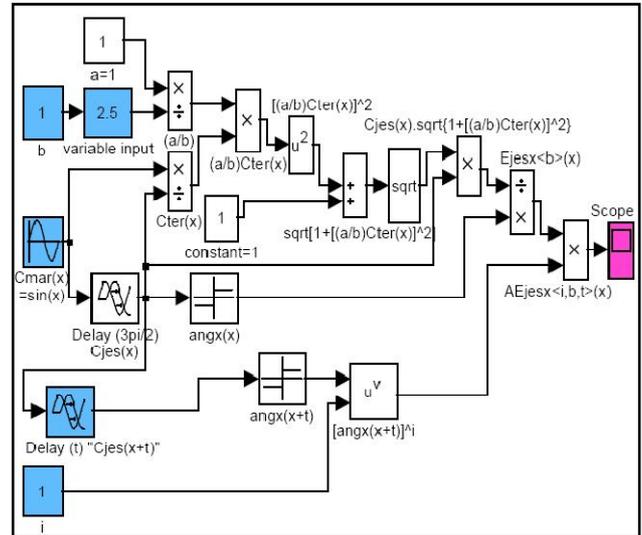
Fig. 17: the circuit model of the function $\bar{E}jes_{x_b^i}(x)$

• Multi form signals made by $\bar{E}jes_{x_b^i}(x)$:

Taking $\gamma = 0$ for this example, figures 18 and 19 represent multi form signals obtained by varying two parameters (i and b). For the figures 18.a to 18.e the value of $i = 2$, for the figures 19.a to 19.f the value of $i = 1$.
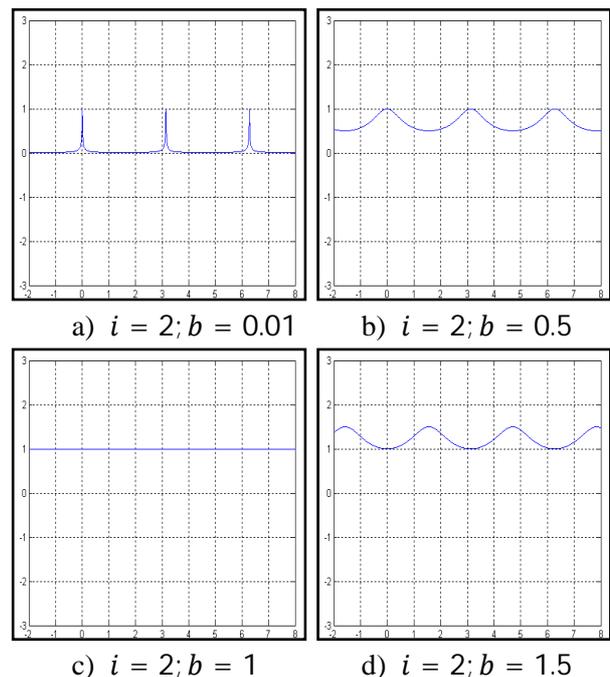
a) $i = 2; b = 0.01$ b) $i = 2; b = 0.5$

c) $i = 2; b = 1$ d) $i = 2; b = 1.5$
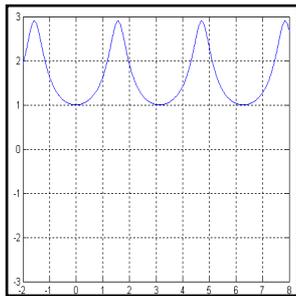
e) $i = 2; b = 2.9$

Fig. 18: multi form signals of the function $\bar{E}jes_{x_b^i}(x)$ for $i = 2$ and for different values of $b > 0$.
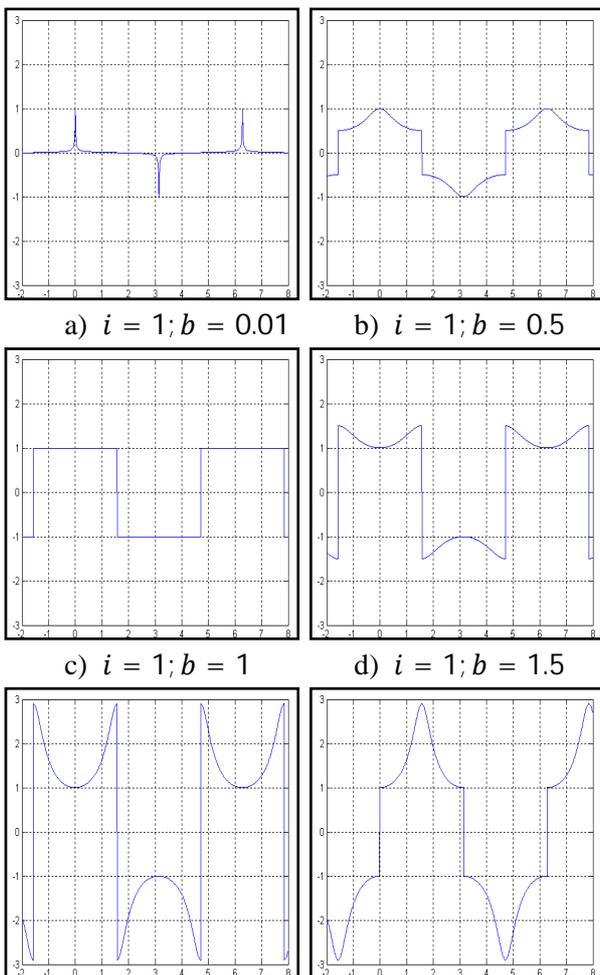
a) $i = 1; b = 0.01$ b) $i = 1; b = 0.5$

c) $i = 1; b = 1$ d) $i = 1; b = 1.5$

e) $i = 2; b = 2.9$ f) $i = 2; b = 2.9; \gamma = \pi/2$

Fig.19: multi form signals of the function $\bar{E}jes_{x_b^i}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function:
Impulse train with positive part only, sea waves, continuous signal, amplified sea waves, impulse train with positive and negative part, square waveform, saw signal …

7.4 programming the Elliptic *Jes-x* function in Matlab

As presented and analyzed in the previous section, the *Elliptic Jes-x* function can be also programmed and written in the Matlab software. Thus, the elliptical trigonometry functions can be used in any industrial applications.

The following program represents the detailed steps in writing the *Elliptic Jes-x* function in Matlab.

```
%------------------------------------------------------------------
%Programming the Absolute Elliptic Jes-x
%Introduced by Claude Ziad Bayeh
clc; close all; a=1; x=-15:0.004:15;
fprintf('---The Absolute Elliptic Jes-x---\n');
repeat='y';
while repeat=='y'
   b=input('determine   the   form   of   the   Elliptic
   trigonometry: b=');
   fprintf('b is a variable can be changed to obtain
   different signals \n');
   %b is the intersection of the Ellipse and the axe y'oy in
   the positive part.
   if b<0,
      b
      error('ATTENTION: ERROR b must be greater than
      Zero');
   end;
   fprintf('AEjesx=Ejesx*(angx(x-T))^i\n');
   i=input('for Absolute Elliptic Jes-x put 1, for Elliptic
   Jes-x put 2: i=');
   T=input('put the translation of the period for angx(x-
   T), T=');
   if i<0,
      i
      error('ATTENTION: ERROR i must be greater or
      equal to Zero');
   end;
   Ejes=(1./(sqrt(1.+((a/b).*tan(x)).^2))).*angx(x);
   Emar=(1./(sqrt(1.+((a/b).*tan(x)).^2))).*angx(x).*tan(x
   ).*a/b;
   % the Absolute Elliptic Jes-x "AEjesx"
   AEjesx=Ejes./cos(x).*(angx(x-T)).^i;
   plot(x,AEjesx); axis([-2 8 -3 3]); grid on;
   fprintf('Do you want to repeat ?\nPress y for "Yes" or
   any key for "No"\n');
   repeat=input('Y/N=','s'); %string input
   close all
end; %End while
%------------------------------------------------------------------
```

8 Conclusion

In this paper, an original study in trigonometry is introduced. The elliptical unit and its trigonometric functions are presented and analyzed. In fact the proposed Elliptical Trigonometry is a new form of trigonometry that permits to produce multiple forms

of signals by varying some parameters; it can be used in numerous scientific domains and particularly in mathematics and in engineering. For the case treated in this paper, 32 elliptical trigonometric functions are defined; only two functions are analyzed and simulated using software as Matlab-Simulink. In general, a connection cable with specific transmission data protocol connects any industrial system to the computer. One can use the studied functions in order to generate control signals in need for power components of the industrial system.

The elliptical trigonometry functions will be widely used in electronic domain especially in power electronics. Thus, several studied will be improved and developed after introducing the new functions of the elliptic trigonometry. Some mathematical expressions and electronic circuits will be replaced by simplified expressions and reduced circuits.

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