Automatic Regulation Time Series for Sampled-data Feedback Control Systems with Limited Control Inputs

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Abstract: - In this literature, a new nonlinear digital controller is proposed for analyses and designs of sampled-data feedback control systems. The controller is derived from the converging characteristic of a specified numerical series. The ratios of neighbourhoods of the series are formulated as a function of the output of the plant and the reference input command, and will be converged to be unities after the output has tracked the reference input command. Limitation of the series can be applied to get better performance. Two kinds of servo system, a time-delay system, one very high order system and a 2x2 multivariable aircraft gas turbine engine are used to illustrate effectiveness of the proposed nonlinear digital controller. Comparisons with other conventional methods are also made.

Key-Words: - Time series, Nonlinear control, Sampled data feedback control system.

1 Introduction

For unit feedback discrete-time control systems, the control sequences are usually functions of the difference between the sampled reference input and output of the plant [1-5]. The discrete-time control sequence can be generated by Finite Impulse Response (FIR) filter or Infinite Impulse Response (IIR) filter. The input of FIR or IIR filter is the difference between the sampled reference input and output of the plant. The output of FIR or IIR will be the input of the plant. In general, they are linear controllers.

In this literature, a nonlinear discrete-time control sequence described by periodic numerical series $G(jT_S)$ with ratios of the reference input and plant output is first proposed for analyses and designs of sampled-data feedback control systems. T_S represents the sampling interval. The ratios of $G((k+1)T_S)$ to $G(kT_S)$ of the series are formulated as a function of the reference input command and the output of the plant. The value of $G(kT_S)$ is the control input of the plant at time intervals between $(k-1)T_S$ and kT_S. Thus, the considered system is closed as a feedback control system with $G(jT_S)$. It will be seen that the output of the plant tracks the reference input command exactly after ratios $G((k+1)T_S)/\ G(kT_S)$ of the series being converged to unities. It implies that $G(kT_S)$ will be converged to a steady-state value for a constant reference input applied. The stability of the closed-loop system is guaranteed by selecting the proper function of ratios $G((k+1)T_S)\ /\ G(kT_S)$. This function can be called as "Regulation Function". It will be proven that the considered system with $G(kT_S)$ becomes a negative feedback control system for a stable plant[4].

Note that it needs not integration to get zero tracking error, and performance of controlled systems are dependent on selected functions of $G((k+1)T_S)\ /\ G(kT_S)$. In this paper, a first-order polynomial and a piecewise-linear input versus output are used for illustrating operating concepts of the proposed nonlinear control system. Naturally, more complicated function(i.e., shape) can be applied to considered systems. The shaping of regulation function around the equilibrium point is corresponding to the loop transfer function shaping at the medium frequency band in the conventional control techniques [4].

Furthermore, an adaptive limitation for $G(kT_S)$ can be applied also to minimize the control effort and get better performance. Controlled results will be compared with conventional famous PI and PID controllers[6-15].

In following sections, basic concepts of the proposed nonlinear discrete-time control sequence is discussed first, and then two kinds of servo system, a time-delay system, a very high order system, and a 2x2 multivariable aircraft gas turbine examples are used to illustrate their tracking behaviour and performance. Simulating results will show that the proposed nonlinear digital controller gives another possible way for analyses and designs of sampled-data feedback control systems. Design results of fifth example give the proposed method can also be applied to multivariable feedback control systems.

2 The Basic Approach

2.1 Automatic regulation time series

A numerical series with time interval T_S [1-5] can be written as in the form of

$$G(jT_S),\ j=1,2,3,...,n,n+1,..., \quad (1)$$

where $G(jT_S)$ represents a constant value between time interval from $(j-1)T_S$ and jT_S. For simplicity, the representation of $G(jT_S)$ will be replaced by $G(j)$ in following evaluations. The ratios $G(j+1)/G(j)$ of the series are defined as in the form of

$$F(j)=G(j+1)/G(j),\ j=1,2,3,...,n,n+1,.... \quad (2)$$

Eq.(2) gives the value of $G(n+1)$ approaches to be a constant value when the value of $F(n)$ approaches to be unity. Now, the problem for closing the considered system is to find the formula of $F(j)$ which is the function of the reference input command *R* and the output of the plant *Y*. $G(n+1)$ is used as the input of the considered system. Considering a series given bellows:

$$G(n+1)=\left[\sum_{i=0}^{m} a_i\left(R(n)/Y_S(n)\right)^i\right]G(n) \quad (3)$$

where $R(n)$ represents the reference input command and $Y_S(n)$ represents the non-zero sampled output of the plant at the sampling interval nT_S. Note that this non-zero constraint will be removed later by level shifting. Eq.(3) is a possible way to close the considered system as a sampled-data feedback control system. Assume the reference input command has been tracked by applying control effort $G(j)$, Eq. (3) becomes

$$G(n+1)=\sum_{i=0}^{m} a_i G(n) \quad (4)$$

For steady-state condition, $G(n+1)$ approaches to be a constant value, it gives

$$\sum_{i=0}^{m} a_i = 1 \quad (5)$$

Rearranging Eq.(3) and taking the derivative of it with respect to $Y_S(n)/R(n)$, one has

$$F(n)=\sum_{i=0}^{m} a_i\left(\frac{Y_S(n)}{R(n)}\right)^{-i} \quad (6)$$

and

$$\frac{\partial F(n)}{\partial(Y_S(n)/R(n))}=-\sum_{i=0}^{m} i a_i\left(\frac{Y_S(n)}{R(n)}\right)^{-1-i} \quad (7)$$

The sufficient but not necessary condition for Eq.(7) less than zero is $a_i>0$ for $Y_S(n)/R(n)\cong 1$ and Eq.(6) can be rewritten as in the form of

$$F(n)=\sum_{i=0}^{m} a_i\left\|\frac{Y_S(n)}{R(n)}\right\|^{-i} \quad (8)$$

$a_i>0$ will be used in following evaluations. Negative value of Eq.(7) represents the closed-loop system with Eq.(3) activated as a negative feedback system around the equilibrium condition; i.e., $Y_S(n)=R(n)$. This statement will be illustrating and discussed by a graph in the next paragraph. The first order polynomial described in Eq.(3) can be written as in the form of

$$G(n+1)=\left[(1-\beta)\frac{R(n)}{Y_S(n)}+\beta\right]G(n)\,; \quad (9)$$

where β satisfies constrains stated above and becomes an adjustable parameter. Thus, the ratios $F(n)$ becomes

$$F(n)=(1-\beta)\frac{R(n)}{Y_S(n)}+\beta \quad (10)$$

$F(n)$ can be called as "Regulation Function" also. Similarly, the third order representation of $F(n)$ is in the form of

$$F(n)=\alpha\left(\frac{Y_S(n)}{R(n)}\right)^{-3}+\gamma\left(\frac{Y_S(n)}{R(n)}\right)^{-1}+1-\alpha-\gamma \quad (11)$$

where $0<\alpha$ and $0<\gamma$.

Taking the derivative of Eq.(10) with respect to $Y_S(n)=R(n)$, one has

$$\frac{\partial F(n)}{\partial(Y_S(n)/R(n))} = -(1-\beta)\left(\frac{Y_S(n)}{R(n)}\right)^{-2} \quad (12)$$

For negative value of Eq.(12), the value of β must be less than one. This implies the range of β is $0<\beta<1$. The suitability of the proposed nonlinear adaptive digital controller is based on this negative regulation characteristic. Fig.1 shows ratios $F(n)$ versus $R(n)/Y_S(n)$ represented by Eq.(9) for β= 0.9, 0.7, 0.5,0.3 and 0.1; respectively.

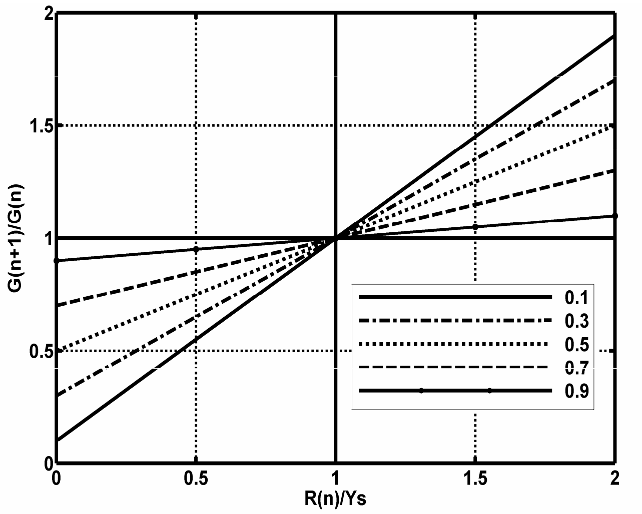

Fig.1. G(n+1)/G(n) Versus R/Y_S for β=0.9, 0.7, 0.5,0.3 and 0.1.

Fig. 1 shows that the value of $F(n)$ is less than one for that of $Y_S(n)$ greater than that of $R(n)$, then the value of $G(n+1)$ will be decreased; and the value of $F(n)$ is greater than one for that of $Y_S(n)$ less than that of $R(n)$, the value of $G(n+1)$ will be increased. This implies that the controlled system connected with Eq.(9) will be regulated to the equilibrium point ($Y_S(n)/R(n)=1$) and gives a negative feedback control system for deviation from the equilibrium point. From Fig.1, it can be seen that one can adjust β to get desired regulating slope; i.e., regulating characteristic. Certainly, other tracking functions can be formulated and proposed also for the considered system, if its derivative with respect to $Y_S(n)/R(n)$ is negative. Similar to the derivation of Eq.(12), Eq.(11) gives

$$\frac{\partial F(n)}{\partial(Y_S(n)/R(n))} = -\left\{\gamma\left(\frac{Y_S(n)}{R(n)}\right)^{-2} + 3\alpha\left(\frac{Y_S(n)}{R(n)}\right)^{-4}\right\} \quad (13)$$

The constraint of non-zero $Y_S(n)$ can be removed by $R(n)/Y_S(n)$ of Eq.(9) replaced by $(R(n)+Y_O)/(Y_S(n)+Y_O)$. Y_O is a positive value and represents the negative maximal control swing, The modified equation of Eq.(9) becomes

$$G(n+1) = \{(1-\beta)\frac{R(n)+Y_o}{Y_S(n)+Y_o} + \beta\}G(n)\,. \quad (14)$$

Eq.(14) implies ratios $G(n+1)/G(n)$ are in the form of

$$F(n) = [(1-\beta)\frac{R(n)+Y_o}{Y_S(n)+Y_o} + \beta],\ \text{n}=1,2,3,.,j,j+1,.. \quad (15)$$

Control inputs of the plant are in the form of

$$u(n+1) = G(n+1) - Y_o / P(0) \quad (16)$$

for the negative swing control with positive values of β, $G(n)$ and $F(n)$. Eq.(14) gives negative regulation characteristics also for $R(n)=Y_S(n)$ is corresponding to $R(n)+Y_O = Y_S(n)+Y_O$. Similar to the evaluation of the Eq.(12), the derivative of Eq.(15) becomes

$$\frac{\partial F(n)}{\partial((Y_S(n)+Y_o)/(R(n)+Y_o))} = -(1-\beta)\left(\frac{Y_S(n)+Y_o}{R(n)+Y_o}\right)^{-2} \quad (17)$$

Fig.2 shows the connected system configuration using Eqs.(14) an (16) in which U is the sampled with hold output of the controller. The values of $G(n)$ and $F(n)$ will be all positive for the summation of $Y_s(n)$ and Y_o (or R and Y_o) is greater than zero with specified values of Y_o. All positive values will give the better continuity and regulating characteristic of the time series. Naturally, absolute value of $(R(n)+Y_O)/(Y_S(n)+Y_O)$ can be used in Eq.(14) to guarantee positive of $G(n)$ and $F(n)$ for negative of $R(n)$.

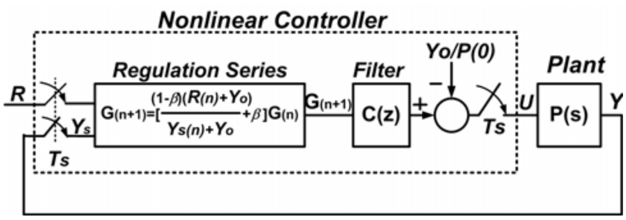

Fig.2. A Nonlinear Digital Controller with Automatic Regulation Time Series.

2.2 Control effort limitation

An adaptive value of Y_o can be selected at $|R(n)|$ for the system is well controlled. Then Eqs.(14) and (16) can be rewritten as

$$G(n+1)=\left\{(1-\beta)\frac{R(n)+|R(n)|}{Y_S(n)+|R(n)|}+\beta\right\}G(n) \qquad (18)$$

and

$$u(n+1)=G(n+1)-|R(n)|/P(0) \qquad (19)$$

respectively. The maximal value of $G(n)$ can be limited by an adaptive constraint $|R(n)+|R(n)||$ to minimize the control effort. The control input U of the plant is now described by Eq.(19).

Note that the singularity of Eq.(18) must be avoided when $Y_S(n)+|R(n)|=0$. It is easy to replace $Y_S(n)+|R(n)|=0$ by a small value. A small value of $G(n)$ is selected also to avoid null the time series.

Fig.3 shows an equivalent block diagram of Fig.2 with constraint of $G(n)$ and singularity avoidance of $Y_S(n)+|R(n)|=0$. The constrain of $G(n)$ cannot only for minimizing the control effort but also for improving system performance. A conventional digital filter $C(z)$ in Fig.3 can be applied for filtering $G(n)$, if it is necessary. In general, phase-lead is used for speeding up the time response.

The proposed control scheme will be applied to four numerical examples in next section.

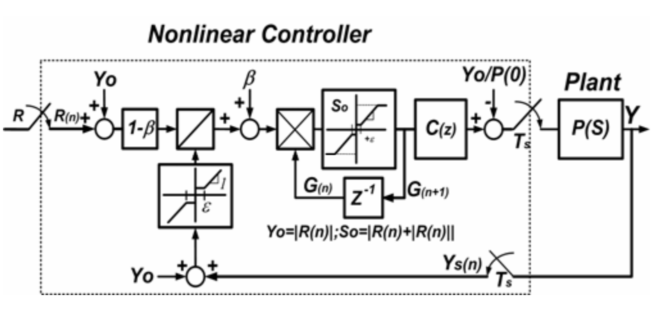

Fig.3. Equivalent Block Diagram of Fig.2 with $G(n)$ limitations and singularity avoidance of Eq.(18).

3 Numerical Examples

Example 1: The first example [6] is shown in Fig.3, in which $P(s)$ is in the form of

$$P_1(s)=\frac{30}{s^2+10s+30} \qquad (20)$$

DC gain of $P_1(s)$ is unity. The sampling period T_S is selected to be equal to 0.1 second for illustrating variations of $G(n)$ and $F(n)$. Time responses of the overall system with the nonlinear digital controller for $\beta=0.5$, $Y_o=|R(n)|$ and $C(z)=1$ are shown in Fig.4. Magnitudes of reference inputs between 0 and 5 seconds are equal to 1; between 5 and 10 seconds are equal to -0.7, between 10 and 14 seconds are equal to 0.5, and between 14 and 17 seconds are equal to -0.3, in which gives reference input $R(n)$ (dash-line), output Y(solid-line), Time series $G(n)$ (dot-line), and ratios $F(n)$ (dash-dot-line) of $G(n)$. Fig.4 shows that all values of $G(n)$ and $F(n)$ are positive while the value of output Y tracking the negative value of the reference input $R(n)$. The value of $R(n)$ can be positive or negative.

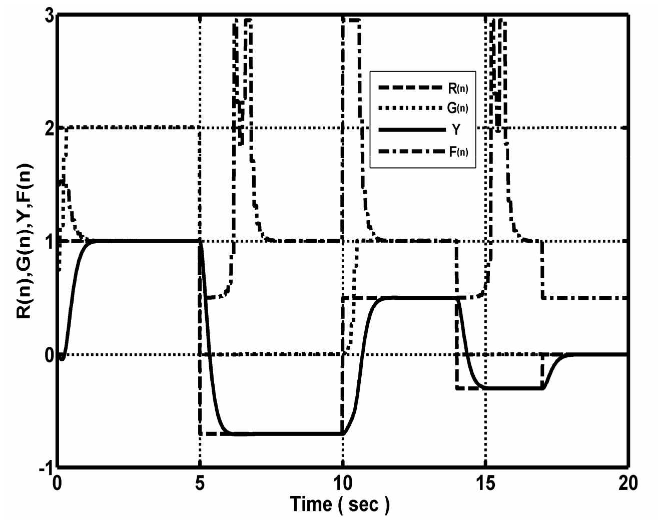

Fig.4. Time Responses of Example 1 for β=0.50 and Ts=0.1sec.

Fig.4 shows also that ratios $F(n)$ are converged to be unities quickly; i.e., the controlled output tracks the reference input quickly. The proposed method gives a good performance and zero steady-state error without integration. Note that maximal values of $G(n)$ are set to be $|R(n)+|R(n)||$ for better performance and minimal the control effort. Eq.(14) gives $F(n)$ will be converged to 0.5 for zero input($R(n)=0$) and $\beta=0.5$. Eq.(14) and Fig.1 give that the less value of β, the larger regulation slope will be. $\beta=0.5$ is the optimal value for the considered system.

Fig.5 shows time responses for $\beta=0.50$ and sampling frequency equal to 100, 40, 20, 10 and

5Hz, respectively. It shows that 40Hz (i.e., T_S=25ms) is fast enough for the considered system. Fig.6 shows comparisons with a phase-lead C(z) is included in the control loop. The phase-lead C(z) is in the form of

$$C(z) = \left. \frac{0.15923\, s + 1}{0.03185\, s + 1} \right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \tag{21}$$

It can speed up the time responses while keeping system performance.

Another way to speed up the transient responses is to use piecewise-linear regulation function $F(n)$. Enlarging the slope of $F(n)$ at the conditions away from the equilibrium point(i.e., $Y_S(n) = R(n)$) to speed up transient responses, and keeping the original slope to maintain system performance(e.g., peak overshoot and settling time, etc.).

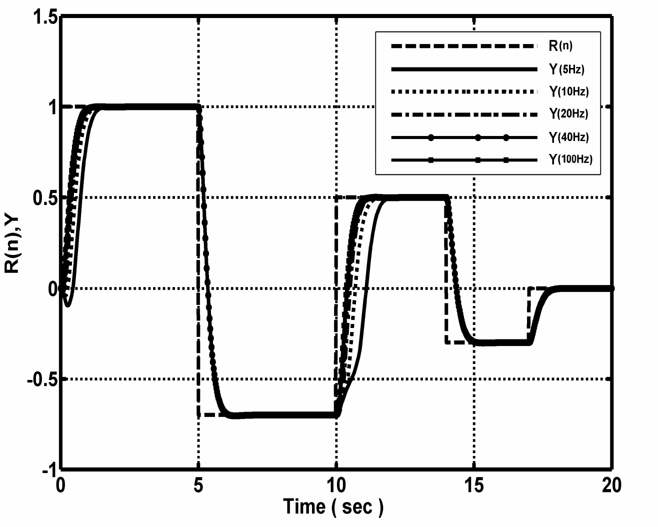

Fig.5. Time Responses of Example 1 for β=0.50 and *Sampling Frequency* Equaling to 100, 40, 20, 10 and 5Hz, respectively.

Fig.7 shows a comparison between a new regulation function $F(n)$ and the original $F(n)$. A piecewise linear function is used. The new function is in the form of

$$F(n) = \begin{cases} 10RDY + (0.2\beta - 7.2) & 0 < RDY < 0.8 \\ (1-\beta)RDY + \beta; & 0.8 < RDY < 1.25 \\ 10RDY - (0.25\beta + 11.25) & 1.25 < RDY \end{cases} \tag{22}$$

where

$$RDY = (R(n) + Y_O)/(Y_S(n) + Y_O) \tag{23}$$

and $\beta = 0.50$ and $T_S = 25ms.$ Eqs.(22) gives large regulation slopes than original $F(n)$ described by Eq.(14). Fig.7 shows that the rise time can be reduced with this new regulation function described by Eq.(22).

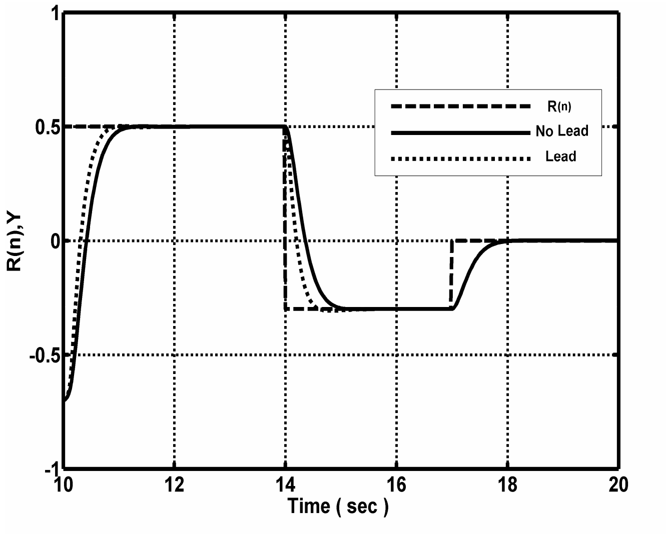

Fig.6. Time responses of Example 1 with/without C(z) for β=0.5; $T_S = 25ms$.

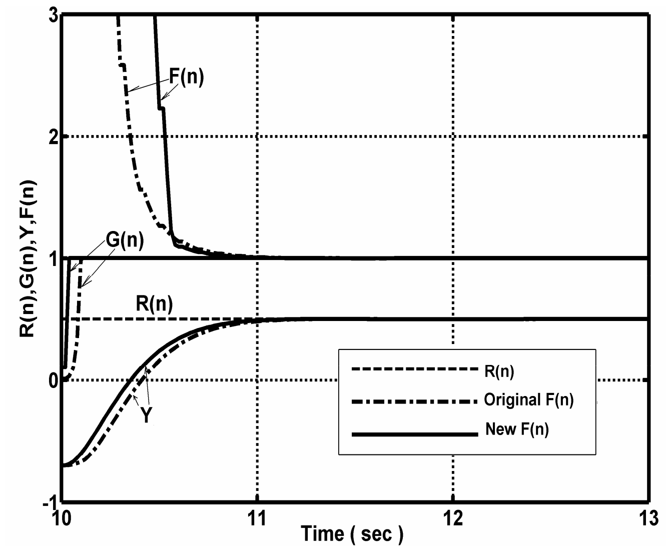

Fig.7. Time Responses Comparison between a New $F(n)$ and the original $F(n)$.

Example 2: Consider a stable plant has the transfer function[7,8]:

$$P_2(s) = \frac{e^{-s}}{(s+1)^2} \tag{24}$$

Parameters of the nonlinear controller are $\beta = 0.5$ and $T_S = 25ms.$ Fig.8 shows time response of the controlled system, in which gives reference input

$R(n)$ (dash-line), output Y(solid-line), Time series $G(n)$ (dot-line), and ratios $F(n)$ (dash-dot-line) of $G(n)$. The phase-lead filter C(z) is in the form of

$$C(z) = \left. \frac{0.4s+1}{0.02s+1} \right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (25)$$

Fig.8 shows the proposed method give good performance and zero steady-state error. Simulation results of the proposed method and four other methods are presented for comparisons. They are Ziegler-Nichols method[9-12] for finding PI and PID compensators, Tan et al[13,14] for finding PID compensator and Majhi[7,8] for finding PI compensator. The controller is in the form of

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{d}{dt} e(t); \quad (26)$$

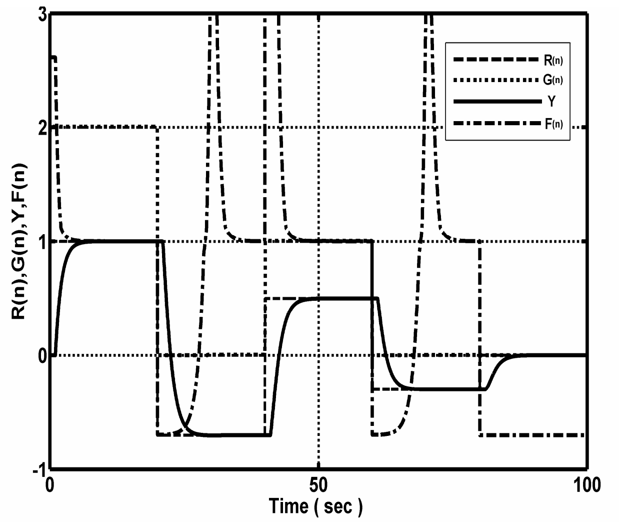

Fig.8. Time responses of Example 2 with C(z) for β=0.5; $T_S = 25ms.$

Parameters of four found compensators are given below:

(1)ZN(PI) : $K_p = 1.240$ and $K_i = 0.251$.

(2)ZN(PID) : $K_p = 1.6367, K_i = 0.4187$ *and* $K_d = 0.5972$.

(3)Tan's(PID): $K_p = 0.620, K_i = 0.5636$ *and* $K_d = 0.1705$.

(4)Majhi's(PI): $K_p = 0.864$ *and* $K_i = 0.3653$.

Integral of the Square Error(ISE), and Integral of the Absolute Error (IAE) are given in Table 1. Time responses are shown in Fig.9. From Table 1 and Fig.9, one can see that the proposed method gives faster and better performance than those of other methods presented.

Table 1. IAE and ISE Errors of Example 2 with Different Control Methods.

Methods	Proposed	ZN(PI)	ZN(PID)	Tan's	Majhi's
IAE	1.5977	2.2675	1.7694	2.2471	2.4654
ISE	2.1471	4.0107	2.8757	3.0725	4.0659

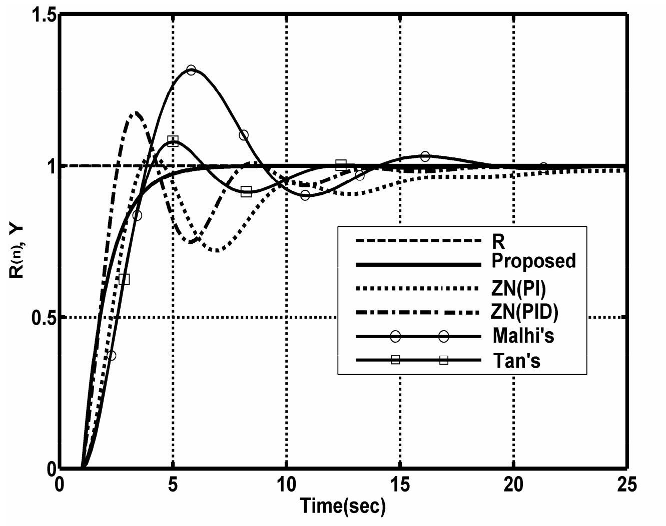

Fig.9. Time Responses of Example 2 with Different Control Methods.

Example 3: Consider the very high order plant[7,8]:

$$P_3(s) = \frac{1}{(s+1)^{20}} \quad (27)$$

Parameters of the nonlinear controller are $\beta = 0.5$ and $T_S = 25ms.$ Fig.10 shows time response of the controlled system, in which gives reference input $R(n)$ (dash-line), output Y(solid-line), Time series $G(n)$ (dot-line), and ratios $F(n)$ (dash-dot-line) of $G(n)$. It gives good performance and zero steady-state errors. The phase-lead filter C(z) is in the form of

$$C(z) = \left. \frac{0.8s+1}{0.02s+1} \right|_{s=\frac{2}{T_S}\frac{z-1}{z+1}} \quad (28)$$

Fig.10 shows the considered plant is a large time-lag system. The high order system model is usually used to describe the industry process for replacing pure time-delay(e.g. $e^{-T_d s}$). Such that conventional analysis and design techniques can be applied[7,8]. Fig.10 shows the proposed method can be applied to a large time-delayed system.

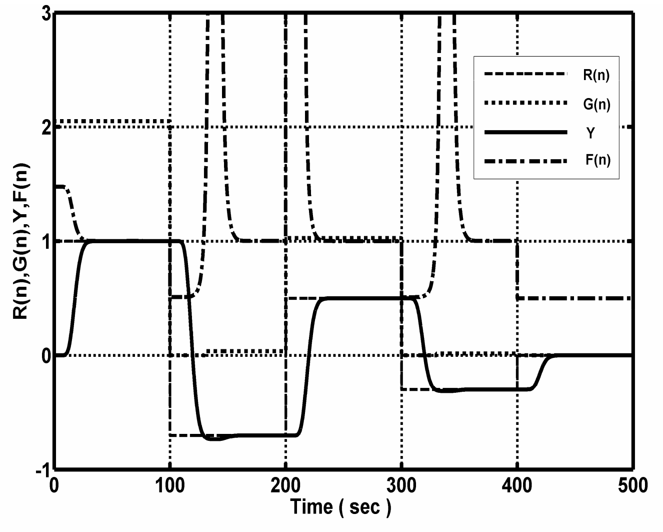

Fig.10. Time responses of Example 3 with C(z) for β=0.5; $T_S = 25ms$.

Final results and four other methods are presented for comparison and show the merit of the proposed method. They are Ziegler-Nichols method[9-12] for finding PI and PID compensators, Zhuang et al. [15] for finding PI compensator and Majhi[7,8] for finding PI compensator. Parameters of four found compensators are given below:

(1)ZN(PI) : $K_p = 0.585$ *and* $K_i = 0.0305$.

(2)ZN(PID): $K_p = 0.77256, K_i = 0.05088$ *and* $K_d = 4.9135$.

(3)Majhi's(PI) : $K_p = 0.5097$ *and* $K_i = 0.0443$.

(4)Zhuang's(PI): $K_p = 0.473$ *and* $K_i = 0.058$.

Time responses are shown in Fig.11. Table 2 gives integration of absolute error(IAE) and integration of square error(ISE) of them. From Table 2 and Fig.11, one can see that the proposed method gives better performance than those of other methods.

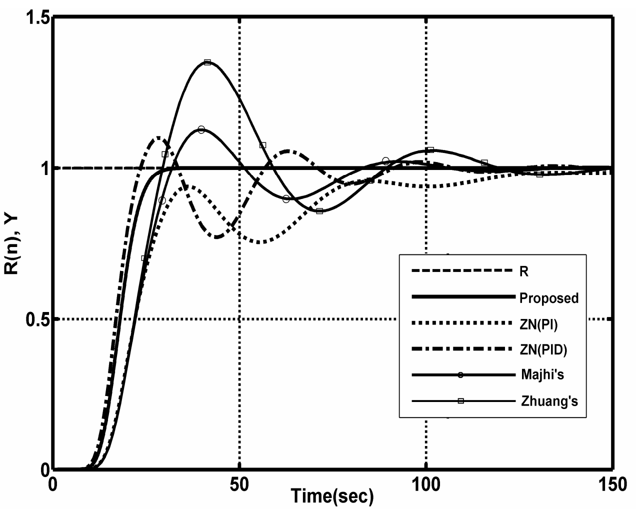

Fig.11. Time Responses of Example 3 with Different Control Methods.

Table 2. IAE and ISE Errors of Example 3 with Different Control Methods.

Methods	Proposed	ZN(PI)	ZN(PID)	Majji's	Zhuang's
IAE	16.0101	21.2271	16.2160	20.1908	21.8142
ISE	18.3378	32.7084	22.9707	26.8295	32.9125

Example 4: Now, consider an electro-hydraulic velocity servo system [16, 17] shown in Fig. 12 with system parameters given below:

$K_S = 2.3\times10^{-7}\sqrt{P_S - sign(X_V)P_L}\ m^2/s$; $K_V = 0.5\ m/v$

$P_S = 1.4\times10^{7}\ N_t/m^2$; $\beta_o = 3.5\times10^{7}\ N_t/m^2$;

$V_t = 3.3\times10^{-5}\ m^2/rad$; $C_{tp} = 2.3\times10^{-11}\ m^5/s/N_t$;

$D_m = 1.6\times10^{-5}\ m^3/rad$; $J = 5.8\times10^{-3}\ Kg\text{-}m\text{-}s^2$;

$B_m = 0.864\ Kg\cdot m\cdot s/rad$;

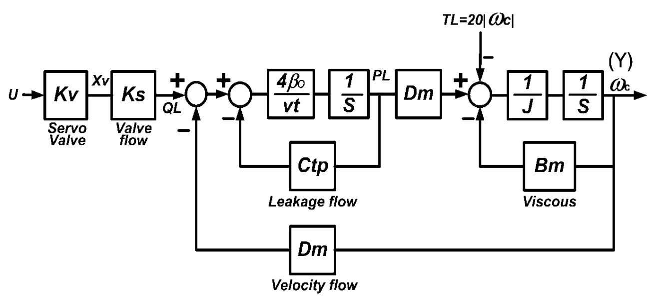

Fig.12. Mathematical Model of Example 4.

The objective of the control is to keep the velocity ω_c of the hydraulic system following the desired reference input. The relation between the valve displacement X_V and the load flow rate Q_L is governed by the well-known orifice law [18]

$$Q_L = X_V K_J \sqrt{P_S - sign(X_V)P_L} = X_V\ K_S \; ; (27)$$

where K_j is a constant for specific hydraulic motor; P_S is the supply pressure; P_L is the load pressure and; K_S is the valve flow gain which varies at different operating points. The following continuity property of the servo valve and motor chamber yields

$$Q_L = D_m\omega_C + C_{tp}P_L + (V_t - 4\beta_o)\dot{P}_L; \quad (28)$$

where D_m is the volumetric displacement; C_{tp} is the total leakage coefficient; V_t is the total volume of the oil; β_o is the bulk modulus of the oil; and ω_c is the velocity of the motor shaft. The torque balance equation for the motor is in the form of

$$D_m P_L = J \dot{\omega}_C + B_m \dot{\omega}_C + T_L ; \qquad (29)$$

where B_m is the viscous damping coefficient and T_L is the external load disturbance which is assumed to be dependent upon the velocity of the shaft or slowly time varying as described by the following equation:

$$T_L = 20|\omega_c|. \qquad (30)$$

Step responses of the Example 4 for $\beta = 0.7$, $T_S = 20ms$, and values of reference inputs $R(n)$ between 0 and 0.3 seconds are equal to 1; between 0.3 and 0.6 seconds are equal to 0.4, between 0.6 and 0.9 seconds are equal to 0.8, and between 0.9 and 1.2 seconds are equal to 0.2, are shown in Fig. 10, in which gives the output Y(solid-line), the time series $G(n)$ (dot-line) and regulation function $F(n)$ (dash-dot-line) of the series. Fig.13 shows that $F(n)$ are converged to be unities quickly, and the control method gives good performance and zero steady-state error.

The regulation function $F(n)$ used in this example is in the form of

$$F(n) = \begin{cases} 1.1R(n)/Y_S(n) + (0.2\beta - 0.08) & 0 < R(n)/Y_S(n) < 0.8 \\ (1-\beta)R(n)/Y_S(n) + \beta; & 0.8 < R(n)/Y_S(n) < 1.25 \\ 1.1R(n)/Y_S(n) - (0.25\beta + 0.125) & 1.25 < R(n)/Y_S(n) \end{cases} \qquad (31)$$

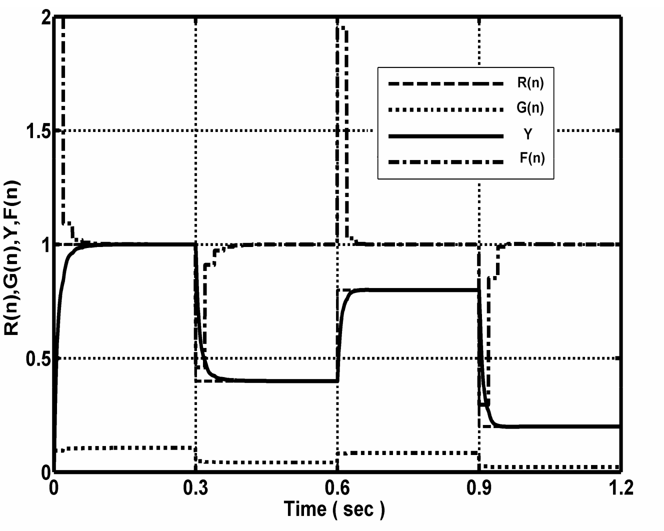

Fig.13. Time Responses of Example 4 for β=0.7 and $T_S = 20ms$.

Fig.14 shows a comparison with the PID control and the proposed method. Parameters of the PID control law are

$$K_p = 3.025\times10^{-3}, K_i = 10.3125 \ and \ K_d = 2.1588\times10^{-2}$$

They are found by the optimizations method toolbox of MATLAB for minimized the integration of absolute errors(IAE). Note that the comparison of two methods is based on the same amount of control efforts (U). It is defined as

$$E(n) = \sum_{j=1}^{n} (u(n))^2 T_S \qquad (32)$$

Fig.14 shows performance of the proposed method is compatible(or slightly better than) with that of the PID control with optimized parameters. This implies that the proposed method is compatible to the optimized PID Controller. But it is much simpler to select parameters of the controller.

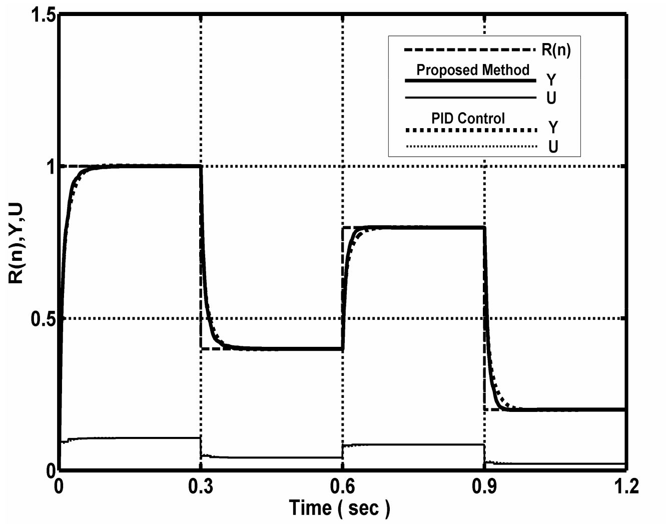

Fig.14. Time Responses Comparison between the Proposed Method and the Optimized PID Control.

Example 5: Consider a gas turbine engine with plant transfer function matrix[19-21].

$$P_5(s) = \frac{1}{\Delta(s)} \begin{bmatrix} 2533 + 1515.33s + 14.9s^2 & 1805947 + 1132094.7s + 95150s^2 \\ 12268.8 + 8642.68s + 85.2s^2 & 2525880 + 1492588s + 124000s^2 \end{bmatrix} \qquad (33)$$

where $\Delta(s) = 2525 + 3502.7s + 1357.3s^2 + 113.22s^3 + s^4$. It is a 2×2 multivariable plant. The steady-state gain of open loop $P_5(s)$ is in the form of

$$P_5(0) = \begin{bmatrix} 1.00316 & 715.2265 \\ 4.85893 & 1000.3485 \end{bmatrix} \quad (34)$$

A pre-compensating matrix $P_5^{-1}(0)$ is first applied to decouple the plant in low-frequency band. Then, two digital filters are used in the diagonal to filter outputs of two time series for speeding up trasient responses. They are in the form of

$$C_1(z) = \left. \frac{0.75s+1}{0.15s+1} \right|_{s=\frac{2}{T_s}\frac{z-1}{z+1}} \quad (35)$$

and

$$C_2(z) = \left. \frac{0.60s+1}{0.25s+1} \right|_{s=\frac{2}{T_s}\frac{z-1}{z+1}} \quad (36)$$

where $T_s = 25ms$ is the sampling period. Fig.15 shows time responses of this controlled system for β=0.5. It shows that the proposed control scheme can be applied to the multivariable feedback control system also.

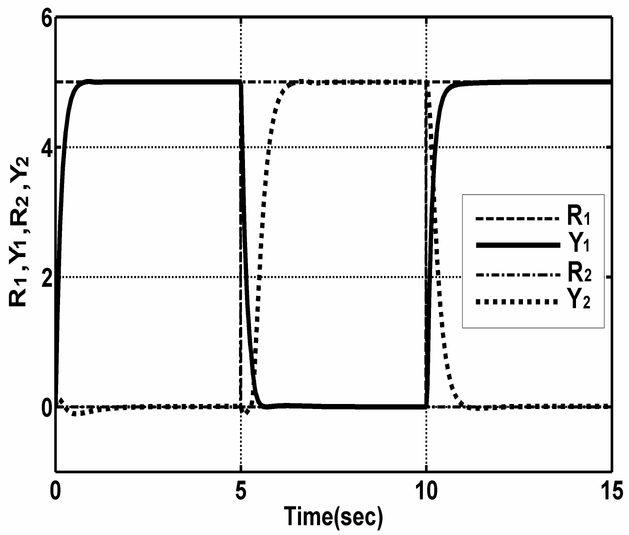

Fig.15. Time Responses of Example 5 for β=0.5 and $T_S = 25ms$.

4 Conclusions

In this literature, a new nonlinear digital controller has been proposed for analyses and designs of sampled-data feedback control systems. It was applied to five simple and complicated numerical examples to get good performance and zero steady-state errors. No integrations of tracking errors are needed to get zero steady-state errors. From simulation and comparison results with other famous control methods, it can be seen that the proposed method provides another possible control scheme for sampled-data feedback control systems, and it is worthwhile to find other regulation $F(n)$ to get better performance.

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